

$$f(x,y) = \begin{cases} 2 & 0 \leq x \leq 1, 0 \leq y \leq 1-x \\ 0 & \text{otherwise} \end{cases}$$

(1)

$$\begin{aligned} F(u,v) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) e^{-j2\pi(ux+vy)} dx dy = \int_0^1 \int_0^{1-x} 2 e^{-j2\pi(ux+vy)} dy dx \\ &= \int_0^1 2 e^{-j2\pi ux} \int_0^{1-x} e^{-j2\pi vy} dy dx = \int_0^1 2 e^{-j2\pi ux} \cdot \left[\frac{e^{-j2\pi vy}}{-j2\pi v} \right]_0^{1-x} dx = \\ &= \int_0^1 2 e^{-j2\pi ux} \left[-\frac{1}{j2\pi v} (e^{-j2\pi v(1-x)} - 1) \right] dx \\ &= -\frac{2}{j2\pi v} \int_0^1 e^{-j2\pi ux} (e^{-j2\pi v(1-x)} - 1) dx = \\ &= -\frac{2}{j2\pi v} \int_0^1 e^{-j2\pi ux - j2\pi v + j2\pi vx} - e^{-j2\pi ux} dx = \\ &= -\frac{2}{j2\pi v} \cdot \int_0^1 e^{-j2\pi v} \cdot e^{-j2\pi x(u-v)} - e^{-j2\pi vx} dx = \\ &= -\frac{2}{j2\pi v} \left[e^{-j2\pi v} \cdot \frac{e^{-j2\pi x(u-v)}}{-j2\pi(u-v)} \Big|_0^1 - \frac{e^{-j2\pi vx}}{-j2\pi v} \Big|_0^1 \right] = \\ &= -\frac{2}{j2\pi v} \left[e^{-j2\pi v} \cdot \left[\frac{e^{-j2\pi(u-v)}}{-j2\pi(u-v)} - 1 \right] - \frac{e^{-j2\pi v}}{-j2\pi v} + 1 \right] \\ &= -\frac{2}{j2\pi v} \left[\frac{e^{-j2\pi u}}{-j2\pi(u-v)} - e^{-j2\pi v} - \frac{e^{-j2\pi v}}{-j2\pi v} + 1 \right] \end{aligned}$$