

## Homework #3

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### I. Part 1: Eigenfaces

#### A. Section 1

We computed the average image from the grayscale training set of face images. The result is presented in Fig. 1. The resulting average image is a vague male face; notice the black lines on the top and bottom of the average image, they are presented in all training set images.



Figure 1: Average image from training set.

#### B. Section 2

From all the training set images, we computed the 25 eigenfaces with the largest eigenvalues. We flattened each  $243 \times 320$  pixel image into a vector with 77760 elements. We concatenated all these vectors into a single  $A \in \mathbb{R}^{77760 \times 150}$  matrix. In order to compute the eigenfaces, we will have to create the matrix  $AA^T \in \mathbb{R}^{77760 \times 77760}$ , the covariance matrix that corresponds to each pixel, and compute its eigenvalues and eigenvectors. This matrix is extremely large and computing its eigenvectors is costly, thus we will compute

the eigenvalues and vectors of  $A^T A$  and use it to compute the eigenvectors of  $AA^T$  instead:

$$A^T A x = x \lambda \quad (1)$$

where  $\lambda$  is the eigenvalue of  $A^T A$  and  $x$  is the corresponding eigenvector. Then:

$$(AA^T)(Ax) = (Ax)\lambda \quad (2)$$

thus  $\lambda$  is a (non-zero) eigenvalue of  $AA^T$  as well, and its corresponding eigenvector is  $Ax$ . Therefore we only need to compute the eigenvectors of  $A^T A \in \mathbb{R}^{150 \times 150}$ , which is a significantly more manageable task.

In Fig. 2 we present the 5 eigenfaces with the largest eigenvalues. The images are normalized such that the 0 value is 50 percent gray.

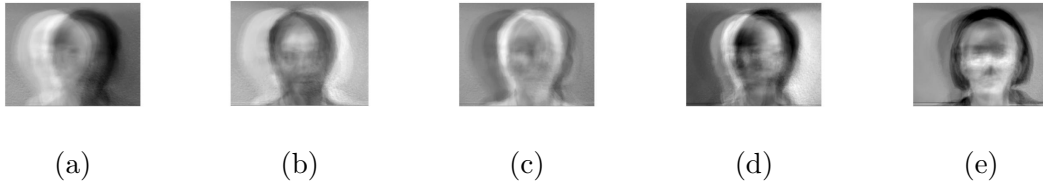


Figure 2: From left to right: eigenfaces with largest eigenvalues in descending order.

### C. Section 3

In this section we reconstruct the training set images using the reduced representation, and reconstruction errors. Note that for the reconstruction we normalized the  $W$  matrix of eigenfaces such that  $W^T W = I$ .

In Fig. 3 we present training set images on the left column, compared to a reconstruction using 25 highest eigenvalue eigenfaces. We can see that the reconstruction is lacking detail, but overall pretty close to the training set. If we were to take all 150 eigenvalues, we would have a perfect reconstruction.

In Fig. 4 we present a graph of errors relative to the serial number of the image. We can see that dynamic range RMSE error has a significantly smaller value than RMSE error, as it tends to normalize large value swings. The average RMSE error is  $6.4949 \cdot 10^{-3}$ , while the average Dynamic Range RMSE error is  $0.2725 \cdot 10^{-3}$ .

### D. Section 4

In this section we reconstruct the test set images using the reduced representation, and reconstruction errors.

In Fig. 5 we present test set images on the left column, compared to a reconstruction using 25 highest eigenvalue eigenfaces trained on the original 150 images. We can see

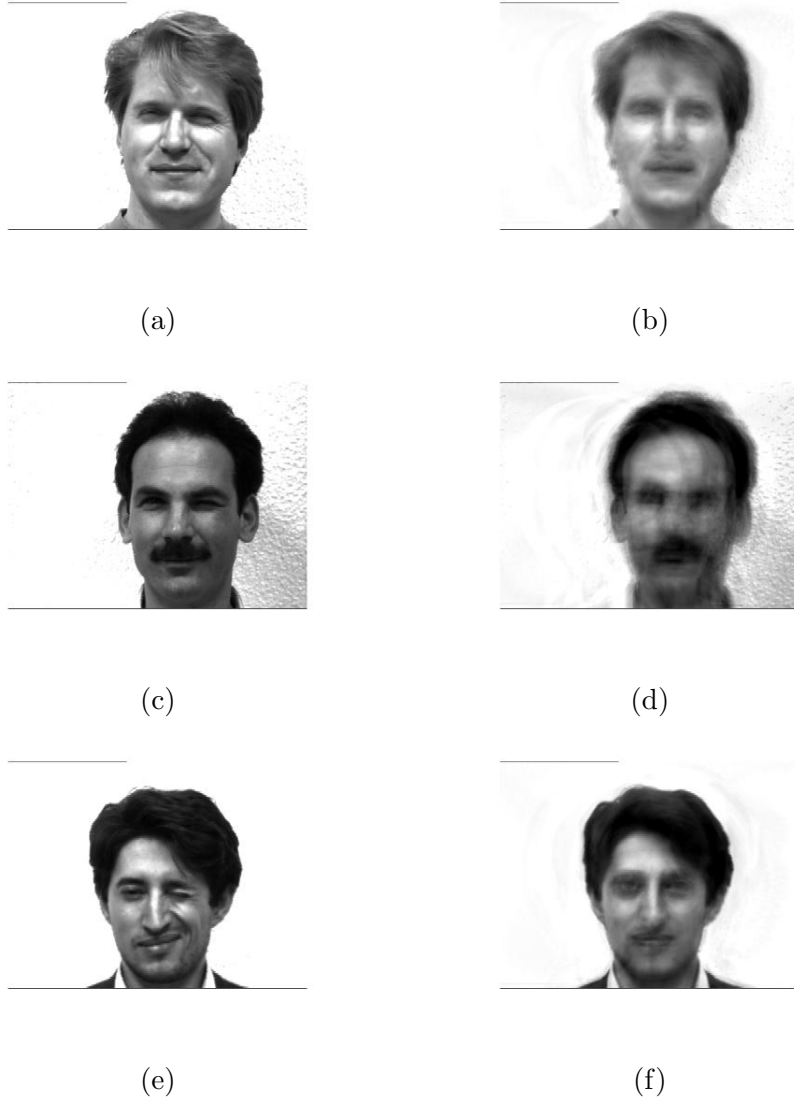


Figure 3: Training set; Left column: original images from training set. Right column: image reconstruction based on 25 largest eigenvalue eigenfaces.

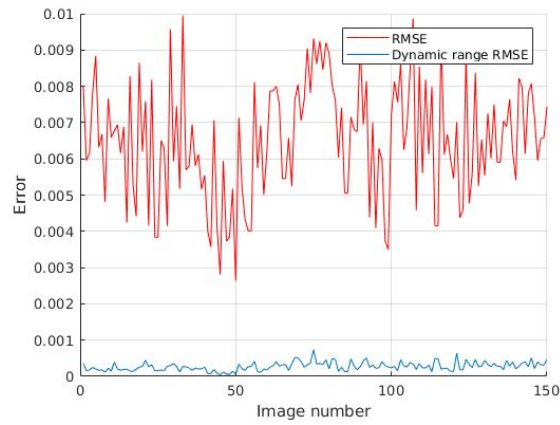


Figure 4: Training set; RMSE and Dynamic Range RMSE errors vs. image serial number.

that the reconstruction captures faces' general form well but fails to reconstruct objects that are not faces, such as the Google Chrome symbol. This reconstruction is enough for classifying faces relatively well, as the classification accuracy we reached for the eigenfaces algorithm is **81.82 percent**. We performed the classification over all face images, and ignored the non-face images as the inferred class has no meaning for these cases.

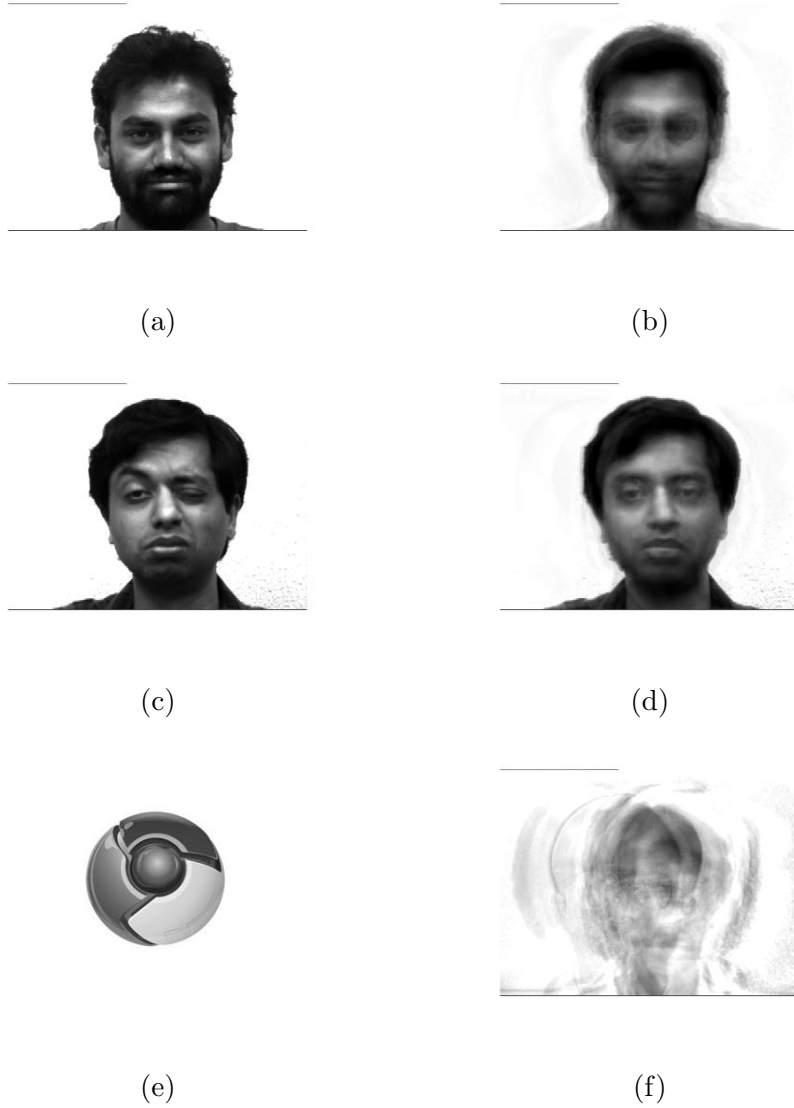


Figure 5: Test set; Left column: original images from training set. Right column: image reconstruction based on 25 largest eigenvalue eigenfaces.

In Fig. 6 we present a graph of errors relative to the serial number of the image. The average RMSE error is  $11.748 \cdot 10^{-3}$ , while the average Dynamic Range RMSE error is  $0.5626 \cdot 10^{-3}$ . As expected, the reconstruction error for the test set is significantly larger than the training set, also due to the fact that the test set contains images that are very far from the training set (high model uncertainty).

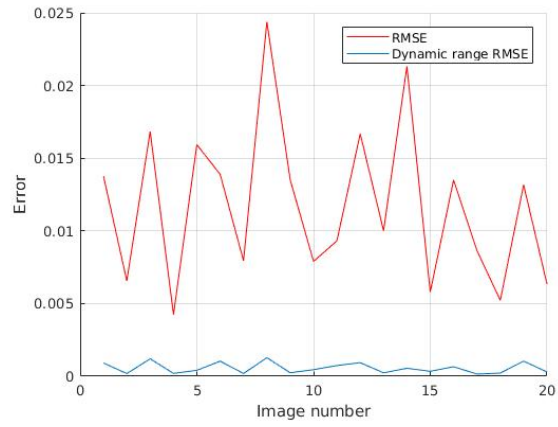


Figure 6: Test set; RMSE and Dynamic Range RMSE errors vs. image serial number.