Vision-Aided Navigation (086761)
Homework #1

Submission in pairs by: 12 November 2017, 13:30. Electronic submission is preferred.

Basic probability

1. Consider a random vector x with a Gaussian distribution:

$$x \sim N\left(\mu_x, \Sigma_x\right).$$

   (a) Write an explicit expression for $p\left(x\right)$.

   (b) Consider a linear transformation $y = Ax + b$. Show y has a Gaussian distribution, $y \sim N\left(\mu_y, \Sigma_y\right)$, and find expressions of μ_y and Σ_y in terms of μ_x and Σ_x.

2. Let $p\left(x\right) = N\left(\hat{x}_0, \Sigma_0\right)$ be a prior distribution over $x \in \mathbb{R}^n$ with known mean $\hat{x}_0 \in \mathbb{R}^n$ and covariance $\Sigma_0 \in \mathbb{R}^{n\times n}$. Consider a given measurement $z \in \mathbb{R}^m$ with a corresponding linear measurement model $z = Hx + v$, where H is a measurement matrix and v is Gaussian noise $v \sim N\left(0, R\right)$ with covariance R. The matrices $H \in \mathbb{R}^{m\times n}$ and $R \in \mathbb{R}^{m\times m}$ are known.

   (a) Write an expression for the a posteriori probability function (pdf) over x, $p\left(x|z\right)$, in terms of solely the prior $p\left(x\right)$ and measurement likelihood $p\left(z|x\right)$.

   (b) Derive analytically an expression for the maximum a posteriori (MAP) estimate $x^\star$ and the associated covariance Σ or information matrix $I = \Sigma^{-1}$ such that $p\left(x|z\right) = N\left(x^\star, \Sigma\right)$.
   Useful relation: $\|a\|_\Sigma^2 = \left\|\Sigma^{-1/2}a\right\|^2$ where $\Sigma^{-1} = \Sigma^{-T/2}\Sigma^{-1/2}$.

Hands-on Exercises - Please print and submit your code.

1. Rotations. Implement transformation from rotation matrix to Euler angles and vice versa

   (a) Implement a function that receives as input Euler angles (roll angle ϕ, pitch angle θ,and yaw angle ψ) and calculates the corresponding rotation matrix assuming roll-pitch-yaw order from Body to Global: $R = R_Z\left(\psi\right) R_Y\left(\theta\right) R_X\left(\phi\right)$.

   (b) What is the rotation matrix from Body to Global for $\psi = \pi/7$, $\theta = \pi/5$, and $\phi = \pi/4$?

   (c) Implement a function that receives as input a rotation matrix and calculates the corresponding Euler angles assuming roll-pitch-yaw order.

   (d) What are the Euler angles <u>in degrees</u> for the following rotation matrix (Body to Global, assuming roll-pitch-yaw order):

$$R_B^G = \begin{bmatrix} 0.813797681 & -0.440969611 & 0.378522306 \\ 0.46984631 & 0.882564119 & 0.0180283112 \\ -0.342020143 & 0.163175911 & 0.925416578 \end{bmatrix}$$

2. 3D rigid transformation. The coordinates of a 3D point in a global frame are

$$l^G = \left(450, 400, 50\right)^T.$$

   This 3D point is observed by a camera whose pose is described by the following rotation and translation with respect to the global frame:

$$R_G^C = \begin{bmatrix} 0.5363 & -0.8440 & 0 \\ 0.8440 & 0.5363 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$t^{G}_{C\to G} = (-451.2459, 257.0322, 400)^{T}$$

Calculate the 3D point coordinates in a camera frame ($l^{C} =?$). Write an explicit expression for the appropriate 3D transformation.

3. Pose composition. An autonomous *ground* vehicle (robot) is commanded to move forward by 1 meter each time. Due to imperfect control system, the robot instead moves forward by 1.01 meter and also rotates by 1 degree.
Remark: In this exercise we consider a 2D scenario, where pose is defined in terms of x-y coordinates and an orientation (heading) angle.

   (a) Write expressions for the corresponding commanded and actual transformations - note these are relative to the robot frame.

   (b) Assuming robot starts moving from the origin, calculate evolution of robot pose (in terms of x-y position and orientation angle) for 10 steps using pose composition. Draw the commanded and actual robot pose for 10 steps. What is the dead reckoning error at the end?