Question 1.

Part a.

We develop the equations to calculate the throughput following the examples in the lectures.

The protocol used in simple ALOHA. We divide the time into slot as in the lecture.

Given 2 groups of stations, we can calculate the throughput for each station:

$$S_A = P(1\ transmission\ from\ group\ A\ in\ slot\ i) \cdot P\big(0\ transmissions\ from\ group\ A\ in\ slot\ (i-1)\big) \cdot P(0\ transmissions\ from\ group\ B\ in\ slots\ \{i, i-1\})$$

The transmissions are distributed in the Poisson distribution:

$$P(k, \lambda, t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

Thus,

$$S_A = G_A e^{-G_A} \cdot e^{-G_A} \cdot e^{-2G_B} = G_A e^{-2G_A} \cdot e^{-2G_B}$$

In the similar way, we can calculate the throughput for the group B. In fact, everything is almost similar:

$$S_B = P(1\ transmission\ from\ group\ B\ in\ slot\ i) \cdot P\big(0\ transmissions\ from\ group\ B\ in\ slot\ (i-1)\big) \cdot P(0\ transmissions\ from\ group\ A\ in\ slots\ \{i, i-1\})$$

$$S_B = G_B e^{-2G_B} \cdot e^{-2G_A}$$

The requirement:

$$S_B = 3S_A$$

$$G_B e^{-2G_B} \cdot e^{-2G_A} = 3G_A e^{-2G_A} \cdot e^{-2G_B}$$

$$G_B = 3G_A$$

Part b.

The calculation for the reservation ALOHA is similar to CSMA/CD. The collisions may occur during the reservation slot, where multiple stations may try to reserve a slot. If only 1 station sends the reservation request, it will be granted a collision-free window of T slots.

We define:

- Group 1: n stations which want to transmit at probability $P_{t_1} = \frac{1}{n}$
- Group 2: $2n$ stations which want to transmit at probability $P_{t_2} = \frac{2}{n}$
- Probability of error white sending request: $P_e = p$

The throughput is calculated in a similar to CSMA/CD:

$$S = \frac{T}{T + E(contention\ interval)}$$

Where the contention interval includes the reservation, and all the re-transmissions of the reservation. It will include AT LEAST 1 interval where the reservation will take place.

We can calculate the probability of a successful reservation by a certain station:

$$P(successful\ revervation) = P_A$$

$$P_A = \underbrace{\left((1-P_e)P_{t_1} \cdot \left(1-P_{t_1}\right)^{n-1} \cdot \binom{n}{1}\right)}_{\substack{1\ successful\ reservation \\ transmit\ from\ Group\ 1}} \cdot \underbrace{\left(\left(1-P_{t_2}\right)^{2n}\right)}_{\substack{0\ reservation\ transmits \\ from\ Group\ 2}} +$$

$$\underbrace{\left((1-P_e)P_{t_2} \cdot \left(1-P_{t_2}\right)^{2n-1} \cdot \binom{2n}{1}\right)}_{\substack{1\ successful\ reservation \\ transmit\ from\ Group\ 2}} \cdot \underbrace{\left(\left(1-P_{t_1}\right)^{n}\right)}_{\substack{0\ reservation\ transmits \\ from\ Group\ 1}}$$

$$P_A = \left((1-P_e)P_{t_1} \cdot \left(1-P_{t_1}\right)^{n-1} \cdot n\right) \cdot \left(\left(1-P_{t_2}\right)^{2n}\right) +$$

$$\left((1-P_e)P_{t_2} \cdot \left(1-P_{t_2}\right)^{2n-1} \cdot 2n\right) \cdot \left(\left(1-P_{t_1}\right)^{n}\right)$$

$$P_A = \left((1-p)\frac{1}{n}\cdot\left(1-\frac{1}{n}\right)^{n-1}\cdot n\right)\cdot\left(\left(1-\frac{2}{n}\right)^{2n}\right)+$$

$$\left((1-p)\frac{1}{2n}\cdot\left(1-\frac{2}{n}\right)^{2n-1}\cdot 2n\right)\cdot\left(\left(1-\frac{1}{n}\right)^{n}\right)$$

Remembering the definition for the Euler's number:

$$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{n} = \frac{1}{e}$$

We can rewrite:

$$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{n-1} = \lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{n}\cdot\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^{-1} = \frac{1}{e}\cdot 1 = \frac{1}{e}$$

$$\lim_{n\to\infty}\left(1-\frac{2}{n}\right)^{2n-1} = \frac{1}{e^2}$$

And obtain:

$$P_A = \left((1-p)\frac{\cancel{1}}{\cancel{n}}\cdot\frac{1}{e}\cdot\cancel{n}\right)\cdot\left(\frac{1}{e^2}\right)+\left((1-p)\frac{\cancel{1}}{\cancel{2n}}\cdot\frac{1}{e^2}\cdot\cancel{2n}\right)\cdot\left(\frac{1}{e}\right) = (1-p)\cdot\left(\frac{1}{e^3}+\frac{1}{e^3}\right) = (1-p)\cdot\frac{2}{e^3}$$

We define τ to be the smallest interval.

Thus, the contention interval:

$$E(contention\ interval) = \tau + \sum_{k=1}^{\infty}(1-P_A)^k P_A\cdot k\cdot\tau = \tau\cdot\left(1+P_A\sum_{k=1}^{\infty}(1-P_A)^k\cdot k\right)$$

$$= \tau\cdot\left(1+P_A(1-P_A)\sum_{k=1}^{\infty}(1-P_A)^{k-1}\cdot k\right) = \tau\cdot\left(1+P_A(1-P_A)\frac{1}{\left(1-(1-P_A)\right)^2}\right)$$

$$= \tau\cdot\left(1+\frac{P_A(1-P_A)}{{P_A}^2}\right) = \tau\cdot\left(1+\frac{1-P_A}{P_A}\right) = \tau\left(\frac{1}{P_A}\right)$$

Which fits, meaning we need only 1 reservation slot before message transmit.

Since the time units here are given in units of 1, $\tau = 1$

So, we obtain:

$$S = \frac{T}{T+E(contention\ interval)} = \frac{T}{T+\tau\left(\frac{1}{P_A}\right)} = \frac{T}{T+\left(\frac{1}{P_A}\right)}$$

$$S = \frac{TP_A}{TP_A+1}$$

Sanity check:

$$P_A = 1 : \; S = \frac{T}{T+1}$$

Which is logical, we need 1 reservation slot.

$$P_A = 0 : S \to 0$$

Which is logical, if we can't succeed to send reservation, throughput is 0.

The expression for P_A was found before. Inserting:

$$P_A = \frac{2(1-p)}{e^3}$$

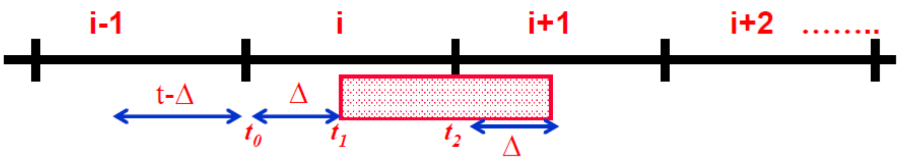
$$S = \frac{2T\,(1-p)e^{-3}}{2T\,(1-p)e^{-3} + 1}$$

Question 2.

Using the illustrations from the lecture to answer this question.

a.

ALOHA will send the frame at any time.



Accounting only for the group A of stations (which operate at ALOHA), the probability of successful transmission is:

$$P(succ) = P(1\ transmission\ at\ slot\ i) \cdot P(0\ transmissions\ at\ an\ interval\ of\ t)$$
$$= P(1, G, 1) \cdot P(0, G, 1)$$

Where the interval is divided into 2:

1. $[t_0 - (t - \Delta), t_0)]$
2. $[t_2, t_2 + \Delta]$

And the total length of interval is length of 1 slot.

We have to account for the group B of stations, which operate at Slotted ALOHA, specifically add the term:

$$P(0\ transmissions\ at\ slots\ i, i+1) = P(0, G', 2)$$

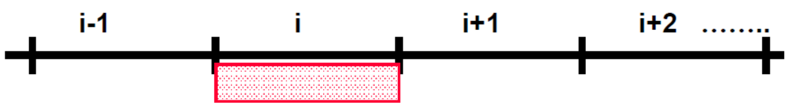
Because a transmission during any of those slots will interrupt the current transmission.

$$P(k, \lambda, t) = \frac{(\lambda t)^k}{k!} e^{-\lambda t}$$

$$P(succ_A) = P(1, G, 1) \cdot P(0, G, 1) \cdot P(0, G', 2) = Ge^{-G} \cdot e^{-G} \cdot e^{-2G'} = Ge^{-2G} \cdot e^{-2G'}$$

b.

Accounting only for slotted ALOHA, we require that only 1 appearance will be at slot i, other slots don't have effect:



Now, when we also have ALOHA operating, we put further restrictions:

1. No frame should be transmitted during i-1
2. No frame should be transmitted during i

Thus we expect 0 transmission from group A for 2 slots.

Putting this all together:

$$P(succ_B) = P(1, G', 1) \cdot P(0, G, 2) = G'e^{-G'}e^{-2G}$$

c.

We wish both success chances to be equal. Thus:

$$P(succ_A) = P(succ_B)$$

$$Ge^{-2G} \cdot e^{-2G'} = G'e^{-G'}e^{-2G}$$

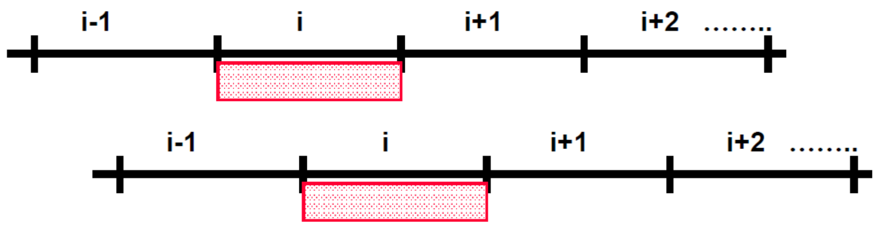
$$Ge^{-G'} = G'$$

$$G = G'e^{G'}$$

Question 3.

a.

The groups of stations are not synced. This may look like this:



The packets transmission is distributed via Poisson distribution $\sim Pois(G)$

Given 2 groups of stations, and their sizes, we can calculate their rate.

1. Group 1 of size $N_1 = \frac{1}{3}N$ will transmit at rate $G_1 = \frac{1}{3}G$
2. Group 2 of size $N_2 = \frac{2}{3}N$ will transmit at rate $G_2 = \frac{2}{3}G$

A frame from Group 1 will succeed in slot i if :

1. only 1 frame from Group 1 will be in this slot
2. and no frame from Group 2 will be in slots which intersect this slot (on the picture above – those are i-1 and i in the bottom timeline)

Thus:

$$P(succ_A) = Pr\left(1, \frac{1}{3}G, 1\right) \cdot \Pr\left(0, \frac{2}{3}G, 2\right) = \frac{G}{3}e^{-\frac{G}{3}} \cdot e^{-2 \cdot \frac{2}{3}G} = \frac{G}{3}e^{-\frac{5}{3}G}$$

b.

In the same way, calculating the success probability for a slot for group 2:

$$P(succ_B) = Pr\left(1, \frac{2}{3}G, 1\right) \cdot \Pr\left(0, \frac{1}{3}G, 2\right) = \frac{2G}{3}e^{-\frac{2G}{3}} \cdot e^{-2 \cdot \frac{1}{3}G} = \frac{2G}{3}e^{-\frac{4}{3}G}$$

As expected, the probability for success in a certain slot for Group 2 is higher, since more stations obey to the same protocol rules and will not interrupt the message in the middle.

c. As there is no overlap between them the answer is simply the sum of the two previous questions:

$$P(suuc_A) + P(succ_B) = \frac{1}{3}Ge^{-\frac{5}{3}G} + \frac{2}{3}Ge^{-\frac{4}{3}G}$$

d. As the protocol allows us either to send one message or send no message we arrive at the same solution as the previous answer that the utilization is the same as the likelyhood of sending a message:

$$\frac{1}{3}Ge^{-\frac{5}{3}G}+\frac{2}{3}Ge^{-\frac{4}{3}G}$$

שאלה 4

א. ההסתברות שתחנה כלשהי תצליח בשידור הראשון היא ההסתברות שתחנה אחת תשדר ושתי התחנות האחרות לא ישדרו בפרק זמן השידור T_i (מובטח לנו שתהיה התנגשות בחריץ הראשון)

$$P_{suc}\cdot P_{fail}\cdot P_{fail}$$

עבור ניסיון ראשון יש לכל תחנה 0.5 סיכוי לשידור ו0.5 סיכוי להמתנע לכן הסיכוי לתרחיש הוא:

$$\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2}=\frac{1}{8}$$

ב. בהינתן כי היה שידור מוצלח עבור אחת התחנות, מובטח לנו כי 2 התחנות האחרות ניסו לשדר בחריץ הנותר ויצרו התנגשות

ההסתברות לבחירת חריץ כלשהו מבין החריצים $[1,2,\dots,N]$ כאשר $N=2^i-1$ עבור התנגשות ה-i היא $P=\frac{1}{N}$ ובאופן כללי התוחלת של לשידור מוצלח היא $E(c)=\frac{1}{1+N}\sum_{i=0}^{N} i=\frac{1}{1+N}\frac{N(N+1)}{2}=\frac{N}{2}=\frac{2^c-1}{2}$ עבור התנגשות מספר c. במקרה שלנו $c=2$.

$E(2)=\frac{2^2-1}{2}=\frac{3}{2}\tau$ נסכום יחד עם זמן השידור לקבלת זמן הזמן עד לקבלת שקט ברשת שוב, סה"כ: $\frac{3}{2}\tau+T_i$

ג. עבור סעיף א כעת יתכנו 3 מקרים, התחנה הראשונה תגריל את החריץ הראשון בהסתברות $P=\frac{1}{4}$ במקרה זה לתחנות האחרות קיימת אפשרות לבחור אחת מבין 3 החריצים הנותרים $P=\frac{3}{4}$. מקרה אחר שקיים, התחנה המשדרת הגרילה את החריץ השני גם בהסתברות $P=\frac{1}{4}$ וכעת נותר לשאר התחנות בחירה מבין 2 תחנות $P=\frac{1}{2}$. מקרה אחרון הוא שהתחנה המשדרת הגרילה את חריץ השלישי $P=\frac{1}{4}$ ושאר התחנות נדרשות להגריל את החריץ האחרון $P=\frac{1}{4}$.

לסיכום ההסתברות:

$$P=\frac{1}{4}\cdot\left(\frac{3}{4}\right)^2+\frac{1}{4}\cdot\left(\frac{1}{2}\right)^2+\frac{1}{4}\cdot\left(\frac{1}{4}\right)^2=\frac{7}{32}$$

עבור סעיף ב, כעת $N=2^i$ לכן התוחלת תהיה $E(2)=\frac{2^2}{2}=2\tau$ סה"כ: $2\tau+T_i$

ד. ניתן לראות כי קיים הסתברות גבוהה יותר להצלחת שידור ראשון של אחד התחנות (יתרון) אבל מנגד זמן הממוצע (תוחלת) עד שתוכל התחנה האחרונה לשדר גדולה עבור השינוי שנעשה.