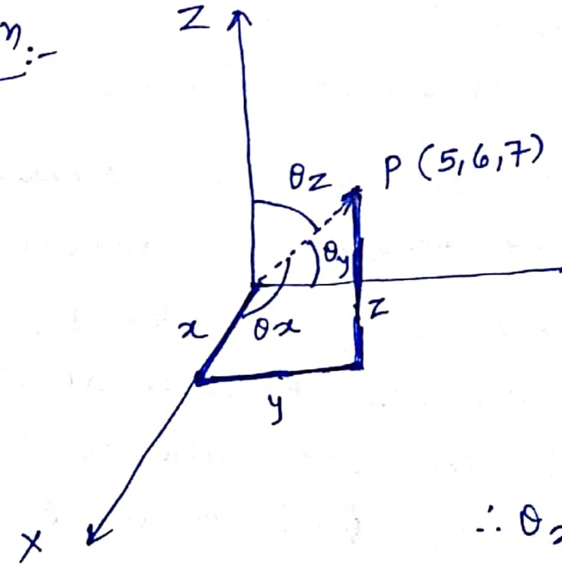
Ex. on Cartesian frame of reference :-

A point (5,6,7) is located in a cartesian frame of reference. Calculate θ_x, θ_y and θ_z and also the length of the ~~tip~~ tip of this vector from origin.

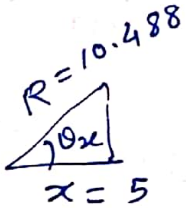
Soln :-



$$R = \sqrt{5^2+6^2+7^2}$$

$$= 10.488 \text{ units.}$$

$\theta_x \rightarrow$



$$\therefore \theta_x = \cos^{-1}\frac{x}{R} = \cos^{-1}\frac{5}{10.488}$$

$$= 61.527^\circ$$

|||ly $\theta_y =$

$$\therefore \theta_y = \cos^{-1}\frac{y}{R} = \cos^{-1}\frac{6}{10.488}$$

$$= 55.104^\circ$$

|||ly $\theta_z =$

$$\therefore \theta_z = \cos^{-1}\frac{z}{R} = \cos^{-1}\frac{7}{10.488}$$

$$= 48.131^\circ$$

R is actually $\rightarrow$ $\sqrt{5^2+6^2}$ = R of x & y = R_{xy}

$$R = \sqrt{R_{xy}^2 + z^2}$$

θ_x is the angle made by R with x axis

θ_y ——— " ——— R —— y ——"——

θ_z ——— " ——— R —— z ——"——

Rotation Matrix :-

Concept :- • Robots work in a 3-D environment.

• The base of Robot is generally stationary at a perticular position but the links and joints are movable. They undergo ---.

1) Rotary motion - Revolute
2) Linear motion - Prismatic (Telescopic)

e.g. Consider that there is a 2-D Robot working in a x-y frame of reference.

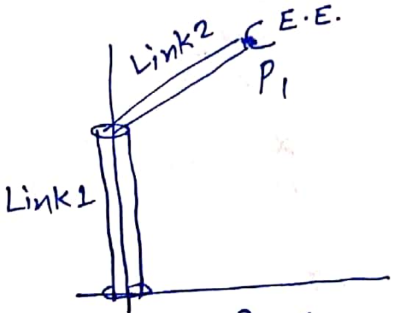


fig-1

It has only 2 links and there is an end-effector at the end. Let the co-ordinates of the E.E. are given by $P_1(x_1, y_1)$

<u>Case 1</u>

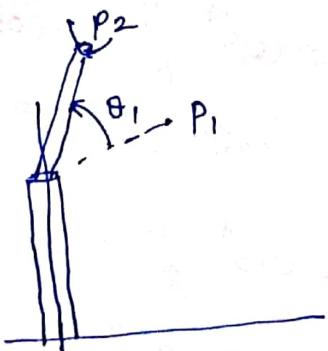


fig. 2

Now if the Link2 is rotated through θ, then the co-ordinates of the new position of P_2 can be found out using simple geometry w.r.t. P_1 if θ is known. (Here Link 1 is not moved)

<u>Case 2</u> Now if the Link 1 is rotated as shown in fig. 3. Even if Link2 is not rotated, it has to turn because Link 1 rotates ~~at~~ through an angle of θ_2 (cw)

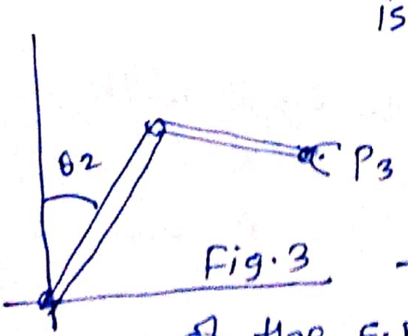
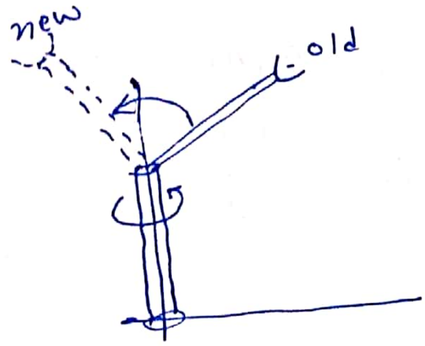


Fig. 3

Thus position and orientation of the E.E. changes. Can we find new position P_3 as easily as case 1 ? No !

case 3 If Link 2 is allowed to spin about the y axis, then also position and orientation of E-E. will change.

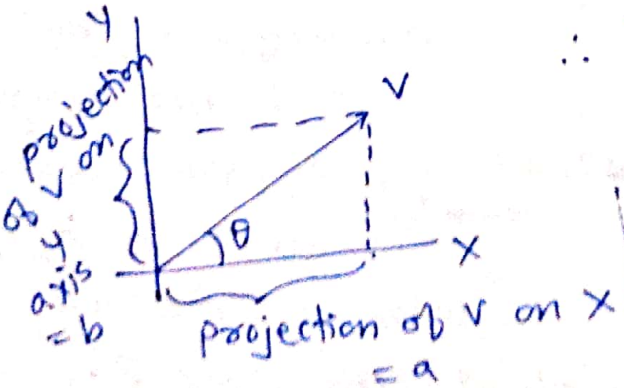


There should be some methodology developed to find co-ordinates of E.E. w.r.t. the base/body of the Robot.

In actual practice the turning/rotations are in a 3-D space and that too with 3 or 4 links working simultaneously,

The process is called as Rotational Transformation. Using this the location of E.E. can be found out w.r.t. any other frame or link.

- The positions are represented by displacement vectors and
- Rotations are represented by Rotation matrices.

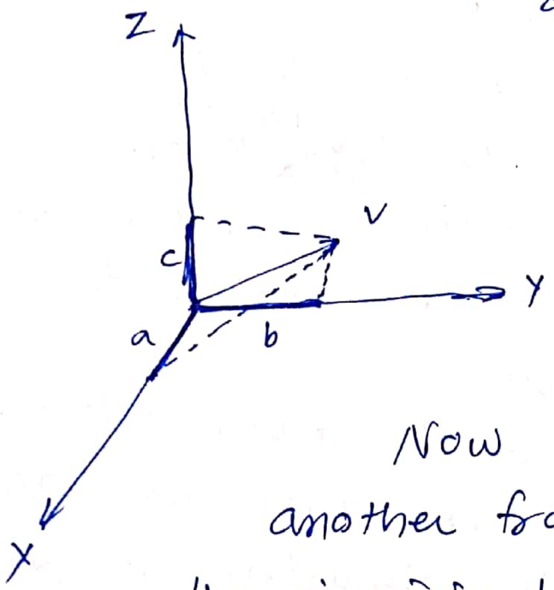


∴ V is shown as V(a,b) where a & b are projections of vector V on X & Y axes respectively

This projection can be shown as $\begin{bmatrix} a \\ b \end{bmatrix}$

and $\theta = \tan^{-1} \frac{b}{a}$

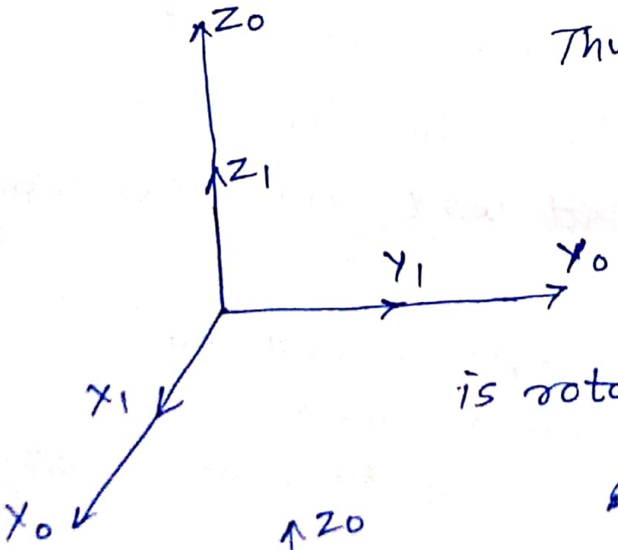
In actual practice, Robots work in a 3D environment ∴ Z axis comes in picture.
& projection on Z is c



∴ $V(a,b,c)$

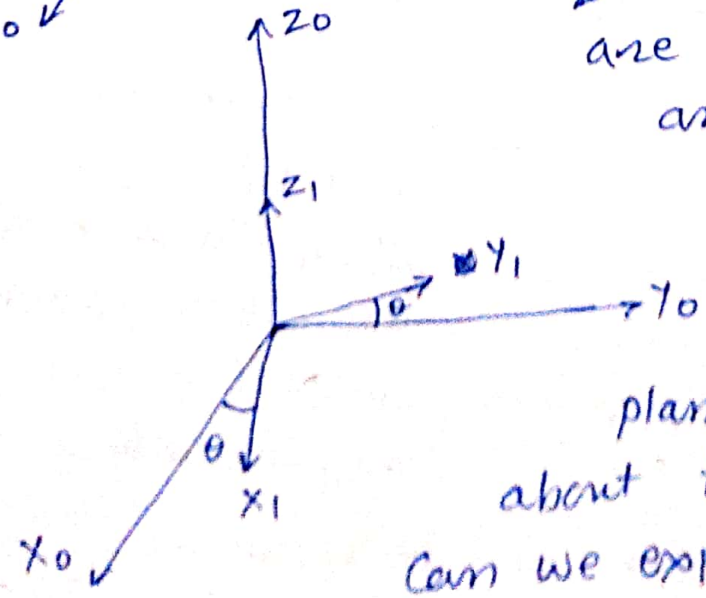
$$\therefore V = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

Now let us consider another frame $X_1Y_1Z_1$ and rename the original frame as $X_0Y_0Z_0$.



Thus initially both the frames are matching exactly as shown.

Now, if the frame1 is rotated as shown,
i.e. Both X_1 and Y_1 are rotated through an angle θ, but Z_1 is not moved.
Thus we can say that X_1Y_1 plane is rotated about the axis Z.



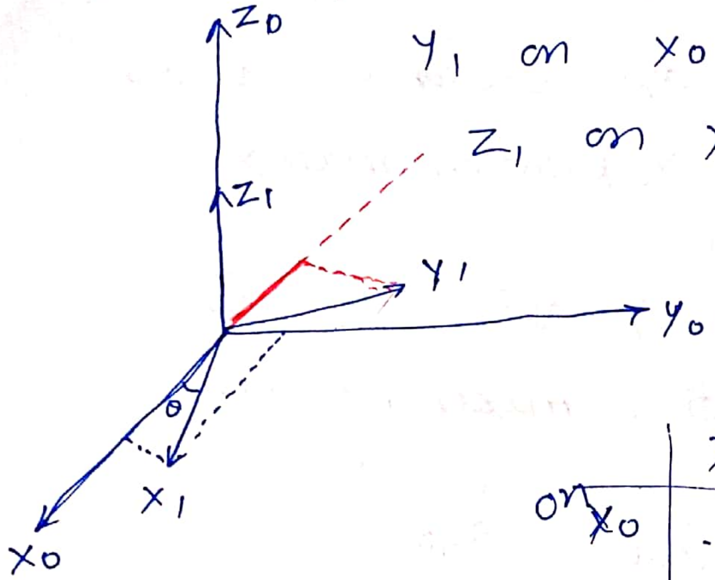
Can we express Rotation of frame 1 w.r.t. frame 0 ?

∴ Let us find projection of ~~x~~ ....

X_1 on X_0, Y_0 & Z_0

Y_1 on X_0, Y_0 & Z_0 &

Z_1 on X_0, Y_0 & Z_0

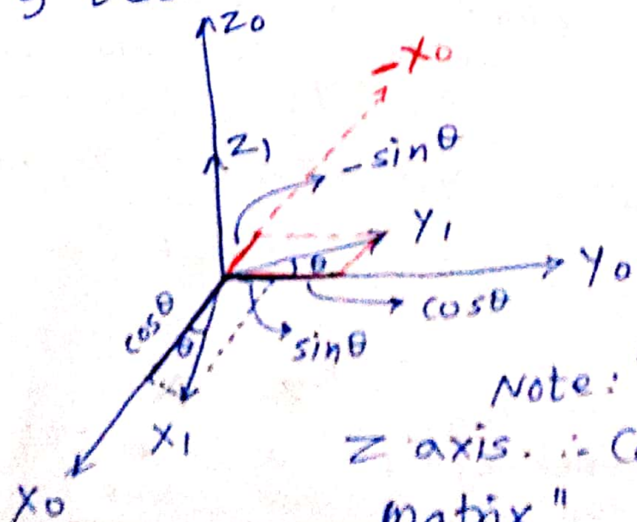


projections of

on	X_1	Y_1	Z_1
X_0	---	---	---
Y_0	---	---	---
Z_0	---	---	---

If the above 9 terms are found out then we can find the relation between frame 1 w.r.t. frame 0.

If X_1 Y_1 and Z_1 are assumed to have a unit length (just for the sake of understanding) then the above 9 terms will be as shown.



$$
\begin{array}{c|c}
 & \begin{array}{ccc} X_1 & Y_1 & Z_1 \end{array} \\
\hline
\begin{array}{c} X_0 \\ Y_0 \\ Z_0 \end{array} & \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}
\end{array}
$$

Note: Rotation is around Z axis. ∴ Called as "Z Rotation matrix"

⑥

$\therefore$ written as $R_1^0 \Rightarrow R_{12}^0 = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\parallel$'y keeping X_1 axis stationary as X_0 we get "X Rotation matrix"

and

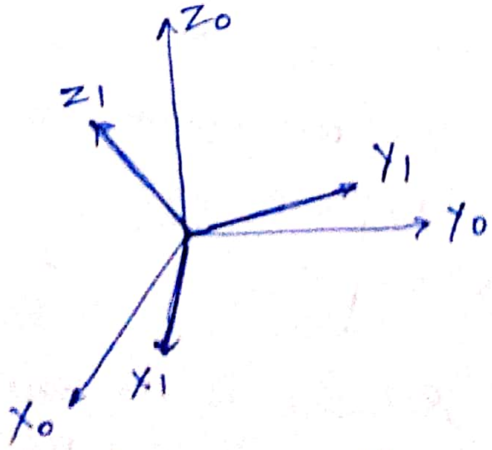
Rotation around Y axis we get "Y Rotation ~~matx~~ matrix."

$$\therefore R_X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_Y = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_Z = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now, if Z_1 is also rotated along with X_1 and Y_1 we can go stepwise split the task and have a combination as...



rotate	rotate	stationary
X_1	Y_1	$Z_1 \to$ about Z_1
Y_1	Z_1	$X_1 \to$ about X_1

$$\therefore R_2^0 = R_1^0 R_2^1$$

$$= R_Z R_X$$

$$\therefore R = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

END