4) Varignon's Theorem:- Algebric sum of moments of all forces about any point in their plane is equal to the moment of their resultant about the same point.

1) Polygon Law of forces:- If a number of coplaner, concurrent forces are acting simultaneously at a point, which are represented by sides of a polygon, taken in order, then their resultant is represented in M & Φ by the closing side of the polygon, taken in opposite order

2) Moment of force:- Tendency to produce rotation in a body i.e. the turning effect produced by the force.

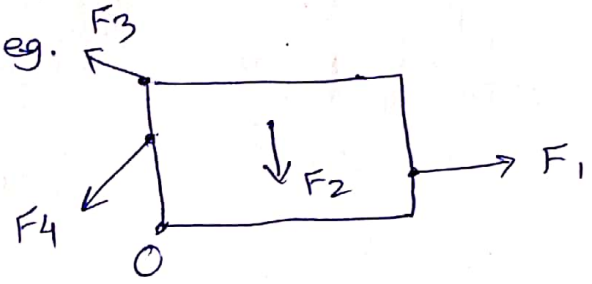
1.3.13

M. of force = Force x ⊥r dist. betn the Line of action & the point about which moments are to be taken.

3) Couple:- It is pair of two, equal, opposite & parallel forces acting on a body.

(★) To find resultant for a Non-concurrent & non-parallel Coplaner force system: - at a particular point say 'O'

for eg.



Steps:- 1) Resolve all forces into x & y compo.

2) Find Σx & Σy for the Resultant R

~~3) For a given point 'O' find moments (at 'O') & add them considering cw & ccw~~

3) Find $R = \text{---}$ & $\theta = \text{---}$

4) For a given point 'O', find moments (at 'O') & add them considering cw & ccw. ΣM_O

5) Position of resultant can be obtained by either of the following 3 eqns.

a) Perpendicular distance from 'O'.

$$d = \frac{\Sigma M_O}{R}$$

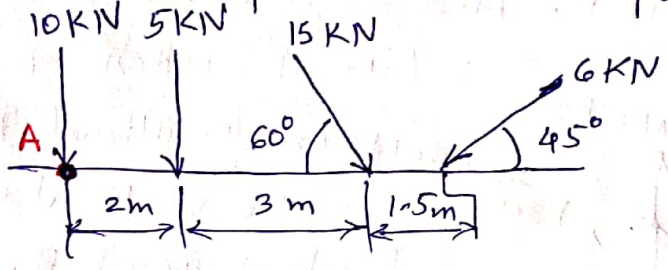
b) Horizontal distance from 'O'.

$$x = \frac{\Sigma M_O}{\Sigma F_y}$$

c) Vertical distance from 'O'.

$$y = \frac{\Sigma M_O}{\Sigma F_x}$$

ex 4.5.33 Find magnitude, direction & Line of action of the resultant for following planer system. Consider point A for Line of action



Soln - Find X-components of all forces.

$\Sigma X = 0 + 0 + 15\cos 60 - 6\cos 45 = 3.2573\,KN$

$\Sigma Y = -10 - 5 - 15\sin 60 - 6\sin 45 = -32.233\,KN$

$R = 3.2573 - j\,32.233 = 32.3971\,KN \angle -84.2295^\circ$

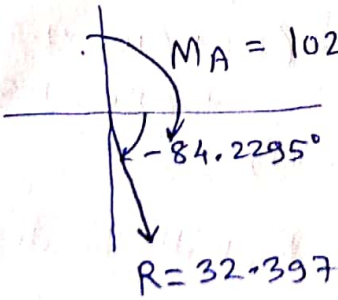
Now to find line of action of the Resultant.

Find Moments about the point A (cw & ccw)

$\Sigma M_A = (10 \times 0) + (-5 \times 2) + (-15\sin 60 \times 5) + (-6\sin 45 \times 6.5)$

$= -102.529\,KN\text{-}m$) CW

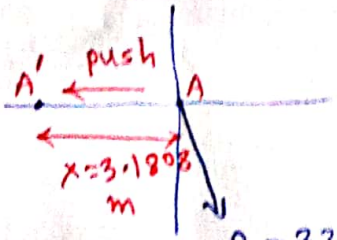
Thus the given system can be replaced by a single force (R) and a single moment at point A as follows ---



Now when R & M_A are combined, the resultant R will be displaced ~~horizontal~~ horizontally by a distance

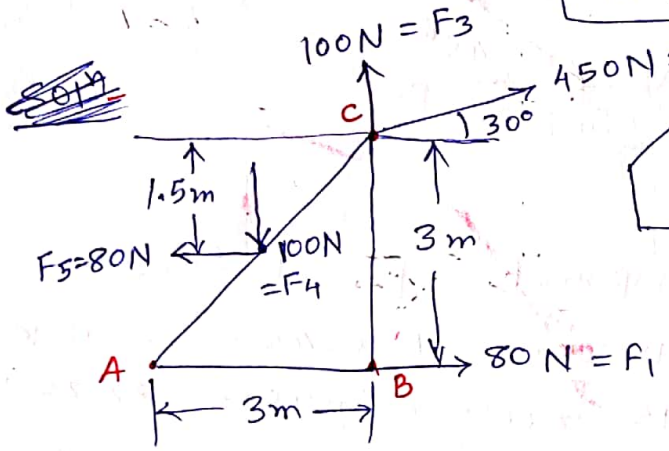
$x = \left|\frac{\Sigma M_A}{\Sigma Y}\right| = \left|\frac{102.529}{32.233}\right| = 3.1808\ m$

& vertically by $y = \left|\frac{\Sigma M_A}{\Sigma X}\right| = \frac{102.529}{3.2573} = 31.4766\ m$



& 1^{st} dist by $d = \frac{\Sigma M_A}{R} = \frac{102.529}{32.3971} = 3.1647\ m$

Ex (2) / 4.5.34 Replace the given force system acting on a triangular plate ABC by a single Resultant Force R. Find the distance betⁿ the Line of action of the Resultant R on the vertical edge BC about point B.

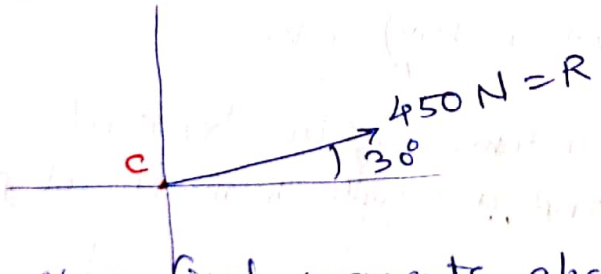


Soln - Find components X & Y.

$\Sigma X = +80 + 450 \cos 30 + 0 + 0 - 80 = 389.7114\ N$

$\Sigma Y = 0 + 450 \sin 30 + 100 - 100 + 0 = 225\ N$

$R = 389.7114 + j\ 225 = 450\ N \angle 30°$



Now find moments about point B.

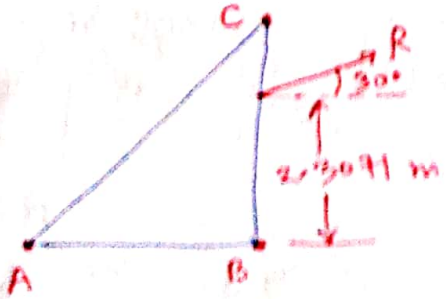
$\Sigma M_B = 0 + (-450 \cos 30 \times 3) + (0) + (100 \times 1.5) + (80 \times 1.5)$

$= -899.1343\ N\text{-m}\ (CW)$

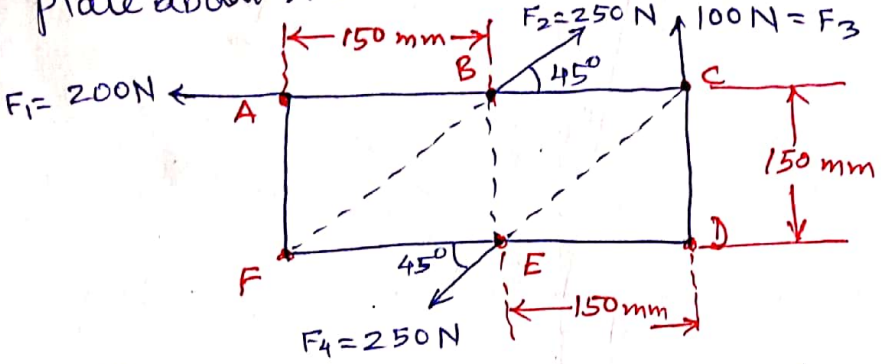
Vertically the Resultant R will be displaced by a distance

$y = \left| \frac{\Sigma M_B}{\Sigma X} \right| = \frac{899.1343}{389.7114}\ \frac{N\text{-m}}{N}$

$= 2.3091\ m$



x(3)
11.5.37 A plate of 150 x 300 mm is subjected to four loads as shown. Find the resultant R and the two points where the line of action of the resultant R would intersect the edge of the plate about A.

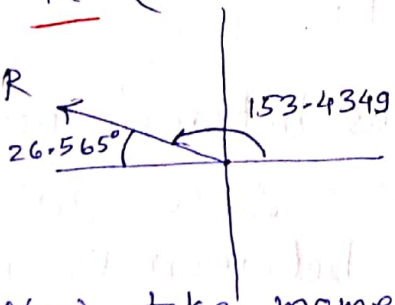


Soln - Find X & Y components

$\Sigma X = -200 + 250\cos 45 + 0 - 250\cos 45 = -200\ N$

$\Sigma Y = 0 + 250\sin 45 + 100 - 250\sin 45 = +100\ N$

$\therefore R = (-200 + j\,100) = 223.6067\ N \angle -26.565°$



Now, take moments about A.

$\Sigma M_A = 0 + 0 + (250\sin 45 \times 150) + (100 \times 300) + (-250\cos 45 \times 150) + (250\sin 45 \times 150)$

$= 26516.50429 + 30000 - 26516.50429 - 26516.50429$

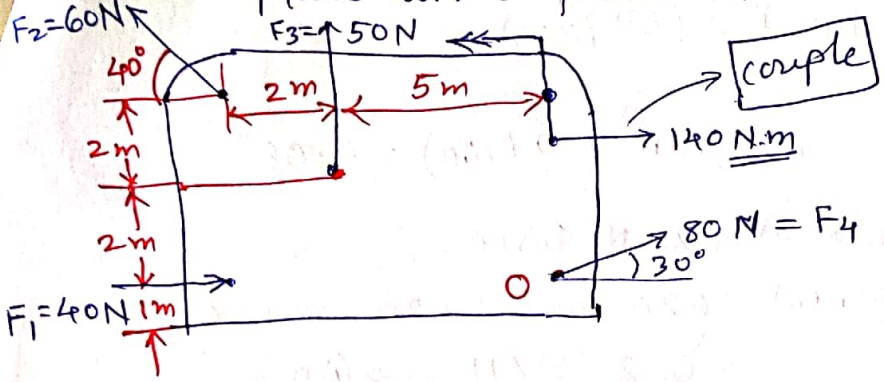
$= 3483.4957\ N$

$\therefore$ Line of action of R will be produced at a distance from A

$y = \left|\frac{\Sigma M_A}{\Sigma Y}\right| = \frac{3483.4957}{100} = 34.83.49\ mm$

$x = \left|\frac{\Sigma M_A}{\Sigma X}\right| = \frac{3483.4957}{200} = 17.41744\ mm$

Ex ④ / 4.5.41 Determine the resultant of the four forces and the couple that acts on the plate w.r.t point 'O'.

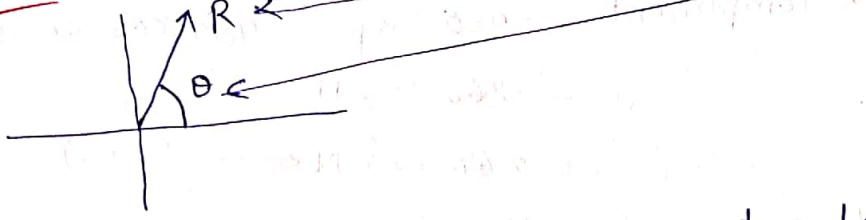


Soln Find X & Y compo. of all forces:

$\Sigma X = +40 - 60\cos 40 + 0 + 80\cos 30 = +63.3193\ N$

$\Sigma Y = 0 + 60\sin 40 + 50 + 80\sin 30 = +128.5672\ N$

$\therefore R = 63.3193 + j128.5672 = 143.3138\ N \angle 63.7797^\circ$



Now to find couple, take moments about 'O'.

$\Sigma M_O = 0 + (60\cos 40 \times 4) - (60\sin 40 \times 7) + 140 + 0$

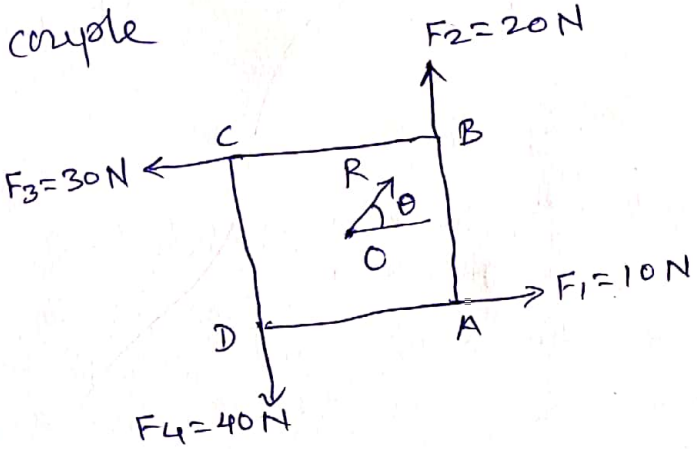
$= -196.1201\ N\text{-}m\ \ CW$

The line of action of R wrt O will be at a distance of ....

$x = \frac{|\Sigma M_O|}{|\Sigma Y|} = \frac{196.1201}{128.5672} = 1.5254\ m$ horizontally

& $y = \frac{|\Sigma M_O|}{|\Sigma X|} = \frac{196.1201}{63.3193} = 3.0973\ m$ vertically

ex 5 / 4.5.43 Four forces of 10N, 20, 30 & 40N are acting as shown below on a square of 100 mm x 100 mm. A fifth force when acts through the centre of the square at 'O' reduces the force system into a couple. Find magnitude & direction of the 5th force and the sense of the couple



Soln - Assume that there is a 5th force at 'O' with magni. R & angle θ (both unknown)
When there is a couple, the resultant force in any direction is = to zero.
∴ ∑X = 0 and ∑Y = 0.
∴ Write components along X & Y

$\Sigma X = 0 \quad \therefore +10 - 30 + R\cos\theta = 0$

$\therefore R\cos\theta = 20 \quad -(1)$

Also, $\Sigma Y = 0 \quad \therefore +20 - 40 + R\sin\theta = 0$

$\therefore R\sin\theta = 20 \quad -(2)$

$\therefore \tan\theta = 1 \quad \therefore \theta = 45^\circ$ ..... from (2)/(1)

As, $R\cos 45 = 20 \quad \therefore R = 28.2842\ N\angle 45^\circ$

Now take moments about 'O'.

$\Sigma M_O = (10\times 50) + (20\times 50) + (30\times 50) + (10\times 50) = 5000\ N\text{-}m$ CCW ↑