8.1 Equilibrium of Force Systems in Plane

8.1.1 Equilibrium :

- A rigid body is said to be in equilibrium, when the resultant of the force system on it is zero.
- The resultant of the force system can be a force or a moment.
- If the resultant is zero, it implies that the resultant force ($\sum$ F) and resultant moment ($\sum$ M) both are zero.
- If the resultant force i.e. $\sum$ F i.e. zero, there is no translation and if the resultant moment i.e. $\sum$ M = 0. There is no rotation. Then the body is said to be in complete static equilibrium.

8.1.2 Conditions of Equilibrium :

When the body is in equilibrium;

(i) Resultant force, $\sum F = 0$ and

(ii) Resultant moment, $\sum M = 0$

Resultant force $\sum$ F can be resolved into two perpendicular components in x and y directions.

If $\sum F = 0$, then its x-component ($\sum F_x$) and y-component ($\sum F_y$) both are zero.

$\therefore$ The necessary conditions or equations of equilibrium for coplanar force system are :

$$\sum F_x = 0 \quad ...(1)$$

$$\sum F_y = 0 \quad ...(2)$$

And $$\sum M = 0 \quad ...(3)$$

($\sum$ M about any point must be zero)

8.2 Particular Cases of Equilibrium

(i) Equilibrium with a Single Force :

Let a force 'F' is acting on a body at 'A' at an angle 'θ' w.r.t. horizontal.

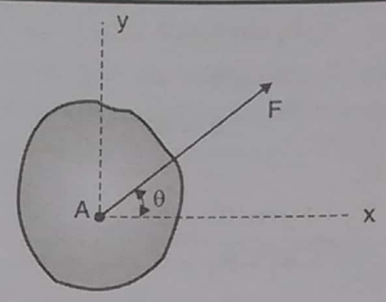


Fig. 8.2.1

To maintain equilibrium ;

$$\sum F_x = 0$$

$$\therefore \quad F \cos\theta = 0 \quad ...(1)$$

$$\sum F_y = 0$$

$$F \sin\theta = 0 \quad ...(2)$$

Suppose, $F \neq 0$, then $\sin\theta = 0$

$$\therefore \quad \theta = 0°$$

Putting in Eq^n (1), $F \cos 0° = 0$

$$F(1) = 0$$

$$\therefore \quad F = 0$$

$\therefore$ **When one force is acting on a body, equilibrium is not possible unless that force itself is zero.**

(ii) Equilibrium with two forces :

Let two non parallel F_1 and F_2 are acting on the body as shown in Fig. 8.2.2.

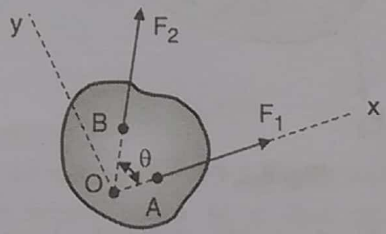


Fig. 8.2.2

Let the lines of action of two forces F_1 and F_2 intersect at point 'O' and 'θ' be the angle between two forces.

Considering x-axis along F_1 and y-axis perpendicular to it and resolving F_1 and F_2 into x and y components.

For equilibrium,

$$\sum F_x = 0$$

$$F_1 + F_2 \cos\theta = 0 \quad ...(1)$$

$$\sum F_y = 0$$

$$F_2 \sin\theta = 0 \quad \text{...(2)}$$

But $F_2 \neq 0$, $\therefore \sin\theta = 0$

$$\theta = 0°$$

From Eq^n (1),

$$F_1 + F_2 \cos 0° = 0$$

$$F_1 + F_2 = 0$$

$$F_1 = -F_2$$

$\therefore$ **When two non parallel force are acting to maintain equilibrium, those two forces must be equal in magnitude, opposite in direction and collinear in action.**

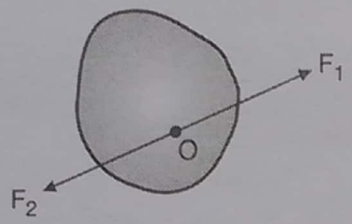


Fig. 8.2.3

(iii) Equilibrium under three forces :

Let F_1, F_2 and F_3 be the three non parallel forces acting on the body as shown in Fig. 8.2.4.

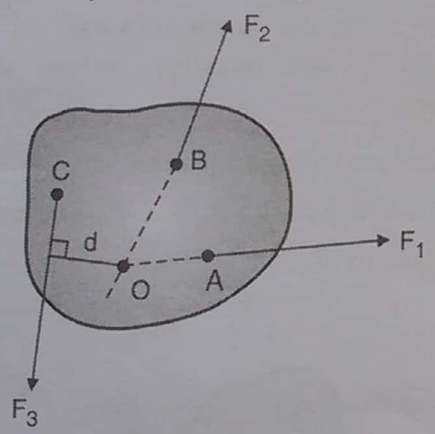


Fig. 8.2.4

Let 'O' be the point where the lines of action of forces F_1 and F_2 intersect.

'd' is the perpendicular distance from the line of action of force F_3 to the point 'O'.

For equilibrium,

$\sum M$ about any point must be zero.

Taking moments about point 'O'

$$\sum M_o = 0$$

$$F_3 \times d = 0$$

(Moments of F_1 and F_2 at 'O' are zero as their lines of action passing through point 'O').

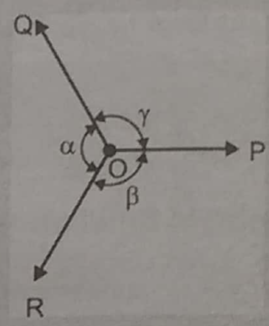
but, $F_3 \neq 0$

$\therefore$ $\mathbf{d = 0}$

Which means that the line of action of force F_3 is also passing through point 'O'.

$\therefore$ **When three non parallel forces are acting on the body, to maintain equilibrium, these three forces must be concurrent.**

Lami's Theorem : It states that when three concurrent forces are acting at a point which is in equilibrium, then the magnitude of each force is proportional to the sine of angle between other two forces.



As per Lami's theorem :

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

8.3 Support and Support Reaction :

***Support** is defined as the structure which tends to maintain equilibrium of the body.*

*While maintaining equilibrium under the action of applied forces or self weight of the body the support exerts a force or moment on the body known as **"support reaction".***

- When a particular displacement of the body is prevented corresponding reaction will develop.
- If the linear displacement is prevented, support will offer a reaction force and if the angular displacement or rotation of the body is prevented, support will offer a reaction moment.

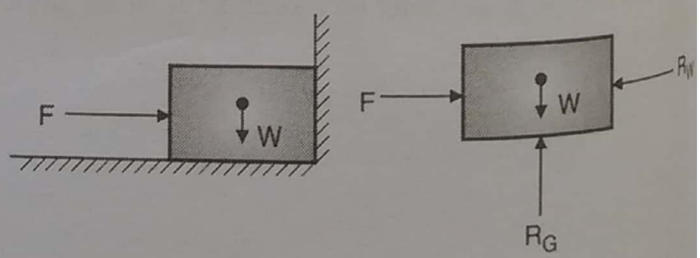


Fig. 8.3.1

Because of weight, W body will have vertical downward displacement which is prevented by the ground surface by offering a support reaction R_G in the opposite direction.

Similarly, applied force 'F' will cause displacement of the body towards right which is prevented by vertical wall by offering a reaction force R_W in the opposite direction.

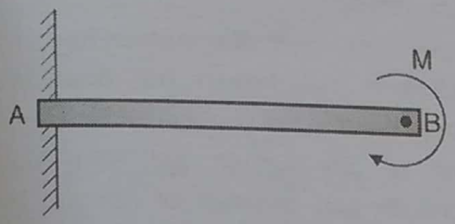


Fig. 8.3.2

Let a clockwise moment 'M' is applied to member 'AB' which tends to rotate the member in CW direction. To maintain equilibrium support at 'A' will offer reaction moment (M_R) in ACW direction i.e. opposite to the direction of rotation.

Fig. 8.3.3

8.4 Types of Supports

In engineering, basically three types of supports are used :

(i) Simple support

  a) Sliding b) Roller

(ii) Hinge or pin support

(iii) Fixed support

(i) **Simple supports :** Simple support is the support in which there is no link or connection between the body and the support.

There are two types of simple support :

a) **Sliding support :** If the body rests against a surface whether it is horizontal, vertical or inclined, then it is known as sliding support.

b) **Roller support :** If the body rolls over the surface instead of sliding then it is known as roller support.

Sliding or roller support will prevent the displacement in the direction perpendicular to the plane of surface. Hence sliding or roller support always offer reaction normal to the plane of surface.

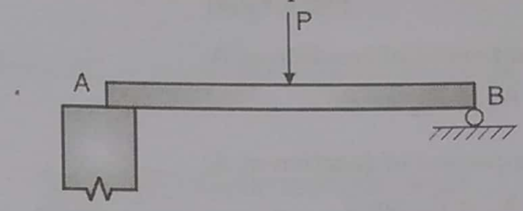


Fig. 8.4.1(a)

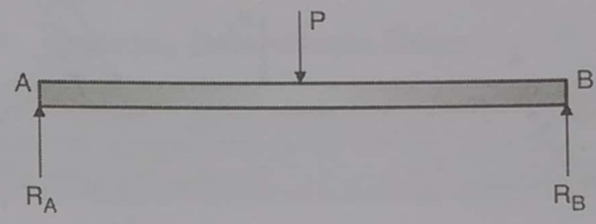


Fig. 8.4.1(b)

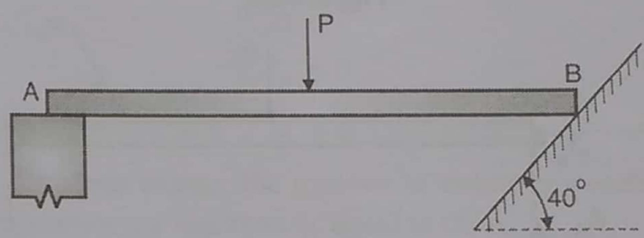


Fig. 8.4.1(c)

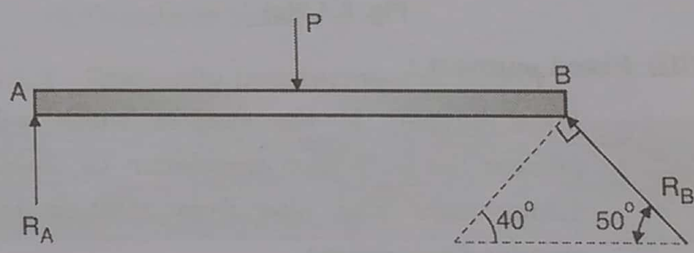


Fig. 8.4.1(d)

(ii) **Hinge or pin support :** Hinge support is the support where there is a link or connection between the body and the support, but it allows the rotation of the body.

Hinge support will prevent linear displacement and allows angular displacement.

∴ Hinge support will offer reaction force by preventing linear displacement but the reaction moment is zero.

The reaction force will have magnitude 'R' and direction 'θ'. It can be resolved in two reaction components R_x and R_y along horizontal and vertical directions.

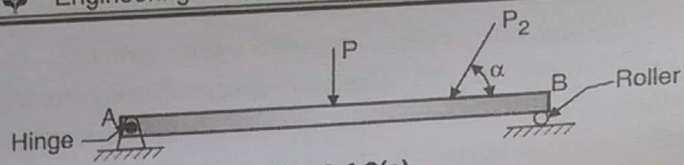


Fig. 8.4.2(a)

x-component of reaction at A,

$$A_x = R_A \cos \theta$$

y-component of reaction at A,

$$A_y = R_A \sin \theta$$

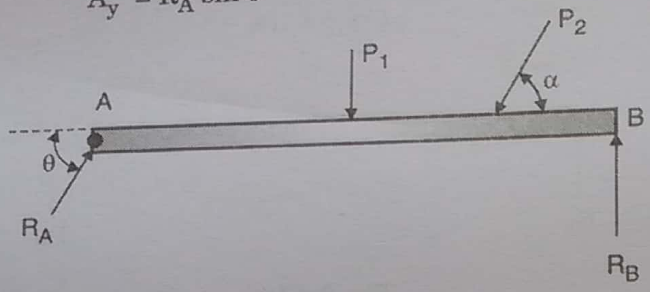


Fig. 8.4.2(b)

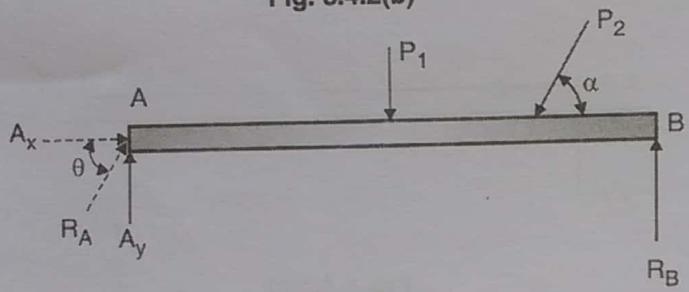


Fig. 8.4.2(c)

(iii) Fixed support :

- Fixed support is the support which will prevent the body from translation i.e. linear displacement and also from rotation i.e. angular displacement.
- Hence fixed support will offer reaction force and reaction moment.
- In fixed support, there is a rigid connection between the body and the support.

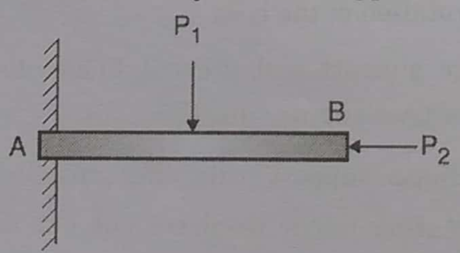


Fig. 8.4.3(a)

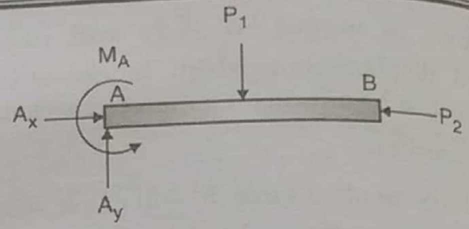


Fig. 8.4.3(b)

Important notes :

(i) Simple support will offer reaction force. Magnitude of reaction is not known but direction is always perpendicular to the plane of surface.

(ii) Hinge or pin support will offer reaction force. Magnitude and direction of reaction both are not known. It can be resolved into x and y components.

(iii) Fixed support will offer reaction force and reaction moment.

(iv) The number of unknowns at

Simple support :	1.	i.e. magnitude of reaction
Hinge/pin support :	2.	i.e. magnitude and direction of reaction or x and y components of reaction force
Fixed support :	3.	i.e. magnitude and direction of reaction and reaction moment. OR x and y components of reaction force and reaction moment.

8.5 Beam

A beam is defined as a structural member which is subjected to transverse loading. i.e. the load acting perpendicular to the longitudinal axis of the member.

Due to applied loads, reactions will develop at the supports and the system of forces consisting of applied loads and reactions keep the beam of equilibrium.

The nature of reaction depends on the type of supports.

8.5.1 Types of beams

Depending on the type of supports, beams are classified into the following types :

(i) **Simply supported beam :** One end is supported by hinge and the other end roller.

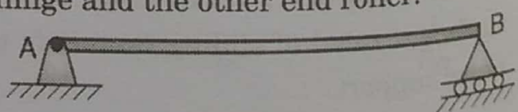


Fig. 8.5.1

(ii) **Cantilever beam :** One end of the beam is fixed and the other end free.

Fig. 8.5.2

(iii) **Fixed beam :** Both ends of the beam are fixed.

Fig. 8.5.3

(iv) **Propped cantilever :** One end of the beam is fixed and the other end is simply supported.

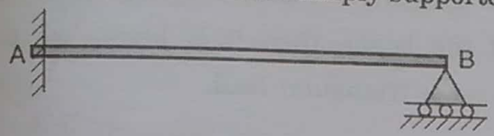


Fig. 8.5.4

(v) **Overhanging beam :** Beam is having projection beyond the support.

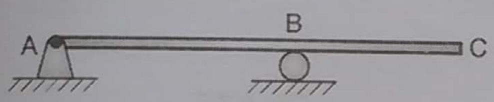


Fig. 8.5.5 : Singly overhanging beam (Having projection on one side)

Fig. 8.5.6 : Doubly overhanging beam

(Having projection on both sides)

(vi) **Continuous beam** : Beam supported by more than two supports.

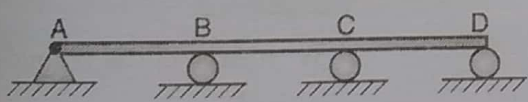


Fig. 8.5.7

(vii) **Compound beam :** It is the combination of two simple beams.

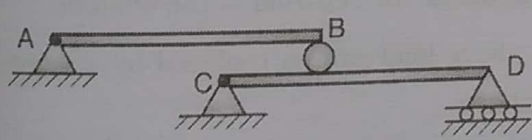


Fig. 8.5.8

Beam AB rests on beam CD.

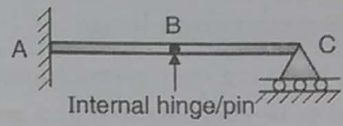


Fig. 8.5.9

Beam AB is connected to beam BC by an internal hinge.

8.6 Statically Determinate and Indeterminate Beams

8.6.1 Statically Determinate Beam :

*A beam which can be analysed using three equations of equilibrium is known as **statically determinate beam.***

The equilibrium equations are :

$$\sum F_x = 0,$$

$$\sum F_y = 0$$

$$\text{and } \sum M = 0$$

In these beams, the number of unknown reaction components are less than or equal to three.

E.g. (i) Simply supported beam

(ii) Cantilever beam, etc.

8.6.2 Statically Indeterminate Beam :

*The beam which cannot be analysed using 3 equations of equilibrium is called **statically indeterminate beam.***

In this type of beam, the number of unknown reaction components are more than three equilibrium equations.

E.g. (i) Fixed beam

(ii) Propped cantilever

(iii) continuous beam, etc.

8.7 Types of Loads

Beam is subjected to the following types of loads:

(i) Point or concentrated load

(ii) Uniformly Distributed Load (UDL)

(iii) Uniformly Varying Load (UVL)

(iv) Variation is nonlinear

(i) Point or concentrated load :

If the load is acting over a small area or length, it can be assumed to be concentrated at a point known as point load or concentrated load.

It is represented by an arrow head.

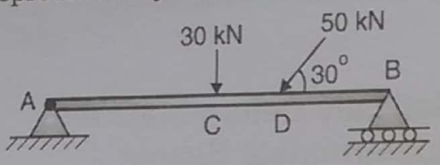


Fig. 8.7.1

Point load of 30 kN is acting at point 'C' vertically and 50 kN load is acting at point 'D' and having inclination of 30° w.r.t. horizontal.

(ii) Uniformly Distributed Load (UDL) :

If the load is spread or occupied over a considerable length of the beam it is known as distributed load. If the intensity of load i.e. load per unit length is uniform throughout, then it is known as Uniformly Distributed Load (UDL).

It is represented as below :

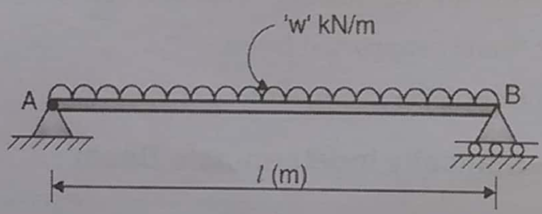


Fig. 8.7.2

Magnitude of UDL,

$$W = (\text{Intensity of load}) \times (\text{Length})$$

Total load, $W = w.l$ kN

Total load will act the midpoint of the length i.e. AB.

UDL can be replaced by the point load as below :

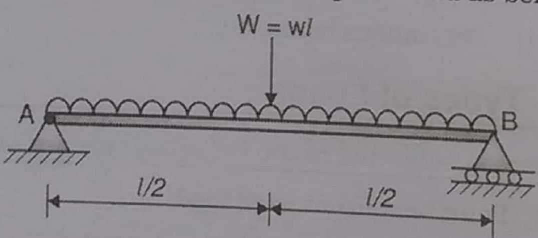


Fig. 8.7.3

Uniformly distributed load is also called rectangular load as it can be represented as a rectangle shown below :

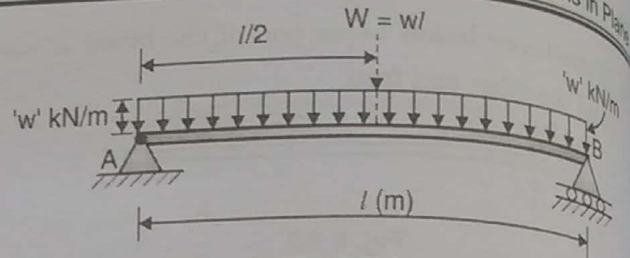


Fig. 8.7.4

Area of rectangle = Total magnitude of load (W)

$\therefore$ W = 'wl' kN

It will act at the centroid of the rectangle.

(iii) Uniformly varying load (UVL) :

If the intensity of load is varying uniformly over a length of the beam, then it is known as uniformly varying load or triangular load.

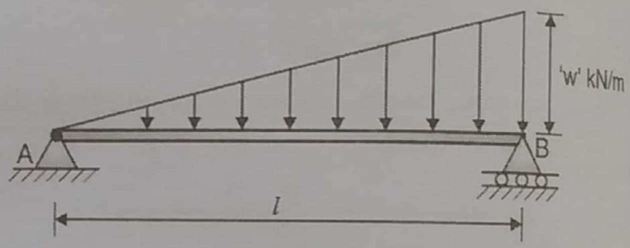


Fig. 8.7.5

Here intensity of load is varying uniformly from zero at end A to 'w' kN/m at end B.

Magnitude of total load, W = (Average intensity of load) × Length

$$\therefore \quad W = \left(\frac{0+w}{2}\right) l$$

$$= \left(\frac{wl}{2}\right) \text{kN}$$

OR

$$W = \text{Area of the triangle}$$

$$= \frac{1}{2} \times l \times w$$

$$= \left(\frac{wl}{2}\right) \text{kN}$$

It will act at the centroid of the triangle.

The above load can be replaced by a point load as shown below :