Moment of Inertia (MI)

Moment of force M_F = Force × ⊥ar distance from line of action.

If moment of force is multiplied by d again, we get Moment of Inertia – MI. called as second moment of force. $(F \times d) \times d$ [moment of moment of force]

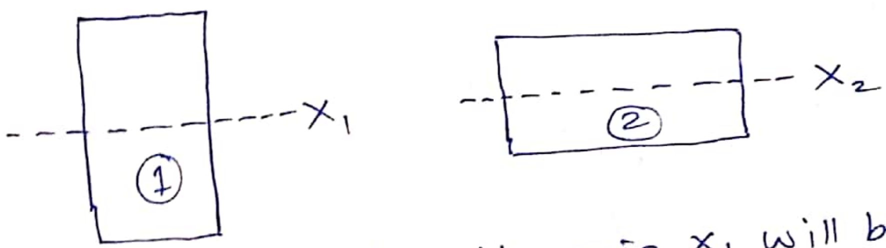
–1) Area moment of inertia – unit mm^4 or m^4

–2) Mass moment of inertia – unit $kg\text{-}mm^2$, $kg\text{-}m^2$

→ M.I. tells about ability of a beam to resist bending

→ Larger the M.I. less is the bending of the beam.

→ M.I depends upon the reference axis.



M.I. of the body with axis X_1 will be more than the MI of the same body with axis X_2.

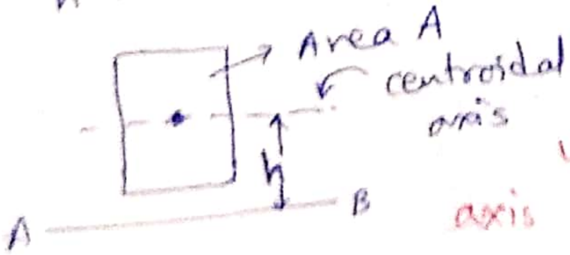
<u>Parallel Axis Theorem</u>:– MI of a plane area A about any axis AB which is parallel to the centriodal axis located at a distance h is given by ....

$$MI_{AB} = MI_{cen.Axis} + Ah^2$$

where $MI_{cent.Axis}$ = MI about centroidal axis

A = Area of the plane

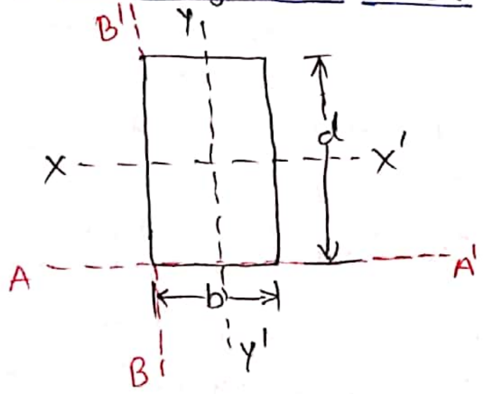
h = Distance betn centroidal axis & the parallel axis AB.



[This theorem can be used when MI @ a different axis is asked, which is not as per standard list given]

M.I. of standard shapes

1) Rectangular Area -



about $XX' = \frac{bd^3}{12}$

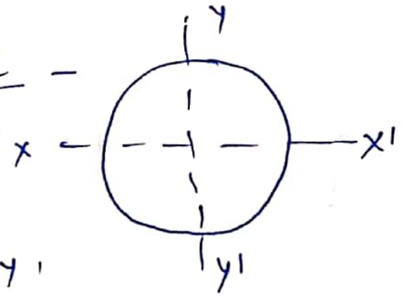
about $YY' = \frac{b^3 d}{12}$

about $AA' = \frac{bd^3}{3}$

about $BB' = \frac{b^3 d}{3}$

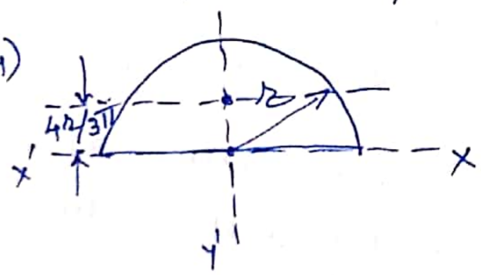
2) Square - $I_{xx} = I_{yy} = \frac{a^4}{12}$

3) Circle -



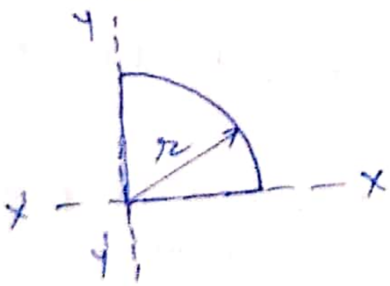
$I_{xx'} = I_{yy'} = \frac{\pi D^4}{64}$

4)



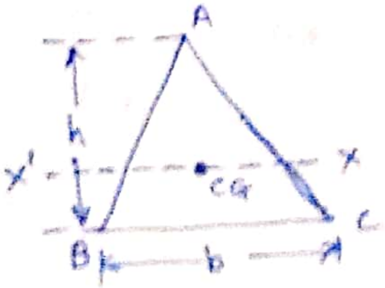
$I_{xx'} = I_{yy'} = \frac{\pi D^4}{128}$

5) Quarter Circle -



$I_{xx'} = I_{yy'} = \frac{\pi D^4}{256}$

6) Triangle -



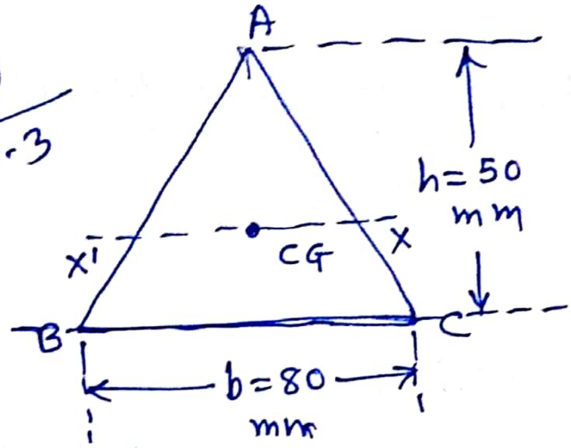
$I_{BC} = \frac{bh^3}{12}$

$I_{xx'} = \frac{bh^3}{36}$

Procedure to find M.I. :-

1) Divide into standard sections.
2) Find C.G. of each section.
3) Find M.I. of each section.
4) Using parallel axis theorem, find M.I. of each section @ the centroidal axis.
5) Addition of M.I. of each section = answer = M.I. of the whole section.

Ex-1
6.10.3



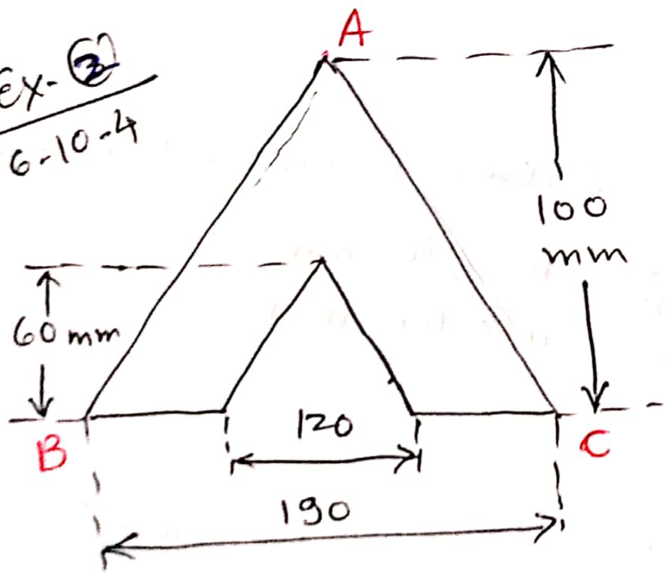
Calculate M.I. ob the isoseeles Δ about XX' and BC axis.

Soln –

$$I_{xx} = \frac{bh^3}{36} = \frac{80 \times 50^3}{36} = 277777.\overline{77}\ mm^4$$

$$I_{BC} = \frac{bh^3}{12} = \frac{80 \times 50^3}{12} = 833333.\overline{33}\ mm^4$$

Ex-2
6-10-4



Find M.I. about BC

I_{BC} = MI of Δ − MI of Δ

$$= \frac{BH^3}{12} - \frac{bh^3}{12}$$

$$= \frac{190 \times 100^3}{12} - \frac{120 \times 60^3}{12}$$

$= 15833333.33$

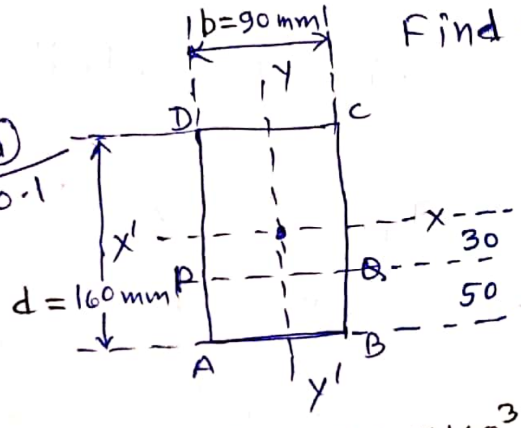
$- 2160000$

$= 13673333.\overline{33}\ mm^4$

~~Rectangle~~ Procedure to find M.I.:-

1) Divide into standard sections.
2) Find C.G. of each section.
3) Find M.I. of each section.
4) Using parallel axis theorem, find M.I. of each section @ the centroidal axis.
5) Addition of M.I. of each section = answer = M.I. of the whole section.

Ex 3 / 6.10.1



Find MI @ XX', YY' & AB & PQ

Area $= 90 \times 160 = 14400\ mm^2$

$b = 90$ mm

$d = 160$ mm

$$I_{xx} = \frac{bd^3}{12} = \frac{90 \times 160^3}{12} = 30720000\ mm^4$$

$$I_{yy} = \frac{b^3 d}{12} = \frac{90^3 \times 160}{12} = 9720000\ mm^4$$

using parallel axis theorem ----

$$I_{AB} = I_{xx} + Ah^2$$
$$= (30.72 \times 10^6) + (14400 \times 80^2)$$
$$= 122.88 \times 10^6\ mm^2$$

OR

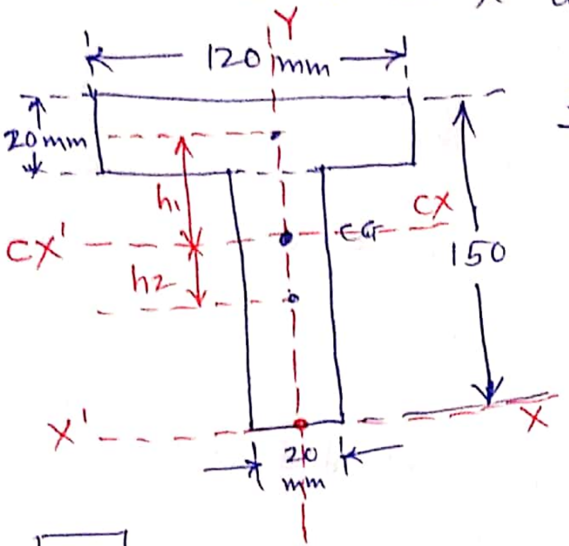
$$I_{AB} = \frac{bd^3}{3} = \frac{90 \times 160^3}{3} = 122.88 \times 10^6\ mm^4$$

using ||el axis thm....

$$I_{PQ} = I_{xx} + Ah^2$$
$$= (30.72 \times 10^6) + (14400 \times 30^2) = 43.68 \times 10^6\ mm^4$$

Answer

Ex (4) Find MI of the T section about the centroidal X and Y axes.
6-10-6



Soln – First find location of Centroid of the I section.

▭ and ▯

$Area_1 = 2400 \ mm^2$ ▭

$Area_2 = 2600 \ mm^2$ ▯

▭ $x_1 = 0$, $y_1 = 140$ mm

▯ $x_2 = 0$, $y_2 = 65$ mm

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{(2400 \times 140) + (2600 \times 65)}{2400 + 2600}$$

$$= \frac{336000 + 169000}{5000} = 101 \ mm \quad (y \text{ of centroid})$$

and $\bar{x} = 0$ mm

Now, MI about centroidal axis – YY'

for ▭ $I_{G_1} = \frac{bd^3}{12} = \frac{120 \times 20^3}{12} = 80000 \ mm^4$

for ▯ $I_{G_2} = \frac{bd^3}{12} = \frac{20 \times 130^3}{12} = 3660000 \ mm^4$

Find h_1 (distance betn CX'-CX and X-X' of upper part)
& h_2 (— " — — " — lower part)

$\therefore h_1 = 49 - 10 = 39$ mm, $h_2 = 101 - 65 = 36$ mm.

Using parallel axis theorem,

$I_{XX_1} = I_{G_1} + A h_1^2 = 80000 + (2400 \times 39^2)$
↳ (for upper) $= 3730000 \ mm^4$

$I_{XX_2} = I_{G_2} + A_2 h_2^2 = 3660000 + (2600 + 36^2)$
$= 7030000 \ mm^4$

Total MI $I_{XX} = I_{XX_1} + I_{XX_2} = 3730000 + 7030000$
$= 10760000 \ mm^4$

contd.

To find MI @ centroidal axis (Y)

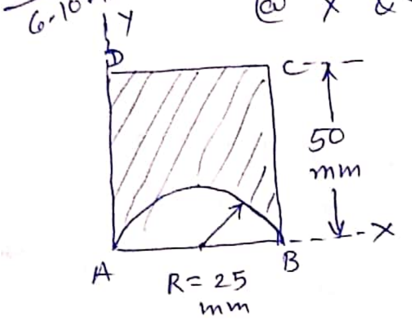
$\therefore I_{yy} = I_{yy_1} + I_{yy_2}$

$= \dfrac{120^3 \times 20}{12} + \dfrac{20^3 \times 130}{12}$

$= 2.88 \times 10^6 + 0.087 \times 10^6$

$= 2967000 \text{ mm}^4$

Ex (5) 6-10-11 Calculate MI of the shaded area @ X & Y axes.



$2083333.\overline{33}$
$- 153398.08$
$\boxed{1929935.255 \text{ mm}^4}$

Sol^n.

MI @ X axis

$= I_{\square} - I_{\frown}$

$= \dfrac{b^4}{3} - \dfrac{\pi D^4}{128}$

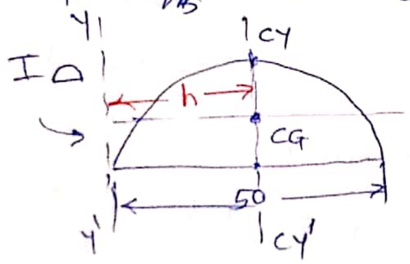
$= \dfrac{50^4}{3} - \dfrac{\pi\, 50^4}{128}$

$=$

MI @ Y axis $\therefore I_y = I_{\square} - I_{\frown}$

$I_{\square} = \dfrac{b^4}{3} = \dfrac{50^4}{3} = 2083333.\overline{33} \text{ mm}^4$

$I_{\frown}$



MI @ cy-cy' axis

$= \dfrac{\pi D^4}{128} = \dfrac{\pi \times 50^4}{128}$

$= 153398.0788 \text{ mm}^4$

but we want @ Y-Y' axis $\therefore$ use $\parallel^{el}$ axis thm.

$\therefore I_{yy'} = I_{cy-cy'} + Ah^2$

$= 153398.0788 + \left[\left(\dfrac{\pi \times 25^2}{2}\right) \times 25^2\right]$

$= 766990.394 \text{ mm}^4$

$\therefore$ I about Y-Y'

= I of square − I of semicircle

$= 2083333.\overline{33} - 766990.394$

$= 1316342.939 \text{ mm}^4$

Answer