Random Variables

Random Variables

Study Material for Week 5

Lecture One

Random Variables

Let Ω be a sample space associated with given random experiment. A function defined from Ω to a number set is known as **Random Variable**. Thus a random variable is a function whose domain set is a sample space of some random experiment.

The collection of values actually taken by a random variable is known as **Range** of the random variable. Depends on the range; random variables get classified into the following two types.

- **Discrete Random Variables::** Random variable whose range set is a discrete set or the set of natural numbers, i.e., a countable set is known as discrete random variable.

e. g. Number of accidents on Mumbai-Pune expressway, Number of Neutrons emitted by a radioactive isotope, Number of students in a class.

- **Continuous Random Variables::** Random variable whose range set is an interval is known as continuous random variable.

e.g. Lifetime of an electrical or electronic components, Height of students, Amount of cosmic radiation, Time between arrivals of two flights at certain airport, Time taken for surgery at a certain hospital.

Note : Random variables arising through the counting processes are usually discrete in nature while those which arises through measuring processes are continuous in nature.

Examples of Random Variables

1. Consider the experiment of tossing of two fair coins simultaneously. The sample space is

$\Omega = \{HH, HT, TH, TT\}$. Let $X_1 =$ count of heads in a single toss. By this every sample point get assign by a real number viz $X_1(HH) = 2, X_1(HT) = X_1(TH) = 1, X_1(HH) = 0$.

Thus X_1 is a random variable defined on Ω. Here Range of $X_1 = \{0,1,2\}$. Since the range of X_1 is a subset of discrete set, X_1 is a discrete random variable.

Similarly, $X_2 =$ count of tails in a single toss is also a discrete random variable defined on Ω

2. Consider the experiment of tossing of two fair dice simultaneously.

Sample space $\Omega = \{(i, j) / 1 \le i, j \le 6\}$.

i) $X_1 =$ Sum of the numbers appear on the two faces, i.e., $X_1((i, j)) = i + j$.

Random Variables

Range of $X_1 = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$.

ii) $X_2 =$ Product of the numbers appear on the two faces, i.e., $X_2((i, j)) = ij$

Range of $X_2 = \{1, 2, 3, \cdots, 36\}$.

iii) $X_3 =$ Minimum or Maximum of the two numbers appear on the two faces, i.e., $X_3((i, j)) = \min(i, j) \, or \max(i, j)$. Range of $X_3 = \{1, 2, 3, 4, 5, 6\}$.

Note : Range of all X_1, X_2 and X_3 is a subset of set of Natural number set (Discrete set)

Hence X_1, X_2 and X_3 are discrete random variables (d.r.v).

iv) $X_4 =$ Quotient of the two numbers appear on the two faces, i.e., $X_4((i, j)) = i / j$. Range of $X_4 = \left\{\frac{i}{j} / 1 \leq i, j \leq 6\right\}$. Here X_4 is taking integer as well as rational values. Therefore range of X_4 is an interval $\left(\frac{1}{6} \quad 6\right)$. But as the set is countable, X_4 is a discrete random variable (d.r.v.).

3. Consider the experiment of measurement of height of all First year students. $X =$ count of height of student. This X can take any positive value between 3.5 upto 7., Assuming least measured height is 3.5 feet and maximum 7.00 feet. Thus range of X is $(3.5, 7)$. Therefore X is a continuous random variable.

4. A point is chosen at random in a circle of radius r. Let X be the distance of the point from the centre of the circle. Then the range of X is the closed interval with end points 0 and r, i.e., range is $[0, r]$. Here X is a continuous random variable.

We first discussed discrete random variables (d.r.v).