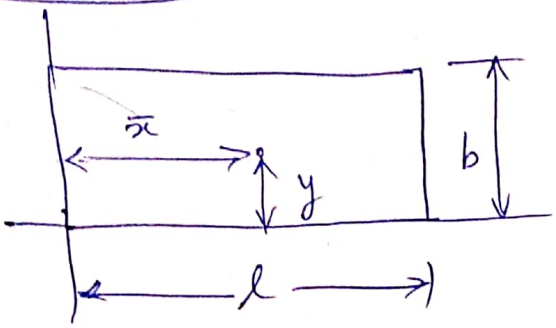
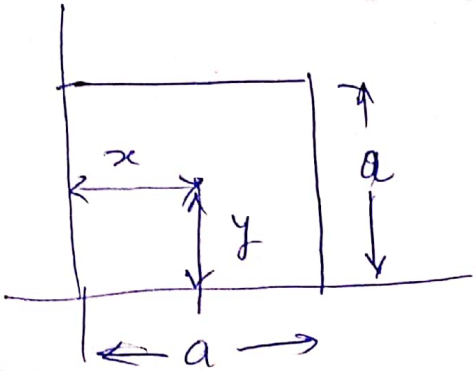
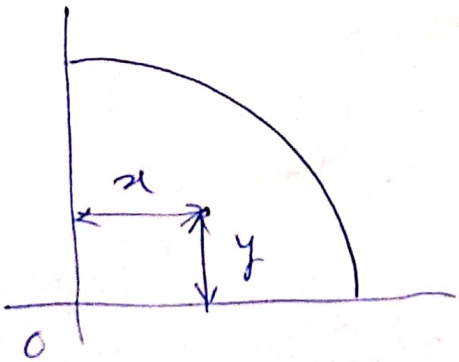
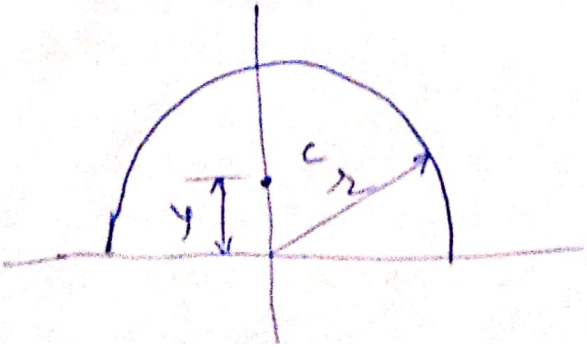
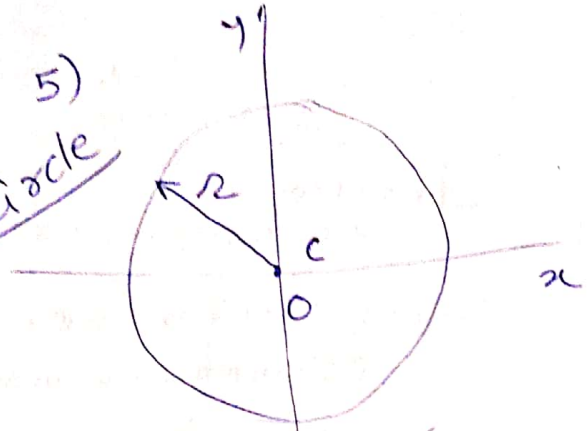
Centroids & Areas – formulae

(no derivation)

	Area	$\bar{x}$	$\bar{y}$
1) Rectangle – 	$l \times b$	$\frac{l}{2}$	$\frac{b}{2}$
2) square – 	a^2	$\frac{a}{2}$	$\frac{a}{2}$
3) Quarter circle – 	$\frac{\pi r^2}{4}$	$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$
4) Semi-circle – 	$\frac{\pi r^2}{2}$	0	$\frac{4r}{3\pi}$

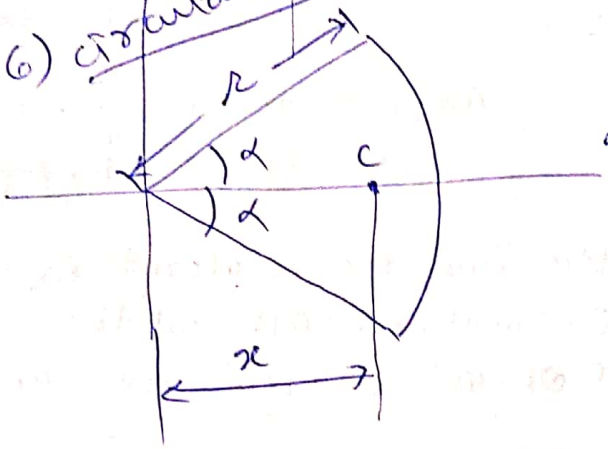
5) circle



πr^2 | O | O

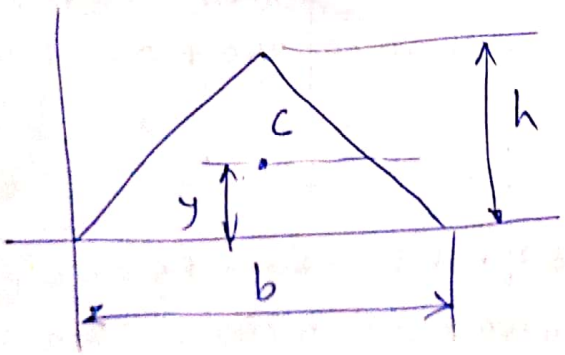
symmetrical at y axis | Symm at X axis

6) circular sector



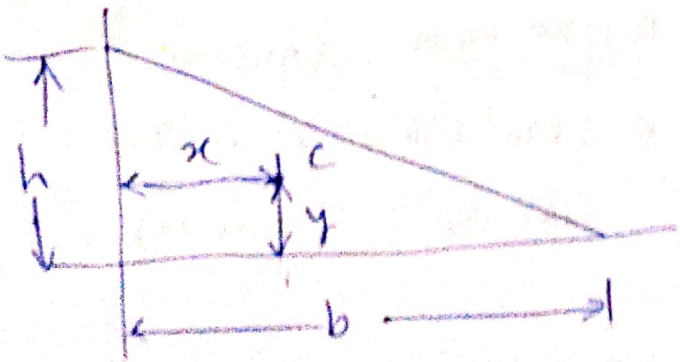
$\alpha^c r^2$ | $\frac{2r \sin \alpha^\circ}{3\alpha}$ → deg, Rad | O

7) Triangle -



$\frac{1}{2}bh$ | $\frac{b}{2}$ | $\frac{1}{3}h$

8) ⊥ triangle -

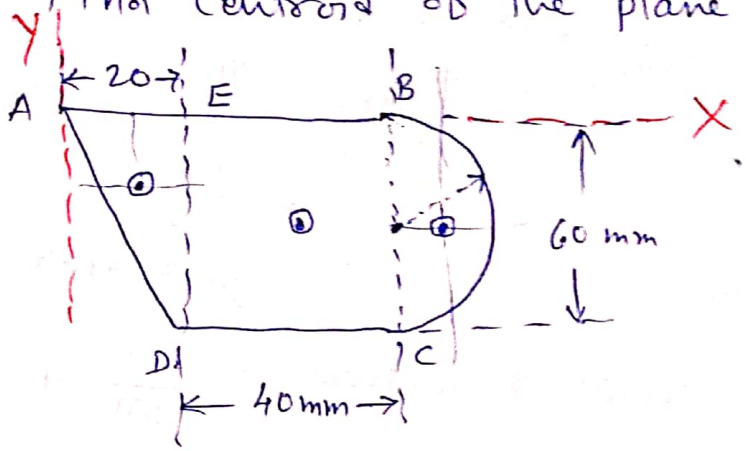


$\frac{1}{2}bh$ | $\frac{1}{3}b$ | $\frac{1}{3}b$

Centroid of areas :-

Q.1) Find centroid of the plane lamina.

5.8.15



Soln :

Divide into sections

1) Triangle AED
2) Rectangle EBCD
3) Semicircle BC

→ Find Area, distance of Centroid from Y & X
Moments about the two axes X & Y

1) Triangle AED

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2} \times 20 \times 60 = 600 \text{ mm}^2$$

Centroid is at $x = \frac{2b}{3}$ $y = \frac{h}{3}$

$\therefore$ X-compo. of Centroid $= \frac{20 \times 2}{3} = 13.\overline{33}$ mm

Y- —u— $= \frac{-60}{3} = -20$ mm

$A \cdot X = 600 \times 13.\overline{33} = 8000 \text{ mm}^2$

$A \cdot Y = 600 \times -20 = -12000 \text{ mm}^2$

2) Rectangle EBCD → Area $= 60 \times 40 = 2400 \text{ mm}^2$

$x = \frac{l}{2}$ $y = \frac{b}{2}$

$\therefore$ e $x = \frac{40}{2} = 20$ mm $y = \frac{-60}{2} = -30$ mm

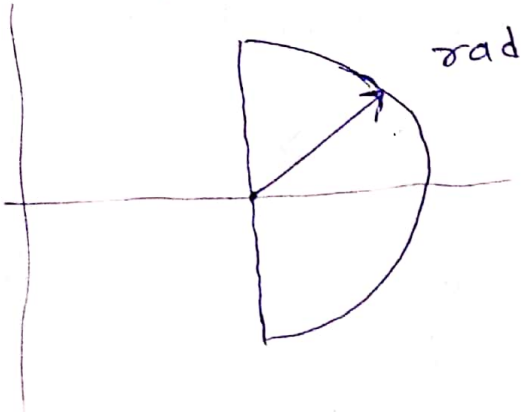
$+20$

40 mm

$A \cdot X = 2400 \times 40 = 96000 \text{ mm}^2$

$A \cdot Y = 2400 \times -30 = -72000 \text{ mm}^2$

3) Semi Circle BC



rad = 30 m

$$\text{Area} = \frac{\pi r^2}{2}$$

$$= \frac{\pi \times 30^2}{2}$$

$$= 1413.7167 \text{ mm}^2$$

X - compo. of centroid $= \frac{4r}{3\pi} + 60$

$$= \frac{4 \times 30}{3\pi} + 60 = 72.7324 \text{ mm}$$

$$Y = \frac{-60}{2} = -30 \text{ mm}$$

$$A \cdot X = 1413.7167 \times 72.7324$$

$$= 102823.002 \text{ mm}^2$$

$$A \cdot Y = 1413.7167 \times -30 = -42411.501 \text{ mm}^2$$

$$\rightarrow \quad \Sigma A = 600 + 2400 + 1413.7167$$

$$= 4413.7167 \text{ m}^2$$

$$\Sigma A \cdot X = 8000 + 96000 + 102823.002$$

$$= 206823.002 \text{ mm}^2$$

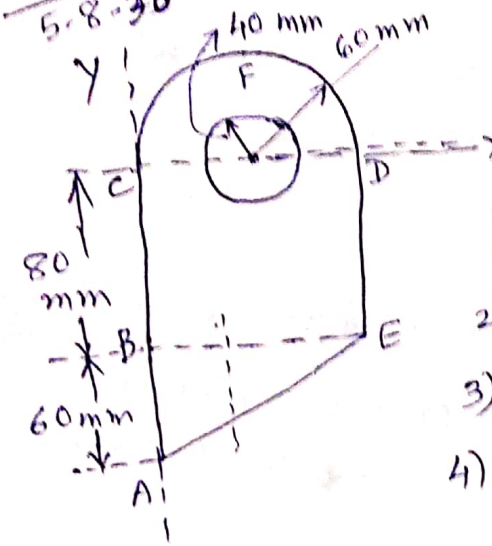
$$\Sigma A \cdot Y = -12000 - 72000 - 42411.501$$

$$= -126411.501 \text{ mm}^2$$

$$\bar{x} = \frac{\Sigma Ax}{\Sigma A} = \frac{206823.002}{4413.7167} = 46.8591 \text{ mm}$$

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{-126411.501}{4413.7167} = -28.6406 \text{ mm}$$

(2) 5.8.36 Find centroid of the plane lamina



Soln:- Decide sections

1) Triangle ABE 60 × 120
2) Rectangle BCDE 80 × 120
3) Semicircle CFDC r = 60 mm
4) Circle r = 40 mm.

1) Triangle ABE :-

$$\text{Area} = \frac{1}{2}bh = \frac{1}{2} 60 \times 120 = 3600 \text{ mm}^2$$

X-component of centroid of section ABE $= \frac{+h}{3} = \frac{+120}{3}$

$= +40$ mm.

Y-component of centroid of section ABE $= -\left(80 + \frac{b}{3}\right)$

$= -80 - \frac{60}{3} = -100$ mm

$AX = 3600 \times 40 = 144000 \text{ mm}^2$

$AY = 3600 \times -100 = -360000 \text{ mm}^2$

2) Rectangle BCDE

$\text{Area} = l \times b = 80 \times 120 = 9600 \text{ mm}^2$

X-component of centroid = 60

Y- —h— = −40

$AX = 9600 \times 60 = 576000 \text{ mm}^2$

$AY = 9600 \times -40 = -384000 \text{ mm}^2$

3) Semicircle → Radius = 60 mm

$\text{Area} = \frac{\pi r^2}{2} = \frac{\pi \times 60^2}{2} = 5654.8667 \text{ mm}^2$

X-compo. = 60 mm

Y-compo. $= \frac{4r}{3\pi} = \frac{4 \times 60}{3 \times \pi} = 25.4647$ mm

$AX = 5654.8667 \times 60$

$= 339292 \text{ mm}^2$

$AY = 5654.8667 \times 25.4647$

$= 144000 \text{ mm}^2$

4) Circle of radius 40 mm (to be removed)

Area $= \pi r^2 = \pi \times 40^2 = 5026.5482\ mm^2$

X-compo $= 60$, Y-compo $= 0$

$AX = 301592.892$ $\quad AY = 0$

$$\bar{X} = \frac{\Sigma AX}{\Sigma A} = \frac{144000 + 576000 + 339292 - 301592.892}{3600 + 9600 + 5654.8667 - 5026.5482}$$

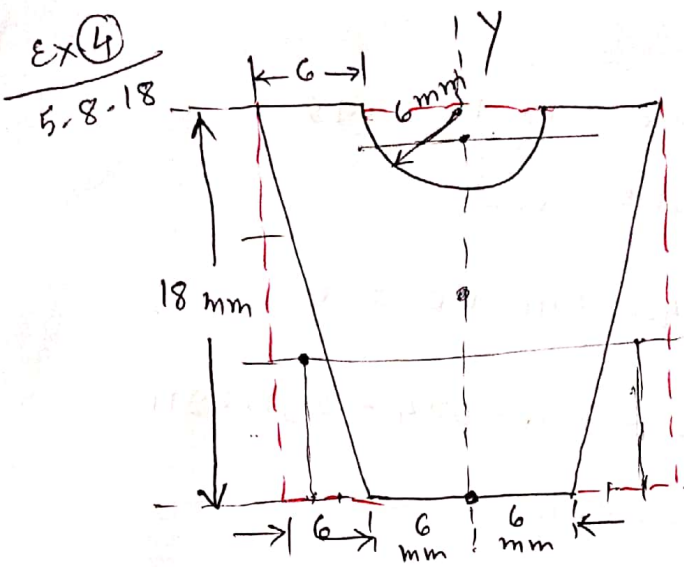
$$= 54.7932\ mm$$

$$\bar{Y} = \frac{\Sigma AY}{\Sigma A} = \frac{-360000 - 384000 + 144000 + 0 \ldots}{13828.3185}$$

$$= \frac{+600000}{13828.3185}$$

$$= -43.3892 \quad \text{Ans.}$$

Ex (4)
5.8.18



find x & y co-ordinates of the centroid of the area given.

Soln- find ~~cent~~ Area of the 18 × 24 rectangle and subtract 2 Δ's & semi circle.

(1) Rectangle 18 × 24 → Area = 432 mm^2

x at = 0 , y at = $\frac{h}{2} = \frac{18}{2} = 9$ mm

$AX = 432 \times 0 = 0\ mm^3$ & $AY = 432 \times 9 = 3888\ mm^3$

(2) Left Δ Area = $\frac{1}{2} bh = \frac{1}{2} \times 6 \times 18 = 54\ mm^2$

x at $\frac{b}{3}$ ∴ $x = \frac{6}{3} = 2$ mm ∴ $x = -10$

y at $\frac{h}{3}$ ∴ $y = \frac{18}{3} = 6$ mm ∴ $y = +6$

$AX = 54 \times -10 = -540$ & $Ay = 54 \times 6 = 324$

(3) Right Δ Area = $\frac{1}{2} bh = \frac{1}{2} \times 6 \times 18 = 54\ mm^2$

x at $\frac{b}{3}$ ∴ $x = \frac{6}{3} = 2$ mm ∴ $x = +10$ mm

y at $\frac{h}{3}$ ∴ $y = \frac{18}{3} = 6$ m ∴ $y = 6$ mm

$AX = 54 \times 10 = 540$ & $Ay = 54 \times 6 = 324$

(4) Area = $\pi r^2/2 = \pi \times 6^2/2 = 56.5486\ mm^2$

x at 0 , y at $\frac{4r}{3\pi} = \frac{4 \times 6}{3\pi} = 2.5464\ mm^2$

∴ $y = 18 - 2.5464 = 15.4535$ mm

$AX = 0$, $Ay = 56.5486 \times 15.4535 = 873.8737$

$$\Sigma A = 432 - 54 - 54 - 56.5486$$

$$= 267.4514 \text{ mm}^2$$

$$\Sigma Ax = 0 - 540 + 540 + 0 = 0$$

$$\Sigma Ay = 3888 - 324 - 324 - 873.8737$$

$$= 2366.1263 \text{ mm}^3$$

$$\therefore \bar{x} = 0$$

Ans

$$\bar{y} = \frac{\Sigma Ay}{\Sigma A} = \frac{2366.1263}{267.4514}$$

$$= 8.8469$$

Ans