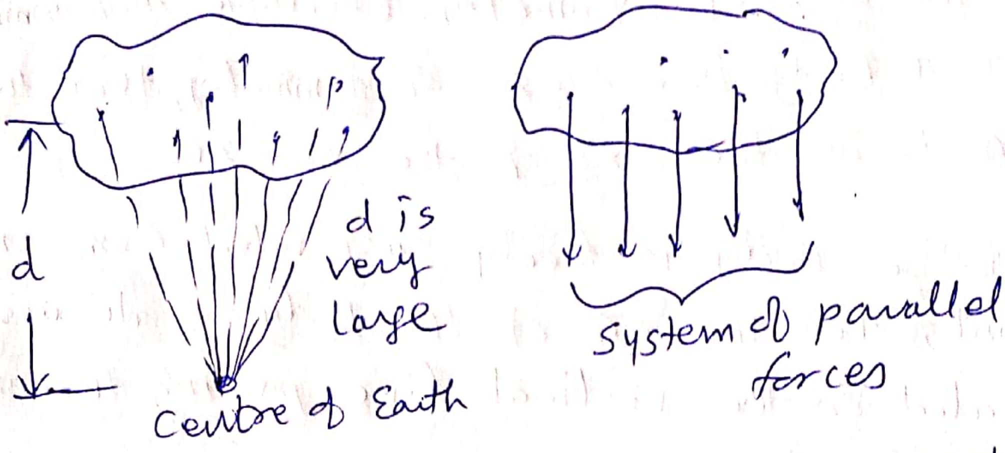
Centre of Gravity / Centroid etc.



C.G. - The point through which the resultant of all the parallel forces passes is called as C.G. of the body.

C.M.:- C.M. of a body is that point at which the whole mass of the body can be supposed to be concentrated.

cm & CG will be same if ...
1) The material is homogeneous
2) —— " —— is uniform

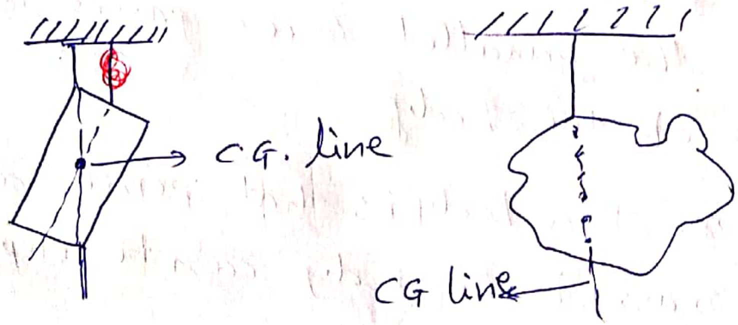
→ Lines, curves, areas ~~to~~ are not affected by Earth's attraction as they do not posses mass or volume.
Therefore they have 'Centroid'
e.g. centroid of line, centroid of area etc.

→ Axis of symmetry:- It is that line which divides the body or figure in two parts, so that the moments of these parts about the line are equal and opposite.
C.G. or centroid is always located on the axis of symmetry

→ ∴ Any line passing thro' the C.G. can be called as the axis of symmetry. (multiple A.of.Symm)

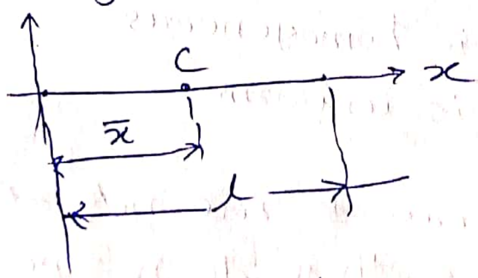
→ If a body has 2 axes of symmetry, then the CG is at the X^n of the two A.O.S.

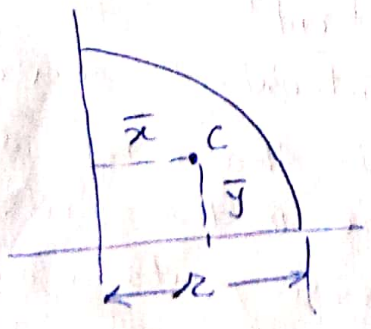
→ If a body is freely suspended from any point, then the CG of the body is always located on the vertical line passing through the point of suspension.



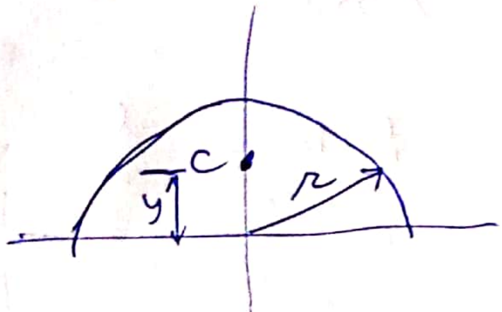
Centroid of lines

	length	X (distance of centroid from Y axis)	Y (from X axis)
1) straight Line	l	$\frac{l}{2}$	0
2) Quarter circular Arc	$\frac{\pi r}{2}$	$\frac{2r}{\pi}$	$\frac{2r}{\pi}$



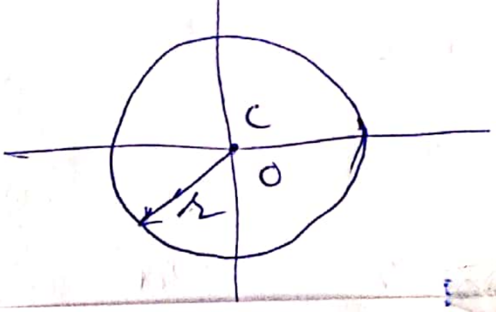


3) Semi circular Arc



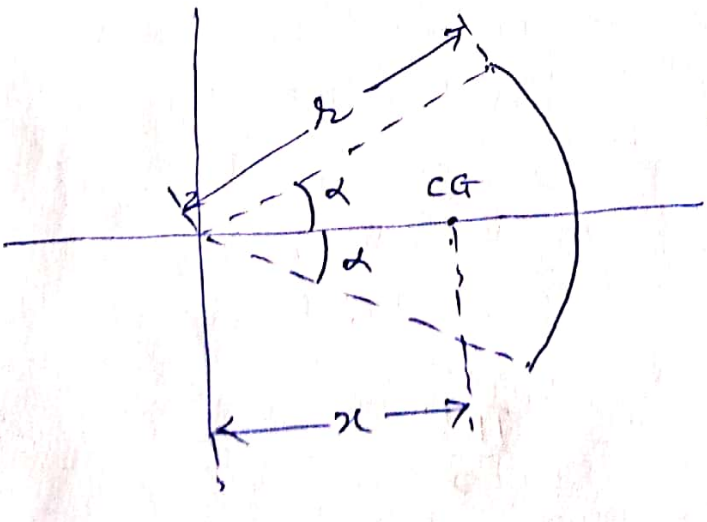
	length	x	y
Semi circular Arc	πr	0	$\frac{2r}{\pi}$

4) Circle:-



	length	x	y
Circle	$2\pi r$	0	0

5) Arc of a circle:-



length	x	y
$2r\alpha^c$	$\frac{r\sin\alpha^\circ}{\alpha^c}$	0
Radians	Radians degrees	

Scanned by CamScanner

4

Procedure for finding co-ordinates of Centroid

1) Select reference point and axes. (x & y)
2) Check the symmetry about x or y.

   e.g. if symmetrical @ x, then $y=0$....

   @ y, then $x=0$.
3) Consider different sections of the object.
4) Calculate & tabulate

   l — i) l = length of the segment

   x & y — ii) Position of centroid of segment w.r.t. y axis & x axis

   lx & ly — iii) Moments of lengths of segments about x & y axes.

   lx & ly → moments of length @ X axis mm^2; → moments of length about Y axis (mm^2)

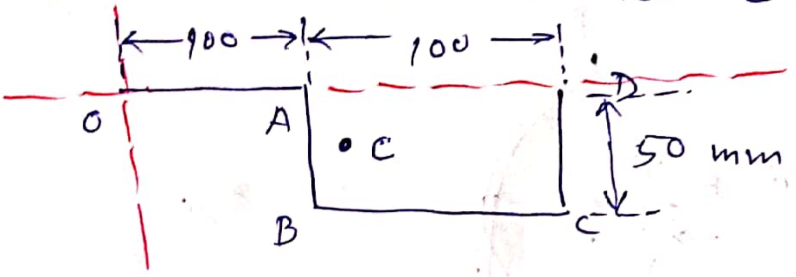
   after this, find

   iv) Σl, Σlx, Σly
5) The position coordinates are given by

$$x = \frac{\Sigma lx}{\Sigma l} \quad \& \quad y = \frac{\Sigma ly}{\Sigma l}$$

Ex. 1
5.8.3

A rod is bent as shown. Find centroid of the new shape wrt O (5)



Soln

Select x-y system with O as origin.

		l	x	y	lx	ly
1)	O —100— A	100	50	0	5000	0
2)	A | 50 | B	50	100	−25	5000	−1250
3)	B —100— C	100	150	−50	15000	−5000
4)	D | 50 | C	50	200	−25	10000	−1250
		300			35000	−7500

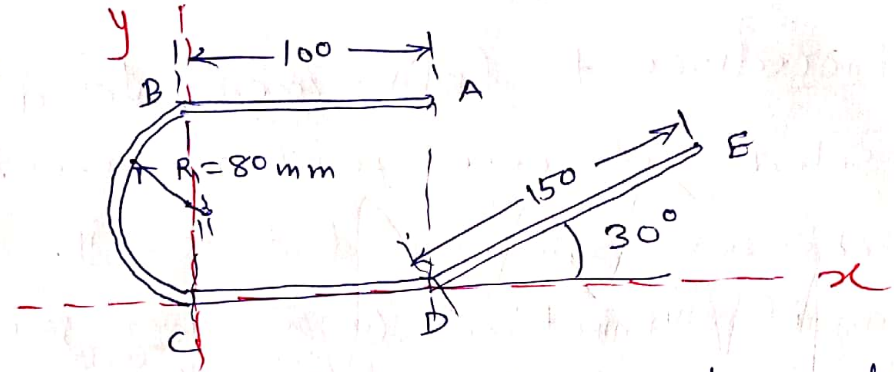
find

$$\bar{x} = \frac{\Sigma lx}{\Sigma l} = \frac{35000}{300} = 116.6666 \text{ m}$$

from 'O'

$$\bar{y} = \frac{\Sigma ly}{\Sigma l} = \frac{-7500}{300} = -25 \text{ mm}$$

(6) Q.2) / 5.8.1



A uniform wire is bent as shown, determine (portion) coordinates of C.G. of the wire w.r.t. point C

Soln – consider AB – BC – CD – DE separately.

		l	x	y	lx	ly
1) AB	B 100 A	100	50	160	5000	16000
2) BC	B, 80, C	πr $= 251.3274$	$-\frac{2r}{\pi}$ -50.9295	80	-12800	20106.1929
3)	C 100 D	100	50	0	5000	0
4	D 100 30° E	150	$100 + 75\cos 30$ $= 164.9519$	$75 \sin 30$ 37.5	24742.785	5625
		601.3274			21942.785	41731.1929

Co-ordinates CG wrt point C are

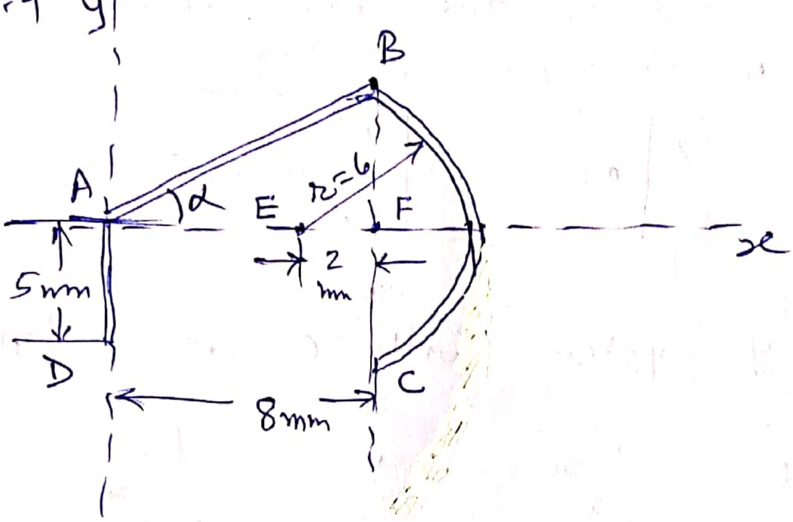
$$x = \frac{\Sigma lx}{\Sigma l} = \frac{21942.785}{601.3274} = 36.4905 \text{ mm}$$

$$y = \frac{\Sigma ly}{\Sigma l} = \frac{41731.1929}{601.3274} = 69.3984 \text{ mm}$$

from 'C'

Q.3) find centroid (co-ordinates) 7

5-8-4



1) Section AD $\quad l = 5$ mm , $x = 0$

$y = -\frac{5}{2} = -2.5$ mm.

$lx = 0$, $\quad ly = -12.5$

2) Section AB $\quad l = ? = BF$

$EF = 2$, $EB = 6$ $\quad EFB$ is $\perp$

$BF = \sqrt{6^2 - 2^2} = 5.6568$ mm

$\therefore \quad AB = \sqrt{AE^2 + EB^2} = (8 + j\,5.6568)$

$AB = l = 9.7979$ m $\angle 35.2641° \rightarrow \alpha°$

$\rightarrow r$

x of AB $= 4$ mm

y — " — $= \frac{BF}{2} = \frac{5.6568}{2} = 2.8284$ m

~~circular arc BC~~

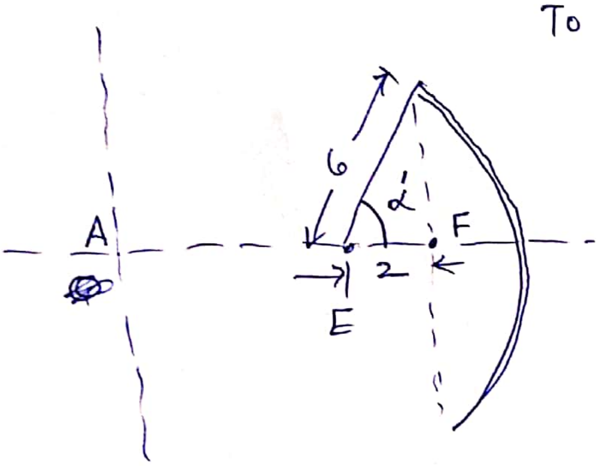
$lx = 9.7979 \times 4$

$= 39.1916$

$ly = 9.7979 \times 2.8284$

$= 27.7123$

3) Circular Arc BC



To find AE + EF

$\therefore$ 6 + ?

To find EF,

$\alpha' = \cos^{-1}\frac{2}{6} = 70.5287^{\circ}$ (deg.)

$= 1.2309^{c}$ (Rad)

length $l = 2r\alpha'^{c}$

$= 2 \times 6 \times 1.2309$

$= 14.7708$ mm

EF is the distance of centroid.

$x = AE + EF$

$= 6 + \frac{r\sin\alpha'^{\circ}}{\alpha'^{c}}$

$= 6 + \frac{6\sin 70.5287}{1.2309}$

$= 10.5957$ mm.

$y = 0$ (symmetrical)

$lx = 14.7708 \times 10.5957 = 156.5069$ mm

$ly = 0$

$\Sigma lx = 0 + 39.1916 + 156.5069 = 195.6985$ m

$\Sigma ly = -12.5 + 27.7123 + 0 = 15.2128$ mm

$\Sigma l = 5 + 9.7999 + 14.7708 = 29.5687$ m

Co-ordinates of Centroid are....

$\therefore x = \frac{\Sigma lx}{\Sigma l} = \frac{195.6985}{29.5687} = \underline{6.6184}$ mm

$y = \frac{\Sigma ly}{\Sigma l} = \frac{15.2128}{29.5687} = \underline{0.51448}$ mm