Random Variables

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Study Material for Week 5

Lecture Zero

In these sessions you will learn probability, a **random variable** or **stochastic variable.**

A **random variable** or **stochastic variable** is a variable whose value is subject to variations due to chance ,i.e. ,randomness. In this topic you will learn random variables, types of random variable and standard probability distributions. You will learn these concepts through ample practical situations and applications.

Basic Preliminaries: -

- **Random Experiment::**Experiment whose outcomes are not predictable.
- **Sample Point::** Each and every outcome of random experiment.
- **Sample Space::** Totality or aggregate of sample points. It is denoted by symbol S or Ω
- **Event::** A subset of sample space.
- **Impossible Event::** Event which does not contain any sample point.
- **Certain Event::**Event which contains all sample points**.**
- **Mutually Exclusive events::** The happening of any one event excludes the happening of the other event.
- **Exhaustive Event::**The events $\{A_1, A_2, \cdots, A_n\}$ are said to be exhaustive if $\bigcup_{i=1}^{n} A_i = S$.
- **Independent Events::** The occurrence or non occurrence of one does not affect the occurrence or non occurrence of the other.

 # Probability

If a random experiment results in 'n' mutually exclusive and equally likely outcomes of which 'm' are favorable to event A then probability of event A, denoted as

$p(A)$ and is defined as $p(A) = \frac{m}{n}$.

Standard Results ::

1. $0 \le p(A) \le 1$
2. $p\left(A^C\right) = p\left(\overline{A}\right) = 1 - p\left(A\right)$
3. $p(\phi) = 0,\ p\left(S\right) = 1$
4. If $A \subseteq B$ then $p(A) \le p(B)$
5. $p\left(A \cup B\right) = p\left(A\right) + p\left(B\right) - p\left(A \cap B\right)$

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6. If A and B are **mutually exclusive** events then $p(A\cap B)=0$
7. In particular, A and B are mutually exclusive $p(A\cup B)=p(A)+p(B)$
8. **Conditional Probability:** $p(A/B)=\dfrac{p(A\cap B)}{p(B)}$ **,** $p(B/A)=\dfrac{p(A\cap B)}{p(A)}$**.**

   Thus $p(A\cap B)=p(A)\,p(B/A)$ **,** $p(A\cap B)=p(B)\,p(A/B)$

For n events $A_1, A_2, \cdots, A_n$ the probability of intersection event is

$$p(A_1\cap A_2\cap A_3\cap\cdots\cap A_n)=p(A_1)\,p(A_2/A_1)\,p(A_3/A_1\cap A_2)\cdots p(A_n/A_1\cap A_2\cap\cdots A_{n-1})$$

Sufficient condition for above result to hold is $p(A_1\cap A_2\cap\cdots A_{n-1})>0$.

9. If A and B are **independent** events then $p(A\cap B)=p(A)\,p(B)$.
10. $\{A_1, A_2, \cdots, A_n\}$ is a set of mutually exclusive events then $\sum_{i=1}^{n} p(A_i)=p(S)=1$.
11. **Binomial Probability ::** An experiment is performed n number of times repeatedly. A is an event known as success with probability p. If event A occurs r times among n trials then $P(r\,successes)={}^nC_r\,p^r q^{n-r}$, where p is the probability of success and $q=1-p$ the probability of failure.

1. Product Rule ::Two experiments are to be performed then if then if experiment 1 can result in any one of m possible outcomes and for each outcome of experiment 1 there are n possible outcomes of experiment 2 then together there are mn possible outcomes of the both the experiments.
2. Sum Rule :: Two experiments are to be performed then if experiment 1 can result in any one of m possible outcomes and for each outcome of experiment 1 there are n possible outcomes of experiment 2 then there are m + n possible outcomes for at least one of the experiments.
3. There are $n(n-1)(n-2)\cdots 2\cdot 1=n!$ different ways to arrange n distinct objects.
4. Out of given n distinct objects if n_1 are of one kind, n_2 are of second kind and so on n_r are of rth kind then the number of dierent permutations is $\dfrac{n!}{n_1!\,n_2!\cdots n_r!}$
5. The number of arrangements of r objects out of given n objects without repetition is $${}^nP_r=\frac{n!}{(n-r)!}$$

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6. The number of arrangements of r onjects out of given n with repetition is n^r
7. The number of distributions of r objects out of which r_1 are of one kind, r_2 are of second kind, and so on r_k are of k^{th} kind into n distinct boxes is given by $\dfrac{P(n,r)}{r_1!r_2!\cdots r_k!}$
8. The number of ways of placing r objects of the same kind in n number of boxes is ${}^nC_r$