ISC202: Assignment 5: Due in class on Wed, 4 Mar 2015

1. Let u be a variable which can assume any of the M discrete values, u_i, $i = 1,\ 2,\ ....,M$.

   (a) Show that $\bar{u^2} > \bar{u}^2$, where $f(\bar{u})$ is the mean value of $f(u)$ over the distribution of values.

   (b) Obtain $\bar{u}$ and $\bar{u^2}$ for a random walk problem with $M = 2$ and the probabilities for the values u_1(say, stepping left) and u_2(say, stepping right) being p and q respectively; and prove the inequality above.

   (c) For $p = q$; $m =$ displacement form the mean position; obtain $\bar{m}$; $\bar{m^2}$; $\bar{m^3}$; $\bar{m^4}$ after N steps

2. A sample consisting of five molecules has a total energy 5ϵ. Each molecule is able to occupy states of energy $j\epsilon$, with $j = 0,\ 1,\ 2, ...$ (a) Calculate the weight of the configuration in which the molecules are distributed evenly over the available states. (b) Draw up a table with columns headed by the energy of the states and write beneath them all configurations that are consistent with the total energy. Calculate the weights of each configuration and identify the most probable configurations.

3. Consider a system with energy levels $\epsilon_j = j\epsilon$ and N molecules. (a) Show that if the mean energy per molecule is $a\epsilon$, then the temperature is given by $\beta = \frac{1}{\epsilon}\ln\left(1 + \frac{1}{a}\right)$. Evaluate the temperature for a system in which the mean energy is equivalent to 50cm^{-1}. (b) Calculate the molecular partition function for the system when its mean energy is $a\epsilon$. (c) Show that the entropy of the system is $\frac{S}{k_B} = (1 + a)\ln(1 + a) - a\ln a$, and evaluate this expression for a mean energy ϵ.