1a) $f(t) = t + 2\cos t \quad , -2\pi \leq t \leq 2\pi$

i) Domain cos functions the domain is all real numbers $\therefore$ domain $(-2\pi, 2\pi)$

ii) No VA as function oscillates

HA: $-2 \leq 2\cos t \leq 2$ $\qquad \lim_{t \to -\infty} t = -\infty \qquad \lim_{t \to \infty} t = \infty$

$-t \leq t + 2\cos t \leq t$

$\therefore$ No vertical or horizontal asymptotes

iii) $-2\sin t + 1 = 0 \qquad f'(t) = \frac{d}{dt} t - \frac{d}{dt} 2\cos t \qquad f'(t) = -2\sin t + 1$

$\sin t = -\frac{1}{2}$

$t = -\frac{7\pi}{6}$ and $t = -\frac{11\pi}{6}$ and $t = \frac{5\pi}{6}$ and $t = \frac{\pi}{6}$

$\left(-2\pi, -\frac{11\pi}{6}\right) \left(-\frac{11\pi}{6}, -\frac{7\pi}{6}\right) \left(-\frac{7\pi}{6}, \frac{\pi}{6}\right) \left(\frac{\pi}{6}, \frac{5\pi}{6}\right) \left(\frac{5\pi}{6}, 2\pi\right)$

Increasing decreasing increasing decreasing Increasing

iv) $f'(t) = -2\sin t + 1$

Critical numbers (seen above)

$t = \frac{-11\pi}{6}, \frac{-7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$

-2π		$-\frac{11\pi}{6}$		$-\frac{7\pi}{6}$		$\frac{\pi}{6}$		$\frac{5\pi}{6}$		2π
	+		−		+		−		+	

local min: $x = -\frac{7\pi}{6}$

$-\frac{7\pi}{6} + 2\cos\left(-\frac{7\pi}{6}\right) = -\frac{7\pi}{6} + 2\frac{\sqrt{3}}{2} = -\frac{7\pi}{6} + \sqrt{3} = \left(\frac{-7\pi - 6\sqrt{3}}{6}\right)$

local min: $\left(-\frac{7\pi}{6}, \frac{-7\pi - 6\sqrt{3}}{6}\right)$

local max: 2π

$2\pi + 2\cos(2\pi) = 2\pi + 2$

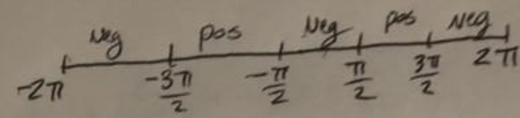
$(2\pi, 2\pi + 2)$

v) $f''(t) = \frac{d}{dt}1 - \frac{d}{dt}2\sin t \quad f''(t) = 0 - 2\cos t$

$0 = -2\cos t$

$0 = \cos t$

$t = -\frac{3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$



Concave down: $(-2\pi, -\frac{3\pi}{2}) \cup (-\frac{\pi}{2}, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi)$

Concave up: $(-\frac{3\pi}{2}, -\frac{\pi}{2}) \cup (\frac{\pi}{2}, \frac{3\pi}{2})$

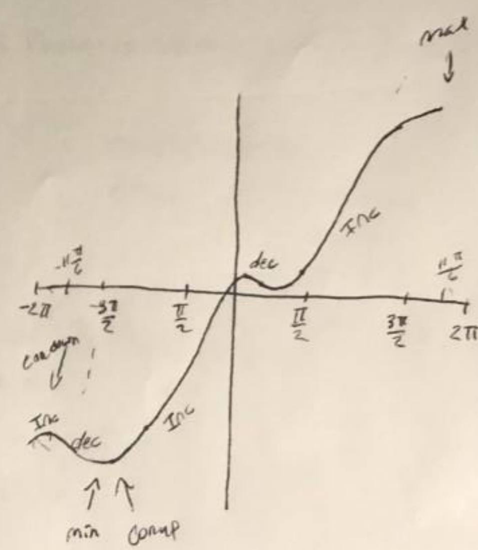
vi) $t = -\frac{3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}$

Inflection points: $-\frac{3\pi}{2} + 2\cos(\frac{-3\pi}{2}) = -\frac{3\pi}{2} + 0$

$\frac{-\pi}{2} + 2\cos(\frac{\pi}{2}) = \frac{\pi}{2} + 0$

$(-\frac{3\pi}{2}, -\frac{3\pi}{2}), (-\frac{\pi}{2}, \frac{-\pi}{2}), (\frac{\pi}{2}, \frac{\pi}{2}), (\frac{3\pi}{2}, \frac{3\pi}{2})$

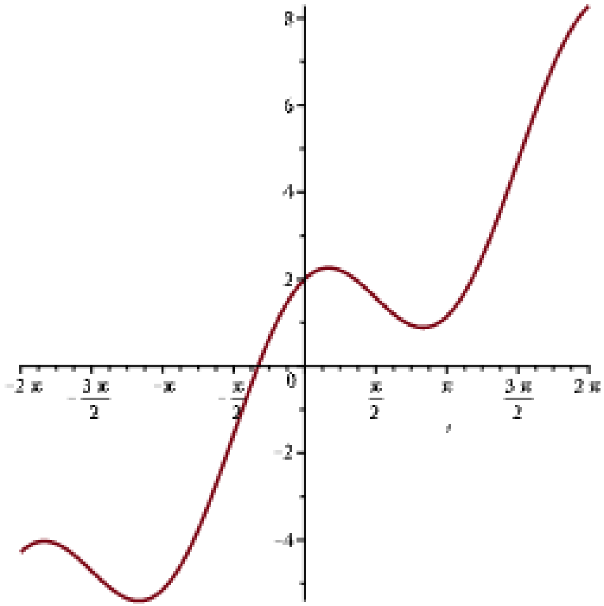
Domain	$(-2\pi, 2\pi)$
V.A	None
H.A	None
inc.	$(-2\pi, -\frac{11\pi}{6}) \cup (-\frac{7\pi}{6}, \frac{\pi}{6}) \cup (\frac{5\pi}{6}, 2\pi)$
dec.	$(\frac{-11\pi}{6}, \frac{-7\pi}{6}) \cup (\frac{\pi}{6}, \frac{5\pi}{6})$
local max	$(2\pi, 2\pi + 2)$
local min	$(\frac{-7\pi}{6}, \frac{-7\pi - 6\sqrt{3}}{6})$
con up	$(-\frac{3\pi}{2}, -\frac{\pi}{2}) \cup (\frac{\pi}{2}, \frac{3\pi}{2})$
con down	$(-2\pi, -\frac{3\pi}{2}) \cup (\frac{-\pi}{2}, \frac{\pi}{2}) \cup (\frac{3\pi}{2}, 2\pi)$
infl points	$(-\frac{3\pi}{2}, -\frac{3\pi}{2}), (\frac{-\pi}{2}, \frac{-\pi}{2}), (\frac{\pi}{2}, \frac{\pi}{2}), (\frac{3\pi}{2}, \frac{3\pi}{2})$



```
> f:=t+2*cos(t);
```

$$f := t + 2\cos(t) \tag{1}$$

```
> plot([f], t = -2*Pi .. 2*Pi);
```



1b) Domain: x^{-2} domain is $x \in R$ e^x domain is $x \in R$

$\therefore$ $g(x)$ domain $(-\infty, \infty)$

ii) $g(x) = \frac{1}{(1+e^x)^2}$ $\lim_{x \to \infty} \frac{1}{(1+e^x)^2} = 0$ $\lim_{x \to -\infty} \frac{1}{(1+e^x)^2} = \frac{1}{1^2} = 1$

$\therefore$ HA = 0 and 1

$0 = (1+e^x)^2$ $0 = 1+e^x$ $-1 = e^x$ $\ln(-1) = x$

$\therefore$ There are no VA as $\ln(-1)$ is undefined

iii) $g'(x) = \frac{(1+e)^2 + 0 - 1(2(e^x+1) \cdot e^x)}{((1+e^x)^2)^2}$ $g'(x) = \frac{(1+e^x)^2 - 2(e^x+1)(e^x)}{(1+e^x)^4}$

$g'(x) = \frac{-2e^x}{(1+e^x)^3}$

$0 = -2e^x$

$\ln(0) = x$

$\therefore$ because $\lim_{x \to -\infty} = 1$ and $\lim_{x \to \infty} = 0$ and there are no other intervals, the function is only decreasing for $(-\infty, \infty)$

iv) The local max and min do not exist as the function is decreasing from $(-\infty, \infty)$

v) $g''(x) = \frac{\frac{d}{dx}(1+e^x)^3 \cdot -2e^x - (\frac{d}{dx} - 2e^x \cdot (1+e^x)^3)}{(1+e^x)^6}$

$= \frac{-2(+3(e^x+1)^2(e^x+0) \cdot e^x - e^x(e^x+1)^3)}{(1+e^x)^6}$

$= \frac{2e^x(2e^x-1)}{(e^x+1)^4}$

$0 = 2e^x(2e^x - 1)$

$e^x = 0$ or $2e^x - 1 = 0$

$2e^x = 1$

$e^x = \frac{1}{2}$

$x = \ln(\frac{1}{2})$

$(-\infty, \ln(\frac{1}{2}))$ $(\ln(\frac{1}{2}), \infty)$

$= -$ $= +$

Concave down $(-\infty, \ln(\frac{1}{2}))$

concave up $(\ln(\frac{1}{2}), \infty)$

vi) inflection points $x = \ln(\frac{1}{2})$

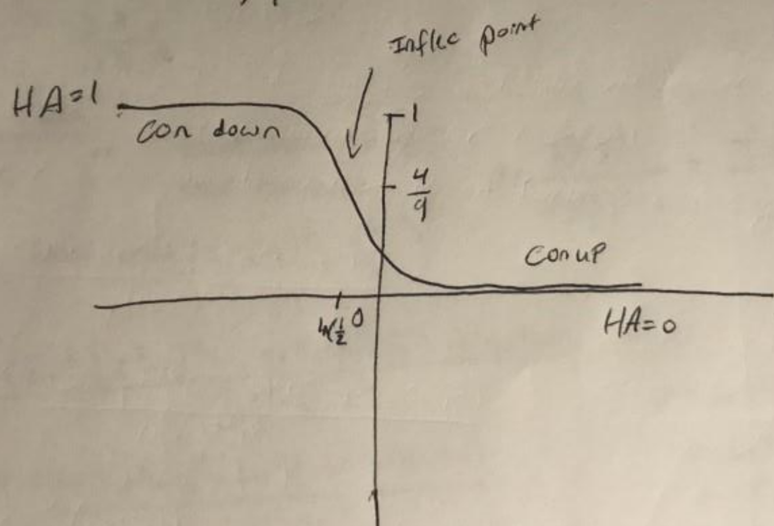
$g(x) = (1+e^{\ln(\frac{1}{2})})^{-2}$

$g(x) = (1+\frac{1}{2})^{-2}$

$g(x) = \frac{4}{9}$

$(\ln(\frac{1}{2}), \frac{4}{9})$

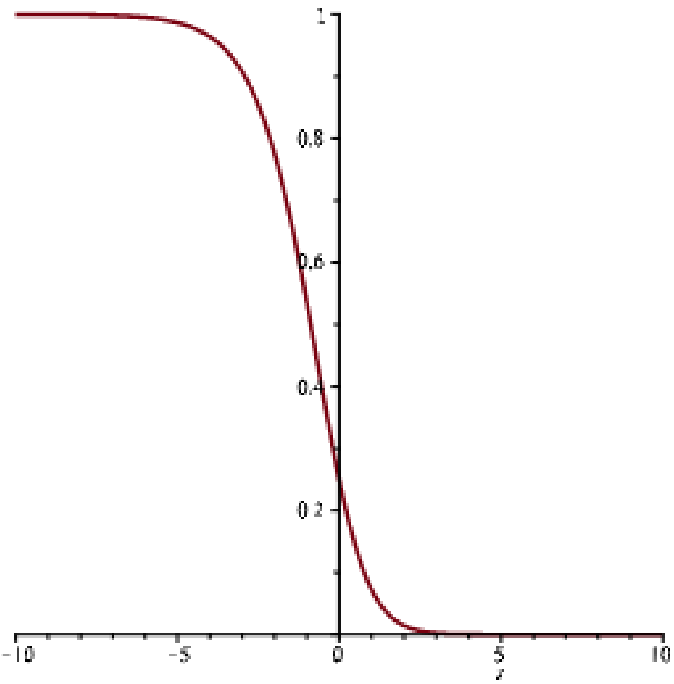
Domain	$(-\infty, \infty)$
V.A	None
H.A	$y=0$ and $y=1$
inc.	None
dec.	$(-\infty, \infty)$
local max	None
local min	None
con up	$(\ln(\frac{1}{2}), \infty)$
con down	$(-\infty, \ln(\frac{1}{2}))$
infl. points	$(\ln(\frac{1}{2}), \frac{4}{9})$



```
> g:= (1+exp(x))^(-2);
```

$$g := \frac{1}{\left(1 + e^{x}\right)^{2}} \tag{2}$$

```
> plot([g], x = -10 .. 10);
```



1c) $R(x) = \dfrac{x}{x^3-1}$

Domain: $x^3-1=0 \quad x^3=1 \quad x=1$

$\therefore$ domain is $\{x \in \mathbb{R} : x \neq 1\}$ $(-\infty,1)\cup(1,\infty)$

ii) $VA: 1=x$

$HA: x^3 > x \therefore HA$ is $y=0$

iii) $R'(x) = \dfrac{(x^3-1)(1)-x(3x^2)}{(x^3-1)^2}$

$= \dfrac{x^3-1-3x^3}{(x^3-1)^2}$

$= \dfrac{-2x^3-1}{(x^3-1)^2}$

$0 = -2x^3-1$

$1 = -2x^3$

$x = -\sqrt[3]{\frac{1}{2}}$

Increasing $(-\infty, \sqrt[3]{\frac{1}{2}})$ decreasing $(-\sqrt[3]{\frac{1}{2}}, \infty)$

iv) Inc dec

$-\infty \quad \sqrt[3]{\frac{1}{2}} \quad \infty$

local max: $x = -\sqrt[3]{\frac{1}{2}}$

local min: None

$R(x) = \dfrac{-(\sqrt[3]{\frac{1}{2}})}{-(\sqrt[3]{\frac{1}{2}})^3-1} = \dfrac{-(\frac{1}{2})^{\frac{1}{3}}}{-\frac{1}{2}-1} = \dfrac{-(\frac{1}{2})^{\frac{1}{3}}}{-\frac{3}{2}} = \dfrac{2^{\frac{2}{3}}}{3}$

local max: $\left(-(\frac{1}{2})^{\frac{1}{3}}, \dfrac{2^{\frac{2}{3}}}{3}\right)$

v) $R''(x) = \dfrac{(6x^2)(x^3-1)^2-(2x^3-1)\cdot 2(x^3-1)(3x^2+0)}{((x^3-1)^2)^2}$

$= \dfrac{-6x^2(x^3-1)^2-6x^2(-2x^3-1)(x^3-1)}{(x^3-1)^4}$

$= \dfrac{-6x^8+12x^5-6x^2+12x^8-6x^5-6x^2}{(x^3-1)^4}$

$R''(x) = \dfrac{6x^2(x^6+x^3-2)}{(x^3-1)^4}$

$0 = 6x^2(x^6+x^3-2)$

$0 = 6x^2$ or $0 = x^6+x^3-2$

$0 = x^2$ or $2 = x^6+x^3$

$0 = x \qquad x = -2^{\frac{1}{3}}$

$(-\infty, -2^{\frac{1}{3}})\ (-2^{\frac{1}{3}}, 0)\ (0, \infty)$

$+ \qquad - \qquad +$

Concave up $(-\infty, -2^{\frac{1}{3}}), (0, \infty)$

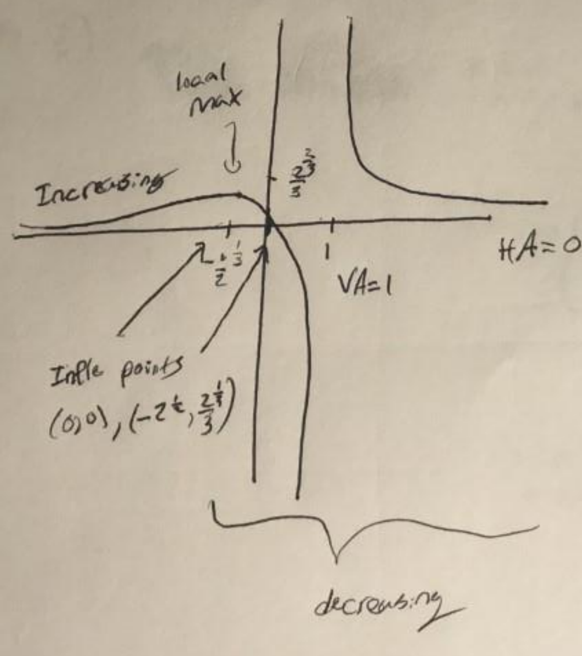
Concave down $(-2^{\frac{1}{3}}, 0)$

vi) Inflation point $x=0$, $x = -2^{\frac{1}{3}}$

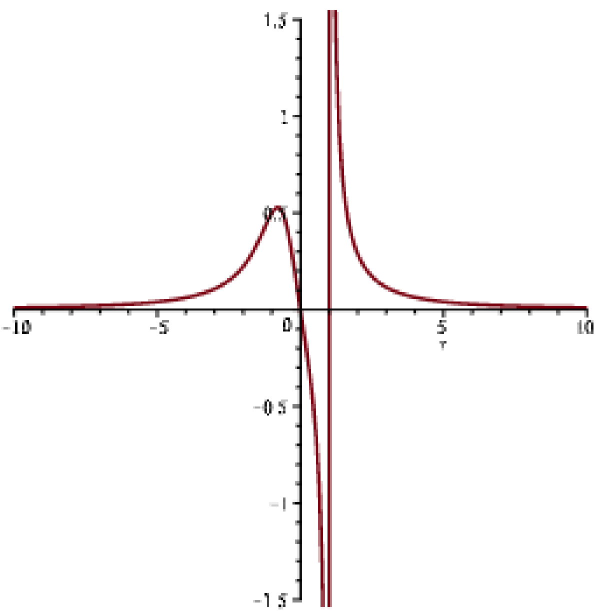
$R(0) = 0 \quad R(-2^{\frac{1}{3}}) = \dfrac{2^{\frac{1}{3}}}{3}$

$(0,0)$ and $\left(-2^{\frac{1}{3}}, \dfrac{2^{\frac{1}{3}}}{3}\right)$

Domain	$(-\infty,1)\cup(1,\infty)$
V.A	$x=1$
H.A	$y=0$
inc	$(-\infty,-\frac{1}{2}^{\frac{1}{3}})$
dec	$(-(\frac{1}{2})^{\frac{1}{3}},\infty)$
localmax	$(-(\frac{1}{2})^{\frac{1}{3}},\frac{2^{\frac{2}{3}}}{3})$
local min	None
Con up	$(-\infty,-2^{\frac{1}{3}})\cup(0,\infty)$
con down	$(-2^{\frac{1}{3}},0)$
infl points	$(0,0),(-2^{\frac{1}{3}},\frac{2^{\frac{1}{3}}}{3})$



```
> plot([r], x = -10 .. 10);
```




```
> disA:=sqrt((3-2)^(2)+x^2);
```

2) $$\lim_{x\to 0} \frac{x^2 \sin x}{\sin x - x} = \lim_{x\to 0} \frac{2x\cdot \sin x + x^2 \cos x}{\cos x - 1} \quad \text{(use l'hopital rule)}$$

$$= \lim_{x\to 0} \frac{-x^2 \sin x + 2\sin x + 4\cos x}{-\sin(x)} \quad \text{(use rule again)}$$

$$= \lim_{x\to 0} \frac{-6x\sin x - x^2\cos x + 6\cos x}{-\cos x} \quad \text{(use again)}$$

$$= \lim_{x\to 0} \frac{6(-x\sin x + \cos x) - x^2 \cos x}{-\cos x} = \frac{-6(1) - 0}{+1} = -6$$

$$\therefore \lim_{x\to 0} \frac{x^2 \sin x}{\sin x - x} = -6$$

3) $$\lim_{x\to -\infty} x\ln\left(1 - \frac{1}{x}\right) = \lim_{x\to -\infty} \frac{\ln\left(1-\frac{1}{x}\right)}{1/x}$$

$$= \lim_{x\to -\infty} \frac{\left(\dfrac{\frac{1}{1-\frac{1}{x}} \cdot 0 + \frac{1}{x^2}}{1 - \frac{1}{x}}\right)}{-\frac{1}{x^2}}$$

$$= \lim_{x\to \infty} \frac{\dfrac{\frac{1}{x^2}}{1-\frac{1}{x}}}{\frac{-1}{x^2}}$$

$$= \lim_{x\to -\infty} \frac{1x^2}{-x(x-1)}$$

$$= \lim_{x\to -\infty} \frac{-x}{x-1} = \frac{-1}{1} = -1$$

4) $\lim_{x \to \infty} (e^x + 10x)^{\frac{1}{x}}$

$= \lim_{x \to a} [P(x) + q(x)]$

$= \lim_{x \to \infty} e + 10x^{\frac{1}{x}}$

$= \lim_{x \to \infty} (10x)^{\frac{1}{x}} + e = 1 + e$

5) $f(x) = 2x\sin x$ $\quad g(x) = \sec x - 1$ $\quad f'(x) = 2(1 \cdot \sin x + x \cdot \cos x)$

$f'(x) = 2(\sin x + x\cos x)$

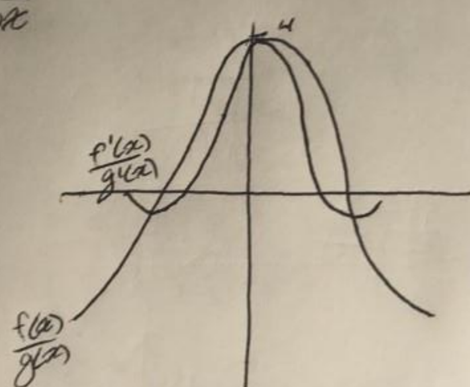
$g'(x) = \sec x \tan x + 0$

$g'(x) = \sec x \tan x$

$\frac{f(x)}{g(x)} = \frac{2x\sin x}{\sec x - 1}$

$\frac{f'(x)}{g'(x)} = \frac{2(\sin x + x\cos x)}{\sec x \tan x}$

$\frac{f(x)}{g(x)}$	$\frac{f'(x)}{g'(x)}$	
3.97	3.95	-0.1
3.99	3.99	-0.01
3.99	3.99	0.01
3.97	3.95	0.1



$\frac{f(x)}{g(x)}$	$\frac{f'(x)}{g'(x)}$	
-1.06	-0.029	-2
1.97	-0.958	-1
1.97	0.958	1
-1.06	0.029	2

using l'hopitals rule

$\frac{f'(x)}{g'(x)} = \frac{2(\sin x + x\cos x)}{\sec x \tan x}$

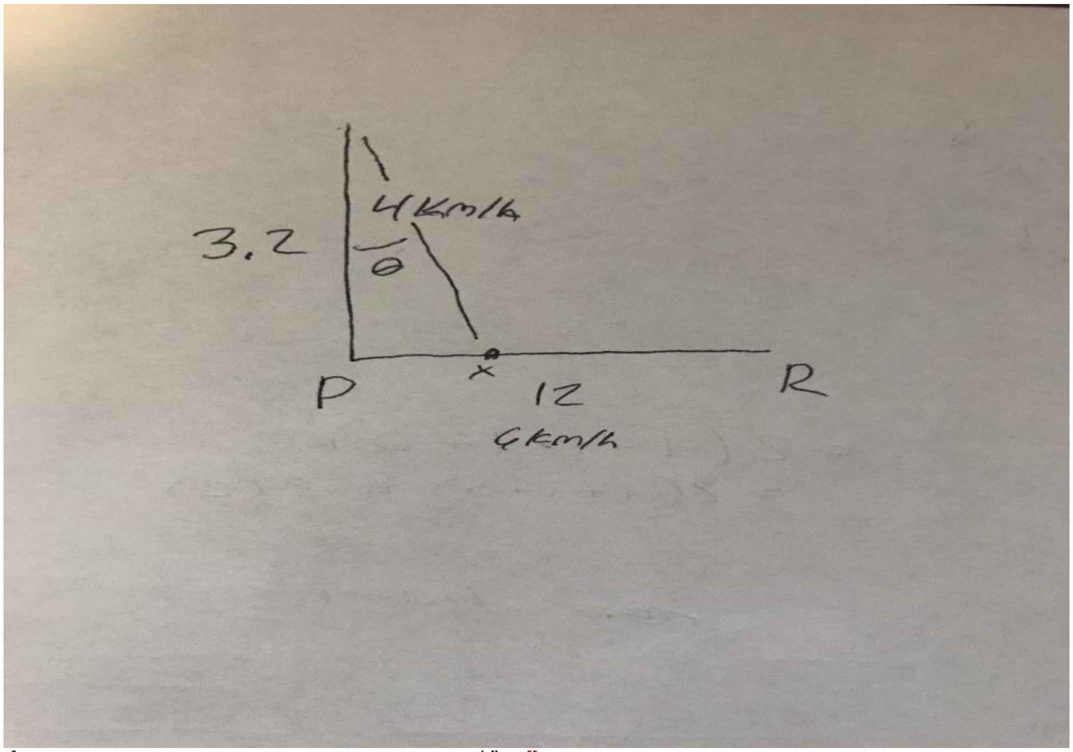
$\frac{f''(x)}{g''(x)} = \frac{2(\cos x + 1 \cdot \cos x + x \cdot -\sin x)}{\sec x \tan x + \sec x \cdot \sec^2 x}$

$\frac{f''(0)}{g''(0)} = \frac{2(\cos 0 + 1 \cdot \cos 0 + 0 \cdot -\sin 0)}{\sec(0)\tan(0) + \sec(0)\sec^2(0)} = \frac{2(1+1 \cdot 1+0)}{1 \cdot 0 + 1}$

$= \frac{2(2)}{1}$

$= 4$

6)




```
> disA:=sqrt((3.2)^(2)+x^2);
```

$$disA := \sqrt{10.24 + x^2} \qquad (4)$$

```
> disB:=12-x;
```

$$disB := 12 - x \qquad (5)$$

$$> tA := \frac{disA}{4}$$

$$tA := \frac{\sqrt{10.24 + x^2}}{4} \qquad (6)$$

$$> tB := \frac{disB}{6}$$

$$tB := 2 - \frac{x}{6} \qquad (7)$$

$$> timeTotal := tA + tB$$

$$timeTotal := \frac{\sqrt{10.24 + x^2}}{4} + 2 - \frac{x}{6} \qquad (8)$$

```
> der:=diff(timeTotal,x);
```

$$der := \frac{x}{4\sqrt{10.24 + x^2}} - \frac{1}{6} \qquad (9)$$

```
> solve(der=0,x);
```

$$2.862167011 \qquad (10)$$

Therefore the distance from point p that the hiker must walk is 2.862167011 km to get to the restaurant as quickly as possible

7)

$A_{sector} = r^2(\theta/2)$ $\quad$ $\text{perimeter} = 2r + 2\pi r\left(\frac{\theta}{360}\right)$

$16 = 2r + \text{Arc length}$ $\quad$ $16 = 2r + 2\pi r\left(\frac{\theta}{360}\right)$

$A = r^2(\theta/2)$

let $\frac{\theta}{360}$ be y

$$16 = 2r + 2\pi r y$$

$$16 = 2r(1+\pi y)$$

$$r = \frac{8}{2(1+\pi y)}$$

$$A = \frac{4}{1+\pi\left(\frac{\theta}{360}\right)} \cdot (\theta/2)$$

$$A'(x) = \frac{1}{2}\left(\frac{16(-2)\left(\frac{\pi\theta}{360}+1\right)^{-3}\left(\frac{\pi}{360}\cdot 1+0\right)}{\left(1+\pi\frac{\theta}{360}\right)^3}\right) + 1\cdot\left(\frac{4}{\left(1+\pi\left(\frac{\theta}{360}\right)\right)}\right)^2$$

$$= \frac{1}{2}\left(\frac{32\left(\frac{\pi}{360}\right)}{-\left(\frac{\pi\theta}{360}+1\right)^3}\right)\theta + \left(\frac{4}{1+\pi\left(\frac{\theta}{360}\right)}\right)^2$$

$$0 = \frac{1}{2}\left(\frac{-4\pi}{45\left(\frac{\pi\theta}{360}+1\right)^3}\right)\theta + \left(\frac{4}{1+\pi\frac{\theta}{360}}\right)^2$$

$$0 = \left(\frac{4147200\pi\theta}{(\pi\theta+360)^3} + \frac{2073600}{(360+\pi\theta)^2}\right)\frac{1}{2}$$

$$0 = \frac{7464960000 - 2073600\pi\theta}{(\pi\theta+360)^3}\left(\frac{1}{2}\right)$$

$$0 = 7464960000 - 2073600\pi\theta$$

$$\theta = \frac{360}{\pi} \qquad 16 = 2r + 2\pi r\left(\frac{\left(\frac{360}{\pi}\right)}{360}\right)$$

$$2r + 2r = 16$$

$$r = 4$$

∴ The circle with a radius of 4 will have the most burning area

8)

```
> Area:=2*Pi*rad^2+(2*Pi*x*height);
```

$$Area := 2\,\pi\, x\, height + 2\,\pi\, rad^2 \tag{11}$$

```
> elim:=Pi(rad^(2))(height);
```

$$elim := \pi(rad^2)(height) \tag{12}$$

```
> newR:=(750)/(Pi(rad^2));
```

$$newR := \frac{750}{\pi(rad^2)} \tag{13}$$

```
> Area:=2*Pi*rad^2+(2*Pi*rad*newR);
```

$$Area := \frac{1500\,\pi\, rad}{\pi(rad^2)} + 2\,\pi\, rad^2 \tag{14}$$

```
> Area:=(1500/rad)+2*Pi*rad^2
```

$$Area := \frac{1500}{rad} + 2\,\pi\, rad^2 \tag{15}$$

```
> derArea:=diff(Area,rad);
```

$$derArea := -\frac{1500}{rad^2} + 4\,\pi\, rad \tag{16}$$

```
> solve(derArea=0,rad);
```

$$\frac{5\, 3^{1/3}\left(\pi^2\right)^{1/3}}{\pi},\ -\frac{5\, 3^{1/3}\left(\pi^2\right)^{1/3}}{2\,\pi} + \frac{5\, I\, 3^{5/6}\left(\pi^2\right)^{1/3}}{2\,\pi},\ -\frac{5\, 3^{1/3}\left(\pi^2\right)^{1/3}}{2\,\pi} - \frac{5\, I\, 3^{5/6}\left(\pi^2\right)^{1/3}}{2\,\pi} \tag{17}$$

```
> radius := 5*3^(1/3)*(Pi^2)^(1/3)/Pi
```

$$radius := \frac{5\, 3^{1/3}\left(\pi^2\right)^{1/3}}{\pi} \tag{18}$$

```
> height:=750/(Pi*radius)
```

$$height := \frac{50\, 3^{2/3}}{\left(\pi^2\right)^{1/3}} \tag{19}$$

1) a) $2\cos x - \sqrt{e^{2x}}$

$F'(x) = -2(\sin x) - e^{\frac{2}{2}x}$ $\therefore$ The most general antiderivative is

$F(x) = +2\sin x - e^{x} + C$

$F'(x) = 2\cos x - e^{x}$

$F'(x) = 2\cos x - e^{\frac{2}{2}x}$

$F'(x) = 2\cos x - \sqrt{e^{2x}}$

b) $g(t) = \dfrac{t^2+2}{t^2+1}$

$\int \dfrac{t^2+2}{t^2+1}$

$G(x) = \int \dfrac{t^2}{t^2+1} + \dfrac{2}{t^2+1}$

$= \int \left(\dfrac{1+t^2}{1+t^2} - \dfrac{1}{1+t^2}\right) dx + 2\int \dfrac{1}{t^2+1}\,dx$

$= \int \left(1 - \dfrac{1}{1+t^2}\,dx\right) + 2\int \dfrac{1}{t^2+1}\,dx$

$= x - \tan^{-1}x + C + 2(\tan^{-1}x + c)$

$= x - \tan^{-1}x + C + 2\tan^{-1}x + 2C$

$G(x) = x + \tan^{-1}x + 3C$

10) $F''(t) = \sqrt[3]{t} - \cos t \qquad F(0)=2 \quad F(1)=2$

$= t^{\frac{1}{3}} - \cos t$

$= \int t^{\frac{1}{3}} dt - \int \cos t \, dt$

$= \int \left(\frac{t^{\frac{4}{3}}}{\frac{4}{3}} - \sin t \right) dt$

$= \frac{t^{\frac{4}{3}}}{\frac{4}{3}} - \sin t + C$

$f'(x) = \frac{3t^{\frac{4}{3}}}{4} - \sin t + C$

$= \int \left(\frac{3t^{\frac{4}{3}}}{4} - \sin t + C \right) dt$

$= \frac{3}{4} \int t^{\frac{4}{3}} - \int \sin t \, dt + C \int 1 \, dt$

$= \int \left(\frac{3}{4} \left(\frac{t^{\frac{7}{3}}}{\frac{7}{3}} \right) - (-\cos t) + Ct \right) + d$

$= \frac{3}{4} \left(\frac{3t^{\frac{7}{3}}}{7} \right) + \cos t + Ct + d$

$= \frac{9t^{\frac{7}{3}}}{28} + \cos t + Ct + d$

$f(0) = 0 + 1 + 0 + d = 2 \qquad d = 1$

$f(1) = \frac{9}{28} + \cos(1) + C + 1 = 2 \qquad \frac{9}{28} + 0 + C = 1 \qquad C = \frac{19}{28}$

$\therefore F(x) = \frac{9t^{\frac{7}{3}}}{28} + \cos t + \frac{19}{28}t + 1$