Student Answer Sheet Analysis

Automated Processing

July 5, 2025

Student Responses

Question 1

Student Answer:

Convert the incidence matrix to adjacency matrix representation.

Given incidence matrix.

Column - an edge Row - a node

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Column-1: Connects node 1 3 Column-2: Connects node 2 3 Column-3: Connects node 2 4

Adjacency matrix is a symmetric 4x4 matrix for 4 vertices We set 1 for each pair of vertices that share an edge.

$$\begin{array}{ccccc} & v_1 & v_2 & v_3 & v_4 \\ v_1 & [0 & 1 & 1 & 0] \\ v_2 & [1 & 0 & 1 & 1] \\ v_3 & [1 & 1 & 0 & 0] \\ v_4 & [0 & 1 & 0 & 0] \end{array}$$

$$\text{Adjacency matrix} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Question 1 continued

Student Answer:

We analyzed each edge in the incidence matrix to identify the two vertices it connects and then marked those pairs in adjacency matrix with 1 since the graph is undirected.

Question 2

Student Answer:

The model where edges form with uniform probability, independent of others:

Answer: B: Erdos-Renyi (Random network model)

Justification:

The Erdos-Renyi model assumes that each possible edge between a pair of nodes in includes in the graphs with a uniform probability p, independently of other edges. The matches the condition in the question: uniform probability independent of other edges.

-Barabasi-Albert model: Based preferential attachment, new nodes are more likely to connect to high-degree nodes.

-Watts-Strogatz model: Begins with a regular lattice and re-wires edges with some probability - edges are not formed independently.

-Configuration model: Generates graphs with a given degree sequence - not based on uniform independent edge probability.

So, option-B fits the description.

(c)

Student Answer:

Option-C: Nash Equilibrium.

A Nash Equilibrium occurs in game theory when no player can benefit by changing their strategy unilaterally, assuming all other players keep their strategies unchanged it represents a stable state where no one has an incentive to deviate.

(d)

Student Answer:

Option-B: Assortative Mixing.

Assortative mixing refers to the tendency of nodes in networks to connect with others that are similar to - age, gender, profession or degree (number of connections), in social networks this typically manifests of people forming bonds with those who are like themselves.

(e)

Student Answer:

Why betweenness centrality is better than degree of centrality in path based info networks.

Answer D: Because it quantifies how often a node lies on the shortest paths between other nodes.

(f)

Student Answer:

Answer C: The presence of mixing nodes with very high degrees (hubs) that maintain connectivity.

Question (a)

Student Answer:

Option - A:

Modularity measures the strength of division of a network into communities. Optimizing for high modularity means finding partitions where more edges fall within communities that would be expected by chance. This helps reveal meaningful groupings in the network.

(h)

Student Answer:

Jaccard Coefficient.

$Jaccard(x, y) = \frac{|N(x) \cap N(y)|}{|N(x) \cup N(y)|}$

Neighbors of x: A, B, C, D

Neighbors of y: C, D, E

Intersection: C, D =¿ size = 2

Union = A, B, C, D, E =¿ size = 5

Option B = 2/5

(i)

Student Answer:

The fundamental difference between the Independent Cascade Model (ICM) Linear Threshold Model (LTM) lies in how nodes get activated.

-ICM: Each active node gets one chance to activate each part of its inactive neighbors with a given independent edge probability.

ICM -¿ edge based probability

LTM -¿ Threshold based activation using weighted influence.

Question (j)

Student Answer:

Ans: Option A

Question 1

Student Answer:

In networks with high heterophily, connected nodes often belong to different classes or have dissimilar features. Standard GCNs works by aggregating features from neighbors, assuming that neighbors are similar.

But in heterophilious settings, this leads to feature mixing from different classes, which blurs the distinctiveness of a node's representation and harms classification accuracy.

Ans: Option B

Question 2

Student Answer:

To effectively minimize total infections when only 3

1) Degree of centrality:

- Degree centrality measures how may direct connections a node has.

- Nodes with high degree centrality interact with many others so they are likely to spread the infection widely.

-Identify and vaccinate individuals with higher degree centrality as a immunization these "hub" at all large parts of potential transmission chains.

2) Betweenness centrality:

- Betweenness centrality measures how often a node lies on the shortest path between other nodes.

- Nodes with high betweenness centrality act as bridges between communities or clusters. If infected they can spread disease across different groups.

-Vaccinate nodes with high betweenness centrality to prevent cross-community transmission, especially important in networks with modular or community structures.

- Combining Two concepts:

- Strategy is to use hybrid approach. 1) Calculate degree betweenness centrality for all nodes. 2) Rank nodes by a combined score 3) Select top 3

Question 2 continued

Student Answer:

-Degree of centrality reduces local transmission.

-Betweenness centrality reduces global spread across cluster

-Together they provide broad protection both with between communities.

Question 3

Student Answer:

To improve the suggested collaborative feature on an academic platform, we can design a system that combines link prediction algorithms with node embedding techniques like NodeVec, leveraging the structural and semantic properties of citation and coauthorship networks.

1) Node Embedding with Node2Vec:

- Learns low dimensional vector representations of researchers based on their network neighbors.

- Node2Vec performs biased random walks to capture both local and global relationships.

- Researchers with similar vectors are likely to be connected or similar in research focus.

2) Link prediction with embeddings:

-After training Node2Vec, we use similarity metrics between node embeddings to predict potential future collaborations.

Question 3 continued

Student Answer:

Link prediction algorithms can estimate the probability of link forming between two researchers based on,

-Embedding similarity

-Common neighbors

-P(si co-authorship pattern

System workload:

1) Train Node2Vec on the co-authorship/citation graph.

2) For each researcher, compute similarity scores with others.

3) Recommend top-k researchers with highest predicted link across link scores.

Role of Homophily in collaboration:

-Researchers tend to collaborate with others in similar fields, reflected by topics - knit community in the network.

-This is captured well by Node2Vec and aids in recommending relevant collaborations.

-However, it can reinforce academic silos by mostly suggesting people from the same discipline.

Question 4

Student Answer:

Promoting cross-disciplinary collaboration:

-Introduce diversity aware ranking in the recommendation system:

-Cluster researchers based on field or topic

-Top-N within field collaborators

-Top-M out of field collaborators with moderate similarity but complementary research areas (eg via topic modeling or citation patterns)

Another idea is to add a filter flag in the UI for ”Explore outside your field” to promote ”unintentional exploration”.

By combining Node2Vec embeddings with link prediction, we recommend collaborations based on both network proximity and topical similarity. Focusing on homophily helps ensure article relevance, while deliberately promoting connection outside a researchers' cluster can spark inter-disciplinary innovation.

Question 4 (a)

Student Answer:

Core idea behind the Girvan-Newman Algorithm:

The Girvan-Newman Algorithm detects communities by progressively removing edges that are most likely to connect different communities. The core idea is that the edges between community have high edge betweenness centrality - they lie on many shortest paths between nodes in different graphs. By removing the edges that bridges the network breaks apart into tightly connected subgroups, revealing community structure.

Question 4 (b)

Student Answer:

Use of edge betweenness centrality. Algorithm works iteratively:

1) Compute edge betweenness centrality for all edges

2) Remove the edge with the highest betweenness

3) Recalculate betweenness centrality after each removal

4) Repeat until the network splits into meaningful communities.

Question 4 (c)

Student Answer:

Major computational limitations:

The biggest limitation is computational cost. Calculating edge betweenness centrality is expensive O(nm) time for each iteration.

n-number of nodes

m- number of edges

And it has to be recalculated after every edge removal. For large networks this makes the algorithm very slow and impractical.

Question 4 (d)

Student Answer:

The Louvain method is greedy, hierarchical algorithm with that optimizes modularity efficiently.

1) Local optimization: Initially each node is its own community. Nodes are moved into neighboring communities if it increases modularity.

2) Community aggregation: Communities are grouped into super nodes, forming a smaller network.

3) Repeat the process on the new network.

Because it avoids costly betweenness calculations and works hierarchically, Louvain is fast scalable, even for very large networks.

Question 5 (a)

Student Answer:

Intuition behind the PageRank Algorithm:

PageRank measures node importance based on the idea that a node is important if it is linked to by other important nodes.

-Think of a random surfer who randomly clicks links on the web.

-Nodes (web pages for en) that are visited often are considered more important.

-Links from highly ranked pages contribute more than links from low-ranked ones.

It's essentially a form of voting, where votes from authoritative nodes count more.

Question 5 (b)

Student Answer:

Role of the damping factor cd:

The damping factor cd 0.85 represents the probability that the random surfer follows a link from the current page with probability 1-d, the surfer jumps to a random node instead (teleportation).

-This prevents the algorithm from getting stuck in loops or dead ends, ensuring that any node has a chance of being visited.

It balances link-following behavior with random exploration and is crucial for PageRank's stability and convergence.

Question 5 (c)

Student Answer:

Dangling nodes problem handling:

Dangling nodes are nodes with no outgoing links.

(Ex: webpage with no hyperlinks)

-During each iteration, treat dangling nodes as if they link to all nodes uniformly.

-This is done by redistributing their link equally across the entire network.

-Mathematically it modifies the transition matrix to ensure it remains stochastic (sums to 1), preserving convergence and avoiding rank sinks.

Question 6 (a)

Student Answer:

Game payoff matrix:

player-1 = row, player-2 = column

$$\begin{array}{ccc} & A & B \\ U & (3,2) & (0,1) \\ L & (2,3) & (3,2) \end{array}$$

Question 6 (b)

Student Answer:

Pure strategy Nash Equilibria:

A pure strategy Nash Equilibrium is a strategy profile where no player has an incentive to unilaterally deviate, given the other players choice.

(1) (U, A) player 1 gets 3 -¿ would switching to L give more? L vs A = 2 -¿ no (3¿2) player 2 gets 2 -¿ switching to B gives 1 -¿ no improvement. (U, A) is a Nash Equilibrium

(2) (U, B) Player 1 get 0 -¿ switching to L gives 2 -¿ would switch x Not a NE

(3) (L, A) Player-1 gets 2 -¿ switching to U gives 3 -¿ would switch x Not a NE

(4) (L, B) Player 1 gets 3 -¿ U vs B gives 0 -¿ no improvement Player 2 gets 2 -¿ A gives 3 -¿ worse (L, B) is also a Nash Equilibrium

Pure strategy Nash Equilibria:

(U, A), (L, B)

Both are stable because no player can improve their payoff by deviating unilaterally.

Question 6 (c)

Student Answer:

Expected Payoffs for Player 2:

Expected payoffs for player 2 assuming player 1 mixes strategies

plays U with probability P plays L with probability (1-P)

- If player 2 plays A: payoffs for player 2:

If p1 plays U: 2 If p1 plays L: 3

Expected payoff = 2P + 3(1-P) = 3 - P

If player 2 plays B: payoffs for player 2:

If p1 plays U: 1 If p1 plays L: 2

Expected payoff = 1P + 2(1-P) = P + 2 - 2P = 2-P

Player 2's expected payoff for A: 3-P player 2's expected payoff for B: 2-P

Question 7

Student Answer:

node B: A -¿ B, C -¿ B, D -¿ B

[Student drew a diagram]

[Student drew a diagram]

Given a Graph Neural Network (GNN) Layer and seed to compute the updated feature vector for node B, $h_B^{(1)}$ using one GNN layer.

Question 7 continued

Student Answer:

Step-1: Aggregate Neighbor Features:

Neighbors of B: N(B) = A, C, D

$h_A^{(0)} = \begin{bmatrix}1\\1\end{bmatrix}$ $h_C^{(0)} = \begin{bmatrix}0\\3\end{bmatrix}$ $h_D^{(0)} = \begin{bmatrix}2\\2\end{bmatrix}$

Average of neighbour features: $h_B^{(0)} = \sigma(w(\frac{1}{|N(B)|}\sum_{u \in N(B)} h_u^{(0)}))$

$h_B^{(0)} = \frac{1}{3}(h_A^{(0)} + h_C^{(0)} + h_D^{(0)}) = \frac{1}{3}\begin{bmatrix}1\\1\end{bmatrix} + \begin{bmatrix}0\\3\end{bmatrix} + \begin{bmatrix}2\\2\end{bmatrix} = \frac{1}{3}\begin{bmatrix}3\\6\end{bmatrix} = \begin{bmatrix}1\\2\end{bmatrix}$

Step-2: Transform using weight matrix:

Weight matrix $w = \begin{bmatrix}0.5 & 0\\0.1 & 0.2\end{bmatrix}$

$wh_{N(B)}^{(0)} = \begin{bmatrix}0.5 & 0\\0.1 & 0.2\end{bmatrix} \times \begin{bmatrix}1\\2\end{bmatrix} = \begin{bmatrix}0.5 \times 1 + 0.2 \times 2\end{bmatrix} = \begin{bmatrix}0.5\\0.5\end{bmatrix}$

Step 3: Apply activation function (ReLU)

ReLU(x) = max(0, x)

$h_B^{(1)} = ReLU\begin{bmatrix}0.5\\0.5\end{bmatrix} = \begin{bmatrix}0.5\\0.5\end{bmatrix}$