Student Answer Sheet Analysis

Automated Processing System

July 7, 2025

Questions and Student Responses

Question 1

Question: Short answer/objective question. For objective questions writing justification is mandatory, otherwise you will get 0.

(a) Consider the following incidence matrix of a simple undirected graph. Convert this into an adjacency matrix representation. [2 marks] Matrix: 1 0 0 1 1 1 0 1 0 0 0 1

(b) Which network model assumes that edges are formed between pairs of nodes with a uniform probability, independent of other edges? [2 marks] A. Barabási-Albert Model B. Erdős–Rényi (Random Network) Model C. Watts-Strogatz (Small-World) Model D. Configuration Model

(c) In game theory, a situation where no player can improve their outcome by unilaterally changing their strategy, given the strategies of other players, is known as: [2 marks] A. Zero-Sum Game B. Dominant Strategy C. Nash Equilibrium D. Mixed Strategy

(d) The tendency for individuals in a social network to associate and bond with similar others is defined as: [2 marks] A. Structural Equivalence B. Assortative Mixing C. Regular Equivalence D. Network Density

(e) Why might betweenness centrality be a more relevant measure than degree centrality for identifying critical nodes in a network transmitting information that must follow specific paths? [2 marks] A. Because it measures the total number of connections a node has. B. Because it prioritizes nodes with high clustering coefficients. C. Because it is easier to calculate for large graphs. D. Because it quantifies how often a node lies on the shortest paths between other nodes.

(f) A key finding about scale-free networks (like those generated by the Barabási-Albert model) is their robustness to random node failures but vulnerability to targeted attacks on hubs. What underlying property best explains this? [2 marks] A. Low average path length. B. A uniform degree distribution. C. The presence of many nodes with very high degrees (hubs) that maintain connectivity. D. A high clustering coefficient across all nodes.

(g) In community detection, optimizing for high modularity aims to find partitions where: [2 marks] A. The number of intra-community edges is significantly higher than expected in a random network with the same degree sequence. B. The number of inter-community edges is maximized. C. The number of communities is maximized. D. The size of all communities is perfectly balanced.

(h) Consider two nodes, X and Y. The neighbors of X are A, B, C, D. The neighbors of Y are C, D, E. What is the Jaccard Coefficient for link prediction between X and Y? [2 marks] A. $\frac{2}{7}$ B. $\frac{2}{5}$ C. $\frac{2}{4}$ D. $\frac{3}{4}$

(i) In the context of information cascade models, how does the activation mechanism differ fundamentally between the Independent Cascade Model (ICM) and the Linear Threshold Model (LTM)? [2 marks] A. ICM uses edge probabilities independently; LTM uses a weighted sum of active neighbors compared to a node threshold. B. LTM uses edge probabilities independently; ICM uses a weighted sum of active neighbors compared to a node threshold. C. Both models rely solely on the number of active neighbors, ignoring edge weights or probabilities. D. ICM activates nodes based on global network properties, while LTM uses only local information.

(j) A standard Graph Convolutional Network (GCN) aggregates information from a node's immediate neighbors. Why might this standard message-passing approach be suboptimal for tasks like node classification in networks with high heterophily (where connected nodes tend to be dissimilar)? [2 marks] A. Because GCNs can only be applied to undirected graphs. B. Because aggregating features from dissimilar neighbors can blur the node's own representative features, making classification harder. C. Because GCNs require nodes to have features, which is not always possible. D. Because GCNs are computationally too expensive for heterophilic graphs.

Student Answer:

(a) Cohesity Matrix:

0 1 0
0
1 0 1
0
0 1 0
1
0 0 1
0

Adjacency Matrix:

1 0 0
1 1 1
0 1 0
0 0 1

Tests

(b) B -¿ Erdos - Random network model connects nodes with fixed probability and generates random graphs without considering node properties.

(c) C Nash equilibrium (640s) Optimal combination of strategies for both players. Unilateral change of strategy reduces chances of swing

(d) B. Assortative mixing. Preference of individuals to connect with similar ones like same interest, age group, profession, in social networks.

(e) D. because it quantifies how often a node lies on the shortest paths between other nodes (centrality) Measurs frequency at which node appears on definition of shortest paths

(f) C. Presence of many nodes with high degrees that maintain connectivity.

(g) A. No of intra community edges is expert in random N with same degree

(h) B (2/5). Jaccard co-of = Intersection / Size of Union = (CIDE) = 2 sentences / Neigh-sets

(i) A Nodes in ICM are activated based on independent edge probabilities. But LTM uses threshold depending on sums of neighbor influences.

(j) B As connected nodes are dissimilar it leads to less accurate feature aggregation poor classification.

Question 1(e) - Reanswered

Question: (Repeated Question Text from 1(e)) Why might betweenness centrality be a more relevant measure than degree centrality for identifying critical nodes in a network transmitting information that must follow specific paths? [2 marks] A. Because it measures the total number of connections a node has. B. Because it prioritizes nodes with high clustering coefficients. C. Because it is easier to calculate for large graphs. D. Because it quantifies how often a node lies on the shortest paths between other nodes.

Student Answer:

Ans: D. For identifying critical nodes in a network, betweenness centrality is more relevant because it quantifies how often a node lies on the shortest paths between other nodes as this indicates the node's importance in communication information flow across the network.

Question 1(f) - Reanswered

Question: (Repeated Question Text from 1(f)) A key finding about scale-free networks (like those generated by the Barabási-Albert model) is their robustness to random node failures but vulnerability to targeted attacks on hubs. What underlying property best explains this? [2 marks] A. Low average path length. B. A uniform degree distribution. C. The presence of many nodes with very high degrees (hubs) that maintain connectivity. D. A high clustering coefficient across all nodes. **Student Answer:**

Ans: C. "Presence of hubs" with many nodes with high degrees that maintain connectivity gives scale-free networks their robustness because hubs are critical for network connectivity random failure affects less connected nodes, but the hubs targeted attacks on hubs make them vulnerable.

Question 1(g) - Reanswered

Question: (Repeated Question Text from 1(g)) In community detection, optimizing for high modularity aims to find partitions where: [2 marks] A. The number of intra-community edges is significantly higher than expected in a random network with the same degree sequence. B. The number of inter-community edges is maximized. C. The number of communities is maximized. D. The size of all communities is perfectly balanced. **Student Answer:**

Ans: A. High modularity indicates strong community structure with dense intra community connections, which is an important goal in community detection. Hence, optimization finds partition with a high No. of intra-community edges.

Question 2

Question: A novel influenza strain (following an SIR - Susceptible, Infected, Recovered - model) is spreading in a city. You have access to a network graph representing close social contacts (nodes=people, edges=contacts). Resources are limited, allowing you to preemptively vaccinate (move directly to the 'Recovered' state) only 5

Student Answer:

Strategy to identify individuals that might face most effectively use limited vaccines for minimizing infections.

(1) Identify individuals that might act as bridges to spread across groups using **Betweenness centrality**.

(2) Vaccinate individuals with high degree of connectivity -¿ high no. of direct contact using **Degree centrality**.

This minimizes spread within and across groups effectively.

Question 3

Question: You are tasked with improving the 'suggested collaborators' feature on a platform for academic researchers. Explain how you could combine link prediction algorithms with node embedding techniques (e.g., Node2Vec trained on paper citation/co-authorship networks) to generate recommendations. Discuss the role of homophily (researchers collaborating within similar fields) in this context and suggest one potential way to promote cross-disciplinary collaborations using your proposed system. [10 marks]

Student Answer:

Combining link prediction algorithms with node embedding to generate recommendations

(1) Use Node2Vec to capture structural semantic relationships in network

(2) Predict potential collaboration within existing networks using link prediction

(3) Homophily in context of collaboration among researchers.

While generating recommendations for collaboration, use the factor that the suggested researcher be from a different field but complimenting disciplines. This can create a crosswalk to suggest which disciplines other than your own are complimentary for collaboration for each researcher looking to collaborate.

Question 4

Question: (a) Describe the core idea behind the Girvan-Newman algorithm for community detection. [3 marks]

(b) Explain how it uses edge betweenness centrality iteratively. [2 marks]

(c) What is a major computational limitation of this algorithm? [2 marks]

(d) Briefly explain how the Louvain method provides a more scalable alternative for optimizing modularity. [3 marks]

Student Answer:

(a) Girvan-Newman Algorithm for community detection - Iteratively removes edges with highest betweenness centrality to discover community structure.

(b) It identifies and removes edges that act as bridges between communities by using edges betweenness centrality.

(c) Computational limitation of Girvan is its high cost of computation, which becomes prohibitive for large networks.

(d) For optimizing modularity Louvain method uses hierarchical clustering -¿ As a scalable alternative.

Question 5

Question: (a) Explain the intuition behind the PageRank algorithm for determining node importance. [3 marks]

(b) Describe the role of the 'damping factor' (d) for random surfer based PageRank algorithm. [3 marks]

(c) What problem arises from 'dangling nodes' (nodes with no outgoing links), and how is this typically handled in the PageRank calculation to ensure convergence? [4 marks]

Student Answer:

(a) Intuition in PageRank algorithm. Node importance in determined based on number quality links of a node.

(b) In the surfer-based Page-Rank algorithm (d) damping factor accounts for probability that of a random surfer continuing following links preventing sink

(c) Problem with nodes that have no outgoing links is they disrupt convergence of ranks of such dangling nodes can be redistributed to other nodes to ensure convergence of algorithm.

Question 6

Question: Two players on a network edge play a game with the following payoff matrix. Row player is Player 1 and Column player is Player 2.

	Strategy A	Strategy B
Strategy U	$(3, 2)$	$(0, 1)$
Strategy L	$(2, 0)$	$(2, 3)$

(a) Identify all Pure Strategy Nash Equilibria in this game. Briefly explain why they are equilibria. [3 marks]

(b) Suppose Player 1 chooses 'Strategy U' with probability p and 'Strategy L' with probability 1 – p. Calculate the expected payoff for Player 2 for each of the strategy. [4 marks]

(c) What will be the expected outcome if $p = 0.7$? [2 marks]

Student Answer:

Nash equilibria = optimal = best strategy combo for both players. Any one switching will only do worse.

Finding Nash equilibria - identifying if a player's outcome can improve by changing strategy. (a) (1) Player 1's strategy "U" v/s Player 2's A

	A	B
U	(3, 2)	(0, 1)
L	(2, 0)	(2, 3)

If Player 1 switching from U to L, Player 2 switches from A to B -¿ Player 2 decreases from (3 to 2), decreases payoff from (2 to 1). (U, A) is a Nash equilibrium.

(2) U v/s B. Play 2 from B to A, Player payoff increases to 3. Doesn't qualify. Play 2 switching from B to A payoff increases 1 to 2. Not Nash.

(3) (L, A) Play 1 Switch L to U. Doesn't qualify. increases payoff to 2-¿3. Play 1 A to B, increases, doesn't qualify, payoff 0-¿3. Not Nash.

(4) (L, B) Players switching from (L to U) equilibrium. Pay off decreases 2 to 0. Didn't do so player switching from B to A, 3 -¿ 0.

(a Continued) So, Pure Strategy Nash Equilibria are (U, A) and (L, B) where unilateral change of strategy reduces change of payoff for both players.

(b) When player 1 chooses U with prob p. Player 2's strategy A payoff. (P*2) + (1-P)*0 = 2P

Player 2's strategy B payoff. P*1 + (1-P)*3 = 3-2P

(c) When p = 0.7, 2P = 1.4 Playoff for player 2 with strategy A

Playoff for Player 2 = 3-2P with strat B = 3-1.4 = 1.6

Question 7

Question: Consider the following simple directed graph where edges point towards node B: A → B, C → B, D → B. We want to compute the updated feature vector for node B, denoted as $\mathbf{h}_B^{(1)}$, using one layer of a simple Graph Neural Network. [10 marks]

The initial (layer 0) feature vectors for the nodes are $\mathbf{h}_A^{(0)}$, $\mathbf{h}_B^{(0)}$, $\mathbf{h}_C^{(0)}$, and $\mathbf{h}_D^{(0)}$. (Note: Specific vectors are provide at the end of the question).

The GNN layer performs the following steps to update node B's features:

1. Aggregate neighbor features: Calculate the average of the initial feature vectors of B's neighbors (A, C, D). Let this aggregated vector be $\mathbf{h}_{\mathcal{N}(B)}^{(0)}$, where $\mathcal{N}(B) = \{A, C, D\}$.

2. Transform: Apply a linear transformation using the weight matrix W to the aggregated neighbor vector.

3. Activate: Apply the ReLU (Rectified Linear Unit) activation function, where ReLU(x) = max(0, x) element-wise.

The formula for this update is:

$$\mathbf{h}_B^{(1)} = \sigma\left(W \cdot \left(\frac{1}{|\mathcal{N}(B)|}\sum_{u\in\mathcal{N}(B)} \mathbf{h}_u^{(0)}\right)\right)$$

Where σ is the ReLU activation and the weight matrix is given by:

$$W = \begin{pmatrix} 0.5 & 0 \\ 0.1 & 0.2 \end{pmatrix}$$

Calculate the updated feature vector $\mathbf{h}_B^{(1)}$ for node B. Show your steps for aggregation, transformation, and activation clearly.

Consider initial feature vectors are $\mathbf{h}_A^{(0)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\mathbf{h}_C^{(0)} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$, $\mathbf{h}_D^{(0)} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$.

Student Answer:

Layer 0: $\mathbf{h}_A^{(0)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\mathbf{h}_C^{(0)} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$, $\mathbf{h}_D^{(0)} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$ Step 1: Avg of initial feature vectors of neighbors

$\mathbf{h}_{\mathcal{N}(B)}^{(0)} = \frac{1}{3}[\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \end{pmatrix}]$

$= \frac{1}{3}\begin{pmatrix} 3 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

Step 2: Transformation $\mathbf{h}_B^{(1)} = W \cdot \mathbf{h}_{\mathcal{N}(B)}^{(0)}$ Transform weight Matrix $= \begin{pmatrix} 0.5 & 0 \\ 0.1 & 0.2 \end{pmatrix}$ $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$= \begin{pmatrix} 0.5 * 1 + 0.2 \\ 0.1 * 1 + 0.2 * 2 \end{pmatrix} = \begin{pmatrix} 0.5 + 0 \\ 0.1 + 0.4 \end{pmatrix}$

$= \begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix}$

Step 3: Activation $\sigma \begin{pmatrix} 0.5 & 0 \\ 0.1 & 0.2 \end{pmatrix} \frac{1}{1/2}\mathbf{h}^{(0)}$

$\sigma \begin{pmatrix} 0.5 & 0 \\ 0.1 & 0.2 \end{pmatrix} \frac{1}{1/2}[\begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 0 \\ 3 \end{pmatrix} + \begin{pmatrix} 2 \\ 2 \end{pmatrix}]$