DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING
Indian Institute of Technology Jodhpur
Social Network Analysis (Code: CSL 7390)
Instructor: Dr. Suman Kundu
Apr 19, 25
Time: 3H | Major | Marks: 80

- The exam is closed book, use of notes, digital content and internet is strictly prohibited. If identified, paper will be canceled. Do not use AI, if identified will be marked zero.
- Write clearly and mention all the assumptions (if any) in your answer.

1. Short answer/objective question. For objective questions writing justification is mandatory, otherwise you will get 0.

   (a) Consider the following incidence matrix of a simple undirected graph. Convert this into an adjacency matrix representation. [2]

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

   (b) Which network model assumes that edges are formed between pairs of nodes with a uniform probability, independent of other edges? [2]

   A. Barabási-Albert Model **B. Erdős–Rényi (Random Network) Model** C. Watts-Strogatz (Small-World) Model D. Configuration Model

   Note: The Watts-Strogatz model is a method for generating random graphs that exhibit 'small-world' properties: high clustering coefficients and short average path lengths. It starts with a regular network structure, typically a ring lattice where each node connects to its nearest neighbors. Then, with a certain probability, each edge is randomly 'rewired' to connect to a different node, introducing shortcuts while largely preserving local clustering. Rest of the model already studied in our class.

   (c) In game theory, a situation where no player can improve their outcome by unilaterally changing their strategy, given the strategies of other players, is known as: [2]

   A. Zero-Sum Game B. Dominant Strategy **C. Nash Equilibrium** D. Mixed Strategy

   (d) The tendency for individuals in a social network to associate and bond with similar others is defined as: [2]

   A. Structural Equivalence **B. Assortative Mixing** C. Regular Equivalence D. Network Density

   (e) Why might betweenness centrality be a more relevant measure than degree centrality for identifying critical nodes in a network transmitting information that must follow specific paths? [2]

   A. Because it measures the total number of connections a node has.

   B. Because it prioritizes nodes with high clustering coefficients.

   C. Because it is easier to calculate for large graphs.

Question 1 continues on the next page. . .

D. Because it quantifies how often a node lies on the shortest paths between other nodes.

(f) A key finding about scale-free networks (like those generated by the Barabási-Albert model) is their robustness to random node failures but vulnerability to targeted attacks on hubs. What underlying property best explains this? [2]

A. Low average path length.

B. A uniform degree distribution.

C. The presence of many nodes with very high degrees (hubs) that maintain connectivity.

D. A high clustering coefficient across all nodes.

(g) In community detection, optimizing for high modularity aims to find partitions where: [2]

A. The number of intra-community edges is significantly higher than expected in a random network with the same degree sequence.

B. The number of inter-community edges is maximized.

C. The number of communities is maximized.

D. The size of all communities is perfectly balanced.

(h) Consider two nodes, X and Y. The neighbors of X are A, B, C, D. The neighbors of Y are C, D, E. What is the Jaccard Coefficient for link prediction between X and Y? [2]

A. $\frac{2}{7}$ **B.** $\frac{2}{5}$ C. $\frac{2}{4}$ D. $\frac{4}{3}$

(i) In the context of information cascade models, how does the activation mechanism differ fundamentally between the Independent Cascade Model (ICM) and the Linear Threshold Model (LTM)? [2]

A. ICM uses edge probabilities independently; LTM uses a weighted sum of active neighbors compared to a node threshold.

B. LTM uses edge probabilities independently; ICM uses a weighted sum of active neighbors compared to a node threshold.

C. Both models rely solely on the number of active neighbors, ignoring edge weights or probabilities.

D. ICM activates nodes based on global network properties, while LTM uses only local information.

(j) A standard Graph Convolutional Network (GCN) aggregates information from a node's immediate neighbors. Why might this standard message-passing approach be suboptimal for tasks like node classification in networks with high heterophily (where connected nodes tend to be dissimilar)? [2]

A. Because GCNs can only be applied to undirected graphs.

B. Because aggregating features from dissimilar neighbors can blur the node's own representative features, making classification harder.

C. Because GCNs require nodes to have features, which is not always possible.

D. Because GCNs are computationally too expensive for heterophilic graphs.

2. A novel influenza strain (following an SIR - Susceptible, Infected, Recovered - model) is spreading in a city. You have access to a network graph representing close social contacts (nodes=people, edges=contacts). Resources are limited, allowing you to preemptively vaccinate (move directly to the 'Recovered' state) only 5% of the population. Describe a strategy using at least two distinct network analysis concepts that can be applied together to identify the individuals to vaccinate to most effectively minimize the total number of infections. Justify why your chosen concepts are appropriate and how they would be applied together. [10]

3. You are tasked with improving the 'suggested collaborators' feature on a platform for academic researchers. Explain how you could combine link prediction algorithms with node embedding techniques (e.g., Node2Vec trained on paper citation/co-authorship networks) to generate recommendations. [10]

   Discuss the role of homophily (researchers collaborating within similar fields) in this context and suggest one potential way to promote cross-disciplinary collaborations using your proposed system.

4. (a) Describe the core idea behind the Girvan-Newman algorithm for community detection. [3]

   Solution:

   (b) Explain how it uses edge betweenness centrality iteratively. [2]

   (c) What is a major computational limitation of this algorithm? [2]

   Solution:

   (d) Briefly explain how the Louvain method provides a more scalable alternative for optimizing modularity. [3]

5. (a) Explain the intuition behind the PageRank algorithm for determining node importance. [3]

   (b) Describe the role of the 'damping factor' (d) for random surfer based PageRank algorithm. [3]

   (c) What problem arises from 'dangling nodes' (nodes with no outgoing links), and how is this typically handled in the PageRank calculation to ensure convergence? [4]

6. Two players on a network edge play a game with the following payoff matrix. Row player is Player 1 and Column player is Player 2.

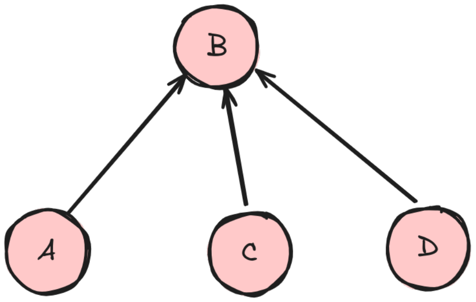
	Strategy A	Strategy B
Strategy U	$(3, 2)$	$(0, 1)$
Strategy L	$(2, 0)$	$(2, 3)$

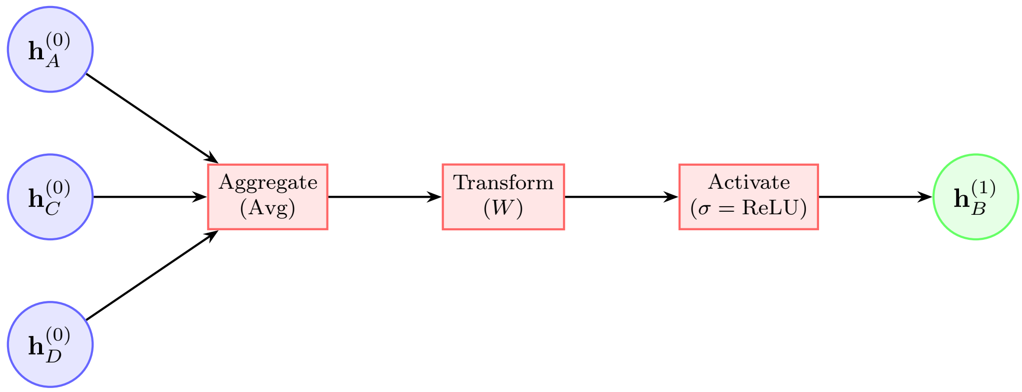
   (a) Identify all Pure Strategy Nash Equilibria in this game. Briefly explain why they are equilibria. [3]

   (b) Suppose Player 1 chooses 'Strategy U' with probability p and 'Strategy L' with probability $1 - p$. Calculate the expected payoff for Player 2 for each of the strategy. [4]

   (c) What will be the expected outcome if $p = 0.7$? [2]

7. Consider the following simple directed graph where edges point towards node B: A $\rightarrow$ B, C $\rightarrow$ B, D $\rightarrow$ B. We want to compute the updated feature vector for node B, denoted as $\mathbf{h}_B^{(1)}$, using one layer of a simple Graph Neural Network. [10]





The initial (layer 0) feature vectors for the nodes are $\mathbf{h}_A^{(0)}$, $\mathbf{h}_B^{(0)}$, $\mathbf{h}_C^{(0)}$, and $\mathbf{h}_D^{(0)}$. (Note: Specific vectors are provide at the end of the question).

The GNN layer performs the following steps to update node B's features:

1. **Aggregate neighbor features:** Calculate the average of the initial feature vectors of B's neighbors (A, C, D). Let this aggregated vector be $\mathbf{h}_{\mathcal{N}(B)}^{(0)}$, where $\mathcal{N}(B) = \{A, C, D\}$.
2. **Transform:** Apply a linear transformation using the weight matrix W to the aggregated neighbor vector.
3. **Activate:** Apply the ReLU (Rectified Linear Unit) activation function, where $\text{ReLU}(x) = \max(0, x)$ element-wise.

The formula for this update is:

$$\mathbf{h}_B^{(1)} = \sigma\left(W \cdot \left(\frac{1}{|\mathcal{N}(B)|} \sum_{u \in \mathcal{N}(B)} \mathbf{h}_u^{(0)}\right)\right)$$

Where σ is the ReLU activation and the weight matrix is given by:

$$W = \begin{pmatrix} 0.5 & 0 \\ 0.1 & 0.2 \end{pmatrix}$$

Calculate the updated feature vector $\mathbf{h}_B^{(1)}$ for node B. Show your steps for aggregation, transformation, and activation clearly.

Consider initial feature vectors are $\mathbf{h}_A^{(0)} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $\mathbf{h}_C^{(0)} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$, $\mathbf{h}_D^{(0)} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$).