

Qa

Given Adjacency Matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Consider rows as nodes - 4 nodes

Consider columns as edges - 3 edges

→ Find adjacency matrix by following below mapping

Edge 1 connects node A & B

Edge 2 connects node B & C

Edge 3 connects node B & D

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

Qb

Ans B. Erdos-Renyi Chandon network Model)

- It creates edges between pairs of nodes is formed independently with equal probability

Q.c

Ans C. Nash Equilibrium

- It states where no player can benefit by unilaterally changing their strategy, assuming all other players are holding their strategies constant.

Q.d

B. Attractive Mixing.

- The tendency of nodes in a network to connect with other nodes that have similar characteristics.

Q.e

D. Because it quantified how often a node lies on the shortest paths between other nodes.

Q.f

C. The presence of many nodes with very high degrees (hubs) that maintain connectivity.

Q.g

A. The number of intra-community edges is significantly higher than expected in random network with the same degree sequence.

Q.h

X: A, B, C, D

Y: C, D, E.

~~A B~~ ~~2~~ 5 XUY 2 5

~~A B~~ XAY 2 2

Jaccard Coefficient = $\frac{2}{5}$

B: $\frac{2}{5}$

Q1

A. BCM uses edge probabilities independently;
LTM uses a weighted sum of active neighbors
compared to a node threshold.

Q2

B. Because aggregating features from dissimilar
neighbors can blur the nodes own representative
features, making classification harder.

Q3 Vaccination Strategy using network analysis

a. Betweenness centrality - It helps to identify
nodes that act as bridges between different
communities. These nodes often lie on the
shortest paths between many pairs of nodes
and control the flow of the infection between
different parts of the network.

b. Degree centrality - It helps to identify
high degree nodes that have many connections and
higher potential to spread the virus.

Qa.

The core idea behind Girvan-Newman algorithm is to identify communities in a network by iteratively removing edges with high betweenness centrality until the network breaks down into smaller, well defined groups.

Qb.

Betweenness centrality measures how often a node lies on the shortest path between other nodes in a network.

Qc.

- It helps to calculate betweenness centrality for all edges in the network.

- Removing the edge with highest betweenness centrality.

Qd.

Major computational limitation of Girvan-Newman algorithm is its high computational complexity.

- For a network n nodes & m edges will calculate with $O(mn)$ time.

- This calculation must be repeated after removing each edge.

- ~~Overall~~ This makes the algorithm impractical for large networks with thousands / millions of nodes & edges.

(5)

4d Louvain method provides a more scalable alternative for optimizing modularity through.

- Local optimization - It starts by assigning each node to its own community. Then iteratively moves individual nodes to communities that result in the largest increase in modularity.
- Hierarchical Aggregation - It creates new network where nodes are the communities found in previous steps.
- Iteration on reduced network - It repeats the process on this smaller network, allowing it to select Multilevel community structure.

5a PageRank Algorithm

- Intuition behind Page-Rank is based on the idea that the popularity of a webpage is determined not only by the number of incoming links but also by the kind of incoming links. Citations from highly ranked pages contribute more than lower ranked webpages.

5.b Rate of Damping factor (d).

Damping factor (d) represents the probability that the random surfer will follow an outgoing link rather than randomly teleporting to another page.

5.c Problems with Dangling nodes.

- Dangling nodes create a leak in page rank calculation.
- It causes all page rank values to approach zero.

6 Given payoff matrix

	Strategy - A	Strategy - B
Strategy - U	(3, 2)	(0, 1)
Strategy - L	(2, 0)	(2, 3)

6.a pure Strategy

(U, A) - player 1 gets 3, player 2 gets 2

(U, B) - player 1 gets 0, player 2 gets 1

(L, A) - player 1 gets 2, player 2 gets 0

(L, B) - player 1 gets 1, player 2 gets 3

6.b For Expected payoffs for A

$$E[A] = p \times 2 + (1-p) \times 0 = \underline{2p}$$

Expected payoffs for B

$$E[B] = p \times 1 + (1-p) \times 3 = p + 3 - 3p = \underline{3-2p}$$

6.c

Expected outcome of $p = 0.7$

$$E[A] = 2p = 2 \times 0.7 = \underline{1.4}$$

$$E[B] = 3 - 2p = 3 - 2 \times 0.7 = \underline{1.6}$$

⑦.

Neighbourhood of B $N(B) = \{A, C, D\}$

Given feature vectors

$$h_A^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, h_C^{(0)} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}, h_D^{(0)} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

Find average of initial feature vectors.

$$h_{N(B)}^{(0)} = \frac{1}{3} (h_A^{(0)} + h_C^{(0)} + h_D^{(0)})$$

$$= \frac{1}{3} \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right)$$

$$= \frac{1}{3} \begin{pmatrix} 3 \\ 6 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 1 \\ 2 \end{pmatrix}}}$$

Given weight Matrix

$$W = \begin{bmatrix} 0.5 & 0 \\ 0.1 & 0.2 \end{bmatrix}$$

$$W \times h_{N(B)}^{(0)} = \begin{bmatrix} 0.5 & 0 \\ 0.1 & 0.2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{pmatrix}$$

$$= \begin{bmatrix} 0.5 \times 1 + 0 \times 2 \\ 0.1 \times 1 + 0.2 \times 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0.5 + 0 \\ 0.1 + 0.4 \end{bmatrix}$$

$$= \underline{\underline{\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}}}$$