Roll NO — Social network analysis
624A11126 Preetam Eklarpur — 19-4-2025
(623A11028 for old tests.) — Pg 1

(a)

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{\text{Converting to Adjacency matrix}} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

(b) B -> Erdos-Random Ntwk model connects nodes with fixed probability & generates random graphs without considering node properties.

(c) C Nash equillibrium (Q. 6 kws) optimal combination of strategies for both players. unilateral change of strat reduces chances of win.

(d) B. Assortive mixing
Preference of individuals to connect with similar ones. like. same interests, Age group, Profession. in social networks.

(e) D. ~~Because it quantifies how often a node lies on the shortest paths b/w other nodes~~ betweenness centrality definition: Measures frequency at which Node appears on shortest paths.

Re. Answered at last page.

(f) ~~C. Presence of many nodes with high degrees that maintain connectivity~~

(g) ~~A. No. of intra community edges > expected in random N with same degree sequence~~

(h) B. (2/5)

$$\text{Jaccard co-ef.} = \frac{\text{intersection}}{\text{Size of Union of neigh. sets}} = \frac{(C, D)}{(A, B, C, D, E)} = \frac{2}{5}$$

Pg2

(1) (A) Nodes in ICM are activated based on independent edge probabilities. But LTM uses threshold depending on sum of Neighbor influences.

(j) (B). As connected nodes are dissimilar it leads to less accurate feat. aggregation & poor classif. Performance

(Q2) strategy to identify individuals for most effectively use limited vaccines for minimizing infection

(1) Identify individuals that might act as bridges to spread across groups using Betweenness centrality.

(2) Vaccinate individuals with high degree of connectivity. -> high No. of direct contact using Degree centrality

This minimizes spread within and across groups effectively

Q3

Combining link prediction algorithm with node embedding to generate recommendations

(1) Use Node2Vec to capture structural & semantic relationships in network

(2) predict potential collaborations within existing network using link prediction

(3) Homophily in context of collaboration (Pg 3)

among researchers.

While generating recommendations for collaborations, use the factor that the suggested researcher be from a different field but complimenting desciplines

- can create a crosswalk to suggest which desciplines other than own are complimentary for collaboration, for each researcher looking to collaborate.

Q4.) (a) Girvan-Newman Algorithm for community detection. - Iteratively removes edges with highest betweenness centrality to discover community structure

(b) (b) It Identifies and removes edges that act as bridges b/w communities by using edges ~~the~~ betweenness centrality

(c) Computational limitation of Girven is its high cost of computation. which becomes prohibitive for large networks

(d) For optimizing modularity Louvain method uses heirarchical clustering -> As a scalable alternative

Q. 5
(a) Intuition in Pagerank algorithm. (Pg 4)

- Node importance is determined based on number of quality links of a Node

(b) In a surfer based Page-Rank algorithm (d) dampingfactor. accounts for probability ~~that~~ of a random surfer continuing following links & preventing sink

(c) Problem with nodes that have no out going links. is they disrupt convergence ranks of of such dangling nodes can be redistributed to other nodes to ensure convergence of Algorithm.

Q. (6)

Player 2

Player 1	strat A	strat B.
stat U	(3,2)	(0,1)
strat L	(2,0)	(2,3).

Nash equillibria
= optimal
= best stratergy combo for both players.
Any one switching will only do worse.

- continued in nxt pg.

Q6 Finding Nash equillibria- [Pg 5]

identifying if a players outcome can improve by changing strategy.

(a) (1) Player 1's strategy "U" v/s Player 2's A.

Player 1's switching from U to L,

if players 1 switching from U to L

Playoff decreases from (3 to 2)

Player 2 switching from A to B

Player 2 switching from decreases playoff from (2 to 1)

Plyr 1 \ Player 2	A	B
U	(3,2)	(0,1)
L	(2,0)	(2,3)

$\boxed{\text{(U,A) is a Nash equillibrium}}$

(2) U vs B. Play 2 from B to A,

(0,1) increases playoff ~~0 to 3~~. 0 to 2

∴ doesnt qualify

Play 2 switching from B to A playoff increases 1 to 2. X No nash.

(3) (L,A). play 1 switch L to U

P.off 2 -> 3 X doesn't qualify increases

Play 2 A to B. increases. doesnt qu.

P.off 0 -> 3 X

(4) (L,B) Player 1 sw. from (L to U) -> Nash equilli brium

Play off decreases 2 to 0.

Player 2 sw from B to A, 3 -> 0

(Q6.) (continued)

(a) So, pure strategy Nash equilibria are (U,A), and (L,B)

where unilateral change of strategy reduces chance of payoff for both players.

(b) When player 1 chooses U with prob P

Player 2's strategy A payoff.

$$(P.2) + (1-P).0 = 2P$$

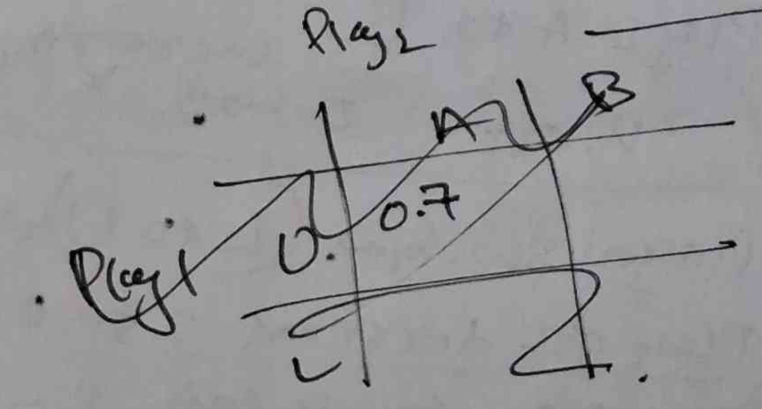
Player 2's strategy B payoff.

$$P.1 + (1-P).3 = 3-2P.$$

(c) When $P = 0.7$, $2P = 1.4$

Payoff for player 2 with strategy A.

Payoff for Player 2 with strat. B $= 3-2P$
$= 3-1.4$
$= 1.6.$



Page 7.

Q(7) GNN — Layer 0 $h_A^{(0)} = \begin{pmatrix}1\\1\end{pmatrix}$, $h_C^{(0)} = \begin{pmatrix}0\\3\end{pmatrix}$ $h_D^{(0)} = \begin{pmatrix}2\\2\end{pmatrix}$

Aggregation of neighbor features

Step 1

Avg of initial feature vectors

$$h^{(0)}_{N(B)} = \frac{1}{3}\left[(1,1)+(0,3)+(2,2)\right]$$

$$= \frac{1}{3}\left[(3,6)\right] = (1,2)$$

Step 2 Transformation

$$h^{(1)}_{NB} = W \cdot h^{(0)}_{NB}$$

Trans. weight matrix

$$= \begin{pmatrix}0.5 & 0\\0.1 & 0.2\end{pmatrix}\begin{pmatrix}1\\2\end{pmatrix}$$

$$= \begin{pmatrix}0.5\times 1 + 0.2\\0.1\times 1 + 0.2\times 2\end{pmatrix} = \begin{pmatrix}0.5+0\\0.1+0.4\end{pmatrix}$$

$$= \begin{pmatrix}0.5\\0.5\end{pmatrix}$$

Step 3 Activation

$$\sigma\begin{pmatrix}0.5 & 0\\0.1 & 0.2\end{pmatrix}\left(\frac{1}{(12)}\sum h_u^{(0)}\right)$$

$$= \sigma\begin{pmatrix}0.5 & 0\\0.1 & 0.2\end{pmatrix}\left(\frac{1}{(12)}\left((1\ 1)+(0\ 3)+(2\right.\right.$$

Q1 re-answered with justification Page 8

Q 1 with justification Ans (D).

(e) For identifying critical nodes in a network, betweenness centrality is more relevant because it quantifies how often a node lies on the shortest paths b/w other nodes as this indicates the nodes importance in communication & information flow across the network.

(f) Ans. "C." Presence of many nodes "hubs" - with high degrees that maintain connectivity gives scale-free networks their robustness because hubs are critical for network connectivity & random failure affects less connected nodes. but the Presence of high No. of hubs & targeted attacks on hubs make them vulnerable.

(g) Ans "A." High modularity indicates strong community structure with dense intra community connections, which is ~~one of~~ important goal in community detection. hence optimization finds Partitions with high No. of intra community edges.