A.1 a] Convert this into d adjacent matrix :

$$
\begin{bmatrix}
1 & 0 & 0 \\
1 & 1 & 1 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
$$

Each column represent an edge, and the rows are node.

Edge 1 connets Node 1 & Node 2

Edge 2 connets Node 2 & Node 3

Edge 3 connets Node 2 & Node 4

$$
\begin{bmatrix}
0 & 1 & 0 & 0 \\
1 & 0 & 1 & 1 \\
0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}
$$

b] B - Erdös -Rényi (Random Network) Model

In this model, each possible edge between a pair of nodes is formed independent with equal probability.

c] C - Nash equilibrium.

A Nash equilibrium occurs when each player's strategy is optimal given the strategies of all other players, thus no one can improve their outcome by changing their strategy alone.

d] B - Assortative Mixing.

This referes to the tendency of nodes
to connect to others that are similar
in some way like age, degree etc.

e] D - Because it quantify how often a node line
on the shortest path's between other nodes.
- Betweenness certainly focuses on how critical
a node is for connecting others, specially when
information has to pass along shortest path
unlike degree centrality, which jo just counts
immidiate connections.

f] C - The presence of many nodes with very high
degree (hubs) that maintain connectivity.
- Scale - free network have a few highly connected
hubs. Random failure usually hit low-degree
nodes, so the network survives. But it if
the hub fails, it can break the whole
structure, making it vulnerable the targeted
attacks.

g] A - The number of intra - community edge is
significantly higher than   - degree required
- High modularity means most connections
are within communities, and very few are
between them - more than what you'd expect
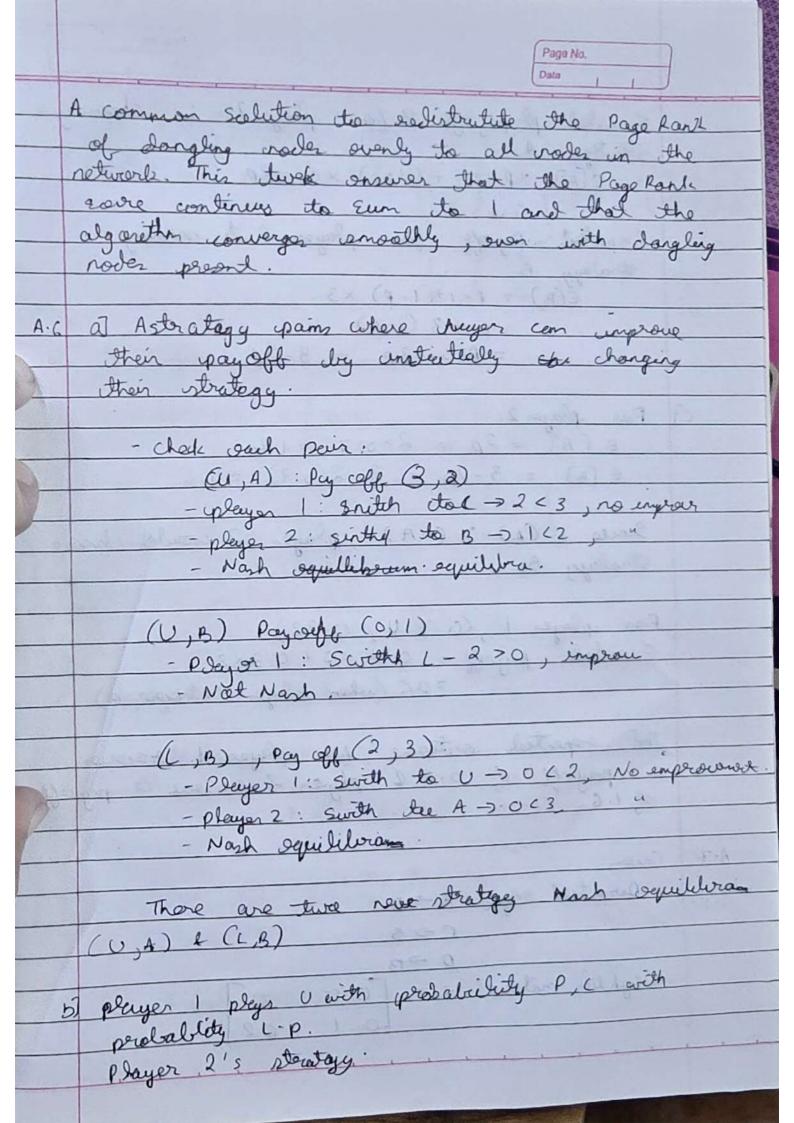in a random setup.

h] B - 2/5
- Neighbours of X : A, B, C, D
  Neighbours of Y : C, D, E
  Intersection : C, D → 2 nodes
  Union : A, B, C, D, E → 5 nodes

  ∴ Jaccard coefficient = $\dfrac{2}{5}$

i] A - ICM uses edge probability independent;
LTM uses a weight of active - threshold.
- ICM - each active neighbour gets one independent chance to activate a node.
LTM - a node becomes active only when the sum of influence from neighbours crosses a threshold.

j] B - Because aggregating feature from dissimilar neighbours can blur the node's ___ harder.
- In heterophilic networks, pulling in feature from neighbours can actually confuse the model. The GCN ends up blending unrelated info which reduces classification accuracy.

A-2 To effectively reduce the spread of a novel influenza strain modeling by the SIR framework, and with the constraint of vaccinating only 5% of the population, we can use a combination of network analysis technique to identify the most strategic individual for vaccination. The goal is to maximize impact with minimal resource by targeting these

most critical to the virus potential spread.

- Using two network concepts:
1. Degree centrality — This metric identifies individual with the highest number of direct connections in the network. These people interact with many others and are more likely to become infected and pass the disease to several others.
Why it works — Vaccinating individuals with high degree centrality can prevent a large number of initial infections.
Sort nodes by degree. Like a person who interacts with 30 people daily poses a high risk of spreading the virus. So, vaccinating them early cuts off multiple potential transmission chains.

- 2. Between centrality — This metric highlights nodes that frequently lie on the shortest path between other nodes. These people serve as a bridge between the parts of the network, enabling the disease to jump between groups or communities.
Vaccinating high-betweenness individuals can stop the spread from moving across communities which is critical in keeping the outbreak localized.
Compute betweenness scores and flags nodes that take risk highly especially those not already chosen by degree critical.
Like even if someone doesn't have a lot of

connections, they may be the only link between
ture large groups. So, if deseen reaches than
it could suddenly infect the entire new section.

- To maximize effectiveness - Rank all individually
& both degree & betweenness centrality.
Then identify the score high on both, as they
are both highly connected and act as critical
badges.
Select the top 5% from this prioritized list
of for vaccination. This ensures that we are
vaccinated not just the socially active
individuals, but also the structural 'gatekeepers'
of the network.

- To justify this - Using just degree centrality
might ignore structurally important nodes with
lower connections but high strategic placement.

Using only betweenness might ignore 'super
spreading' with many contacts.

Combining both ensures a balance strategy
blocking both high frequency and high impact paths.

A.3 To enhance the suggested collaborators feature
for academic researchers we can combine link
prediction algorithm with node embedding
technique like Node 2Vec, using data from
co citation and co-authorship networks. This
would identify meaningful, research-driven connection

that are both relevent and novel.

1 Using link prediction + Node Embedding.
So, link prediction helps forecasting which pairs of reserchers are likely to collaborate in the future based on exesting network patterns. Like - Jaccard coeffient, Adamic - Adar, etc. However these methods are mostly rly on local network structure and may miss deeper patterns.

Node embedding technique like Node 2 Vec solve this ly learning a vector representation of each researcher based on their position in the network. These vectors captures both structural & semantic similarities.

By combining the two - Node 2 Vec provides dense embedding capture the reserchers role & context. Link prediction uses these embddeds to predict future or pro potential links.

- Role of Homophily in collaboration - This is a tendeny for similar individual to colaborate more often.

In practice : A computer sientist is more likely to co-author with another CS than with a biolgist.

Effect on recommendation : Node 2 Vec embedding's

naturally captures homophily, since co-authorship and citation network often group researcher with similar interests.

- While homophily is useful, it can also reinforce academy silos. To encourage cross-disciplinary collaboration we can tweak our system to highlight 'bridge' opportunities.

Add a diversity penalty or bonus to the recommended score:
For example, suggest a data scientist to a & climate researcher if they have both co-authors with someone in environmental modeling.

This promotes novel and interdisciplinary connections without compromising on quality.

A.4 a) The Girvan-Newman algorithm is a method for community detection that works by progressively removing edges to revel the underlying community structure of a network. Think of it like cutting & the weakest link between groups to let natural cluster emerge.

b - At each step compute edge betweenness for all edges and remove the edge with the highest value. Repeat this until desired con correct community structure emerges.

c→ Computing edge betweeness outrality is expensive per calculation, making the algorithm slow for large networks.

d→ The Louvain method is a popular, fast & scalable algorithm that detects communities by optimizing modularity. This allows to it to efficiently find the high-modularity partitions in very large network, often with millions of node, much faster then the Girvan-Newman.

A 5 a) Intution behind Page Rank algorithm - Page rank models the importantce of the node as probablity that makes a random walker lands on it considering both direct link and importance of linking node.

b) Damping facter Role - The damping ffcter is (typecally 0.85) represents probability that a random walker flows a link, 1-d is the probability of of jumping to a random node, ensuring the process does not give stuck.

c) Dangling nodes are node with on outgoind link to any other pade page. If the random surface lands on a dangling node, there's nowhere to go, which can I cuse the PageRank scoree to 'leak' and algorthm may not converge preperly.

A common solution to redistribute the Page Rank of dangling nodes evenly to all nodes in the network. This trick ensures that the Page Rank score continues to sum to 1 and that the algorithm converges smoothly, even with dangling nodes present.

A.6 a] A strategy pairs where neither can improve their payoff by unilaterally changing their strategy.

    - check each pair:
      (U, A) : Pay off (3, 2)
      - player 1: switch to L → 2 < 3, no improve
      - player 2: switch to B → 1 < 2
      - Nash equilibrium. equilibria.

      (U, B) Payoff (0, 1)
      - Player 1 : Switch L - 2 > 0, improve
      - Not Nash.

      (L, B), Pay off (2, 3):
      - Player 1: Switch to U → 0 < 2, No improvement.
      - player 2: Switch to A → 0 < 3
      - Nash equilibrium

    There are two pure strategy Nash equilibria
    (U, A) & (L, B)

b] player 1 plays U with probability P, L with probability 1-P.
Player 2's strategy.

Exacted pt pay off & for player 2 in choosing
strategy A.

$E[A] : p \times 2 + (1-p) \times 0 = 2p$

Expected pay off for player 2 it choosing
strategy B:

$E(B) = p \times 1 + (1-p) \times 3$

$= p + 3 (1-p)$

$= p + 3 - 3p = 3 - 2p$

c] For player 2

$E[A] = 2p = 2 \times 0.7 = 1.4$

$E[B] = 3 - 2p = 3 - 2(0.7) = 1.6$

Since $E(B) > E(A)$, player 2 would choose
strategy B.

For player 1, (0.7 U, 0.32)

Expected pay off $= 0.7 \times 0 + 0.3 \times 2$

$= 0.6$ (when player 2 choose B)

The expected outcome for player 1 recieves
a pay off of 0.6 & player 2 receive a pay off
of 1.6.

A-7   Given:

Directed edges $A \rightarrow B$

$C \rightarrow B$

$D \rightarrow B$

weighted matrix $w = \begin{bmatrix} 0.5 & 0 \\ 0.1 & 0.2 \end{bmatrix}$

2 Aggregate neighbours -

$$h_n^{(0)} = \frac{1}{3} h_A^{(0)} + h_C^{(0)} + h_0^{(0)} = \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix} + \begin{bmatrix} 2 \\ 2 \end{bmatrix} \right) \frac{1}{3}$$

$$= \frac{1}{3} \begin{pmatrix} 3 \\ 6 \end{pmatrix} = \frac{1}{2}$$

b) Transform -

$$W \cdot h_N^{(0)} = \begin{bmatrix} 0.5 & 0 \\ 0.1 & 0.2 \end{bmatrix} \cdot \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

$$= \begin{bmatrix} 0.5(1) + 0(2) \\ 0.1(1) + 0.2(2) \end{bmatrix}$$

$$= 0.5 \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

c) Apply ReLU Activation.

Applying ReLU

$$h_B^{(1)} = ReLU \left(\begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

Final Answer $h_B^{(1)} = \begin{bmatrix} 0.5, & 0.5 \end{bmatrix}$