

Major Exam April
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Sub - Social Network Analysis
(Code: CSL 7390)

Question - 1

Answer - (a)

Incidence Matrix to Adjacency Matrix

The incidence matrix provided is:

$$\begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

The matrix structure has been given the name. Each edge in an incidence matrix corresponds to a row and every node has its designated column. Each entry of value '1' shows that the node has an incidence with the edge. A simple undirected graph contains exactly two adjacent nodes for every single edge within it.

From the incidence matrix

- Edge 1 connects Node 1 and Node 2
- Edge 2 connects Node 2 and Node 3
- Edge 3 connects Node 2 and Node 4

→ The connection, relationship between nodes in a graph structure appear in adjacency matrix format. The adjacency matrix contains value 1 in position (i, j) and (j, i) when node i and Node j having an edge, otherwise both position contain 0.

The adjacency matrix will represent node to node connection.

$$\begin{array}{c}
 \begin{matrix} & 1 & 2 & 3 & 4 \end{matrix} \\
 \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}
 \end{array}$$

Answer - (B)

Ans → Option - (B) → Erdős-Rényi (Random Network) Model.

→ In the Erdős-Rényi model the edges connect randomly because every edge formation remains isolated from other edges.

Answer - (C)

Ans → Option - (C) → Nash Equilibrium

→ Player reach a stable position known as Nash Equilibrium when none of them achieve higher rewards through independent strategy modifications.

Answer - (d)

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Answer → Option - (B) Assortative Mixing

→ Under the Concept of Assortative Mixing, similar nodes establish connects with each other.

Answer - (e)

Answer - Option → (D) Because it quantifies how often a node lies on the shortest paths between other nodes.

→ The measurement technique called Betweenness Centrality evaluate node based on their capacity to link other nodes because it determines information flow efficiencies.

Answer - (f)

Answer → option - (C) → The presence of many nodes with very high degrees (hubs) that maintain connectivity.

→ Scale-free Network (Hubs) operate through central nodes which serve as crucial connectors. So their attack vulnerability becomes particularly pronounced.

Answer - (g)

Answer → Option - (A) The number of intra-community edges is significantly ~~aims~~ higher than expected in a random network with the same degree sequence.

→ The goal of Community detection (Modularity) is to identify 4 Communities through dense Connection within those group Compared to random Chance expectation.

Answer - (H)

Answer → Option - (B) $\frac{2}{5}$

→ Jaccard Coefficient,

Neighbours of X: A, B, C, D

Neighbours of Y: C, D, E.

Intersection: C, D (2 nodes)

Union: A, B, C, D, E (5 nodes)

$$\text{Jaccard's Coefficient} = \frac{\text{Intersection}}{\text{Union}} = \frac{2}{5} = 0.4$$

The Jaccard's Coefficient calculates set (neighbors) similarity by dividing the overlapping intersection by their combined elements.

Answer - (I)

Answer → Option - (B) → LTM uses edge probabilities independently, LTM uses a weighted sum of active neighbors compared to a node threshold.

→ CH operates through independent edge probabilities but LTM adopts a threshold-based method which takes the weighted sum of neighbors impact.

Answer - (1)

Answer \rightarrow Option - (B) \rightarrow Because aggregating features, from dissimilar neighbors can blur the node's own representative features, making classification harder.

\rightarrow When heterophily occurs during GCN and Heterophily operations it becomes difficult to classify nodes because they lose their individual characteristics through the features aggregation process from dissimilar neighbors.

Question - 2

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Answer - 2

Influenza Spread and Vaccination

- For reducing influenza transmission to 5% level a combined approach of Centrality measurement should be used to determine who get vaccinated first.
- A high value of betweenness Centrality reveals that the node function as a "bridge" through which the disease transmit to numerous other connection. The immunization of these specific people break transmission pathways between different segment of the network.
- High, degree Centrality helps identify "hubs" which possess numerous connection since they have the potential to infect multiple other people.
- A Computation betweenness and degree Centrality should be performed for every individual.
 - A list of ranked people forms based on the Centrality measurement result.
 - The group of people selected for vaccination needs a strong ranking in both Centrality measures.
 - Additional individual should be included for vaccination until 5% coverage is reached if the top selected candidates do not suffice.

Justification →

- A combined approach delivers better results since betweenness centrality identifies network bridge which link different network section while degree centrality distinguishes hubs that spread widely.
- Vaccinating key hubs and bridges provides maximum efficiency by controlling both small outbreaks and widespread distribution of the ~~vaccine~~ virus.

Question - 3

Answer.

Suggested Collaborators features

Links Prediction → System utilizes linking prediction algorithms that generate potential collaboration prediction through the analysis of current co-authorship and citation data.

- When researchers A and B demonstrate numerous joint authors link with each other scientist a link prediction method could indicate potential collaboration between them.
- Through Node2Vec the system creates vector representations of researchers based on network structural information. Embedding that align with each other between researchers demonstrates similar research interest and collaboration potential.

Homophily

- Due to the natural human tendency of homophily the system will provide recommendation related to researchers within similar academic fields.
- Due to encouraging diverse collaboration the system requires a diversity matrix within its recommendation functionality. Although the link prediction score may be slightly lower the system will give priority to research links that combine fields from distinct areas.

Question-4

Answer-(a)

Girvan-Newman Core Idea

- The Girvan-Newman algorithm detects communities through a processing method which successively eliminates edges connecting different communities.

Answer-(b)

Edge Betweenness Centrality →

- Edge Betweenness Centrality analyzes the number of times an edge lies between all pairs of nodes to discover these edges.
- Edges with high betweenness values link separate communities according to this method.

Answer - (C)

Computational Limitation

→ The cost of Computational increases substantially when calculating edge betweenness centrality on extensive network especially when they involve frequent edge removal operation.

Answer - (d)

Louvain Method

→ The Louvain method function as a Scalable approach which uses greedy optimization to shift nodes between Communities during its iterative process.

Question - 5

Ans - (a)

PageRank Algorithm

- According to PageRank the importance of nodes depends on both the quantity and quality of incoming links passing through them.
- Page importance derives from other pages that link to it.

Answer - (b)

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Damping Factor (d)

- During random web navigation a surfer has a probability expressed through the Damping factor (d) either to pause their link clicks or to move to an arbitrary page.

Answer - (c)

Dangling Nodes

- The lack of outgoing links from Dangling Nodes makes Page Rank flow through the network. We handle this condition using an equal distribution amongst all network nodes.
- Each iteration distributes the rank of dangling nodes across all network nodes in an equal way.

Question - 6

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	Strategy A	Strategy B
Strategy U	(3, 2)	(0, 1)
Strategy L	(2, 0)	(2, 3)

Answer - (a)

→ A Strategy pair where player can improve their payoff by unilaterally changing their strategy.

→ Check each pair:-

(U, A) : Payoff (3, 2)

→ Player 1: Switch to L $\rightarrow 2 < 3$, no improve

→ Player 2: Switch to B $\rightarrow 1 < 2$, no improve

→ Nash Equilibrium.

(U, B) : Payoff (0, 1)

→ Player 1: Switch L $\rightarrow 2 > 0$, improve

→ Not Nash

(L, B) : Payoff (2, 3)

→ Player 1: Switch to U $\rightarrow 0 < 2$ No improve

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→ Player 2: Switch to A → $0 < 3$, no improvement
→ Nash equilibrium.

∴ There are two pure strategy Nash equilibrium: (U, A) & (L, B) .

Ans-(b)

→ Player 1 play U with probability P , L with probability $1-P$.

→ Player 2's strategy.

→ Expected Payoff for Player 2 if changing Strategy A

$$E[A] : P \times 2 + (1-P) \times 0 = 2P$$

→ Expected payoff for Player 2 if change Strategy B.

$$E[B] = P \times 1 + (1-P) \times 3$$

$$= P + 3(1-P)$$

$$= P + 3 - 3P = 3 - 2P$$

Answer - C

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For Player 2:

$$E[A] = 2p = 2 \times 0.7 = 1.4$$

$$E[B] = 3 - 2p = 3 - 2 \times 0.7 = 1.6$$

Since $E[B] > E[A]$, Player 2 could choose Strategy B.

For Player 1. $(0.7, 0.32)$

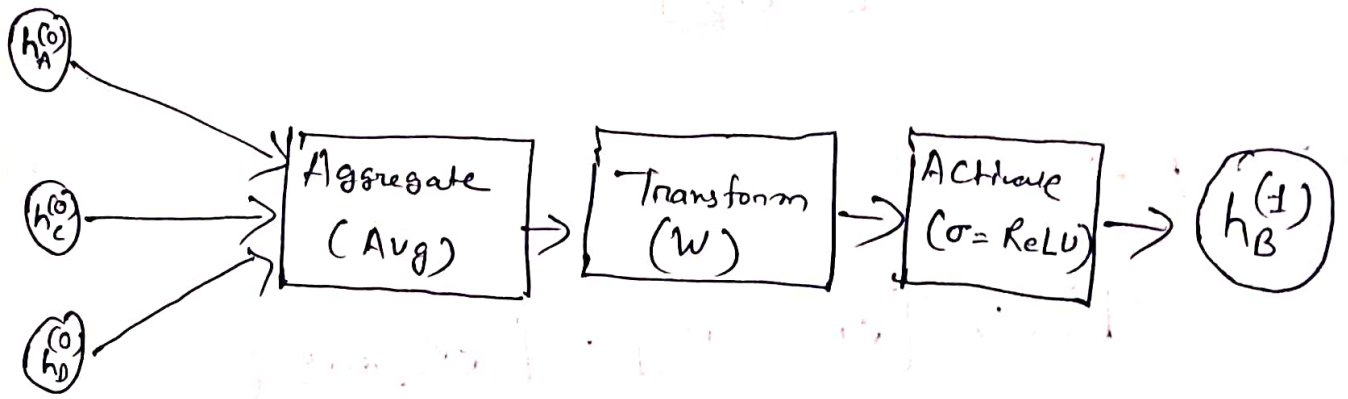
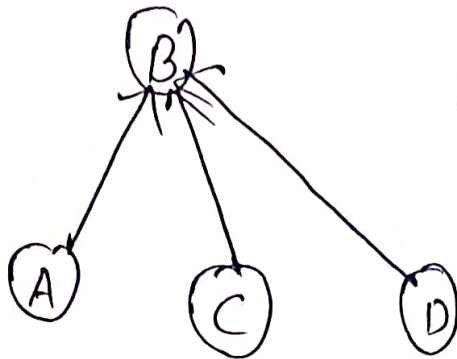
$$\begin{aligned} \text{Expected payoff} &= 0.7 \times 0 + 0.3 \times 2 \\ &= 0.6 \text{ (when Player 2 choose B)} \end{aligned}$$

The expected outcomes for Player 1 receives a payoff of 0.6 & Player 2 receive payoff of 1.6.

Question - 7

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Answer - 7



Graph Neural Network feature update

Given,

Directed edge $A \rightarrow B$

$C \rightarrow B$

$D \rightarrow B$

Weighted matrix $W = \begin{bmatrix} 0.5 & 0 \\ 0.1 & 0.2 \end{bmatrix}$

Step-1 Aggregate neighbor features.

$$h_A^{(0)} + h_C^{(0)} + h_D^{(0)} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 3 \end{bmatrix} + \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$h_{N(B)}^{(0)} = \frac{1}{3} \begin{bmatrix} 3 \\ 6 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad \begin{bmatrix} 3 \\ 6 \end{bmatrix}$$

Step 2. Transform

$$\begin{aligned}
 W \begin{bmatrix} 1 \\ 2 \end{bmatrix} &= \begin{bmatrix} 0.5 & 0 \\ 0.1 & 0.2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} \\
 &= \begin{bmatrix} 0.5 \\ 0.1 \times 1 + 0.2 \times 2 \end{bmatrix} \\
 &= \begin{bmatrix} 0.5 \\ 0.1 + 0.4 \end{bmatrix} \\
 &= \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}
 \end{aligned}$$

Step 3. Activate \rightarrow

$$\text{ReLU} [0.5, 0.5] = [0.5, 0.5]$$

$$\text{Final Answer } h_B^{(1)} = [0.5, 0.5]$$