### The Ballistic Pendulum

AP Physics C – Mechanics

**Learning Intention:** I can calculate unknown quantities in a system by describing the energy and momentum in relation to time and calculating changes in energy and momentum as applicable.

Link to lab: https://ophysics.com/e3.html

**Overview:** The ballistic pendulum is one of the most common systems studied in mechanics because of the several physics concepts that it brings together. A bullet/marble is shot into a block that is suspended by rigid rods of negligible mass. The block and bullet/marble then travel together in a trajectory similar to a vertically hanging pendulum. Your goal is to determine the relationship between the initial velocity of the bullet and the height of the pendulum by plotting data in a graph and utilizing best fit curves. You will then verify the relationship through the theory!

#### **Prelab**

The projectile fires out with a velocity v, then hits a block and sticks to that block. Because it sticks to the block, the collision is perfectly inelastic, which means that kinetic energy is lost while momentum is conserved. Then, the block and marble move together up to a height h following the path of the pendulum. No second collision happens, so energy is conserved through the motion, which means the energy right after the collision and at the highest point in the pendulum is the same. Thus, conservation of energy and conservation of momentum are both used in this experiment.

## **Procedure**

In this online lab, a ball was shot into a pendulum and the max height the pendulum swung to was measured. The mass of the bullet and the mass of the wood block were held constant. The initial velocity of the bullet was varied, and with this variation, the change in height of the wooden block was measured. The change in height was measured by taking the max height through adjusting the meter stick in the online simulation and subtracting the initial height (which was 1 m). The bullet was shot at a total of 7 different speeds and the receptive change in heights were measured.

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#### Data

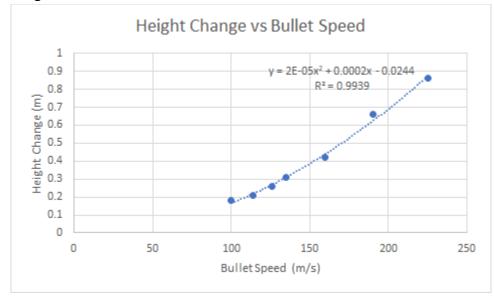
**Table 1.** A table recording the change in height of the pendulum from initial to max height with respect to the bullet speed and bullet speed squared.

Bullet Speed v (m/s)	Max Height y (m)	Height Change h (m)	Bullet Speed
			Squared $v^2\left(rac{m}{s} ight)^2$
160.00	1.42	0.42	25600
190.00	1.66	0.66	36100
225.00	1.86	0.86	50625
135.00	1.31	0.31	18225
126.25	1.26	0.26	15939.0625
113.75	1.21	0.21	12939.0625
100	1.18	0.18	10000

Mass of bullet: 0.075 kg Mass of wood block: 4 kg

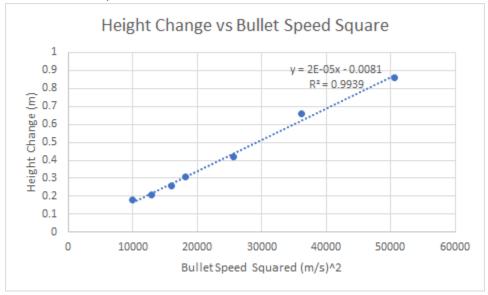
## **Results and Discussion**

1. Choose quantities to graph for on the x- and y-axes of a graph. Include axes names with units as well as a graph title. Determine the best fit curve for the data using a power regression.



Quadratic relation of height change as a function of bullet speed

2. If the best fit curve is *not* a line (approximate power of 1), change the variables plotted so that you *do* get a line. Add any columns to your table above as needed to ensure that the data plotted is available in the table.



Linear relation of height change as a function of bullet speed squared

- 3. Summarize the results of your graph. How are velocity and height related? From the first graph, we get that the change in height is proportional to velocity squared.
- 4. Derive a theoretical expression for the initial velocity of the marble *u* using your answers from the Prelab.

M = mass of block

m = mass of bullet

h = change in height

u = initial velocity of bullet

v = velocity of system immediately following collision

$$(M+m) \cdot g \cdot h = \frac{1}{2}(M+m)v^{2}$$

$$m \cdot u + M \cdot 0 = (M+m) \cdot v$$

$$v = \frac{m \cdot u}{M+m}$$

$$(M+m) \cdot g \cdot h = \frac{1}{2}(M+m)\left(\frac{m \cdot u}{M+m}\right)^{2}$$

$$u^{2} = \frac{2gh(M+m)^{2}}{m^{2}}$$

$$u = \frac{(M+m)}{m}\sqrt{2gh}$$

- 5. Compare your expression with that from the graph. Are the results consistent? Analyze limiting cases for values in your expression from Step 4 to ensure that it makes physical sense.
  - The results from the expression are consistent with the graph, since our expression tells us that velocity is proportional to the square root of change in height. When velocity is 0, h is 0, which makes sense because if the ball has no speed, it never hits the pendulum/causes a change in height. When velocity is infinite, height is also infinite. However, it is realistically not possible because it is bound by a string. As a result, the block would spin indefinitely.
- 6. Derive an expression for the fraction of kinetic energy remaining in the system after the collision.

Variables are same as from Q4.

Initial kinetic energy:  $KE = \frac{1}{2}mu^2$ 

After collision:  $KE = \frac{1}{2}(M+m)v^2 = \frac{1}{2}(M+m)\left(\frac{mu}{M+m}\right)^2$ 

Fraction:  $\frac{m}{M+m}$ 

# **Analysis Questions**

Answer the following questions here:

- You measure the height of a ballistic pendulum in the laboratory, and obtain a value for the initial velocity of the marble using your expression that you found in the Results section. What is another way you could measure the initial velocity to check your answer? List equipment needed and steps to take (Hint: think simple! Don't overcomplicate this).
  - We could also measure the velocity of the block and ball right after the collision, and then use that to find the initial velocity using conservation of momentum. We would need a motion detector set up in a way that it can track the velocity of the block. We then repeat the experiment as per, recording the velocity measured after each collision for each trial.
- 2. You find that the initial velocity obtained from your expression in the Results section is lower than what you measured using the extra equipment. List 2 reasons why this might be. Assume that instead of rigid rods holding the block, you used massless string.
  - 1) Air resistance decreases the change in height of the block
  - 2) The block lost mass because it lost material when the bullet hit it