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Lab 9: Oscillations and Differential Equations
AP Physics C Mechanics

Objective: I can derive a differential equation to describe Newton's Second Law for a spring-mass system by ranking systems with different parameters and explaining the connect between oscillations and Newton's Laws of motion.

To solve for the period of motion for a harmonic oscillator, we can do a force and energy analysis and take into account quantities such as the velocity of the mass at different points in time of the trajectory. This is what you did in AP Physics 1. However, this gets tedious, especially for some of the more complicated examples of harmonic oscillators like the physical pendulum in which you have to use angular momentum and Newton's 2^{nd} Law of rotation to solve for angular speed.

Thankfully, there is a much, much easier method (once you get used to it of course) for finding the period of motion and it helps shed some light on why the motion is described by a sinusoidal function! Please welcome back Differential Equations! After a couple month long hiatus since resistive forces, they are back to help you mathematically solve simple harmonic motion problems.

Today, you will be working with the mass-spring system to conceptually describe the differential equations that we will be using. The differential equations will be differential equations of position for the mass attached to the spring.

In addition, you will also be measuring the period of oscillation for the springs. This will be accomplished by using stopwatches.

Materials

- Motion sensor and PASCO interface
- Springs
- Bronze masses between 5 g and 200 g.
- Ring stand

Procedure

1. Open PASCO Capstone and, if prompted, select "Sensor Data". You need a display (any of them) with a graph. Nothing else is really needed.
2. Go to Hardware Setup on the left side of the screen.
3. Click on the Interface, where the motion sensor is plugged in.
4. Add "Motion Sensor II".
5. Exit out of Hardware Setup.
6. Change the labels of the axes on your graph to show "acceleration" on the vertical and "position" on the horizontal axes.

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Part 1 – Finding spring constant

1. Find the spring constant by hanging a mass from the spring and measuring its displacement from before the mass was placed on the spring. Use Newton's Second Law (you may need paper!) to calculate the spring constant.
   $mg = kx$ (Equilibrium, net force is zero)
   $k = \frac{mg}{x}$

Part 2 – Acceleration vs. Position, Period

1. Pull the mass down slightly and release it from rest.
2. After a few seconds, begin data collection.
3. What shape does the graph have? It should be linear, and if so, then find the slope of the graph.
4. Record in the data table below. Include at least one of your graphs as a screenshot in this report.
5. Measure the period of the spring using a stopwatch. If you are having trouble measuring JUST one period, is there a more accurate way you can measure the time for just one cycle using just a stopwatch?
6. Repeat with a different mass hanging from the spring.

Part 3 – Period vs. Amplitude

1. For one mass only, time the period of oscillation twice, if you pull the spring down different amounts both times. Record the timing and displacements from equilibrium below.

Displacement 1: 0.05 m Period 1: 1.025 s
Displacement 2: 0.10 m Period 2: 0.948 s

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Results and Discussion

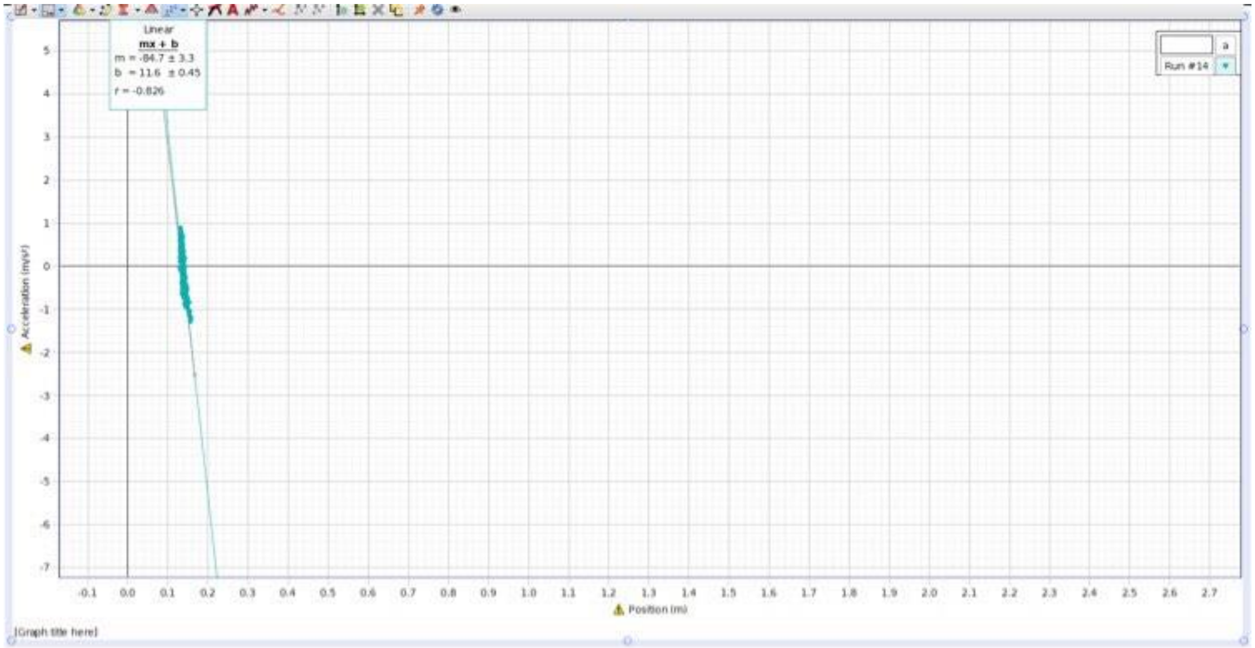

Figure 1. Linear acceleration vs. position graph for mass 0.080 kg

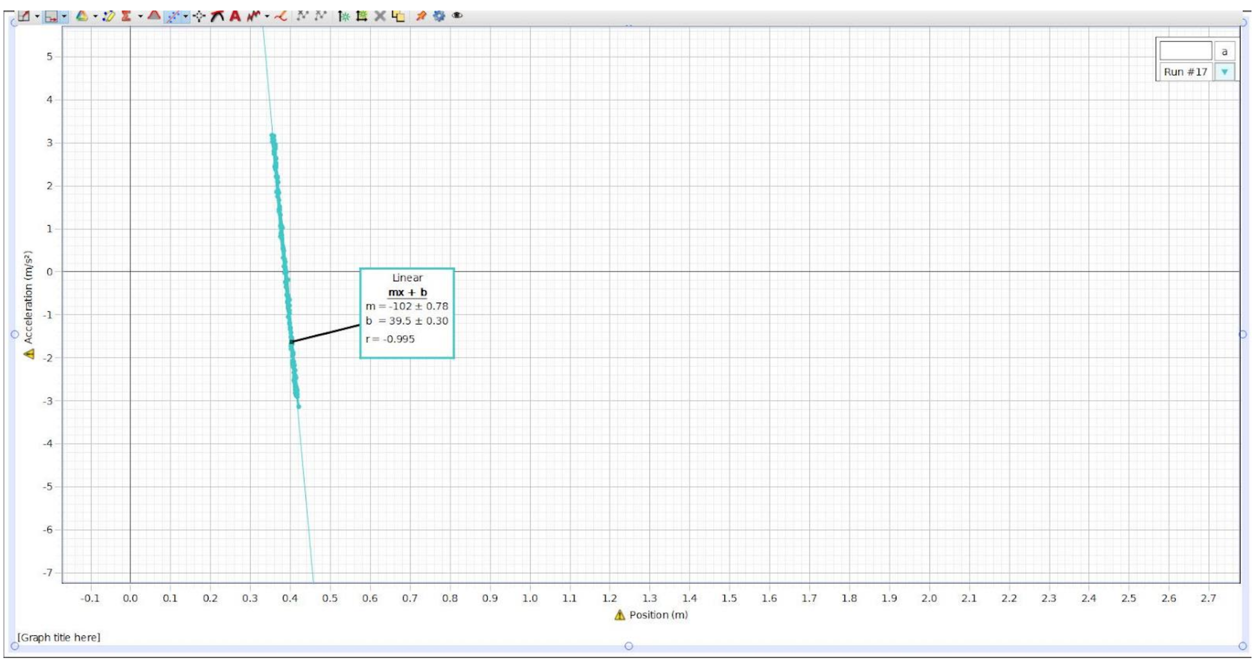

Figure 2. Linear acceleration vs. position graph for mass 0.020 kg

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Table 1. A list of masses used in the lab, along with data obtained from Capstone and the period.

Spring Constant (N/m, same for all rows)	**Mass (kg)**	**Slope of graph ($\frac{1}{s^2}$)**	**Period of oscillation (s)**
3.77307692	0.020	-102	.512
3.77307692	0.050	-54.7	.817
3.77307692	0.070	-72.7	.968
3.77307692	0.080	-84.7	1.020

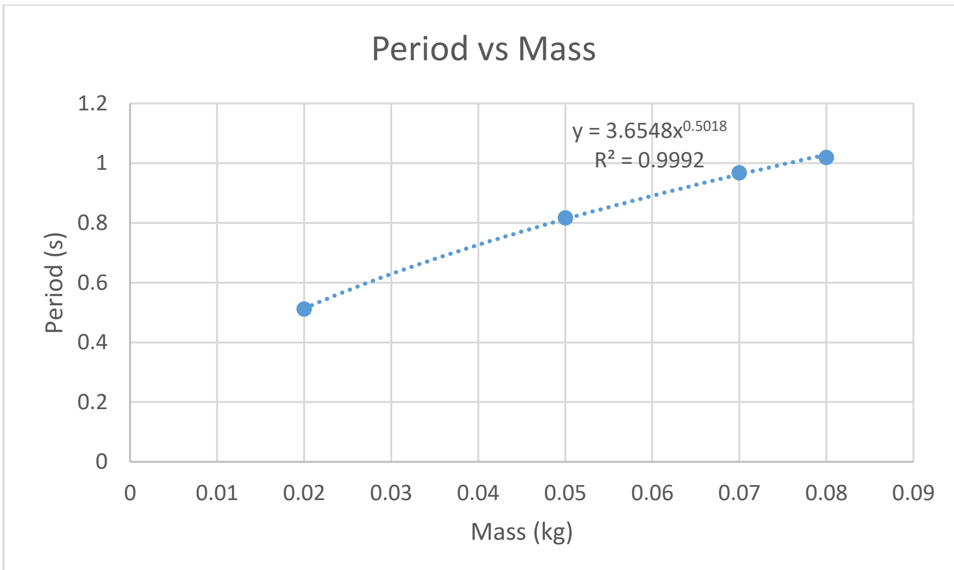

Figure 3. Power regression for period vs. mass

The slope of the graphs represents $-\frac{k}{m}$. We know that the period is $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$. Thus, we know that period should equal $2\pi\sqrt{\frac{1}{-slope}}$. By this slope, we know that $a = -\frac{k}{m}\Delta x$, where Δx is the displacement. The period appears proportional to $\sqrt{\frac{m}{k}}$, because as mass increases, the period increases as well. When a power regression is applied to the graph of period vs. mass, the power is close to 0.5, indicating a square root proportionality. In the graph, the R^2 value for the trendline equation is very close to 1, so the equation is accurate. Given the equation derived in the post-lab questions, this means that as mass increases, the slope of the acceleration vs. position graph should decrease in magnitude, while the period increases. However, from out data, only the period increases as mass increases while the slope of the acceleration vs. position graph does not decrease. This could be due to several sources of error.

In this lab, sources of error include sending the pendulum on a slanted path. This would mean that gravity would affect it differently and could lead to incorrect times being measured. More precision and multiple trials could be used to help reduce this error. Another source of error is air resistance. The lab assumes that there is no air resistance, but there is air resistance which inevitable slows the pendulum down. This can be fixed by using objects with a higher density that are affected less by air resistance. Finally, there is also an assumption made that there that the spring is ideal and massless. This can be prevented by using a spring that is made more “springy” and lighter weight to simulate a more ideal spring. Or calculations could be potentially edited to account for the fact the spring being used is not ideal and not massless (very advanced though). In our data, the trial with a mass of 0.020 kg likely has a very large error, because the slope does not match with the other three points. By removing this point, our graph would be linear. However, in this case the slope does not agree with our calculations, suggesting that our graph follows the opposite trend from our calculated relationship between mass and slope. This means that the 0.020 kg and 0.050 kg are the only two points that follow the trend where slope decreases as mass increases.

Post Lab Questions

1. Write out Newton’s Second Law for a horizontal Mass-spring system (such that gravity cancels with normal force on the mass) at any general point in its motion if the equilibrium is at x = 0 and replace acceleration with $\ddot{x}$ (the second derivative of position). Does the expression for acceleration match your observations?

   $$F = ma$$

   The only force that isn’t balanced is spring force, so $F = -kx$, where k is the spring constant and x is the displacement from 0, which is negative as the force pulls towards x=0.

   $$ma = -kx \to a = -\frac{k}{m}x \to \frac{d^2x}{dt^2} = -\frac{k}{m}x$$

   So, $-\frac{k}{m}$ is the slope of the graph. This kind of matches our observations as the slope is somewhat close to $-\frac{k}{m}$, but the overall trend seems to be wrong as smaller masses result in slopes with lower magnitude, which would be incorrect as the slope should be inversely proportional to the mass. This is likely caused by sources of error both in data collection and in graph analysis.

2. What do you notice about the period in your two trials from Part 3?

   The periods in the two trials are about the same, which means that the displacement does not affect the overall period of oscillation in the spring. The period of the graph should be equal to $2\pi\sqrt{\frac{m}{k}}$, so the displacement has no impact on the period.