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June 9, 2017

Turn-In 19

Problem 1

Describe a polynomial-time reduction from Max Matching to Independent Set:
We define:

G' = For every (u, w) edge in G, we have a vertex V_{uw} or V(u, w) in G'. For every pair of edges (u,w) and (u',w') that share an endpoint in G, create an edge (V_uw, V_u'w') in G.

k' = k

Here is why the reduction can be computed in polynomial time:
We have one iteration when we pass over the graph G and set all edges to be vertices objects in G'. We have another iteration when we check the graph G to see which objects in G' need to be "connected." This is an n^2 runtime and is polynomial.

Here is a proof that the reduction is correct.

Lemma 1. *Given any instance <G, k> of Max Matching where G = (V, E) is a graph and k is an integer k. Then for every (u, w) edge in G, we have a vertex V_{uw} or V(u, w) in G' and for every pair of edges (u,w) and (u',w') that share an endpoint in G, create an edge (V_uw, V_u'w') in G.*

Proof (long form).
1. First we show the "only if" direction.
2. Assume G' has an independent set of size k.
2.1. Let I be an independent set of size k from G'.
2.2. Every vertex in I is represented by an edge in G
2.3. The number of edges between vertices in I is equivalent to the number of vertices that share their endpoint with another edge in G.
2.4. There is a matching of size k in G
3. Next we show the "if" direction.
4. There is a graph G with a matching of size k
4.1. Let M be a matching of size k
4.2. Let I be the set of vertices in G' that correspond to edges in M.
4.3. The number of vertices in graph G that share their endpoint with another edge is equivalent to the number of the independent set size k in G'.
4.4. There is an independent set I of size k
5. By blocks 2 and 4, G' has an independent set of size k if and only if the Max Matching of G has a value of k.

Problem 5

Describe a polynomial-time reduction from Subset Sum to Zero Path:

We define:

Given a subset sum S with input <x, T> there is a matching zero path Z with input <G, s, t>. We set the number of x values to equal a variable n. We create a graph G with n+1 vertices all connected by edges. Each edge takes its corresponding weight value from x, for example edge (u, w) takes x1 weight value. As per the definition of zero path, we need to add an "s" vertex and connect that vertex to all other vertices using edges and we need to add a "t" vertex and connect all vertices besides "s" to "t" using edges.

Here is why the reduction can be computed in polynomial time:

The reduction can be completed by two iterations, each with a nested for loop, creating a polynomial time of $O(n^2)$. The first for loop creates vertices and adds edges between the vertices. The second for loop adds the corresponding weight values to our edges in G.

Here is a proof that the reduction is correct.

Lemma 1. Given a subset sum S with input <x, T> there is a matching zero path Z with input <G, s, t>. Then there is a zero path graph G with |x|+1 vertices connected to each other with edges that hold a corresponding weight value from x.

Proof (long form).
1. First we show the "only if" direction.
2. Assume S has a corresponding graph G with |x|+1 vertices.
2.1. Let G be a graph with |x|+1 vertices connected by edges.
2.2. Each edge in G corresponds to a weight value obtained from its respective x.
2.3. Add s and t vertices to graph G and add an edge between s and every vertex in the graph except t. Add edges between every vertex except s to vertex t.
2.4. There is a zero path graph G.
3. Next we show the "if" direction.
4. There is a zero path graph G with a corresponding subset sum S with input <x,T>.
4.1. Let S be subset sum with input <x,T>.
4.2. G contains n vertices, so S has n-1 x values.
4.3. Each edge in G corresponds to a weight value for each x in S.
5. By blocks 2 and 4, subset sum S has a corresponding zero path Z with a graph G.