Non-Stationary Time Series

EC 421, Set 9

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Prologue

Schedule

Last Time

Autocorrelation

Today

- Brief introduction to nonstationarity

Upcoming

- **Assignment** due Saturday.
- **Assignment** soon after.

Nonstationarity

Nonstationarity

Intro

Let's go back to our assumption of **weak dependence/persistence**

> 1. **Weakly persistent outcomes**—essentially, x_{t+k} in the distant period $t+k$ weakly correlates with x_t (when k is "big").

We're essentially saying we need the time series x to behave.

We'll define this *good behavior* as **stationarity**.

Nonstationarity

Stationarity

Requirements for **stationarity** (a *stationary* time-series process):

1. The **mean** of the distribution is independent of time, *i.e.*,

$$\boldsymbol{E}[x_t] = \boldsymbol{E}[x_{t-k}] \text{ for all } k$$

2. The **variance** of the distribution is independent of time, *i.e.*,

$$\mathrm{Var}(x_t) = \mathrm{Var}(x_{t-k}) \text{ for all } k$$

3. The **covariance** between x_t and x_{t-k} depends only on k—not on t, *i.e.*,

$$\mathrm{Cov}(x_t,\, x_{t-k}) = \mathrm{Cov}(x_s,\, x_{s-k}) \text{ for all } t \text{ and } s$$

Nonstationarity

Random walks

Random walks are a famous example of a nonstationary process:

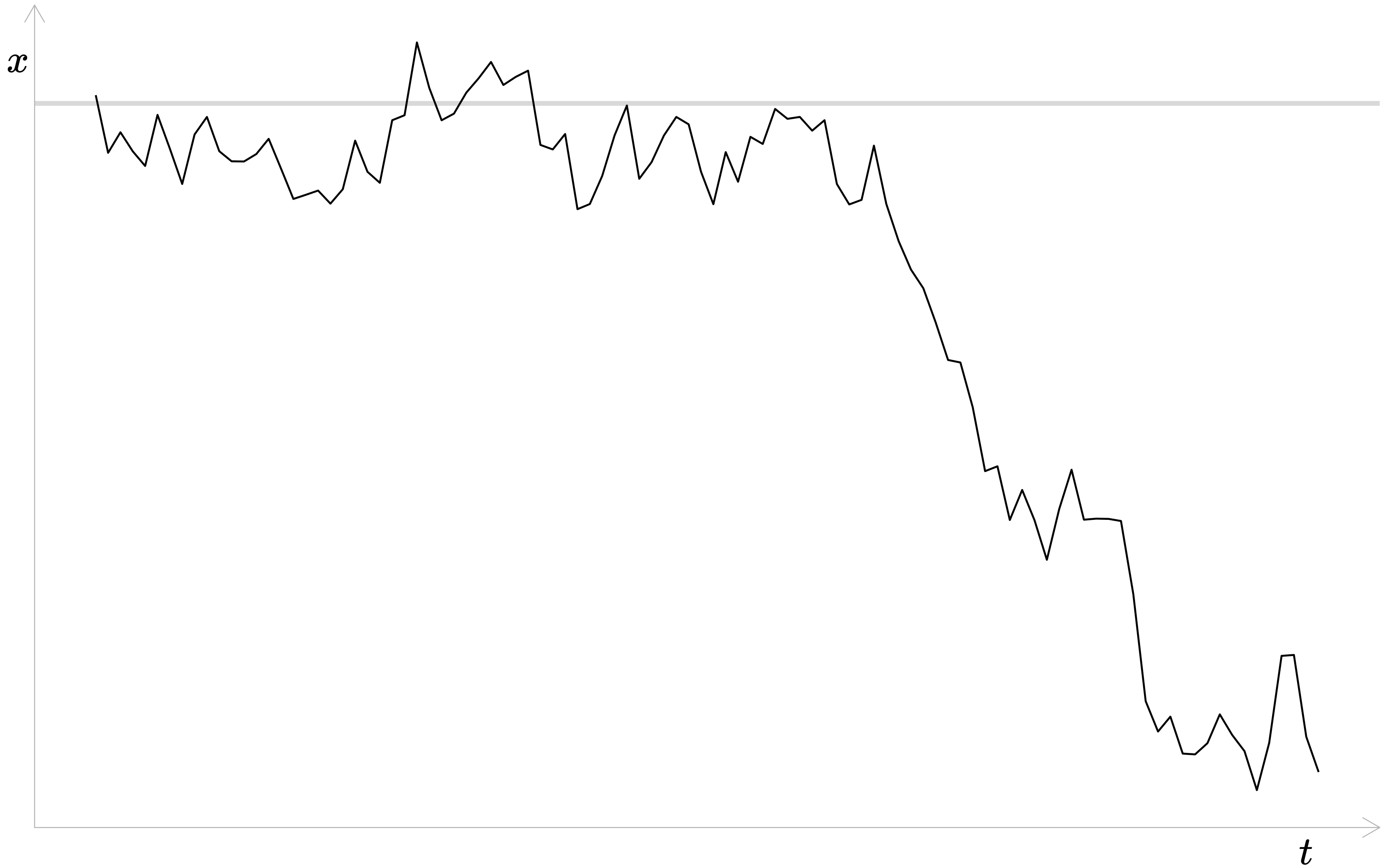
$$x_t = x_{t-1} + \varepsilon_t$$

Why? $\mathbf{Var}(x_t) = t\sigma_\varepsilon^2$, which violates stationary variance.

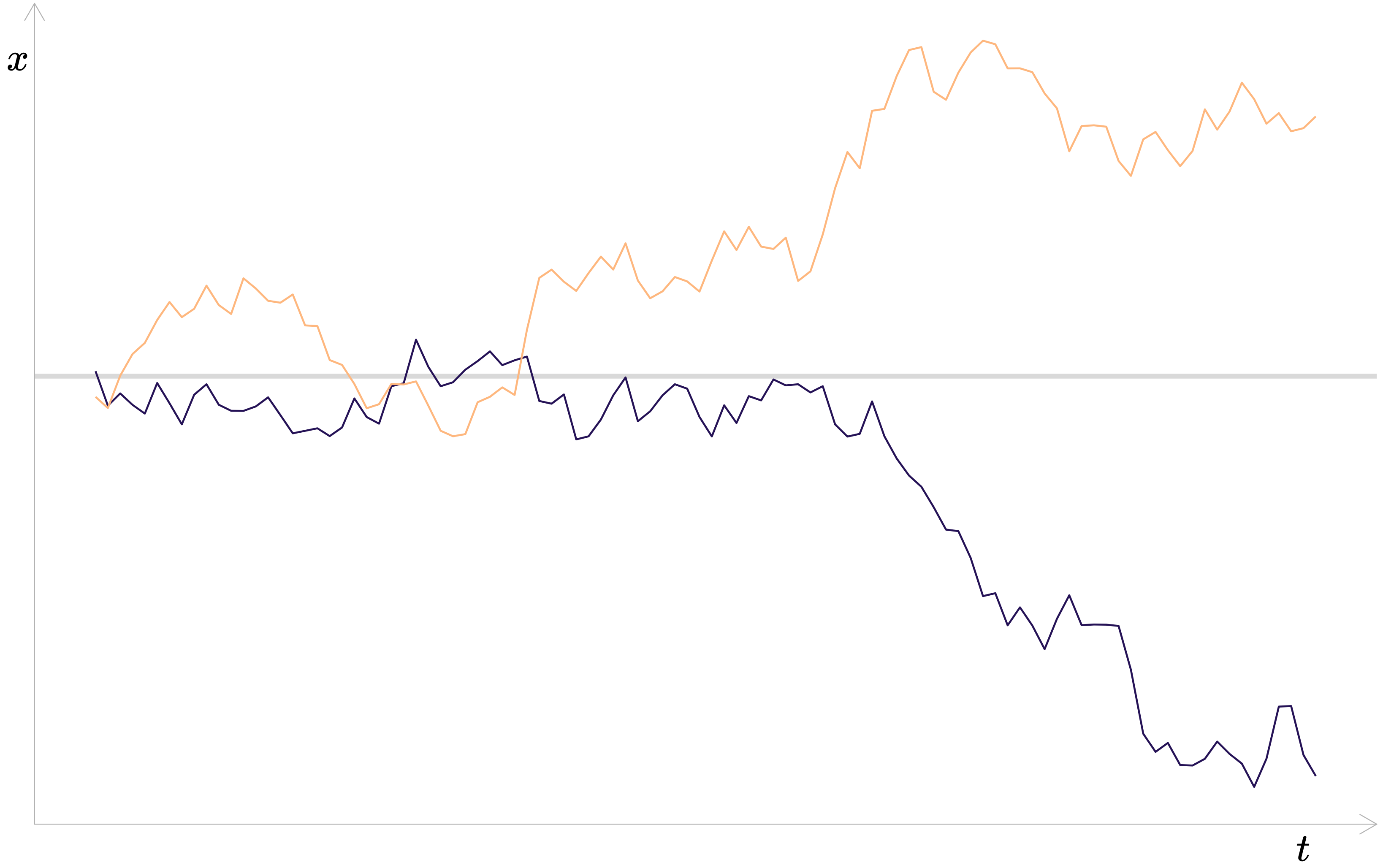
$$\begin{aligned}
\mathbf{Var}(x_t) &= \mathbf{Var}(x_{t-1} + \varepsilon_t) \\
&= \mathbf{Var}(x_{t-2} + \varepsilon_{t-1} + \varepsilon_t) \\
&= \mathbf{Var}(x_{t-3} + \varepsilon_{t-2} + \varepsilon_{t-1} + \varepsilon_t) \\
&\ldots \\
&= \mathbf{Var}(x_0 + \varepsilon_1 + \cdots + \varepsilon_{t_2} + \varepsilon_{t-1} + \varepsilon_t) \\
&= \sigma_\varepsilon^2 + \cdots + \sigma_\varepsilon^2 + \sigma_\varepsilon^2 + \sigma_\varepsilon^2 \\
&= t\sigma_\varepsilon^2
\end{aligned}$$

Q: What's the big deal with this violation?

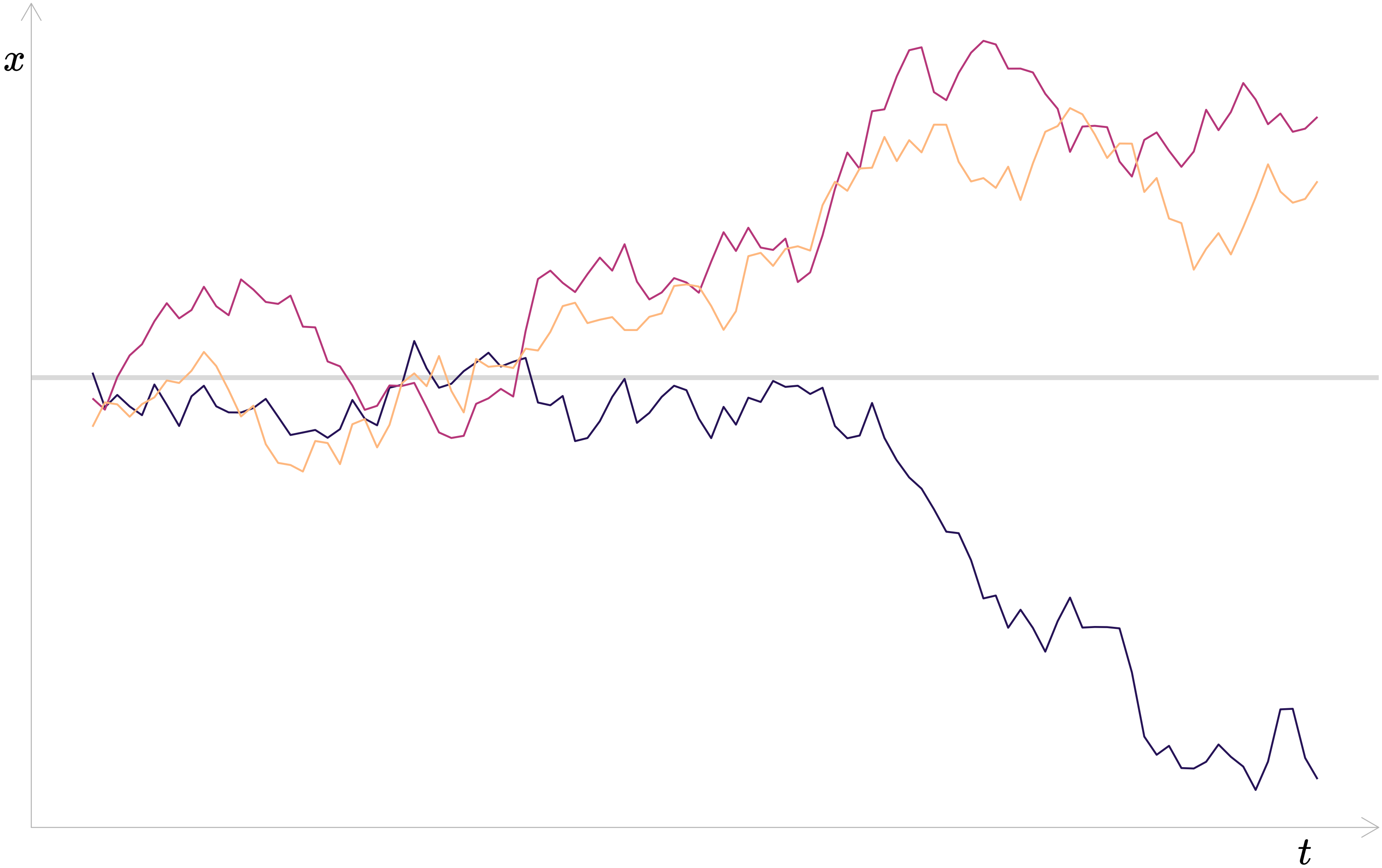
One 100-period random walk



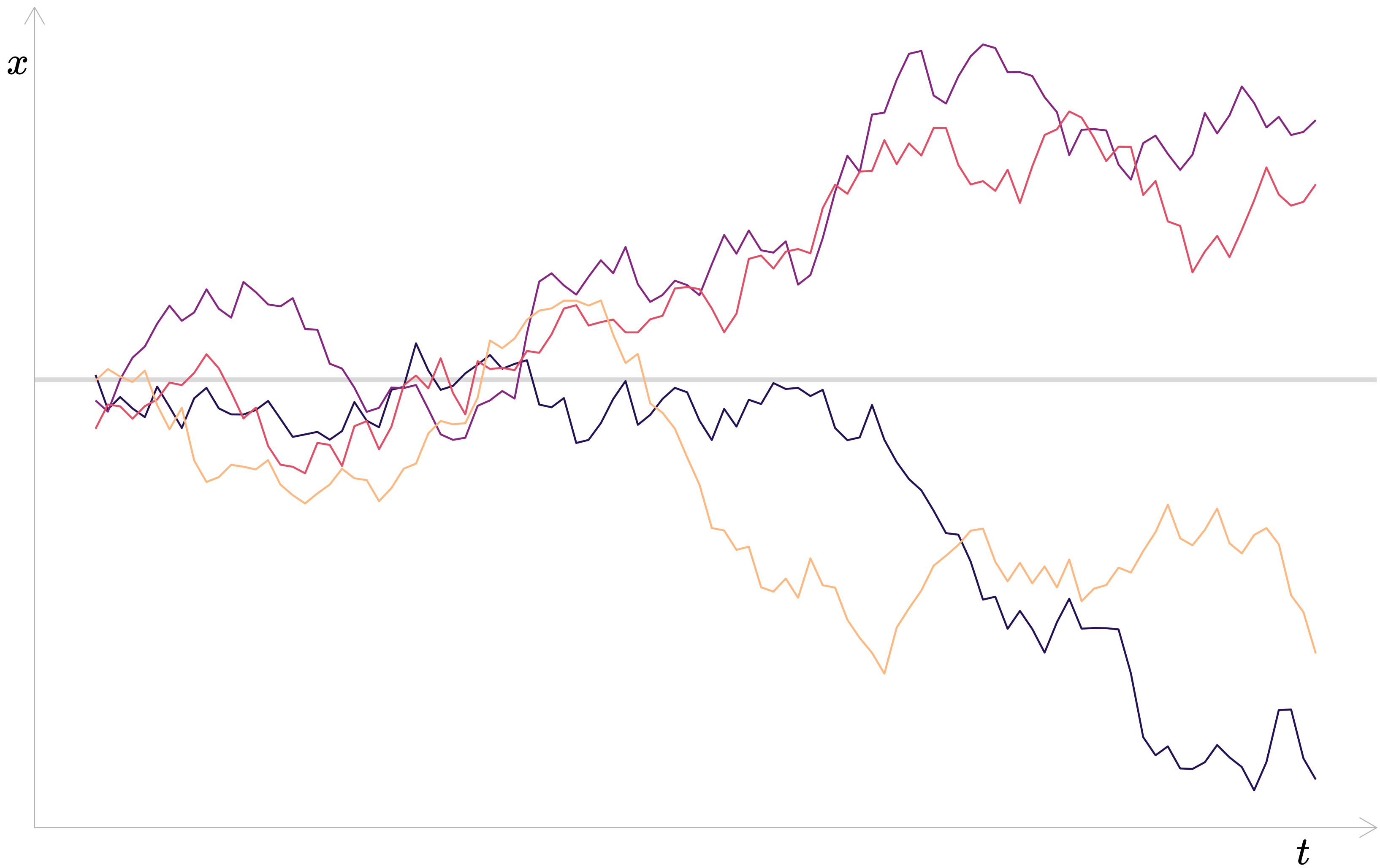
Two 100-period random walks



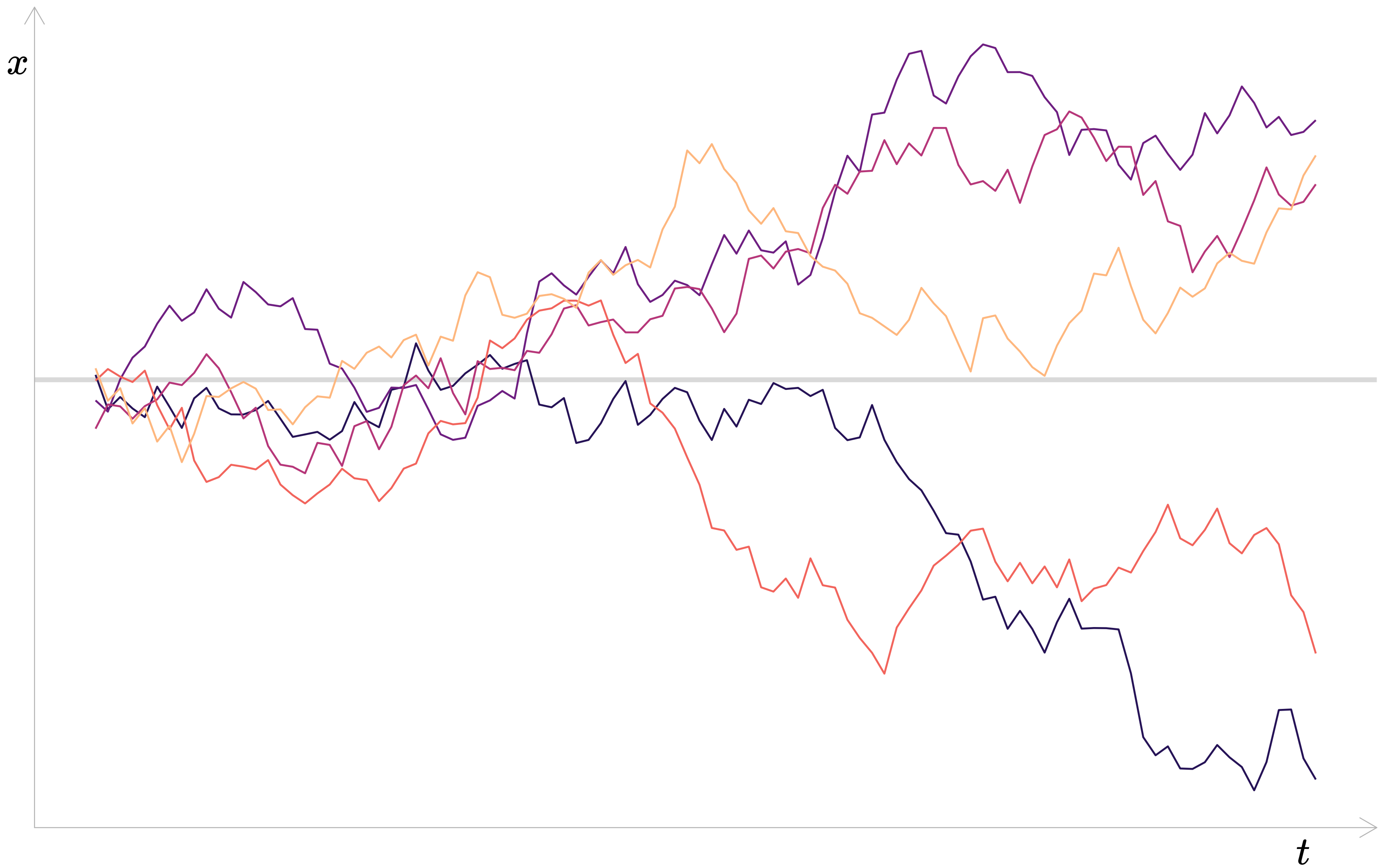
Three 100-period random walks



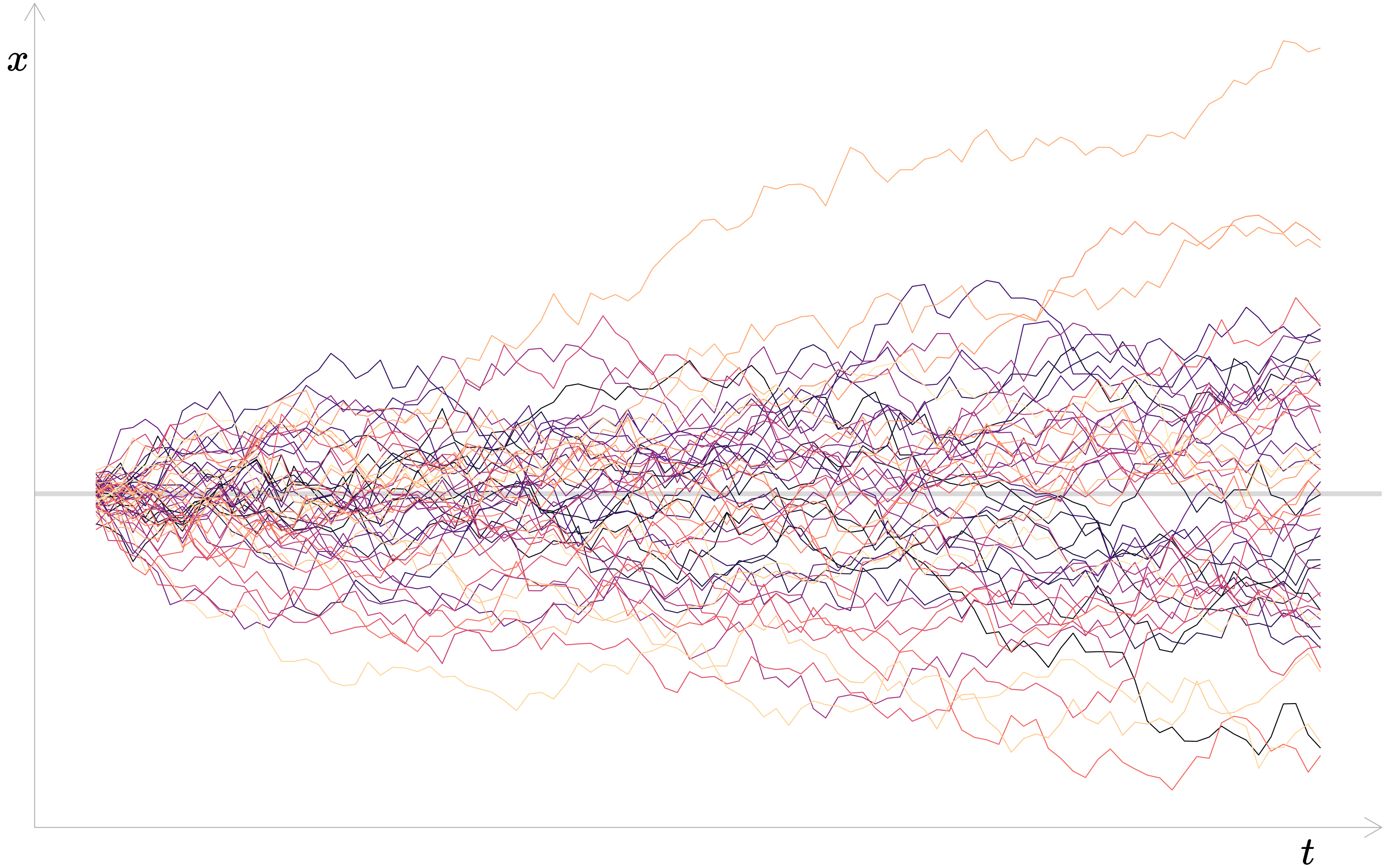
Four 100-period random walks



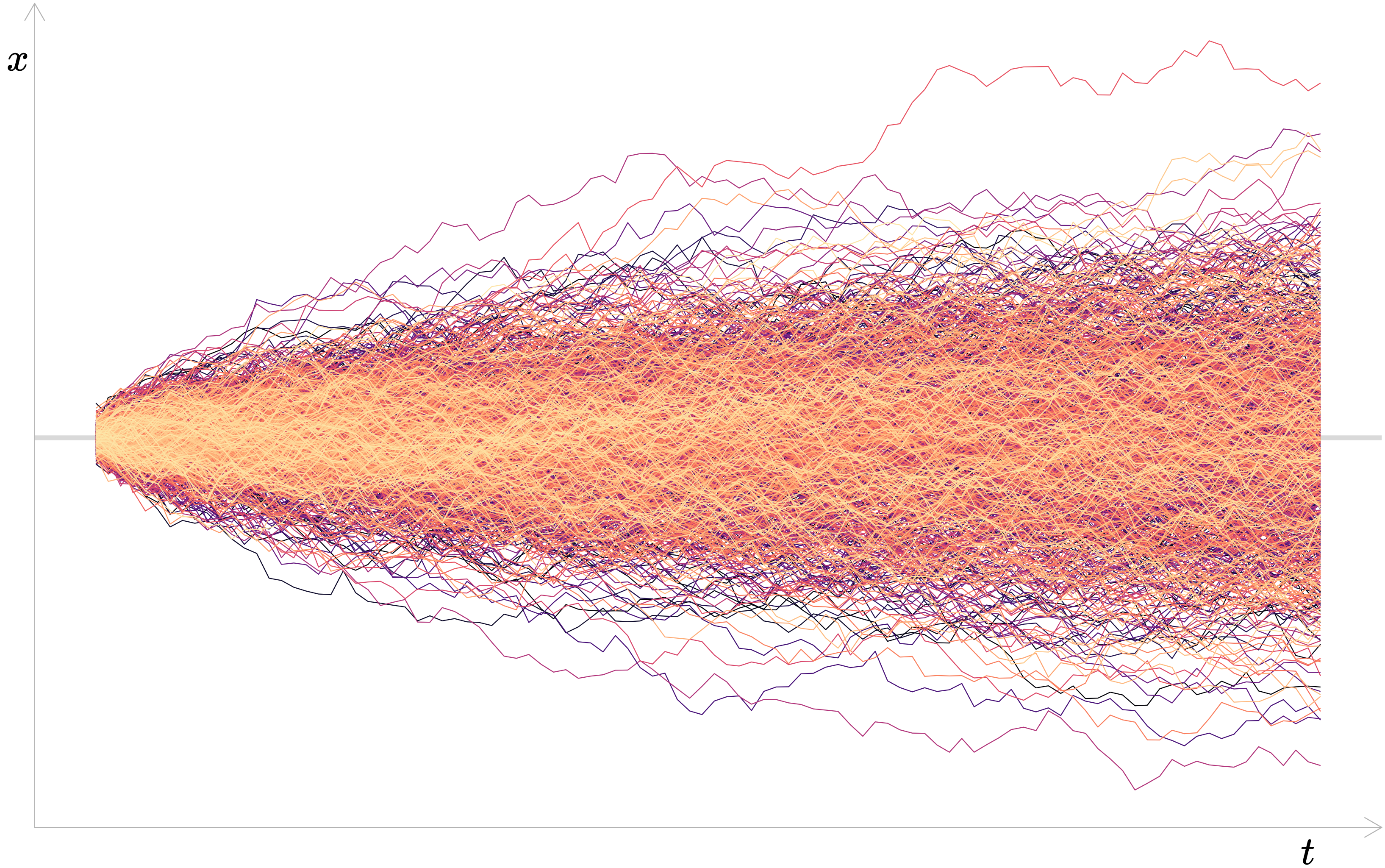
Five 100-period random walks



Fifty 100-period random walks



1,000 100-period random walks



Nonstationarity

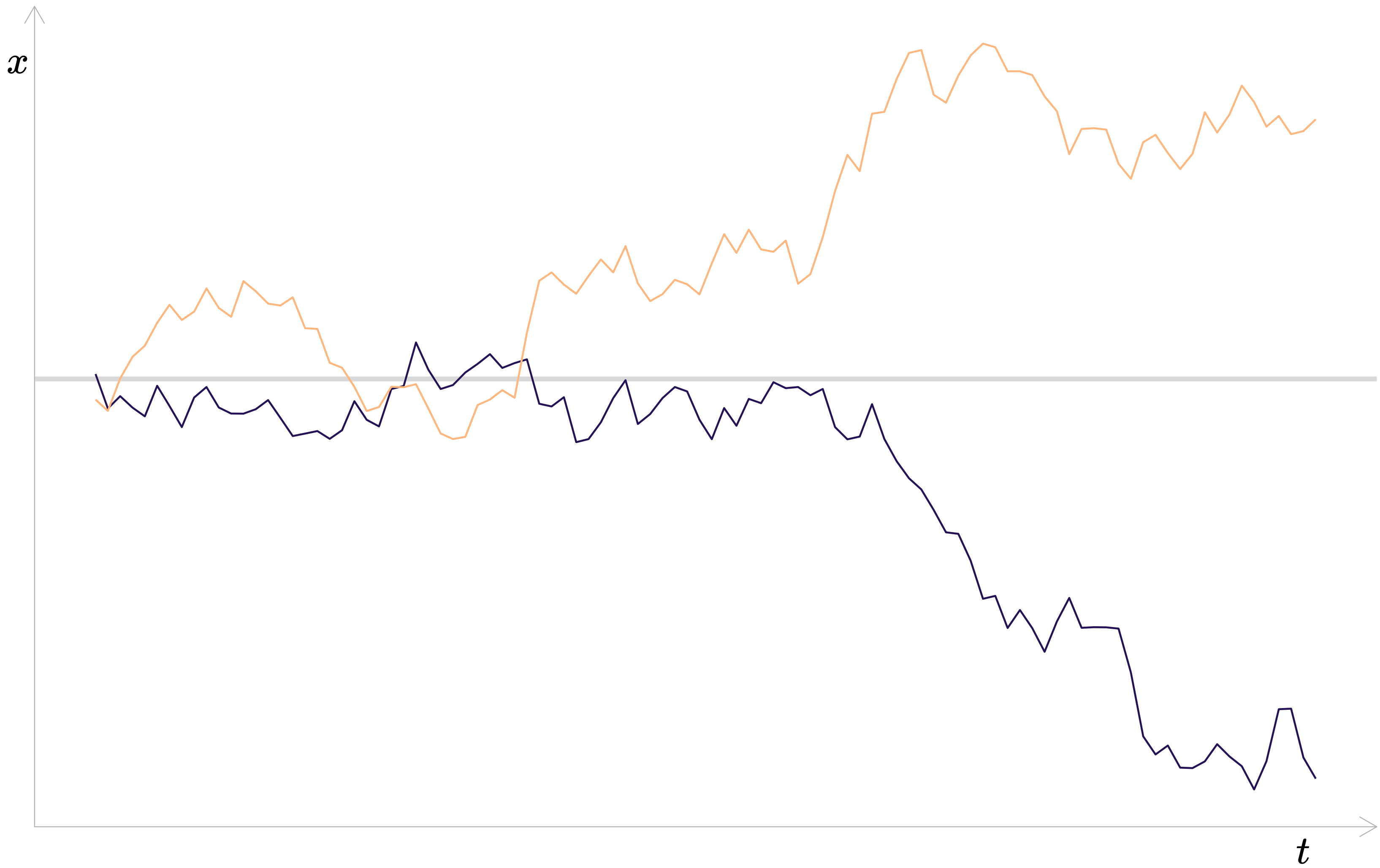
Problem

One problem is that nonstationary processes can lead to **spurious** results.

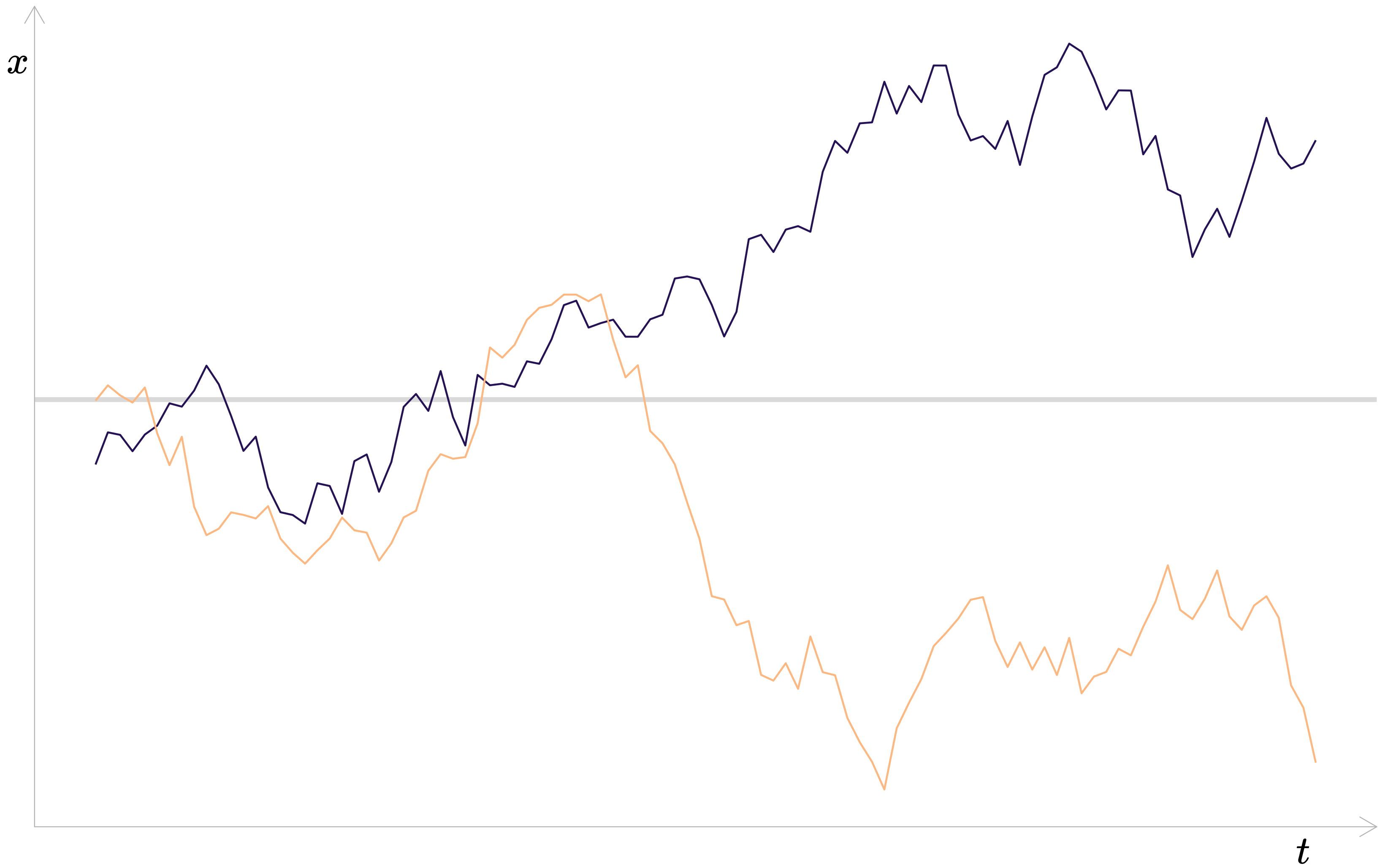
> **Defintion: Spurious**
>
> - not being what it purports to be; false or fake
> - apparently but not actually valid

Back in 1974, Granger and Newbold showed that when they **generated random walks** and **regressed the random walks on each other**, **77/100 regressions were statistically significant** at the 5% level (should have been approximately 5/100).

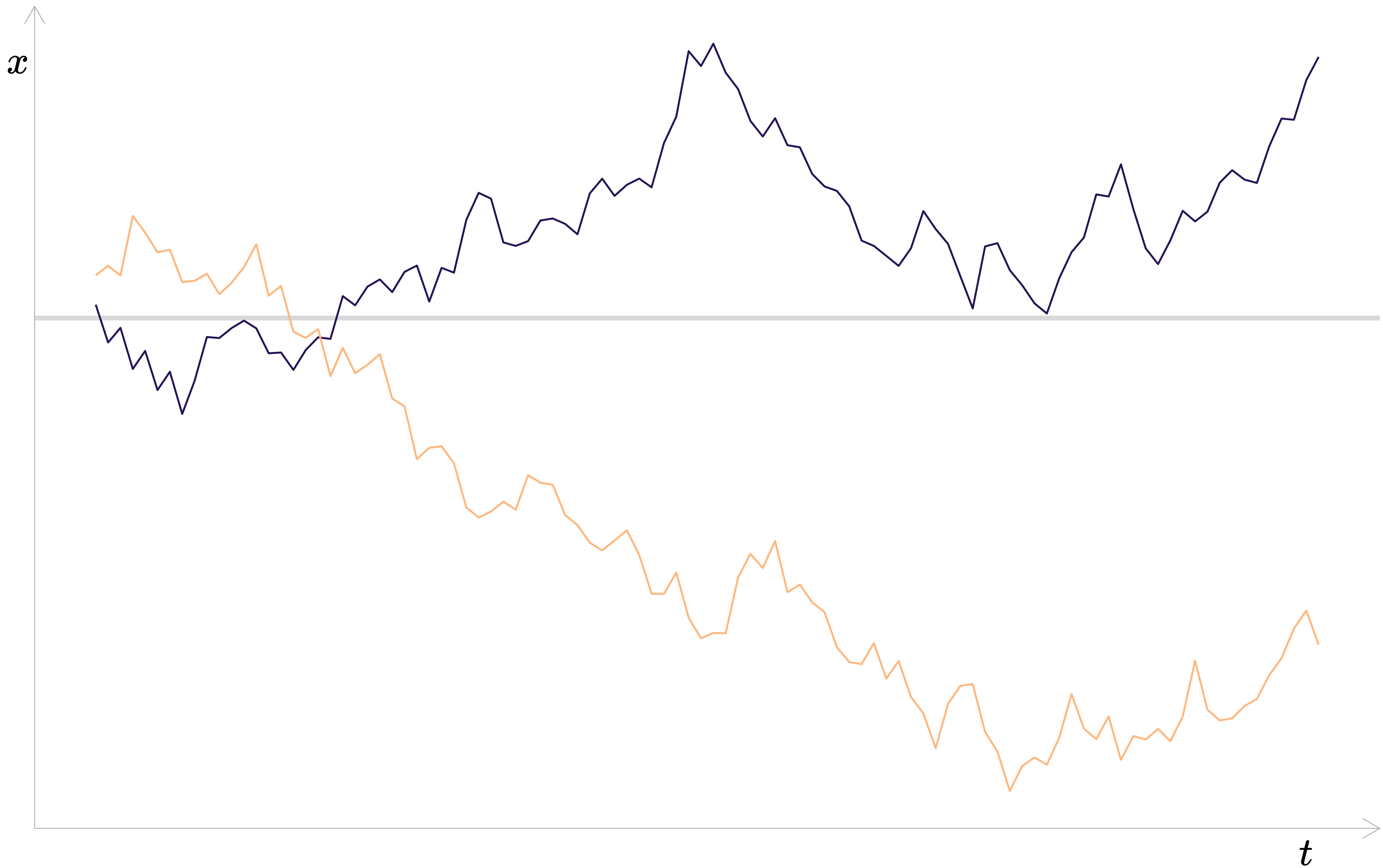
Granger and Newbold simulation example: t statistic ≈ -10.58



Granger and Newbold simulation example: t statistic ≈ -8.92



Granger and Newbold simulation example: t statistic ≈ -7.23



Nonstationarity

Problem

In our data, 74.6 percent of (independently generated) pairs reject the null hypothesis at the 5% level.

The point? If our disturbance is nonstationary, we cannot trust plain OLS.

Random walks are only one example of nonstationary processes...

Random walk: $u_t = u_{t-1} + \varepsilon_t$

Random walk with drift: $u_t = \alpha_0 + u_{t-1} + \varepsilon_t$

Deterministic trend: $u_t = \alpha_0 + \beta_1 t + \varepsilon_t$

A potential solution

Some processes are **difference stationary**, which means we can get back to our stationarity (good behavior) requirement by taking the difference between u_t and u_{t-1}.

Nonstationary: $u_t = u_{t-1} + \varepsilon_t$ (a random walk)
Stationary: $u_t - u_{t-1} = u_{t-1} + \varepsilon_t - u_{t-1} = \varepsilon_t$

So if we have good reason to believe that our disturbances follow a random walk, we can use OLS on the differences, *i.e.*,

$$
\begin{aligned}
y_t &= \beta_0 + \beta_1 x_t + u_t \\
y_{t-1} &= \beta_0 + \beta_1 x_{t-1} + u_{t-1} \\
y_t - y_{t-1} &= \beta_1 \left(x_t - x_{t-1} \right) + \left(u_t - u_{t-1} \right) \\
\Delta y_t &= \beta_1 \Delta x_t + \Delta u_t
\end{aligned}
$$

Nonstationarity

Testing

Dickey-Fuller and augmented Dickey-Fuller tests are popular ways to test of random walks and other forms of nonstationarity.

Dickey-Fuller tests compare

H$_o$: $y_t = \beta_0 + \beta_1 y_{t-1} + u_t$ with $|\beta_1| < 1$ (**stationarity**)
H$_a$: $y_t = y_{t-1} + \varepsilon_t$ (**random walk**)

using a *t* test that $|\beta_1| < 1$.[†]

† People often just test $\beta_1 < 1$.

Table of contents

Admin

1. Schedule

Nonstationarity

1. Introduction
2. Random walks
3. The actual problem
4. A potential solution
5. Dickey-Fuller tests