

# Instrumental Variables

EC 421, Set 11

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# Prologue

# Schedule

## Last Time

Causality

## Today

- Review: Causality
- New: Instrumental variables

## Upcoming

The end?

# Causality

## *Review*

# Causality

## Review

In our last lecture, we returned to the concept of **causality**.

We worked through the *Rubin causal model*, in which we defined

- $y_{1i}$  : the outcome for individual  $i$  if she had received treatment
- $y_{0i}$  : the outcome for individual  $i$  if she had not received treatment

and we referred to individuals who did not receive treatment as *control*.

**If** we were able to know both  $y_{1i}$  **and**  $y_{0i}$ , we could calculate the causal effect of treatment for individual  $i$ , i.e.,

$$\tau_i = y_{1i} - y_{0i}$$

# Causality

## Review

### Fundamental problem of causal inference:

We cannot simultaneously know  $y_{1i}$  and  $y_{0i}$ .

Either we observe individual  $i$  in the treatment group, *i.e.*,

$$\tau_i = y_{1i} - ?$$

or we observe  $i$  in the control group, *i.e.*,

$$\tau_i = ? - y_{0i}$$

but never both at the same time.

# Causality

## Review

If we want to know  $\tau_i$  (or at least  $\bar{\tau}$ ), what can we do?

**Idea:** Estimate the **average treatment effect** as the difference between the average outcomes in the treatment group and the control group, *i.e.*,

$$Avg(y_i \mid D_i = 1) - Avg(y_i \mid D_i = 0)$$

where  $D_i = 1$  if  $i$  received treatment, and  $D_i = 0$  if  $i$  is in the control group.

# Causality

## Review

**Result:** We showed that even when the treatment effect is constant (meaning  $\tau_i = \tau$  for all  $i$ ),

$$\begin{aligned} & Avg(y_i \mid D_i = 1) - Avg(y_i \mid D_i = 0) \\ &= \tau + \underbrace{Avg(y_{0,i} \mid D_i = 1) - Avg(y_{0,i} \mid D_i = 0)}_{\text{Selection bias}} \end{aligned}$$

which says that the difference in the groups' means will give us a **biased estimate** for the causal effect of treatment **if we have selection bias**.



# Causality

## Review

**Q:** What is this **selection bias**?

**A: (Informal)** We have selection bias when our control group doesn't offer a good comparison for our treatment group.

Specifically, the control group doesn't give us a good **counterfactual** for what our treatment group would have looked like if the members had not received treatment. Basically, the groups are different.

**A: (Formal)** The *average untreated* outcome for a member of our **treatment group** (which we cannot observe) differs from the *average untreated* outcome for a member of our **control group**, i.e.,

$$Avg(y_{0,i} \mid D_i = 1) - Avg(y_{0,i} \mid D_i = 0)$$

# Causality

## Review

**Practical problem:** Selection bias is also difficult to observe

$$\underbrace{Avg(y_{0,i} \mid D_i = 1)}_{\text{Unobservable}} - Avg(y_{0,i} \mid D_i = 0)$$

(back to the *fundamental problem of causal inference*)

**Bigger problem:** If selection bias is present, our estimate for  $\tau$  is biased, preventing us from understanding the causal effect of treatment.

Sounds a bit like omitted-variable bias, right? Our **treatment** variable is correlated with something that makes the two groups different.

# Causality

## Review

**Example:** Imagine we have two people—Al and Bri—and a single binary treatment, college. We interested in the effect of college on earnings.

$$\text{Earn}_{1,\text{Al}} = \$60\text{K}$$

$$\text{Earn}_{0,\text{Al}} = \$30\text{K}$$

$$\text{Earn}_{1,\text{Bri}} = \$140\text{K}$$

$$\text{Earn}_{0,\text{Bri}} = \$110\text{K}$$

The selection bias...

If Bri attended college ( $D_{\text{Bri}}=1$ ) and Al did not ( $D_{\text{Al}}=0$ ):

$$\hat{\tau} = \text{Earn}_{1,\text{Bri}} - \text{Earn}_{0,\text{Al}} = \$140\text{K} - \$30\text{K} = \$110\text{K}$$

If Al attended college ( $D_{\text{Al}}=1$ ) and Bri did not ( $D_{\text{Bri}}=0$ ):

$$\hat{\tau} = \text{Earn}_{1,\text{Al}} - \text{Earn}_{0,\text{Bri}} = \$60\text{K} - \$110\text{K} = -\$50\text{K}$$

# Causality

## Review

We have (at least) two problems...

1. Selection bias is difficult to observe
2. If selection bias is present, our estimate for  $\tau$  is biased, preventing us from understanding the causal effect of treatment.

**Solution:** Eliminate/minimize selection bias.

- **Option 1: Distribute treatment** in a way such that the treatment and control groups are essentially identical (experiments).
- **Option 2: Build a control** group that *matches* the treatment group (life with observational data).

# Instrumental variables

# Instrumental variables

## Intro

**Instrumental variables (IV)** is one route econometricians often take toward estimating the causal effect of a treatment/program.

Recall: **Selection bias** means our **treatment** and **control** groups differ on some unobserved/omitted dimension. (**Endogeneity**)

**Instrumental variables** attempts to separate out

- the **exogenous** part of  $x$ , which gives us unbiased estimates
- the **endogenous** part of  $x$ , which biases our results

If we use only the exogenous (*good*) variation in  $x$ , then we can avoid selection bias/omitted-variable bias.

# Instrumental variables

## Introductory example

*Example:* If we want to estimate the effect of veteran status on earnings,

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Veteran}_i + u_i \quad (1)$$

We would love to calculate  $\text{Earnings}_{1i} - \text{Earnings}_{0i}$ , but we can't.

And OLS will likely be biased for (1) due to selection/omitted-variable bias.

# Instrumental variables

## Introductory example

Imagine that we can split veteran status into an exogenous part and an endogenous part...

$$\begin{aligned}\text{Earnings}_i &= \beta_0 + \beta_1 \text{Veteran}_i + u_i \\ &= \beta_0 + \beta_1 \left( \text{Veteran}_i^{\text{Exog.}} + \text{Veteran}_i^{\text{Endog.}} \right) + u_i \\ &= \beta_0 + \beta_1 \text{Veteran}_i^{\text{Exog.}} + \underbrace{\beta_1 \text{Veteran}_i^{\text{Endog.}}}_{w_i} + u_i \\ &= \beta_0 + \beta_1 \text{Veteran}_i^{\text{Exog.}} + w_i\end{aligned}\tag{1}$$

We could use this exogenous variation in veteran status to consistently estimate  $\beta_1$ .

**Q:** What would exogenous variation in veteran status mean?



# Instrumental variables

## Introductory example

**Q:** What would exogenous variation in veteran status mean?

**A<sub>1</sub>:** Choices to enlist in the military that are essentially random—or at least uncorrelated with omitted variables and the disturbance.

**A<sub>2</sub>:** **No selection bias:**

$$Avg(Earnings_{0i} \mid Veteran_i = 1) - Avg(Earnings_{0i} \mid Veteran_i = 0) = 0$$

# Instrumental variables

## Instruments

**Q:** How do we isolate this *exogenous variation* in our explanatory variable?

**A:** Find an instrument (an instrumental variable).

**Q:** What's an instrument?

**A:** An **instrument** is a variable that is

1. **correlated** with the **explanatory variable** of interest (**relevant**),
2. **uncorrelated** with the **disturbance** (**exogenous**).

# Instrumental variables

## Instruments

**Q:** What's an instrument?

**A:** An **instrument** is a variable that is

1. **correlated** with the **explanatory variable** of interest (**relevant**),
2. **uncorrelated** with the **disturbance** (**exogenous**).

So if we want an instrument  $z_i$  for endogenous veteran status in

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Veteran}_i + u_i$$

1. **Relevant:**  $\text{Cov}(\text{Veteran}_i, z_i) \neq 0$
2. **Exogenous:**  $\text{Cov}(z_i, u_i) = 0$

# Instrumental variables

## Instruments: Relevance

**Relevance:** We need the instrument to cause a change in (correlate with) our endogenous explanatory variable.

We can actually test this requirement using regression and a  $t$  test.

**Example:** For the **veteran status**, consider three potential instruments:

1. Social security number

**Probably not relevant**

uncorrelated with military service

2. Physical fitness

**Potentially relevant**

service may correlate with fitness

3. Vietnam War draft

**Relevant**

being draw led to service

# Instrumental variables

## Instruments: Exogeneity

**Exogeneity:** The instrument to be independent of omitted factors that affect our outcome variable—as good as randomly assigned.

$z_i$  must be uncorrelated with our disturbance  $u_i$ . **Not testable.**

**Example:** For the **veteran status**, consider three potential instruments:

1. Social security number

**Exogenous**

Indep. of other factors of service

2. Physical fitness

**Not exogenous**

fitness correlates with many things

3. Vietnam War draft

**Exogenous**

the lottery was random

# Instrumental variables

## Instrumental review

Let's recap...

- Our instrument must be **correlated with our endogenous variable**.
- Our instrument must be **uncorrelated with any other variable that affects the outcome**.

### **In other words:**

The instrument only affects our outcome through the endogenous variable.

# Instrumental variables

## Back to our example

For **veteran status** we considered three potential instruments:

1. Social security number

**Not relevant**

**Exogenous**

2. Physical fitness

**Probably relevant**

**Not exogenous**

3. Vietnam War draft

**Relevant**

**Exogenous**

Thus, only the Vietnam War's draft lottery appears to be a **valid instrument**.

If we have a *valid* instrument (*e.g.*, the draft lottery), how do we use it?



# Instrumental variables

## Estimation

*Recall:* We want to estimate the effect of veteran status on earnings.

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Veteran}_i + u_i$$

Let's consider two related effects:

1. The effect of the **instrument** on the **endogenous variable**, e.g.,

$$\text{Veteran}_i = \gamma_0 + \gamma_1 \text{Draft}_i + v_i$$

2. The effect of the **instrument** on the **outcome variable**, e.g.,

$$\text{Earnings}_i = \pi_0 + \pi_1 \text{Draft}_i + w_i$$

# Instrumental variables

## Estimation

*Recall:* We want to estimate the effect of veteran status on earnings.

$$\text{Earnings}_i = \beta_0 + \beta_1 \text{Veteran}_i + u_i$$

and we know that the draft affected veteran status.

$$\text{Draft} \longrightarrow \text{Veteran status} \longrightarrow \text{Earnings}$$

Using our assumptions on independence and exogeneity:

$$\begin{aligned} &(\text{Effect of the draft on earnings}) = \\ &(\text{Effect of the draft on veteran status}) \times \\ &(\text{Effect of veteran status on earnings}) \end{aligned}$$

# Instrumental variables

## Estimation

We just wrote out an expression for the effect of **the draft** on **earnings**, i.e.,

$$\begin{aligned} (\text{Effect of } \text{the draft} \text{ on } \text{earnings}) = \\ (\text{Effect of } \text{the draft} \text{ on } \text{veteran status}) \times \\ (\text{Effect of } \text{veteran status} \text{ on } \text{earnings}) \end{aligned}$$

but we want to know the effect of **veteran status** on **earnings**. Rearrange!

$$\begin{aligned} (\text{Effect of } \text{veteran status} \text{ on } \text{earnings}) = \\ \frac{(\text{Effect of } \text{the draft} \text{ on } \text{earnings})}{(\text{Effect of } \text{the draft} \text{ on } \text{veteran status})} \end{aligned}$$

Our **instrument** consistently estimates both parts of this fraction!

# Instrumental variables

## Estimation: Bring it all together

By estimating two regressions involving our **instrument**,

1. The effect of the **instrument** on the **endogenous variable**, e.g.,

$$\text{Veteran}_i = \gamma_0 + \gamma_1 \text{Draft}_i + v_i$$

2. The effect of the **instrument** on the **outcome variable**, e.g.,

$$\text{Earnings}_i = \pi_0 + \pi_1 \text{Draft}_i + w_i$$

we can estimate our desired effect:

$$(\text{Effect of } \text{veteran status} \text{ on } \text{earnings}) = \frac{\pi_1}{\gamma_1}$$

# Instrumental variables

## Estimation: Bring it all together

So with instrumental variables, we estimate  $\beta_1$  using

$$\hat{\beta}_1^{\text{IV}} = \frac{\hat{\pi}_1}{\hat{\gamma}_1}$$

where  $\hat{\pi}_1$  and  $\hat{\gamma}_1$  come from the two equations we just discussed.

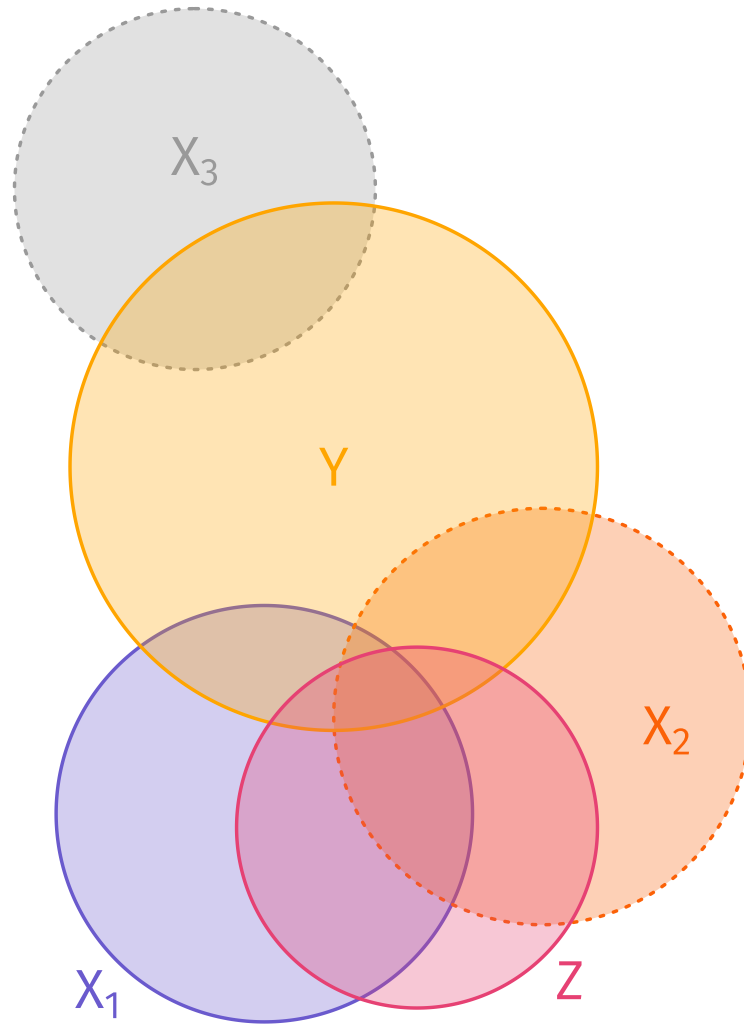
**Q:** Can we trust  $\hat{\beta}_1^{\text{IV}}$ ?

**A:** Yes... **if we have a valid instrument.**

$$\text{plim}\left(\hat{\beta}_1^{\text{IV}}\right) = \beta_1 + \frac{\text{Cov}(\text{Instrument}, u)}{\text{Cov}(\text{Instrument}, \text{Endog. variable})}$$

which equals  $\beta_1$  as long as our instrument is **exogenous** (numerator) and **relevant** (denominator).

Figure 4



# Venn diagram explanation

In these figures (Venn diagrams)

- Each circle illustrates a variable.
- Overlap gives the share of correlation between two variables.
- Dotted borders denote *omitted* variables.

Take-aways

- Figure 1: **Valid instrument** (relevant; exogenous)
- Figure 2: **Invalid instrument** (relevant; not exogenous)
- Figure 3: **Invalid instrument** (not relevant; not exogenous)
- Figure 4: **Invalid instrument** (relevant; not exogenous)

Let's work an example in  $\mathbb{R}$ .



# Instrumental variables

## Example in R

Back to our age-old battle to estimate the returns to education.

```
#> # A tibble: 722 x 4
#>   wage education education_dad education_mom
#>   <int>      <int>      <int>      <int>
#> 1   769         12         8         8
#> 2   808         18        14        14
#> 3   825         14        14        14
#> 4   650         12        12        12
#> 5   562         11        11         6
#> 6   600         10         8         8
#> 7  1154         15         5        14
#> 8  1000         12        11        12
#> 9   930         18        14        13
#> 10  900         15        12        12
#> # ... with 712 more rows
```

# Instrumental variables

## Example in R

OLS for the returns to education with will likely (definitely) be biased...

$$\text{Wage}_i = \beta_0 + \beta_1 \text{Education}_i + u_i$$

### (Likely biased) OLS results:

Term	Est.	S.E.	t stat.	p-Value
Intercept	176.504	89.152	1.98	0.0481
<b>Education</b>	<b>58.594</b>	<b>6.439</b>	<b>9.10</b>	<b>&lt;0.0001</b>

but what if mother's education provides a valid instrument?

# Instrumental variables

## Example in R

We can check/test the *relevance* of **mother's education** for **education**.

This regression is known as the **first stage**:

The effect of the **instrument** on our **endogenous explanatory variable**.

$$\text{Education}_i = \gamma_0 + \gamma_1 (\text{Mother's Education})_i + v_i$$

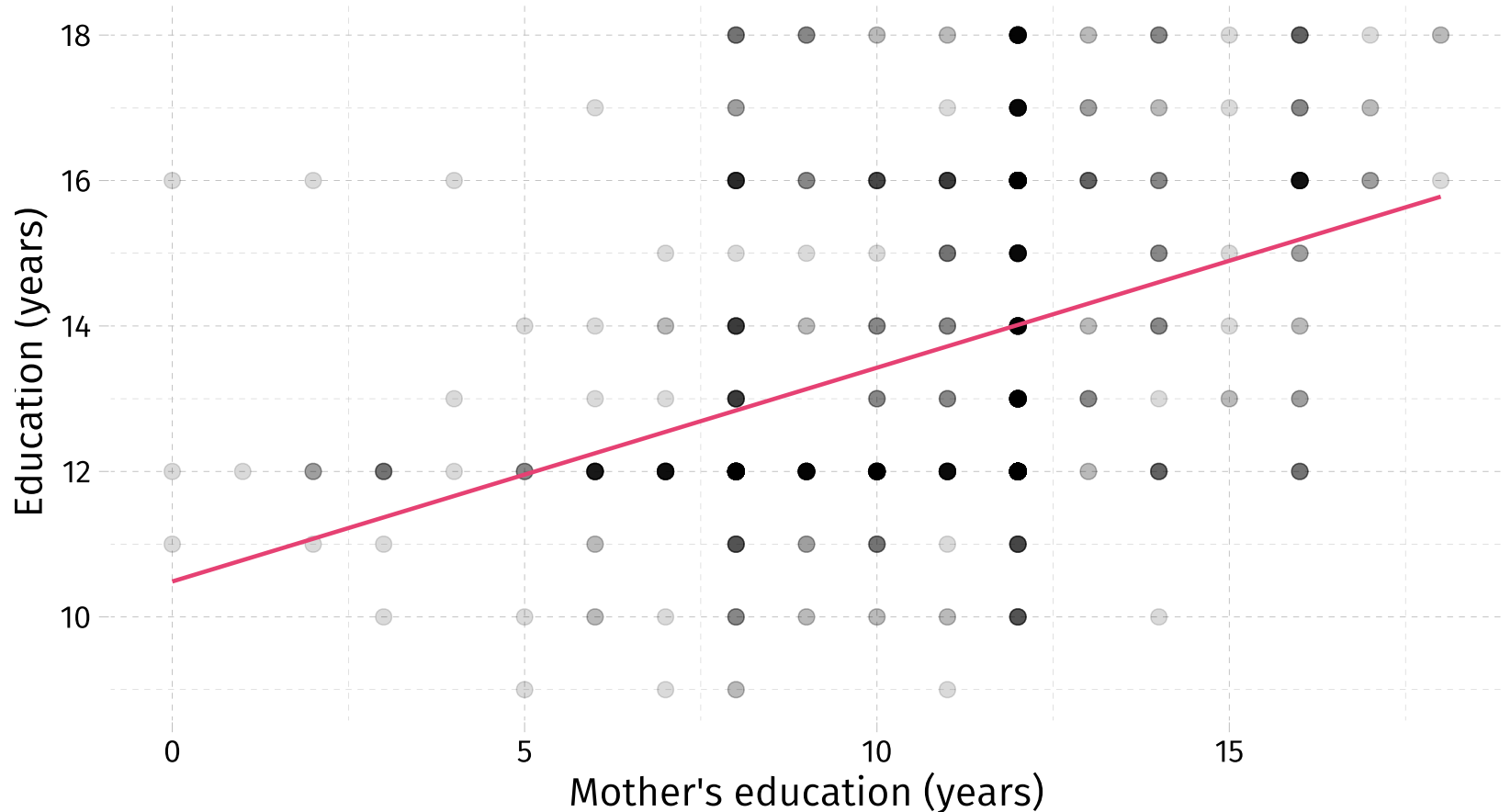
### First-stage results:

Term	Est.	S.E.	t stat.	p-Value
Intercept	10.487	0.306	34.32	<0.0001
<b>Mother's Education</b>	<b>0.294</b>	<b>0.027</b>	<b>10.75</b>	<b>&lt;0.0001</b>

The *p*-value suggests a very strong relationship (very *relevant*).

# Instrumental variables

## Visualizing the first stage



# Instrumental variables

## Exogeneity

**Q:** What does **exogeneity** mean in this case?

**A:** We need two things

1. **Mother's education (our instrument)** must only affect earnings through **education (our endogenous explanatory variable)**.
2. **Mother's education** must be uncorrelated with other factors that affect **wages (our outcome variable)**.

We want to be able to compare two people (*A* and *B*) whose mothers have different levels of education and say that the only differences between the two people (*A* and *B*) are due to their mothers' educational levels.

**Q:** Does *mother's education* seem likely to satisfy exogeneity?

# Instrumental variables

## Example in R

Now, let's estimate the *reduced form*:

The effect of the *instrument* on our *outcome variable*.

$$\text{Wage}_i = \pi_0 + \pi_1(\text{Mother's Education})_i + w_i$$

### Reduced-form results:

Term	Est.	S.E.	t stat.	p-Value
Intercept	633.34	58.58	10.81	<0.0001
<b>Mother's Education</b>	<b>31.81</b>	<b>5.24</b>	<b>6.07</b>	<b>&lt;0.0001</b>

**Q<sub>1</sub>:** How do we interpret this estimated coefficient ( $\hat{\pi}_1$ )?

**Q<sub>2</sub>:** If our instrument is *valid*, can we interpret these estimates as **causal**?

# Instrumental variables

## Example in R

So what is our IV-based estimate for the returns to education?

$$\text{Wage}_i = \beta_0 + \beta_1 \text{Education}_i + u_i$$

We know that the IV estimate for  $\beta_1$  is

$$\hat{\beta}_1^{\text{IV}} = \frac{\hat{\pi}_1}{\hat{\gamma}_1}$$

1. In the **reduced-form equation**, we estimated  $\hat{\pi}_1 \approx 31.81$ .
2. In the **first-stage equation**, we estimated  $\hat{\gamma}_1 \approx 0.294$ .

$$\implies \hat{\beta}_1^{\text{IV}} = \frac{\hat{\pi}_1}{\hat{\gamma}_1} = \frac{31.81}{0.294} \approx 108.2$$

# Instrumental variables

## Example in R

**Alternative:** Use the function `iv_robust()` from the `estimatr` package.

This new function `iv_robust` works very similar to our good friend `lm`:

```
iv_robust(y ~ x | z, data = dataset)
```

- `formula` Specify the regression followed by `|` and your instrument (`z`).
- `data` You still need a dataset.

**Note:** As you might guess by its name, `iv_robust` calculates heteroskedasticity-robust standard errors by default.



# Instrumental variables

## Example in R

In practice...

```
# Estimate our IV regression  
iv_est ← iv_robust(wage ~ education | education_mom, data = wage_df)
```

Term	Est.	S.E.	t stat.	p-Value
Intercept	-501.474	226.476	-2.21	0.0271
<b>Education</b>	<b>108.214</b>	<b>16.810</b>	<b>6.44</b>	<b>&lt;0.0001</b>

# Instrumental variables

## More

So now we know how to "do" instrumental variables *when we have one endogenous variable and one exogenous variable*.

1. Estimate the reduced form (regress **outcome var.** on **instrument**).
2. Estimate the first stage (regress **expl. var.** on **instrument**).
3. Calculate the IV estimate using the estimates from (1) and (2).

Our magical **instrument** isolates the exogenous variation in our **endogenous variable**.

**Q:** What if we want more? (E.g., more instruments or endog. variables)

**A:** ~~Too bad.~~ Extend IV to **two-stage least squares (2SLS)**.

# Two-stage least squares

# Two-stage least squares

## Intro

The intuition and insights of IV carry over into two-stage least squares.

**Plus:** The *first stage* that we've been discussing is actually the *first* of the *two stages* in two-stage least squares.

Endogenous model	$\text{Outcome}_i = \beta_0 + \beta_1 (\text{Endog. var.})_i + u_i$
------------------	---

First stage	$(\text{Endog. var.})_i = \pi_0 + \pi_1 \text{Instrument}_i + v_i$
-------------	--

Second stage	$\text{Outcome}_i = \delta_0 + \delta_1 \widehat{(\text{Endog. var.})}_i + \varepsilon_i$
--------------	---

Reduced form	$\text{Outcome}_i = \pi_0 + \pi_1 \text{Instrument}_i + w_i$
--------------	--

where  $\widehat{(\text{Endog. var.})}_i$  denotes the predicted values (*fitted values*) from the first-stage regression.

# Two-stage least squares

## Intro

Two-stage least squares is very flexible—we include other controls, additional endogenous variables, *and* have multiple instruments.

**But** don't get too distracted by this fancy flexibility.

We still need **valid** instruments.

# Two-stage least squares

In R

Back to our *returns to education* example.

$$\text{Wage}_i = \beta_0 + \beta_1 \text{Education}_i + u_i$$

Imagine that mother's *and* father's education are both valid instruments.

Then our **first-stage regression** is

$$\text{Education}_i = \gamma_0 + \gamma_1 (\text{Mother's education})_i + \gamma_2 (\text{Father's education})_i + v_i$$

which we can estimate via OLS.

**Q:** Why?

# Two-stage least squares

In R

$$\text{Education}_i = \gamma_0 + \gamma_1(\text{Mother's education})_i + \gamma_2(\text{Father's education})_i + v_i$$

```
stage1 <- lm(education ~ education_mom + education_dad, wage_df)
```

## First-stage results:

Term	Est.	S.E.	t stat.	p-Value
Intercept	9.845	0.305	32.31	<0.0001
<b>Mother's Education</b>	<b>0.149</b>	<b>0.032</b>	<b>4.62</b>	<b>&lt;0.0001</b>
<b>Father's Education</b>	<b>0.216</b>	<b>0.028</b>	<b>7.84</b>	<b>&lt;0.0001</b>

Our instruments each appear to be *relevant*.

Formally, we should jointly test them (e.g.,  $F$  test).

# Two-stage least squares

In R

Using our estimated first stage, we grab the *fitted* endogenous variable

$$\widehat{\text{Education}}_i = \hat{\gamma}_0 + \hat{\gamma}_1 (\text{Mother's education})_i + \hat{\gamma}_2 (\text{Father's education})_i$$

```
# Add fitted values from first stage  
wage_df$education_hat ← stage1$fitted.values
```

Now we use OLS (again) to estimate the **second-stage regression**

$$\text{Wage}_i = \delta_0 + \delta_1 \widehat{\text{Education}}_i + \varepsilon_i$$



# Two-stage least squares

In R

$$\text{Wage}_i = \delta_0 + \delta_1 \widehat{\text{Education}}_i + \varepsilon_i$$

```
stage2 <- lm(wage ~ education_hat, wage_df)
```

## Second-stage results:

Term	Est.	S.E.	t stat.	p-Value
Intercept	-454.683	198.149	-2.29	0.022
<b>Fitted Education</b>	<b>104.789</b>	<b>14.462</b>	<b>7.25</b>	<b>&lt;0.0001</b>

## Ordinary least squares

Term	Est.	S.E.	t stat.	p-Value
Intercept	176.504	89.152	1.98	0.0481
<b>Education</b>	<b>58.594</b>	<b>6.439</b>	<b>9.10</b>	<b>&lt;0.0001</b>

## Instrumental variables

Term	Est.	S.E.	t stat.	p-Value
Intercept	-501.474	226.476	-2.21	0.0271
<b>Education</b>	<b>108.214</b>	<b>16.810</b>	<b>6.44</b>	<b>&lt;0.0001</b>

## Two-stage least squares w/ two instruments

Term	Est.	S.E.	t stat.	p-Value
Intercept	-454.683	198.149	-2.29	0.022
<b>Education</b>	<b>104.789</b>	<b>14.462</b>	<b>7.25</b>	<b>&lt;0.0001</b>

# Two-stage least squares

In R

As you probably guessed, R will do both of the stages for you.

```
iv_robust(y ~ x1 + x2 + ... | z1 + z2 + ..., data)
```

In our case, we have

- one explanatory variable (`x`) (`education`)
- two instruments (`z`) (`parents' educations`)

```
iv_robust(wage ~ education | education_mom + education_dad, data = wage_df)
```

Term	Est.	S.E.	t stat.	p-Value
Intercept	-454.683	199.946	-2.27	0.0233
<b>Education, fitted</b>	<b>104.789</b>	<b>14.852</b>	<b>7.06</b>	<b>&lt;0.0001</b>

# Two-stage least squares

## There's more!

Because 2SLS **isolates exogenous variation in an endogenous variable**, we apply it in other settings that are biased from *endogenous* relationships.

### Common applications

- **General causal inference** for observational data (as we've seen).
- **Experiments:** Randomize a treatment that affects an endog. variable.
- **Measurement error:** Regress noisy  $x_1$  on noisy  $x_2$  to capture signal.
- **Simultaneous relationships** (e.g.,  $p$  and  $q$  from supply and demand).

However, in any 2SLS/IV setting, you need to mind the requirements for **valid instruments**—**exogeneity** and **relevance**.

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