Student Information

Name : Abdulkadir Pamukçu

ID : 2237774

Answer 1

a)

First calculate marginal probability of x:

x=0 $->$ 1/12 + 2/12 = 3/12 = 1/4
x=1 $->$ 4/12 + 2/12 = 6/12 = 1/2
x=2 $->$ 1/12 + 2/12 = 3/12 = 1/4

E(x) = $\sum_x P(x) \times x = 1/4 \times 0 + 1/2 \times 1 + 1/4 \times 2 = 1$
So, E(x) = 1

Var(x) $\sum_x (x - E(x))^2 \times P(x) = 1/4 + 0 + 1/4 = 1/2$
So Var(x) = 1/2

b)

To find pmf of X+Y we need to evaluate every case.
Let PMF of X+Y = P(X+Y)

P(0) = P(0+0) = 1/12
P(1) = P(1+0) = 4/12 = 1/3
P(2) = P(2+0) + P(0+2) = 1/12+2/12 = 3/12 = 1/4
P(3) = p(1+2) = 2/12 = 1/6
P(4) = P(2+2) = 2/12 = 1/6

$P(X+Y)$	$Value$
$P(0)$	1/12
$P(1)$	1/3
$P(2)$	1/4
$P(3)$	1/6
$P(4)$	1/6

c)

Cov(X,Y) = E(XY) - E(X)E(Y)
First lets find E(Y) we already know E(X)=1
E(Y) = $\sum_{y} P(Y) \times x = 1/2 \times 0 + 1/2 \times 2 = 1$
So, E(Y) = 1

Then E(XY) = $\sum_{xy} P(x,y) \times x \times y = 1/12 \times 0 \times 0 + 2/12 \times 2 \times 0 + 4/12 \times 1 \times 0 + 2/12 \times 2 \times 1 + 1/12 \times 0 \times 2 + 2/12 \times 2 \times 2 = 12/12 = 1$

So, E(XY) = 1

E(X)E(Y) = 1 $\times$ 1 = 1

So Cov(X,Y) = E(XY) - E(X)E(Y) = 1-1=0

d)

For two independent random variable A and B E(A) $\times$ E(B) should be equal to E(AB) since A and B are independent.

The formula of Covariance is Cov(A,B) = E(AB) - E(A)E(B)
For two independent random variable those two is equal.
Hence we can say that Cov(A,B) = 0 always true for independent A and B

e)

In order to prove independency we need to check every pair of values of X and Y with this equation:
P(X,Y) = P(X) $\times$ P(Y)
If every pair holds the equation we can say that X and Y are independent. But if any of them does not hold it is not independent.

For X=0, Y=0 : P(0,0) = 1/12 and $P_x(0)$ = 3/12 $P_y(0)$ = 6/12
$P_x(0) \times P_y(0)$ = 18/144 = 18
We can see here P(0,0) $\neq$ $P_x(0) \times P_y(0)$

Since (0,0) pair does not hold the equation X and Y are not independent.

Answer 2

a)

This probability is a binomial distribution.

There 12 pen $\rightarrow$ Number of trials = n = 12
A pen is broken with probability 0.2 $\rightarrow$ = Probability of success = p = 0.2
Since question asks us at least 3 pen is broken we can find probability of at most 2 broken pens (at most 2 successful trials) and can subtract it from 1.

At most 2 successful trials from 12 trials with probability of success is 0.2 can be found at binomial distribution table from the book. It is 0.558.

So, 1-0.558=0.442 is the probability at least 3 pen are broken.

b)

Probability of the fifth pen we test will be the second broken pen we find means that it took 5 trials to obtain 2 successes. And it can be found with negative binomial distribution.
5 trials, 2 successes, 0.2 probability of success, it can be found at negative binomial distribution table. It is 0.08192 .

c)

To find how many pens we are going to test to find 4 broken pens we need to use negative binomial distribution, but we are not asked the probability we are asked to average.
Number of success = k = 4 Probability of success = p = 0.2 From the formula of mean in negative binomial distribution:
k/p = 4/0.2 = 20

Hence, the answers is 20

Answer 3

a)

Since getting a phone call every 4 hours an average is an event that happens rarely in a fixed period of time. We'll use the poisson distribution.
The average number of phone call in 2 hour $\rightarrow \lambda = 1/2 = 0.5$

We need to find out probability of getting no phone call in 2 hours.
So number of events occurring $\rightarrow$ x = 0
We can find the probability of this distribution from poisson distribution table from the book. It is 0.607
Hence, the answer is 0.607

b)

We will use poisson distribution here. But this time we'll set the period 10 hours. Since our period 10 hours, average number of phone call in 10 hours $\rightarrow \lambda = 2.5$
We can find the probability of this distribution from poisson distribution table from the book. It is 0.758
Hence, the answer is 0.758

c)

We are given that Bob did not get more than 3 phone calls for the first 10 hours.
Let call the probability of not getting more than 3 phone calls in first 10 hours is P(A).
And we are asked for probability of he does not get more than 3 phone call for first 16 hours given that P(A) and lets call it P(B)

So, basically question is $P(B \mid A)$:

$P(B \mid A) = P(A \cap B) / P(A)$

We already know from P(A) from subsection b, P(A) = 0.758

$P(A \cap B) = P(B)$ since P(B) contains P(A). (Getting no more than 3 phone call for 16 hours means also getting no more than 3 phone call for 10 hours.)
To calculate P(B) we need to look possion distribution table from book. Number of success $\rightarrow$ x = 3 and average number of phone call in 16 hours $\rightarrow \lambda = 4$. It is 0.433

So, $P(B \mid A) = 0.433/0.758 = 0.571$
Hence, the answer is 0.571