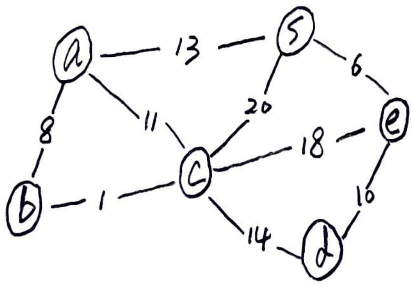
CSE 431/531 HW5 Spring 2022

Feng Wei 50428109

Problem 1.

Prim's Algorithm



$d[v]$ = the weight of the lightest edge between v and S

$\pi[v]$ = the lightest edge between v and S $(\pi[v], v)$.

iteration	Vertex added to S	a: d	a: π	b: d	b: π	c: d	c: π	d: d	d: π	e: d	e: π
1	s	13	s	∞	⊥	20	s	∞	⊥	6	s
2	e	13	s	∞	⊥	18	e	10	e		
3	d	13	s	∞	⊥	14	d				
4	a			8	a	11	a				
5	b					1	b				
6	c										

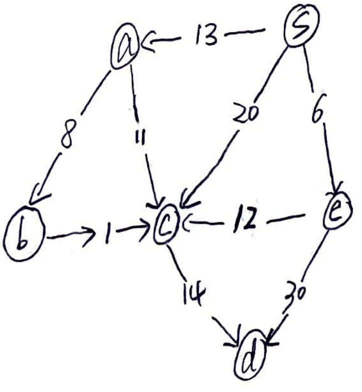
Problem 2.

Dijkstra's Algorithm

iteration	vertex added to S in iteration i	a: d	a: π	b: d	b: π	c: d	c: π	d: d	d: π	e: d	e: π
1	s	13	s	∞	⊥	20	s	∞	⊥	6	s
2	e	13	s	∞	⊥	18	e	36	e		
3	a			21	a	18	e	36	e		
4	c			21	a			32	c		
5	b							32	c		
6	d										

d[v] = the shortest path from s to v

π[v] = the parent of v in the shortest path tree



Problem 3.

Undirected Graph $G=(V,E)$, Non-negative edge weights $(w_e)_{e\in E}$

(3a.) Yes.

proof: Changing the weight of every edge e from w_e to w_e^2 doesn't change the ordering of the edge weights.

Let T be the original MST of G. And T^* be the after MST.

For every cut $(U, V\setminus U)$, T has one edge e and T^* has one edge e^* to connect the cut. Let $e=(u,v)$ and $e^*=(u^*,v^*)$. Assume $e\neq e^*$

According to the definition $\underbrace{W_{(u,v)} < W_{(u^*,v^*)}}_{①}$ and $\underbrace{W^2_{(u,v)} > W^2_{(u^*,v^*)}}_{②}$

Since W_e are non-negative and different.

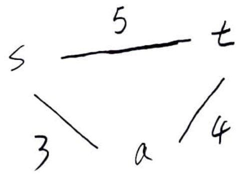
Then ① and ② cannot be correct at the same time.

As a result the assumption is wrong. We have $e=e^* \Rightarrow T=T^*$

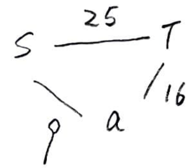
T is still the minimum spanning tree of G.

(3b.) No

Counter - example



⇒



After

original

Assume we only have three vertices s, t, and a.

In the original Graph the shortest path from s to t

is $P(s-t) = 5$

In the After graph, P is no longer unique.

Since $s-t = 25 = sa + aT$

Problem 4.

$$d[u] + w(u,v) < d[v]$$

Assume for every edge $(u,v) \in E$, that $d[u] + w(u,v) \geq d[v]$

Let the negative cycle be $C = (a_1, a_2 \cdots a_n)$ $n > 1$. So we have

$$0 > w(a_1, a_2) + w(a_2, a_3) \cdots + w(a_{n-1}, a_n) + w(a_n, a_1)$$

According to the assumption:

$$d(a_1) + w(a_1, a_2) \geq d(a_2)$$

$$\vdots$$

$$d(a_{n-1}) + w(a_{n-1}, a_n) \geq d(a_n)$$

$$d(a_n) + w(a_n, a_1) \geq d(a_1)$$

$$\Rightarrow [d(a_1) + w(a_1, a_2)] + \cdots [d(a_n) + w(a_n, a_1)] \geq d(a_2) + \cdots + d(a_n) + d(a_1)$$

$$\Rightarrow \underbrace{[d(a_1) + d(a_2) + \cdots d(a_n)]}_{①} + \underbrace{[w(a_1, a_2) + \cdots w(a_n, a_1)]}_{②} \geq \underbrace{[d(a_1) + \cdots d(a_n)]}_{③}$$

Obviously ① = ③ and ② is the negative cycle. So ② < 0

As a result the assumption is wrong. So there exists edge such that $d[u] + w(u,v) < d[v]$