

Homework 2*Instructor: Shi Li***Deadline: 10/14/2021**

Your Name: _____ Your Student ID: _____

Problems	1	2	3	Total
Max. Score	20	30	30	80
Your Score				

Problem 1 Construct the Huffman code (i.e, the optimum prefix code) for the alphabet $\{a, b, c, d, e, f, g\}$ with the following frequencies:

Symbols	a	b	c	d	e	f	g
Frequencies	50	20	27	25	29	85	55

Also give the weighted length of the code (i.e, the sum over all symbols the frequency of the symbol times its encoding length).

Problem 2 We are given an array A of length n . For every integer i in $\{1, 2, 3, \dots, n\}$, let b_i be median of the sub-array $A[1..i]$. (If the sub-array has even length, its the median is defined as the lower of the two medians. That is, if i is even, b_i is the $i/2$ -th smallest number in $A[1..i]$.) The goal of the problem is to output $b_1, b_2, b_3, \dots, b_n$ in $O(n \log n)$ time.

For example, if $n = 10$ and $A = (110, 80, 10, 30, 90, 100, 20, 40, 35, 70)$. Then $b_1 = 110, b_2 = 80, b_3 = 80, b_4 = 30, b_5 = 80, b_6 = 80, b_7 = 80, b_8 = 40, b_9 = 40, b_{10} = 40$.

Hint: use the heap data structure.

Problem 3 A string of “(” and “)” is said to be balanced, if it satisfies the recursive definition:

- The empty string “” is balanced.
- If A is balanced then (A) is balanced.
- If A and B are balanced, then AB is balanced.

For example, “ $((())())$ ” is balanced.

In this problem, you are given a string of “(” and “)”, the goal is to remove the minimum number of characters so that the residual string is a balanced. For example, if the input is “ $((())())()$ ”, then you can remove 3 characters to obtain a balanced string, as follows: “ $((())())$ ”.

Design a greedy algorithm to solve the problem. You need to show the algorithm is correct. To get a full score, your algorithm needs to run in $O(n)$ time, where n is the length of the string.