

Homework 6*Instructor: Shi Li***Deadline: 12/19/2021**

Your Name: _____ Your Student ID: _____

Problems	1	2	Total
Max. Score	50	30	80
Your Score			

Problem 1 For each of the following 5 problems, state (i) whether the problem is known to be in NP, and (ii) whether the problem is known to be in Co-NP. For problems (1c), (1d) and (1e), if your answer for (i) (or (ii)) is yes, you need to give the certificate and the certifier that establishes that the problem is in NP (or Co-NP).

- (1a) Given a graph $G = (V, E)$ and $s \leq |V|$, the problem asks whether G contains an independent set of size s .

NP: yes

Co-NP: no

- (1b) Given two circuits C_1 and C_2 , each with m input variables z_1, z_2, \dots, z_m , decide if the two circuits compute the same function. That is, whether C_1 and C_2 give the same output for every boolean assignment of z -variables.

NP: no

Co-NP: yes

- (1c) Given a graph $G = (V, E)$, decide if G is 3-colorable.

NP: yes

Co-NP: no

Certificate for NP: a valid 3-coloring of V , that is, a coloring of V using 3 colors such that no two adjacent vertices have the same color.

Certifier for NP: Given the graph G and the 3-coloring, the algorithm checks if it is a valid 3-coloring for G .

- (1d) Given a graph $G = (V, E)$, decide if G is 2-colorable.

NP: yes

Co-NP: yes

Deciding if G is 2-colorable is the same as deciding if G is bipartite. So the problem can be solved in polynomial time and thus is in both NP and Co-NP. There is no need to define the certificates. The certifier takes the graph G , and checks in polynomial time if it is (not) 2-colorable.

- (1e) An undirected graph $G = (V, E)$ is called a 1-expander if for every $U \subseteq V$, the number of edges between U and $V \setminus U$ in G is at least $\min\{|U|, |V \setminus U|\}$. Given a graph G , decide if G is a 1-expander.

NP: no

Co-NP: yes

Certificate for Co-NP: a subset $U \subseteq V$, such that the number of edges between U and $V \setminus U$ is less than $|\min\{|U|, |V \setminus U|\}|$.

Certifier for Co-NP: Given the graph G and the set U , the certifier checks if the number of edges between U and $V \setminus U$ is less than $|\min\{|U|, |V \setminus U|\}|$.

Problem 2 Let NPC be the set of NP-Complete problems. Prove the following statements:

- If $P \neq NP$, then $P \cap NPC = \emptyset$.

Proof. We prove the contra-positive of the statement: If $P \cap NPC \neq \emptyset$ then $P = NP$. If $P \cap NPC \neq \emptyset$, then there is a problem X in NPC and in P . Then by our definition of NPC, every problem Y in NP is polynomial time reducible to X and thus can be solved in polynomial time. So $P = NP$. \square

- If $P = NP$, then $P = NPC$.

Proof. If $P = NP$, then every problem in NP has a polynomial time algorithm. This implies that every problem Y in NP is polynomial time reducible to every problem X in NP. (In particular, there is no need to call the black-box algorithm for X in the algorithm for Y .) Therefore every problem X in $P = NP$ is NP-complete. \square