CSE 431/531: Algorithm Analysis and Design	Fall 2021

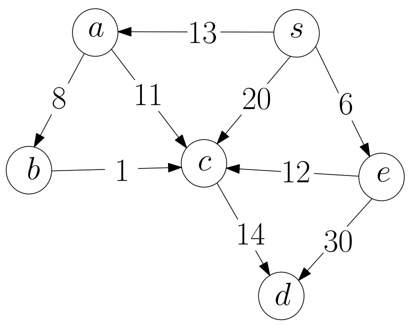
Homework 5

Instructor: Shi Li **Deadline: 11/7/2021**

Your Name: ______________________ Your Student ID: ________________

Problems	1	2	3	Total
Max. Score	24	24	32	80
Your Score				

Problem 1 Consider the following directed graph G with non-negative edge weights. Use Dijkstra's algorithm to compute the shortest paths from s to all other vertices in G.



You just need to fill the following table.

iteration i	vertex added to S in iteration i	a		b		c		d		e	
		d	π	d	π	d	π	d	π	d	π
1	s	13	s	∞	$\perp$	20	s	∞	$\perp$	6	s
2											
3											
4											
5											
6											

Table 1: Dijkstra's algorithm for Shortest Paths

Recall that in the algorithm, we maintain a set S of vertices. For a vertex $v \in S$, $d[v]$ is the length of the shortest path from s to v, and $\pi[v]$ is the parent of v in the shortest path tree. For a vertex $v \notin S$, we have $d(v) = \min_{u \in S:(u,v) \in E} (d(u) + w(u, v))$, and $\pi(v)$ is the vertex $u \in S$ such that $d(v) = d(u) + w(u, v)$. When $d(v) = \infty$, we set $\pi(v) =$"$\perp$".

Problem 2 Consider the minimum spanning tree problem over the graph $G = (V, E)$ with a weight function $w : E \to \mathbb{R}_{\geq 0}$. Assume all the weights are distinct. Let T be the minimum spanning tree of $G = (V, E)$ w.r.t the weight function w. State if each of the following statements is correct or not. If your answer is no for some statement, you need to give a counter example.

(2a) Let C be a cycle in G and e^* be the lightest edge on C. Then e^* is in T.

(2b) Let C be a cycle in G and e^* be the heaviest edge on C. Then e^* is not in T.

(2c) Let $U \subsetneq V, U \neq \emptyset$ be a proper subset of V and e^* be the lightest edge in E between U and $V \setminus U$. Then e^* is in T.

(2d) Let $U \subsetneq V, U \neq \emptyset$ be a proper subset of V and e^* be the heaviest edge in E between U and $V \setminus U$. Then e^* is not in T.

If your answer is "no" for a question, you need to give a counter example. (If your answer is yes, then saying "yes" is sufficient.)

Problem 3 Suppose *there is* a negative cycle over a directed graph $G = (V, E)$ with edge weights $w : \mathbb{E} \to \mathbb{R}$. Show that for any array $d : V \to \mathbb{R}$ over vertices, there exists some edge $(u, v) \in E$ such that $d(u) + w(u, v) < d(v)$.

(So if we do not put an upper bound on the number of iterations the Bellman-Ford algorithm runs, it will run forever when there is a negative cycle.)