$$
\begin{aligned}
\int_0^{\frac{\pi}{2}} \frac{1}{1+\sin(x)^4}\,\mathrm{d}x &= \int_0^{\frac{\pi}{2}} \frac{\frac{1}{\cos(x)^4}}{\frac{1}{\cos(x)^4}+\tan(x)^4}\,\mathrm{d}x \\
&\overset{t=\tan(x)}{\longrightarrow} \int_0^{+\infty} \frac{t^2+1}{\left(t^2+1\right)^2+t^4}\,\mathrm{d}t \\
&= \int_0^{+\infty} \frac{t^2+1}{2t^4+2t^2+1}\,\mathrm{d}t \\
&= \int_0^{+\infty} \frac{1+t^{-2}}{2t^2+2+t^{-2}}\,\mathrm{d}t \\
&= \frac{\sqrt{2}-2}{4}\int_0^{+\infty} \frac{\sqrt{2}-t^{-2}}{\left(\sqrt{2}t+t^{-1}\right)^2+2-2\sqrt{2}}\,\mathrm{d}t + \frac{\sqrt{2}+2}{4}\int_0^{+\infty} \frac{\sqrt{2}+t^{-2}}{\left(\sqrt{2}t-t^{-1}\right)^2+2+2\sqrt{2}}\,\mathrm{d}t \\
&\overset{u=\sqrt{2}t+t^{-1}}{\longrightarrow} \frac{\sqrt{2}-2}{4}\int_{+\infty}^{+\infty} \frac{1}{u^2+2-2\sqrt{2}}\,\mathrm{d}u + \frac{\sqrt{2}+2}{4}\int_0^{+\infty} \frac{\sqrt{2}+t^{-2}}{\left(\sqrt{2}t-t^{-1}\right)^2+2+2\sqrt{2}}\,\mathrm{d}t \\
&= \frac{\sqrt{2}+2}{4}\int_0^{+\infty} \frac{\sqrt{2}+t^{-2}}{\left(\sqrt{2}t-t^{-1}\right)^2+2+2\sqrt{2}}\,\mathrm{d}t \\
&\overset{u=\sqrt{2}t-t^{-1}}{\longrightarrow} \frac{\sqrt{2}+2}{4}\int_{-\infty}^{+\infty} \frac{1}{u^2+2+2\sqrt{2}}\,\mathrm{d}u \\
&= \frac{\sqrt{2}+2}{4}\frac{1}{\sqrt{2+2\sqrt{2}}}\pi \\
&= \frac{\pi\sqrt{1+\sqrt{2}}}{4}
\end{aligned}
$$