$$I(t) = \int_0^\infty \frac{\sin(x)e^{-tx}}{x}\,\mathrm{d}x$$

$$\begin{aligned}\Rightarrow I'(t) &= -\int_0^\infty \sin(x)e^{-tx}\,\mathrm{d}x \\ &= -\mathcal{L}(\sin(x)) \\ &= -\frac{1}{t^2+1}\end{aligned}$$

$$\Rightarrow I(t) = -\arctan(t) + c$$

$$I(\infty) = 0$$

$$\Rightarrow I(t) = -\arctan(t) + \frac{\pi}{2}$$

$$\Rightarrow \int_0^\infty \frac{\sin(x)}{x}\,\mathrm{d}x = I(0) = \frac{\pi}{2}$$