

$$\sin(x) = \Im(e^{ix}) = \frac{e^{ix} - e^{-ix}}{2}$$

$$\begin{aligned} \sin(1) + \sin(2) + \dots + \sin(n) &= \sum_{k=1}^n \sin(k) \\ &= \Im\left(\sum_{k=1}^n e^{ik}\right) \\ &= \Im\left(e^i \frac{e^{in} - 1}{e^i - 1}\right) \\ &= \Im\left(e^i \frac{e^{i\frac{n}{2}}(e^{i\frac{n}{2}} - e^{-i\frac{n}{2}})}{e^{\frac{i}{2}}(e^{\frac{i}{2}} - e^{-\frac{i}{2}})}\right) \\ &= \Im\left(e^{i\frac{n+1}{2}} \frac{e^{i\frac{n}{2}} - e^{-i\frac{n}{2}}}{e^{\frac{i}{2}} - e^{-\frac{i}{2}}}\right) \\ &= \Im\left(e^{i\frac{n+1}{2}} \frac{\sin\left(\frac{n}{2}\right)}{\sin\left(\frac{1}{2}\right)}\right) \\ &= \frac{\sin\left(\frac{n+1}{2}\right) \sin\left(\frac{n}{2}\right)}{\sin\left(\frac{1}{2}\right)} \end{aligned}$$