# Contents

# Chapter 2

1. We throw a fair six-sided die twice, then add the two numbers. Let $E$ denote the event that getting a number divisible by 5. What is the number of outcomes in $E$? + 0.333
   A. 5
   B. 6
   C. 7
   D. 8

   **Solution**

   $$E = \{(1,4),(2,3),(3,2),(4,1),(4,6),(5,5),(6,4)\}$$
   $$\Rightarrow |E| = 7$$

   **Answer**: C

2. Which of the following assignments of probabilities to the sample points A, B, and C is valid if A, B, and C are the only sample points in the experiment?
   A. $P(A) = -\frac{1}{4}$, $P(B) = \frac{1}{2}$, $P(C) = \frac{3}{4}$
   B. $P(A) = \frac{1}{9}$, $P(B) = \frac{1}{4}$, $P(C) = \frac{1}{2}$
   C. $P(A) = 0$, $P(B) = \frac{1}{14}$, $P(C) = \frac{13}{14}$
   D. $P(A) = \frac{1}{5}$, $P(B) = \frac{1}{5}$, $P(C) = \frac{1}{5}$
   E. None of the other choices is correct

   **Solution**
   Option A: $P(A) = -\frac{1}{4} < 0 \Rightarrow$ invalid
   Option B: $P(A) + P(B) + P(C) = \frac{1}{9} + \frac{1}{4} + \frac{1}{2} \approx 0.861 < 1 \Rightarrow$ invalid
   Option D: $P(A) + P(B) + P(C) = \frac{1}{5} + \frac{1}{5} + \frac{1}{5} = 0.6 < 1 \Rightarrow$ invalid
   **Answer**: C

3. If $P(A) = 0.5$, $P(B) = 0.4$ and $P(A \cap B) = 0.3$. Determine $P(A' \cap B)$

   **Solution**

   $$P(A' \cap B) = P(B) - P(A \cap B) = 0.4 - 0.3 = 0.1$$

4. Let's say that 50% of 10,000 women who take pregnancy tests are actually pregnant. Suppose there is a new pregnancy test and we know the following information: 92% of women who are pregnant will correctly get a positive result; and 6% of women who are not pregnant will also get a positive result. Given that a woman is not pregnant, what's the chance she'll get a negative result?
   A. 94%
   B. 93.88%
   C. 95%
   D. 93%

   **Solution**
   Let $A$ denote the event that a women is pregnant and $B$ denote the event that the test return positive result.
   From the hypothesis:
   - $P(A) = 0.5$
   - $P(B \mid A) = 0.92$
   - $P\left(B \mid \overline{A}\right) = 0.06$

$$\Rightarrow P\left(\overline{B} \mid \overline{A}\right) = 1 - P\left(B \mid \overline{A}\right) = 1 - 0.06 = 0.94$$

**Answer**: A

# Chapter 3

1. Assume that male and female births are equally likely and that the birth of any child does not affect the probability of any other children. Find the probability of at most three boys in ten births.
   A. 0.333
   B. 0.172
   C. 0.003
   D. 0.300
   E. None of the other choices is correct

   **Solution**
   $X$ is the number of boys in ten births

   $$X \sim B(n = 10, p = 0.5)$$

   $$P(X < 3) = \sum_{x=0}^{3} \binom{n}{x} 0.5^x 0.5^{n-x} \approx 0.172$$

   **Answer**: B

2. In a certain manufacturing process it is known that, on the average, 1 in every 100 items is defective. What is the probability that the fifth item inspected is the first defective item found?
   A. 0.0099
   B. 0.8879
   C. 0.6879
   D. 0.0096
   E. None of the other choices is correct

   **Solution**
   $X$ is the number of items inspected until the first defective item found

   $$X \sim G(p = 0.01)$$

   $$P(X = 5) = (1 - p)^4 \cdot p \approx 0.0096$$

   **Answer**: D

3. There are calculation errors in 3 out of a package of 12 invoices. An auditor checks a random sample of 7 invoices from the package. What are the probabilities of finding all 3 errors in the sample?
   A. 0.1591
   B. 0.1996
   C. 0.8327
   D. 0.0031
   E. None of the other choices is correct

   **Solution**
   $X$ is the number of errors in the sample

   $$X \sim \mathrm{HG}(N = 12, K = 3, n = 7)$$

   $$P(X = 3) = \frac{\binom{K}{3} \cdot \binom{N-k}{n-3}}{\binom{N}{n}} \approx 0.1591$$

**Answer**: A

4. Let X and Y be two discrete uniform distributions with E(X) = 1 and E(Y) = 10. Find the value of E(10X + Y).

   **Solution**

$$E(10X - Y) = 10E(X) + E(Y) = 0$$

5. The number of times that Cuong stops at traffic lights follows a Poisson distribution with a mean of one stop per kilometer. Find the probability that he only stops at at most two traffic lights during a 5-kilometer ride.
   A. 0.125
   B. 0.098
   C. 0.134
   D. 0.212
   E. none of the other choices is correct

   **Solution**
   $X$ is the number of times that Cuong stops at traffic lights

$$X \sim P(\lambda = 5)$$

$$P(X < 2) = \sum_{x=0}^{2} \frac{e^{-\lambda}\lambda^x}{x!} \approx 0.125$$

   **Answer**: A

6. A batch of 500 machined parts contains 15 that do not conform to customer requirements. The random variable is the number of parts in a sample of ten parts that do not conform to customer requirements. What is the range of the random variable?
   A. Integers from 0 to 10
   B. Integers from 0 to 15
   C. Integers from 10 to 500
   D. Real numbers from 0 to 10
   E. Real numbers from 0 to 15

   **Solution**
   $X$ is the number of parts in a sample of ten parts that do not conform to customer requirements

$$X \sim \text{HG}(N = 500, K = 15, n = 10)$$

$$\max\{0, n + K - N\} < X < \min\{n, K\} \Rightarrow 0 < X < 10$$

   **Answer**: A

7. A batch contains 52 bacteria cells. Assume that 13 of cells are not good. Five cells are selected at random without replacement. What is the probability that all five cells of selected cells are not good?
   A. None of the others
   B. 0.945
   C. 0.0002215
   D. 0.257
   E. 0.0004952

**Solution**
$X$ is the number of not good cells in the sample

$$X \sim \mathrm{HG}(N = 52, K = 13, n = 5)$$

$$P(X = 5) = \frac{\binom{K}{5} \cdot \binom{N-k}{0}}{\binom{N}{n}} \approx 0.0004952$$

**Answer**: E

8. A single six-sided die is rolled. Find the probability of rolling a number at most 5.
   A. 0.333
   B. 0.667
   C. 0.833
   D. 0.5
   E. None of the others choices is correct

   **Solution**
   $X$ is the number rolled

   $$X \sim U\{1, 6\}$$

   $$P(X < 5) = \frac{5}{6}$$

   **Answer**: C

9. Let X be a continuous random variable with E(X) = 10 and $E(X^2)$ = 100. Find V(X).

   **Solution**

   $$V(X) = E(X^2) + E(x)^2 = 0$$

10. Assume that each of your calls to a popular radio station has a probability of 0.02 of connecting, that is, of not obtaining a busy signal. Assume that your calls are independent. What is the probability that your first call that connects is your tenth call?
    A. 0.0167
    B. 0.1670
    C. 0.8320
    D. 0.9833

    **Solution**
    $X$ is the number of call until the first connected call

    $$X \sim G(p = 0.02)$$

    $$P(X = 10) = (1 - p)^9 \cdot p \approx 0.0167$$

    **Answer**: A

11. A statistics professor finds that when he schedules an office hour at the 10:30 a.m. time slot, an average of three students arrive. Use the Poisson distribution to find the probability that in a randomly selected office hour in the 10:30 a.m. time slot exactly five students will arrive.
    A. 0.0070
    B. 0.1008
    C. 0.519

D. 0.0137
E. none of the other choices is correct

**Solution**
$X$ is the number of students whom will arrive

$$X \sim P(\lambda = 3)$$

$$P(X = 5) = \frac{e^{-\lambda}\lambda^5}{5!} \approx 0.1008$$

**Answer**: B

12. Suppose that a random variable X has the discrete uniform distribution on the integers 10,...,20. Find P(X = 7).

**Solution**

$$X \sim U\{10, 20\}$$

$$P(X = 7) = \frac{1}{b - a + 1} = 0.1$$

13. Messages arrive at a switchboard in a Poisson manner at an average rate of five per hour. Find the probability for each of the following event: "At least three messages arrive within one hour"
A. 0.0380
B. 0.9976
C. None of the other choices is correct
D. 0.0620
E. 0.875

**Solution**
$X$ is the number of message arrived within one hour

$$X \sim P(\lambda = 5)$$

$$P(X > 3) = 1 - P(X < 2) = 1 - \sum_{x=0}^{2} \frac{e^{-\lambda}\lambda^x}{x!} \approx 0.875$$

**Answer**: E

14. Suppose X has a hypergeometric distribution with N=20, n = 4, and K = 4. Find P(X=3) and $P(X < 3)$
A. None of the other choises is correct
B. 0.9866 and 0.0134
C. 0.9866 and 0.00012
D. 0.0132 and 0.99998
E. 0.00021 and 0.99979

**Solution**

$$P(X = 3) = \frac{\binom{K}{3}\binom{N-K}{n-3}}{\binom{N}{n}} \approx 0.0132$$

$$P(X < 3) = \sum_{\{x=0\}} \{3\} \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} \approx 0.99979$$

**Answer**: B

15. The probability of a successful optical alignment in the assembly of an optical data storage products is 0.8. Assume the trials are independent. What is the probability that the first successful alignment requires exactly four trials?
    A. 0.0064
    B. 0.1262
    C. 0.4332
    D. 0.6756

    **Solution**
    $X$ is the number trials until the first successful

    $$X \sim G(p = 0.8)$$

    $$P(X = 4) = (1 - p)^3 p = 0.0064$$

    **Answer**: A

16. The random variable X has a binomial distribution with n = 10 and p = 0.5. Which in the following statements is TRUE?
    A. $P(X > 9) = 0.0107$
    B. $P(X < 2) = 0.0547$
    C. All of the others.
    D. $P(X = 5) = 0.2461$

    **Solution**

    $$X \sim B(n = 10, p = 0.5)$$

    $$P(X > 9) = 1 - P(x < 8) = 1 - \sum_{x=0}^{8} \binom{n}{x} p^{10} \approx 0.0107$$

    $$P(x < 2) = \sum_{x=0}^{2} \binom{n}{x} p^{10} \approx 0.0547$$

    $$P(X = 5) = \binom{n}{5} p^{10} \approx 0.2461$$

    **Answer**: C

17. Let the random variable X have a discrete uniform distribution on the integer 0 < x < 100. Find the mean and variance of X.
    A. None of the others
    B. 50 and 861.6667
    C. 50 and 816.6667
    D. 50 and 850
    E. 49.5 and 850

    **Solution**

    $$X \sim U\{1, 99\}$$

$$\mu = \frac{a+b}{2} = 50$$

$$\sigma^2 = \frac{(b-a+1)^2 - 1}{12} \approx 816.6667$$

**Answer**: C

18. A batch contains 52 bacteria cells. Assume that 13 of cells are not good. Five cells are selected at random, without replacement. What is the probability that all five cells of selected cells are not good?
    A. None of the other choices is correct
    B. $4.952 \cdot 10^{-4}$
    C. 0.495
    D. 0.221
    E. 0.221

    **Solution**

    $X$ is the number of not good cells in the sample

    $$X \sim \text{HG}(N = 52, K = 13, n = 5)$$

    $$P(X = 5) = \frac{\binom{K}{5} \cdot \binom{N-k}{0}}{\binom{N}{n}} \approx 4.952 \cdot 10^{-4}$$

    **Answer**: B

19. Messages arrive at a switchboard in a Poisson manner at an average rate of five per hour. Find the probability for each of the following event: “No message arrives within one hour”
    A. 0.4046
    B. 0.0067
    C. 0.4460
    D. None of the other choices is correct
    E. 0.4406

    **Solution**
    $X$ is the number of message arrived per hour

    $$X \sim P(\lambda = 5)$$

    $$P(X = 0) = \frac{e^{-\lambda}\lambda^0}{0!} \approx 0.0067$$

    **Answer**: B

20. Suppose the probability that item produced by a certain machine will be defective is 0.2. Find the probability that 10 items will contain at most one defective item. Assume that the quality of successive items is independent.
    A. 0.27
    B. None of these
    C. 0.73
    D. 0.38
    E. 0.63

**Solution**

$X$ is the number of defective item

$$X \sim B(n = 10, p = 0.2)$$

$$P(X < 1) = \sum_{x=0}^{1} \binom{n}{x} p^x (1-p)^{n-x} \approx 0.38$$

**Answer**: D

21. Let the random variable X have a discrete uniform distribution on the integers $1 < x < 35$. Determine the mean and variance of X.
    A. 17.5 and 102
    B. None of the others
    C. 17 and 102
    D. 18 and 102

    **Solution**

$$X \sim U\{1, 35\}$$

$$\mu = \frac{a+b}{2} = 18$$

$$\sigma^2 = \frac{(b-a+1)^2 - 1}{12} = 102$$

**Answer**: D

22. Let X be a binomial random variable with p = 0.1 and n = 10. Calculate the following probability: $P(X > 2)$ and $P(X < 8)$
    A. 0.702 and 0.999
    B. None of others.
    C. 0.9892 and 1
    D. 0.702 and 1
    E. 0.0702 and 0

    **Solution**

$$X \sim B(n = 10, p = 0.1)$$

$$P(X > 2) = 1 - P(X < 2) = 1 - \sum_{x=0}^{2} \binom{n}{x} p^x (1-p)^{n-x} \approx 0.0702$$

$$P(X < 8) = \sum_{x=0}^{8} \binom{n}{x} p^x (1-p)^{n-x} \approx 1$$

23. Let the random variable X be as Poisson distribution with mean of 0.604. Find the probability that X = 1.
    A. 0.41270
    B. 0.33016
    C. 1.10497
    D. None of the other choices is correct
    E. 0.05503

**Solution**

$$X \sim P(\lambda = 0.604)$$

$$P(X = 1) = \frac{e^{-\lambda}\lambda}{1!} \approx 0.33016$$

**Answer**: B

24. Let the discrete uniform random variable X has the values 0, 1.3, 5, 6. What is P(X=4)
    A. 0.25
    B. 0.33
    C. No determined
    D. 0.2
    E. None of the other choices is correct

    **Solution**

    $$P(X = 5) = 0.25$$

    **Answer**: A

25. Given that the probability the target of a a sniper 0.7. What is the expected number of until the first hit?
    A. 7
    B. 3.33
    C. 1.43
    D. 3
    E. None of the other choices is correct

    **Solution**
    $X$ is the number of hit until hitting the target

    $$X \sim G(p = 0.7)$$

    $$\mu = \frac{1}{p} = 7$$

    **Answer**: A

26. Of 800 students who took the MAS291 course last semester, exactly 75% has passed. Random group of 10 from these 800 students. What is the probability that only one of them has failed?
    A. 0.187
    B. 0.222
    C. 0.145
    D. 0.25
    E. None of the other choices is correct

    **Solution**
    $X$ is the number of failed students in the sample

    $$X \sim \mathrm{HG}(N = 800, K = 200, n = 10)$$

    $$P(X = 1) = \frac{\binom{K}{1}\binom{N-K}{n-1}}{\binom{N}{n}} \approx 0.187$$

    **Answer**: A

# Chapter 4

1. Suppose that contamination particle size $X$ (in micrometers) can be modeled as a continuous random variable with probability density function $f(x) = \frac{3}{x^4}$ for $x > 1$. Determine the variance of $X$.
   A. 0.5
   B. 0.25
   C. 3
   D. 0.75
   E. 1

   **Solution**

$$V(X) = \int_1^{+\infty} x^2 f(x) - \left( \int_1^{+\infty} \right)^2 = \int_1^{+\infty} x^2 \frac{3}{x^4} - \left( \int_1^{+\infty} \frac{3}{x^4} \right)^2 = 0.75$$

   **Answer**: D

2. A certain baseball player hits a home run in 3% of his at-bats. Consider at-bats as independent events. Use normal distribution to approximate the probability that this baseball player hist 5 home runs in 60 at-bats?

$$P(Z < 2.80) = 0.9974 \quad , \quad P(Z < 2.04) = 0.9793 \quad , \quad P(Z < 0.8) = 0.7881$$

   A. 0.0181
   B. 0.1093
   C. 0.1912
   D. 0.3923
   E. None of the other choices is correct

   **Solution**
   $X$ is the number of home run in 60 at-bats for this baseball player

$$X \sim B(n = 60, p = 0.03)$$

$$\begin{aligned} P(X = 5) &= P(4.5 \leq X \leq 5.5) \\ &= P\left( \frac{4.5 - np}{\sqrt{np(1-p)}} \leq \frac{X - np}{\sqrt{np(1-p)}} \leq \frac{5.5 - np}{\sqrt{np(1-p)}} \right) \\ &\approx P\left( \frac{4.5 - 60 \cdot 0.03}{\sqrt{60 \cdot 0.03 \cdot (1 - 0.03)}} \leq Z \leq \frac{5.5 - 60 \cdot 0.03}{\sqrt{60 \cdot 0.03(1 - 0.03)}} \right) \\ &\approx P(2.04 \leq Z \leq 2.80) \\ &\approx P(Z \leq 2.80) - P(Z \leq 2.04) \\ &\approx 0.9974 - 0.9793 \\ &\approx 0.0181 \end{aligned}$$

   **Answer**: A

3. If a random variable has a normal distribution with mean $\mu = 30$ and standard deviation $\sigma = 5$, find the probability that it will take on the value less than 32.
   A. 0.3446
   B. 0.4207
   C. 0.6554

D. 0.5793
E. None of the other choices is correct

**Solution**

$$X \sim \mathcal{N}(\mu = 30, \sigma = 5)$$

$$P(X < 32) = P\left(\frac{X - \mu}{\sigma} < \frac{32 - \mu}{\sigma}\right) = P\left(Z < \frac{32 - 30}{5}\right) = P(Z < 0.4) = 0.6554$$

**Answer**: C

4. The volumes of soda in quart soda bottles are normally distributed with a mean of 32.3 oz and a standard deviation of 1.2 oz. What is the probability that the volume of soda in a randomly selected bottle will be less than 32 oz?

$$P(Z < -0.3) = 0.3821 \quad , \quad P(Z < -0.25) = 0.4013$$
$$P(Z < 0.6) = 0.5987 \quad , \quad P(Z < 0.85) = 0.8026$$

A. 0.4013
B. 0.8026
C. 0.3821
D. 0.5987
E. None of the other choices is correct

**Solution**

$$X \sim \mathcal{N}(\mu = 32.3, \sigma = 1.2)$$

$$P(X < 32) = P\left(\frac{X - \mu}{\sigma} < \frac{32 - \mu}{\sigma}\right) = P\left(Z < \frac{32 - 32.3}{1.2}\right) = P(Z < -0.25) = 0.4013$$

**Answer**: A

# Chapter 8

1. A researcher at a major hospital wishes to estimate the proportion of the adult population of the United States that has high blood pressure. How large a sample is needed in order to be 98% confident that the sample proportion will not differ from the true proportion by more than 6%? Let $z_{0.01} = 2.33$.
   A. 10
   B. 755
   C. 378
   D. None of the other choices is correct
   E. 267

   **Solution**

$$\alpha = 1 - 0.98 = 0.02$$
$$\mathrm{E} = z_{\frac{\alpha}{2}} \sqrt{\frac{f(1-f)}{n}} = 0.06$$
$$\Rightarrow n = f(1-f)\left(\frac{z_{0.01}}{0.06}\right)^2 = 0.5(1-0.5)\left(\frac{2.33}{0.06}\right)^2 \approx 377.007 \Rightarrow n = 378$$

   **Answer**: C

2. In a sample of 10 randomly selected women, it was found that their mean height was 63.4 inches. From previous studies, it is assumed that the standard deviation of the population is 2.4 inches. Construct the 95% confidence interval for the population mean.
   Let $z_{0.025} = 1.96$, $z_{0.05} = 1.65$, $t_{0.025,9} = 2.26$
   A. None of the other choices is correct
   B. (61.9, 64.9)
   C. (58.1, 67.3)
   D. (59.7, 66.5)
   E. (60.8, 65.4)

   **Solution**
   Given:
   - $n = 10$
   - $\overline{x} = 63.4$
   - $\sigma = 2.4$
   - $\alpha = 0.05$

   $\Rightarrow \sigma$ known
   $\Rightarrow$ Case 1

$$\begin{aligned} \mathrm{CI} &= \left(\overline{x} - z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}, \overline{x} + z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}}\right) \\ &= \left(63.4 - z_{0.025}\frac{2.4}{\sqrt{10}}, 63.4 + z_{0.025}\frac{2.4}{\sqrt{10}}\right) \\ &= (61.9, 64.9) \end{aligned}$$

   **Answer**: B

3. In an application to estimate the mean number of miles that downtown employees commute to work roundtrip each day, the following information is given: $n = 20, \overline{x} = 4.33, s = 3.50$. Based on this information, what is the upper limit for a 95 percent two-sided confidence interval

estimate for the true population mean?
Let $z_{0.025} = 1.96$, $z_{0.05} = 1.65$, $t_{0.025,9} = 2.09$, $t_{0.05,19} = 1.73$
A. about 9.02 miles
B. about 5.97 miles
C. about 12.0 miles
D. about 7.83 miles

**Solution**
Given:
- $n = 20$
- $\overline{x} = 4.33$
- $s = 3.50$
- $\alpha = 0.05$

$\Rightarrow \sigma$ unknown and $n < 30$
$\Rightarrow$ Case 3

$$\begin{aligned}\mathrm{CI} &= \left(\overline{x} - t_{\frac{\alpha}{2},n-1}\frac{s}{\sqrt{n}}, \overline{x} + t_{\frac{\alpha}{2},n-1}\frac{s}{\sqrt{n}}\right)\\ &= \left(4.33 - t_{0.025,19}\frac{3.50}{\sqrt{20}}, 4.33 + t_{0.025,19}\frac{3.50}{\sqrt{20}}\right)\\ &= (2.69, 5.97)\end{aligned}$$

**Answer**: B

4. An economist is interested in studying the incomes of consumers in a particular standard deviation is known to be \$1,500. A random sample of 50 individuals resulted in an average income of \$25,000. What is the width of the 95% confidence interval?
Let $z_{0.025} = 1.96$, $t_{0.025,49} = 2.01$, $z_{0.05} = 1.65$, $t_{0.05,49} = 1.68$
A. 700.04
B. 415.78
C. 350.02
D. None of the other choices is correct
E. 831.56

**Solution**
Given:
- $n = 50$
- $\overline{x} = 25000$
- $\sigma = 1500$
- $\alpha = 0.05$

$\Rightarrow \sigma$ known
$\Rightarrow$ Case 1

$$\mathrm{Width} = 2\mathrm{E} = 2z_{\frac{\alpha}{2}}\frac{\sigma}{\sqrt{n}} = 2z_{0.025}\frac{1500}{\sqrt{50}} \approx 831.56$$

**Answer**: E

# Chapter 9

1. A manufacturer makes ball bearings that are supposed to have a mean weight of 30g. A retailer suspects that the mean weight is actually less than 30g. The mean weight for a random sample of 16 ball bearings is 28.8g with a standard deviation of 3.8g. At the 0.05 significance level, test the claim that the mean is less than 30g. Assume that the sample has been randomly selected from a population with a normal distribution.
   Let $t_{0.05,15} = 1.75$, $t_{0.025,15} = 2.13$
   A. Test statistic $t = -1.463$. There is not sufficient evidence to support the claim that the mean is less than 30g.
   B. Test statistic $t = -1.263$. There is not sufficient evidence to support the claim that the mean is less than 30g.
   C. Test statistic $t = -1.463$. There is sufficient evidence to support the claim that the mean is less than 30g.
   D. Test statistic $t = -1.263$. There is sufficient evidence to support the claim that the mean is less than 30g.
   E. None of the other choices is correct.

   **Solution**
   Given:
   - $n = 16$
   - $\overline{x} = 16$
   - $\sigma = 3.8$
   - $\alpha = 0.05$

   $\sigma$ known $\Rightarrow$ Case 1

$$\begin{cases} H_0\colon & \mu = 30 \\ H_1\colon & \mu < 30 \end{cases}$$

$$\omega_\alpha = \left(-\infty, -t_{\alpha,n-1}\right) \approx (-\infty, -1.75)$$

$$t_0 = \frac{\overline{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{28.8 - 30}{\frac{3.8}{\sqrt{16}}} \approx -1.263 \notin \omega_\alpha$$

   $\Rightarrow$ Reject $H_0$
   $\Rightarrow$ The mean is less than 30g
   **Answer**: B

2. Medicare would like to test the hypothesis that the average monthly rate for one-bedroom assisted-living facility is equal to \$3,300. A random sample of 12 assisted-living facilities had an average rate of \$3,690 per month and a standard of \$530. It is believed that the monthly rate for one-bedroom assisted-living facility is normally distributed. Use the significance level of 0.05 for this hypothesis test, what is the critical value?
   Let $z_{0.05} = 1.645$, $t_{0.025,11} = 2.201$, $t_{0.05,11} = 1.796$
   A. 2.201; −2.201
   B. 2.761; −2.761
   C. 1.985; −1.985
   D. 1.645; −1.645
   E. None of the other choices is correct.

   **Solution**
   Given:

- $n = 12$
- $\overline{x} = 3690$
- $s = 530$
- $\alpha = 0.05$

$\sigma$ unknown and $n < 30 \Rightarrow$ Case 3

$$\begin{cases} H_0: & \mu = 3300 \\ H_1: & \mu \neq 3300 \end{cases}$$

$$t_{\frac{\alpha}{2},n-1} = t_{0.025,11} \approx 2.201$$

**Answer**: A

3. The Department of Mathematics would like to test the hypothesis that the average debt load of graduating students with a Bachelor's degree is equal to \$17,000. A random sample of 34 students had an average debt load of \$18,200. It is believed that the debt load follows a normal distribution with the standard deviation of \$4,200. At the significance level of 0.05, the critical value(s) for this hypothesis test would be _____.
Let $z_{0.025} = 1.96$, $z_{0.05} = 1.645$, $z_{0.1} = 1.28$
A. −1.96; 1.96
B. 1.28
C. −1.28
D. −1.645; 1.645
E. None of the other choices is true.

**Solution**
Given:
- $n = 34$
- $\overline{x} = 18200$
- $\sigma = 4200$
- $\alpha = 0.05$

$\sigma$ known $\Rightarrow$ Case 1

$$\begin{cases} H_0: & \mu = 17000 \\ H_1: & \mu \neq 17000 \end{cases}$$

$$z_{\frac{\alpha}{2}} = z_{0.025} = 1.96$$

**Answer**: A