Flight Mechanics

UCK 322E: Term Project
CRN: 12159

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1 Introduction

The flight planning, aircraft motion and some basic research are conducted in this document. The cooperative work of students presented both analytic and numeric solution of flight planning of a preselected aircraft departing from Dallas Love Field, arriving at Frankfurt Main airports. The airport selection are made with respect to the longest distance according to the assignment file. During the route planning, some linearization assumptions are made to lighten the calculation burden, and to ease comparison between two approaches to the flight problem.

1.1 Departure: Dallas Love Field Airport

Dallas Love Field Airport is an internal line airport located at 9.7km northwest from Dallas Downtown. The airfield and airport is named after the death of Moss L. Love in an aircraft incident 4 years prior to 1917 during Military Aviator Test practice. The airport is the major hub for Southwest Airlines which is titled as the world's largest low-cost carrier.

Figure 1: Dallas Love Field Airport (Left), Airport Logo (Right)

The airport has a single terminal with 20 gates for passengers from which 90% of them are claimed by Southwest Airlines. In 2020, There were 170162 flights with over 7.6 million passengers.
The airport has 2 runways around the terminal. 13L, Left-coded, runway has length of 2363 meters and made of concrete. 13R, Right-coded, runway has the length of 2682 meters made of concrete as well [10].

1.2 Arrival: Flughafen Frankfurt Main

Flughafen Frankfurt Main, or Frankfurt Airport, is a major international airport located at 12 km southwest of Frankfurt city. The airport is the main hub of Lufthansa and its subsidiaries. It holds the rank of 4th busiest airport in Europe and 13th busiest worldwide.

Figure 2: Frankfurt Airport (Left), Airport Logo (Right)

The airport has three terminals which Terminal 1 is the first and largest terminal used by mostly Star Alliance where the other one, Terminal 2 is used by other alliances such as Oneworld and SkyTeam. In 2020, There were over 18.7 million passengers even though there was an outbreak of a pandemic disease.
The airport has 4 runways on the field. First one is 07L, which has length of 2.8 km and allows only landings, made of concrete. 07C and 07R have the length of 4 km made of asphalt. The last runway is located at a different spot, which has a direction denoted with 18, length of 4 km [11].

1.3 Departure Time

Based on the flight trajectory, specifications that departure and arrival airports have (runway), how occupied the route and the airport is(capacity) etc. effecting factors of the departure time. Considering business of both departure and arrival airports, 4 pm is suitable for departure time referring to the flight of American Airlines on January 2nd 2022 with a B772 which took off at 15:58.

1.4 Trajectory

The flight path taken from FlightRadar24 shows that the heading of the aircraft changes insignificantly during cruise phase, only changing before leaving Canadian skies and after entering Great Britain's airspace. Therefore the trajectory in Figure 3 is linearized to get rid of the extra calculations which will barely affect the overall fuel consumption, represented in Figure 4.

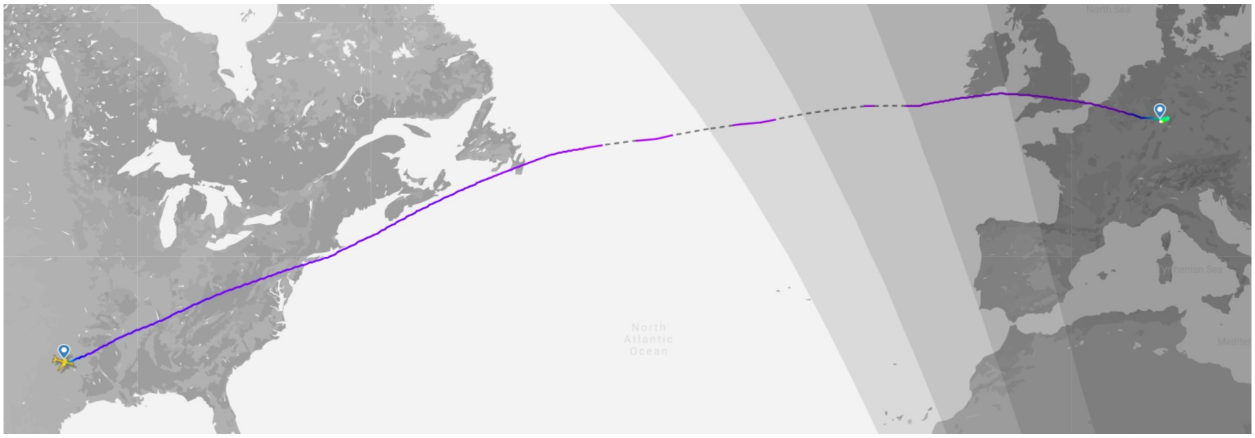

Figure 3: FlightRadar24 Trajectory

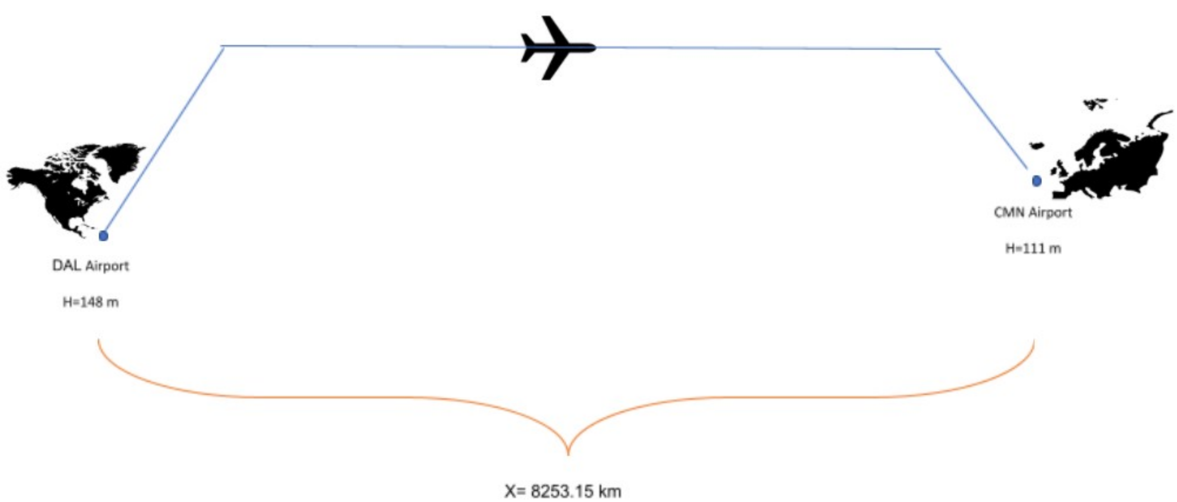

Figure 4: Linearized Trajectory

The required data for the flight path breakpoints are to be defined thanks to the tabulated data represented below about runway specifications, locations and the total distance as a linearized trajectory. Then an aircraft from the American Airlines which has post-Atlantic flights will be chosen in the next title.

	DAL Airport	FRA Airport
Location	32.8459447 N -96.8508767 W	50.033333 N 8.570556 W
Elevation	148 m	111 m
Runway Dimensions	2682 m	4000 m
City	Dallas	Frankfurt
Country	USA	Germany
Airport Type	Large	Large
Distance	8253.15 km	

Table 1: Airport Data

1.5 Aircraft Assignment

Assignment of the aircraft for a specific flight requires consideration of flight range, flight duration, runway distances and amount of payload. Distance between given airports and runway lengths are main choice reasons. The chosen aircraft is Boeing 777-300 with Pratt & Whitney PW4000 Turbofan engine. The basic specifications of the aircraft are presented in the table below.

Aircraft	B777-300		
Range	9,700 km	**Typical Cruise Speed at 35000 ft**	905 km/h
Take-off Distance	2530 m	**AR**	8.68
Landing Distance	1550 m	**Fuel Flow rate**	3730 kg/h
Max. Ceiling	44100 ft	**Wing area**	427.8 m^2
Max. Cruise Speed at 35000 ft	950 km/h	**Max Thrust**	343 kN
Service Ceiling	43100 ft	**Number of Engines**	2
MTOW	247200 kg	**Fuel cons**	0.6 N/h/N

Table 2: Aircraft Specifications Taken from Internet Resources

2 Analytic Approach

2.1 Take off

The take off from Dallas Love Field Airport Runway 13L is represented analytically in this section. The governing equations are taken from the lecture notes with similar assumptions. These assumptions are following as such, overall runway slope is 0, the density ratio is calculated via the runway elevation, throttle is at maximum setting, drag and lift values are calculated as mean values for the aircraft which is stationary initially, then reaches to lift off speed linearly. The used equations are represented below.

$$a_x \approx \frac{g}{W}[T - D_{TO} - W\phi - \mu_r(W - L_{TO})]$$
$$C_{L_{opt}} = \frac{\mu_R}{2K}$$
$$S_G = \frac{1}{\bar{a}}\frac{V_G^2}{2}$$

Some of the used aircraft data is tabulated below, which are required for the later evaluations as well.

c_{D_0}	**V_{LOF}, [m/s]**	**Oswald coeff.**	**Gravit. Accel. [m/s^2]**	**Max. Fines**
0.016	86.427	0.04287	9.81	19.09

Table 3: Performance Characteristics

As the result of these calculations, take off outputs are as such,

V_{avg}, [m/s]	61.113
h_1, [m]	148
σ_{h1}	0.983
μ_R, **[0.02-0.3]**	0.03
Runway Slope, ϕ	0
Acceleration, [$\frac{m}{s^2}$]	2.439
Take off distance, [m]	765.65

Table 4: Take off Data

2.2 Climb

The climb phase is calculated with maximum rate of climb speed. The used equations are represented below as well. Maximum ceiling level and thrust to weight ratio at the sea level are known, E_m can be found. Therefore, the aircraft is climbed to 13 km with steps, data tabulated below for each required parameter.

$$\sigma_c = \frac{1}{E_m(T/W)_{SL}}$$

$$\Gamma = 1 + \left\{1 + \frac{3}{[E_m(T/W)]^2}\right\}^{\frac{1}{2}}$$

$$(R/C)_{max} = V_{(R/C)_{max}} sin\gamma_{(R/C)_{max}}$$

$$(R/C)_{max} = \sqrt{\frac{(T/S)\Gamma}{3\rho_{SL}\sigma c_{D_0}}}\left[\frac{T}{W}\left(1 - \frac{\Gamma}{6}\right) - \frac{3}{2(T/W)E_m^2}\right]$$

$$t_{(R/C)_{max}} = t_{min} = \frac{7254}{(T/W)_{SL}}\sqrt{\frac{27\rho_{SL}c_{D_0}}{8(T/S)_{SL}}}(e^{h_2/7254} - e^{h_1/7254})$$

$$\left(\frac{\Delta W_f}{W}\right) = 1 - e^{\frac{-c}{3600}\sqrt{\frac{27\rho_{SL}c_{D_0}}{8(T/S)_{SL}}}(h_1 - h_2)}$$

$$X_{(R/C)_{max}} = \frac{10.88}{(T/W)_{SL}}(e^{h_2/7254} - e^{h_1/7254})$$

Altitude, [m]	σ	**t**$_{(R/C)_{max}}$**, [s]**	**X, [km]**	$(\frac{\Delta W_f}{W})$	**W, [N]**	**Fuel Cons [N]**
0	1	0	0	0	2425032	0
1000	0.907478	24.34198	5.684	0.00107	2422438	2.594
2000	0.821677	27.93994	6.524	0.00107	2419846	2.592
3000	0.742248	32.06972	7.489	0.00107	2417257	2.589
4000	0.668854	36.80991	8.596	0.00107	2414671	2.586
5000	0.601166	42.25075	9.866	0.00107	2412088	2.583
6000	0.538866	48.4958	11.325	0.00107	2409507	2.581
7000	0.481647	55.66392	12.999	0.00107	2406929	2.578
8000	0.429212	63.89155	14.920	0.00107	2404354	2.575
9000	0.381275	73.3353	17.125	0.00107	2401782	2.572
10000	0.337558	84.17493	19.657	0.00107	2399212	2.570
11000	0.297796	96.61674	22.562	0.00107	2396645	2.567
12000	0.254642	110.8976	25.897	0.00107	2394081	2.564
13000	0.217628	127.2892	29.725	0.00107	2391520	2.561

Table 5: Climb Data (1)

Altitude, [m]	Thrust [N]	$\frac{T}{W}$	Γ	$(\frac{R}{C})_{max}$	$V_{(\frac{R}{C})_{max}}$	$\gamma_{(\frac{R}{C})_{max}}$
0	686000	0.282883	2.05018	40.0741	224.9212	0.179126
1000	622529.6	0.256985	2.06050	36.1375	225.4865	0.160959
2000	563670.2	0.232936	2.07318	32.4410	226.1795	0.143927
3000	509182.2	0.210645	2.08882	28.9710	227.0310	0.127957
4000	458833.8	0.190019	2.10815	25.7135	228.0789	0.112979
5000	412399.7	0.170972	2.13209	22.6537	229.3702	0.098926
6000	369662.2	0.153418	2.16179	19.7765	230.9623	0.085731
7000	330410.1	0.137275	2.19869	17.0659	232.9251	0.073333
8000	294439.6	0.122461	2.24456	14.5055	235.3424	0.061675
9000	261554.7	0.1089	2.30159	12.0784	238.3137	0.050704
10000	231565	0.096517	2.37248	9.7670	241.9556	0.040378
11000	204288	0.085239	2.46048	7.5532	246.4022	0.030659
12000	174684,2	0.072965	2.59570	4.97695	253.0823	0.019667
13000	149292.6	0.062426	2.76421	2.56283	261.1679	0.009813

Table 6: Climb Data (2)

2.3 Cruise

The cruise distance is obtained from The required equations are represented below and results are tabulated.

$$\gamma_{BR;V,c_L} = \frac{8.38c}{V_{BR}E_m}, rad$$
$$\Delta h = 7254ln\left(\frac{1}{1-\xi}\right)$$
$$V_{BR} = \sqrt{\frac{2(W/S)}{\rho_{SL}}\left(\frac{3k}{c_{D_0}}\right)^{\frac{1}{4}}}$$
$$tan\gamma_{BR;V,c_L} = \frac{\Delta h}{X_{BR;step}}$$

Δ h	**Fuel Consumption**	**V$_{BR}$**	γ_{BR}	**X$_{BR;step}$ [km]**	**X [km]**	**Step#**
1000	0.128774969	328	0.000802995	12,453.383	7,814.723	6.27518

Table 7: Cruise Step Specifications

Step#	$X_{BR;step}$ **[km]**	**W$_{fraction}$**	**Time [s]**	**Fuel Used [N]**	**Weight [N]**
1	1,245.338	0.133360579	3,796.737	318934.5	2072585
2	1,245.338	0.133360579	3,796.737	276401.2	1796184
3	1,245.338	0.133360579	3,796.737	239540.2	1556644
4	1,245.338	0.133360579	3,796.737	207595	1349049
5	1,245.338	0.133360579	3,796.737	179910	1169139
6	1,588.032	0.168633056	4,841.527	197155.5	971983.6

Table 8: Data of Each Step

Different cruise methods were also researched, and was tried to implement on the trajectory. The first selection was ideal for fuel consumption compared to method 3 while other two were taking too long to reach the target

Fuel fraction [-]	**Total fuel consumption [N]**	**Time [h]**	**Method**
0.579	1,384,396	6.616	Graded Cruise Flight
0.579	1,384,025	23.82	Method 1
0.678	1,620,587	31.19	Method 2
0.740	1,768,558	6.616	Method 3

Table 9: Data for Different Methods

2.4 Descent

The descent phase of the flight is modelled analytically in this section with respect to gliding flight equations due to the absence of idle throttle settings data, leading the assumption of zero throttle. The descent lasted from the altitude of the last point of the cruise to the sea level, even though the landing runway is 111m above the sea level. Under these assumptions, the calculations are made,

h	**h$_1$-h$_2$**	σ	**R/D**	V_{tot}	**Time [s]**
13000	1000	0.21763	-10.9552	78.3728	82.6844
12000	1000	0.25464	-10.1277	72.4533	88.5847
11000	1000	0.29780	-9.3652	66.998	94.9059
10000	1000	0.33756	-8.7964	62.9286	101.6783
9000	1000	0.38128	-8.2767	59.2111	108.9339
8000	1000	0.42921	-7.8008	55.8067	116.7072
7000	1000	0.48165	-7.3640	52.6815	125.0353
6000	1000	0.53887	-6.9620	49.8061	133.9576
5000	1000	0.60117	-6.5914	47.1548	143.5166
4000	1000	0.66885	-6.2490	44.7051	153.7578
3000	1000	0.74225	-5.9320	42.4373	164.7297
2000	1000	0.82168	-5.6380	40.3341	176.4846
1000	1000	0.90748	-5.3649	38.3800	189.0783
0	0	1.00000	-5.1107	36.5614	0.0000

Table 10: Descent Data

2.5 Landing

The landing runway is chosen as 07L of Frankfurt Main Airport since it is concrete with elevation of 111 m. Approach speed is taken from flight planning, stall speed and touchdown speeds are therefore regulated, Thrust reverser is 75% of T_{max}, Runway had no data, so it was taken as a mean value for friction coefficient. The used equations and approaches are taken from lecture notes as follows,

$$V_{app} = 1.3V_{stall}$$
$$V_{TD} = 1.15V_{stall}$$
$$S_{G_{Landing}} = \frac{W}{g\rho_{h_2}S(C_D - \mu_r C_L)} ln\left[1 + \frac{(C_D - \mu_r C_L)\rho_{h_2} S V_{TD}^2}{2(-T_{reverse} + \mu_r W)}\right]$$

$\mathbf{T}_{reverse}$	μ_R, **0.02-0.3**	**h$_2$, [m]**	**mass$_{last}$**	σ_{h_2}	ρ_{h_2}, **[kg/m^3]**
514500	0.15	111	99080.897	0.987	1,209
$\mathbf{V}_{approach}$, **[m/s]**	$\mathbf{V}_{nom,stall}$, **[m/s]**	$\mathbf{V}_{touchdown}$, **[m/s]**	$\mathbf{c}_{L_g}$	$\mathbf{c}_{D_g}$	$\mathbf{S}_g$, **[m]**
76.652	58.963	67.807	0.81748	0.04465	371.715

Table 11: Landing Data

In the end of analytic evaluations under assumptions, the overall flight time, distance and fuel consumption are tabulated below.

Flight Phase	**Time [h]**	**Distance [km]**	**Fuel Used [N]**
Climb	0.2288	192.38	33,512.09
Cruise	6.6163	7,812.60	1,419,283.31
Descent	0.4439	248.17	0
Total	7.2891	8,253.15	1,452,795.39

Table 12: Overall Flight Data

3 Numerical Approach

In this section, the flight is modelled dynamically with the help of MATLAB and aircraft motion equations. The used functions are taken from homework 3 and modified in order to be used without an operational performance file, then created our own performance parameter variable to store similar parameters with .opf file. The flight plan is first determined as altitudes and cruise time, then functions for each phase is specifically prepared for climb, cruise and descend. Take off and Landing phases could not be modelled since the acceleration is one directional and lift polar was missing to model the vertical equilibrium.

3.1 Trajectory Planning

The aircraft taking off from Dallas Love Field Airfield is assumed to have lift off speed initially when starting to climb, and it was planned to climb to 13 km as calculated in analytic section. The flight simulation will not have heading changes, making the same assumption in analytic part. The climb path angle is calculated through the same way as in the analytic part but instantaneously. The cruise lasted for the distance determined before the evaluation, the difference of total distance and sum of distances dued in climb and descent. The nominal distance for descend was taken as 250 km. The path angle taken as constant 0, throttle is varying with the weight changing because of the fuel consumption. As the cruise ends, the descend starts with zero throttle assumption since throttle data is missing for idle settings, and the path angle is obtained from the same gliding path angle expression.

3.2 Programming Implementations

The previous functions from homework 3 were used and modified for this specific project. All the codes can be found in Appendix-A.

Aircraft motion, acceleration, rate of climb, ground projected airspeed and mass change functions are adjusted to have inputs from aircraft array. Cruise airspeed, Gamma variable, different phases' path angle and thrust functions are written especially for this project to have appropiate control inputs. The array, "aircraft", includes initial mass, total surface area of the wings, parasitic drag coefficient, Oswald drag coefficient factor, sea level thrust value, thrust specific fuel consumption and max fines values of the aircraft, B773.

The flight is discretized as phases, then aircraft motion function is implied for each phase while recording the state and control.

3.3 Results

The outputs of the main body script are represented as charts in figures below for True Airspeed-Time, Rate of Climb-Time, Mass-Time, Thrust-Time, and Path Angle-Time relations.

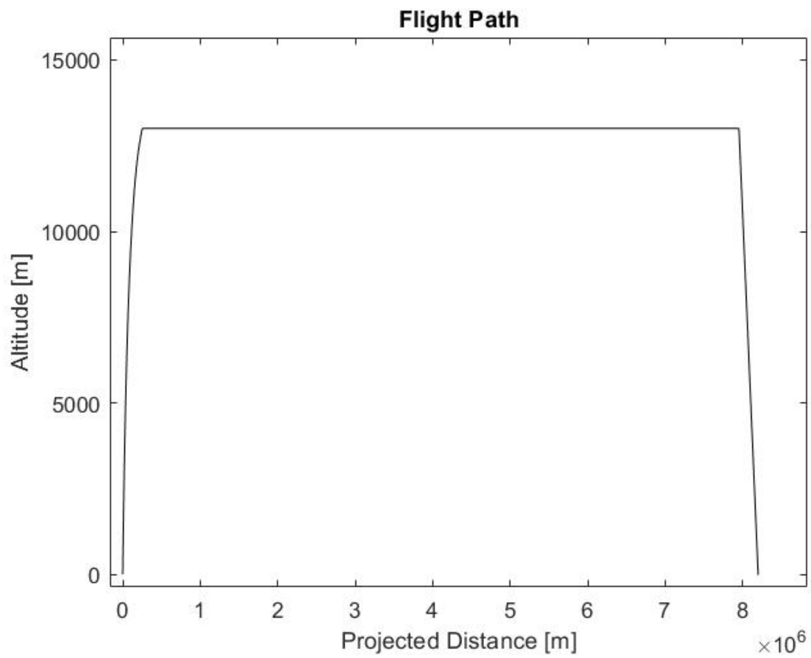

Figure 5: Flight Path of the Aircraft

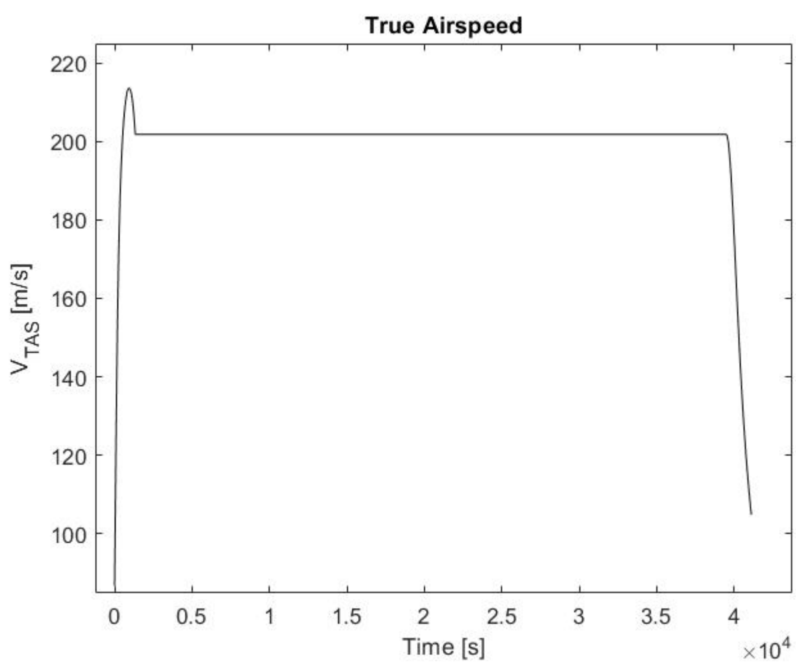

Figure 6: True Airspeed Profile

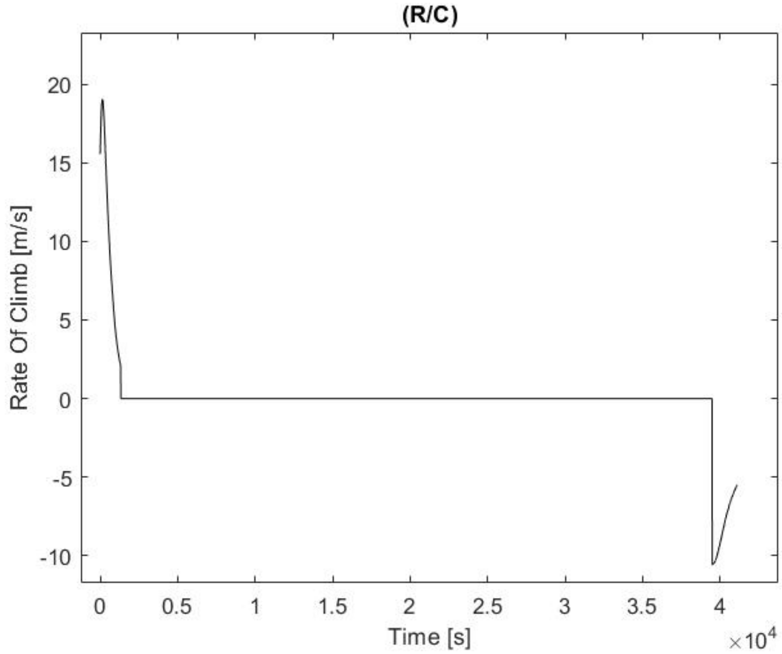

Figure 7: Rate of Climb Over Time

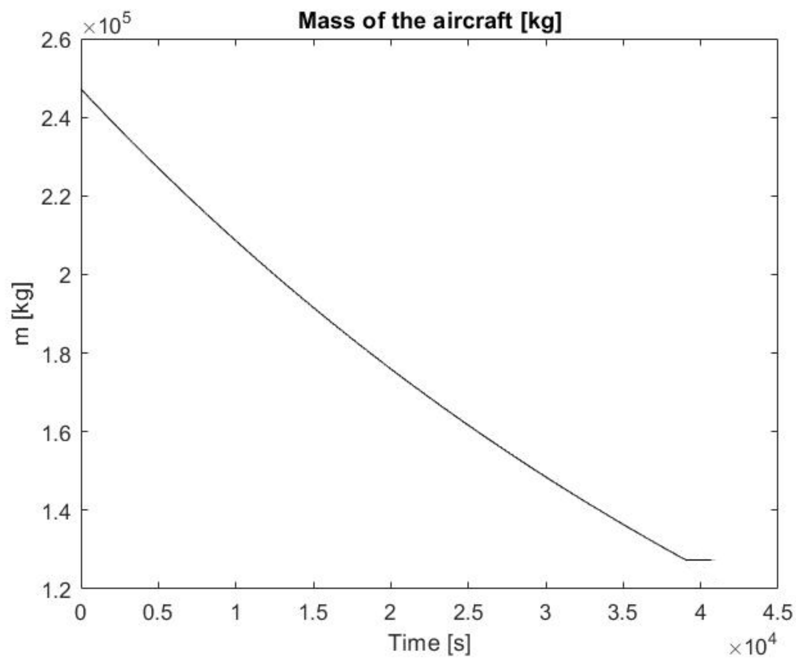

Figure 8: Mass Change Over Time

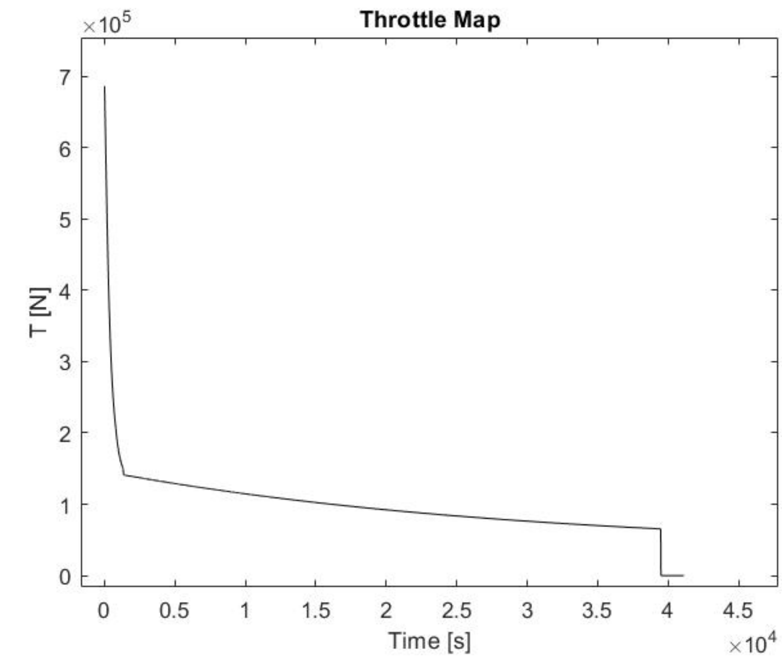

Figure 9: Thrust Map

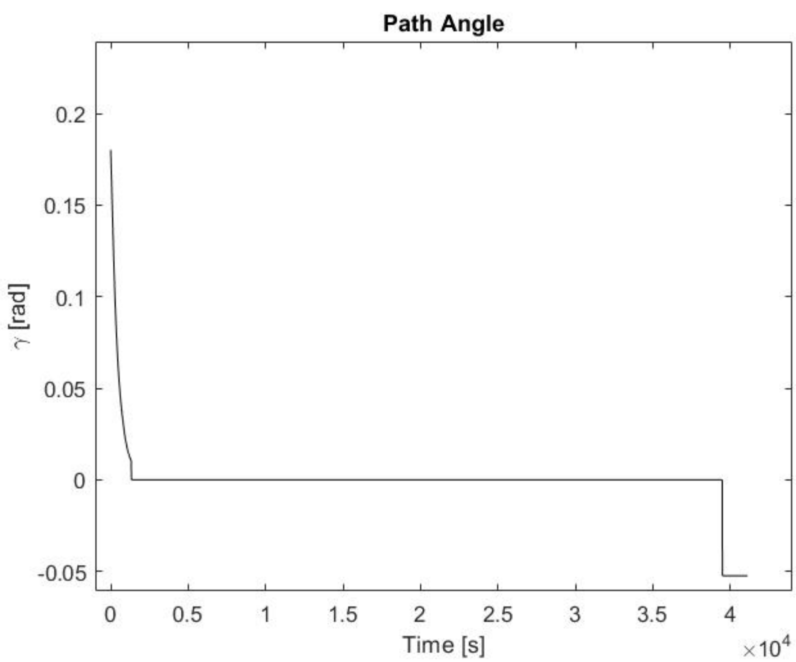

Figure 10: Path Angle Map

The total fuel consumption, flight duration and distances for each phase are displayed on the MATLAB interpreting screen, shown below.

Flight Phase	**Time [h]**	**Distance [km]**	**Fuel Used [kg]**
Climb	0.371111	253.8544	5543.669
Cruise	10.6	7699.404	115289
Descent	0.450278	501.8959	0
Total	11.42139	8455.154	120832.7

Table 13: Flight Data of Numerical Solution

4 Discussion

At the end of numerical and analytic solutions, it can be seen that the analytic approach gives near-exact results for real values of a real flight plan under extreme assumptions while the numerical approach have a different assumption for throttle settings. Therefore, the cruise speed changes with decreasing throttle and weight which extends the flight time by nearly 3 hours. At the end of the day, it was an entertaining project for those who wants to verify their work by using the lecture learned in the class.

4.1 Job Sharing

		Atakan Öztürk	**Hilal Cankurtaran**
Introduction	Airport Research	X	
	Departure Time		X
	Aircraft Selection		X
Analytic Approach	Take off	X	
	Climb		X
	Cruise		X
	Descent		X
	Landing	X	
Numerical Approach	MATLAB Programming	X	
Reporting		X	

Table 14: Team Work

References

[**1**] Flight Trajectory taken from FlightRadar24
https://www.flightradar24.com/data/flights/aa71#2a665ae9
[**2**] The programming language used
MATLAB20a
[**3**] Boeing 777-300 Specifications
https://modernairliners.com/boeing-777/boeing-777-specs/
[**4**] Distance between DAL-FRA
https://www.prokerala.com/travel/airports/distance/from-dal/to-fra/
[**5**] Fuel consumption of B773
https://alliknowaviation.com/2019/12/14/fuel-consumption-aircraft/
[**6**] Lift off speed of B773
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[**8**] Air density with altitude calculator
https://www.omnicalculator.com/physics/air-pressure-at-altitude
[**9**] Thrust Reverser Approach
https://aviation.stackexchange.com/questions/14597/how-much-thrust-can-the-thrust-reversers-create
[**10**] The basic information about Dallas Love Field Airport
https://en.wikipedia.org/wiki/Dallas_Love_Field
[**11**] The basic information about Frankfurt Airport
https://en.wikipedia.org/wiki/https://en.wikipedia.org/wiki/Frankfurt_Airport#First_airport
[**12**] Photograph of Frankfurt Airport
https://structurae.net/en/structures/frankfurt-airport-terminal-1

Appendix-A: MATLAB Codes

Some of the parts of codes are bugged when printing to the LaTeXdue to an encoding issue about UTF-8.

Main Body

```
% Flight Mechanics Term Project, Atakan  zt  rk & Hilal
    Cankurtaran,
% Aircraft Dynamic Model
clc;clear;close;tic;

%% Aircraft Definition
% Required parameters
m_initial=247.2*1000;     % 1,kg
S=427.8;                  % 2,m^2
c_d_0=0.016;              % 3,-
c_d_2=0.04287;            % 4,-
T_sl=343e3*2;             % 5,N
c=0.6;                    % 6,N/h/N
E_m=19.09;                % 7,-
g=9.81;                   % m/s2
V_lof=86.42;              % m/s
headingInit=0;            % Assume heading angle of departing
    runway
pathInit=T_sl/m_initial/g-1/E_m;
total_dist=8253.15e3;     % total distance from DAL to FRA, m
x_des_nom=300e3;        % Nominal descending distance
x=[0 0 0 V_lof headingInit m_initial]; % State
u=[T_sl 0 pathInit]; % Control
aircraft=[m_initial;S;c_d_0;c_d_2;T_sl;c;E_m];  % Aircraft

%% Aircraft Motion
% Initialization and Preallocation for state-control parameters
stepsize=1;
x_log=NaN(1,6);
u_log=NaN(1,3);
iter_i=1;
t_0=0;

% Climb Phase
% Climb to h_2 from h_1
h_2=13e3;
```

```
bank=0;
while x(3) < h_2
    T=thrust(x,u,aircraft);
    path=gamma_cl(x,u,aircraft);
    u=[T bank path];
    [x,u]=aircraft_motion(x,u,aircraft,stepsize);
    x_log(iter_i,:)=x;
    u_log(iter_i,:)=u;
    iter_i=iter_i+1;
end
x_tab(1,:)=[x iter_i];
% Cruise
% For x_cr distance
%{
 %Graded Climb Cruise - Wrong approach
x_cl=sqrt(x(1)^2+x(2)^2);
x_cr=total_dist-x_cl-x_des_nom;
path=gamma_cr(x,u,aircraft); % Defining at the outside of the loop
bank=0;
while sqrt(x(1)^2+x(2)^2) < x_cr+x_cl
    T=thrust(x,u,aircraft);
    u=[T bank path];
    [x,u]=aircraft_motion(x,u,aircraft,stepsize);
    x_log(iter_i,:)=x;
    u_log(iter_i,:)=u;
    iter_i=iter_i+1;
end
%}

 % Constant Height flight, variable T, V, cL
x_cl=sqrt(x(1)^2+x(2)^2);
x_cr=total_dist-x_cl-x_des_nom;
path=0; % Defining at the outside of the loop
bank=0;
while sqrt(x(1)^2+x(2)^2) < x_cr+x_cl
    T=drag(x,u,aircraft);
    u=[T bank path];
    [x,u]=aircraft_motion(x,u,aircraft,stepsize);
    x_log(iter_i,:)=x;
    u_log(iter_i,:)=u;
    iter_i=iter_i+1;
end
x_tab(2,:)=[x iter_i];
```

```
% Descent
bank=0;
path=gamma_des(x,u,aircraft);
while x(3) > 0
    T=0;   % Gliding Flight
    u=[T bank path];
    [x,u]=aircraft_motion(x,u,aircraft,stepsize);
    x_log(iter_i,:)=x;
    u_log(iter_i,:)=u;
    iter_i=iter_i+1;
end
x_tab(3,:)=[x iter_i];
%% Plotting
figure; plot(sqrt((x_log(:,1).^2+x_log(:,2).^2)),x_log(:,3),'k');
title('Flight Path'); xlabel('Projected Distance [m]'); ylabel('
    Altitude [m]');

figure; plot(x_log(:,4),'k'); title('True Airspeed'); xlabel('Time
     [s]');
ylabel('V_{TAS} [m/s]');

figure; plot(x_log(:,4).*sin(u_log(:,3)),'k'); title('(R/C)');
    xlabel('Time [s]');
ylabel('Rate Of Climb [m/s]');

figure; plot(x_log(:,6),'k'); title('Mass of the aircraft [kg]');
xlabel('Time [s]'); ylabel('m [kg]');

figure; plot(u_log(:,1),'k'); title('Throttle Map');
xlabel('Time [s]'); ylabel('T [N]');

figure; plot(u_log(:,3),'k'); title('Path Angle');
xlabel('Time [s]'); ylabel('\gamma [rad]');

fprintf('Total Fuel Consumption is %.2f [kg]\n\n', m_initial-x(6))
    ;
fprintf('Total Flight Duration is %.2f [h]\n\n', iter_i/3600);

toc
```

Acceleration Function

```
function [OUT] = accel(x,u,aircraft)
[D,~]=drag(x,u,aircraft);                    % -
g=9.81;                                      % m/s^2
S=aircraft(2);                               % m^2
OUT=-g*sin(u(3))-D/x(6)+u(1)/x(6);
end
```

Drag Function

```
function [D,cD] = drag(x,~,aircraft)
% D=1/2*ro*v^2*s*cd
% V is true airspeed
% dT is temperature difference from isa
% H is altitude
dT=0;
g=9.81;
cD_0=aircraft(3);
cD_2=aircraft(4);
[~,~,rho,~]=non_ISA(dT,x(3));

S=aircraft(2);                               % m^2
c_L=x(6)*g/(1/2*rho*x(4)^2*S);                 % -

cD=cD_2*c_L^2+cD_0;
D=1/2*rho*x(4)^2*cD*S;
end
```

Fuel Consumption Function

```
function [c] = fuelcons_jet(~,~,~)
c=0.6/3600;
end
```

Γ Function

```
function [OUT]=gamFunc(x,u,aircraft)
E_m=aircraft(7);
W=9.81*x(6);
T=u(1);
OUT=1+sqrt(1+3/(E_m*T/W)^2);
end
```

Climb Path Angle

```
function [OUT] = gamma_cl(x,u,aircraft)
Gamma=gamFunc(x,u,aircraft);
T=u(1);
W=x(6)*9.81;
E_m=aircraft(7);
OUT=asin((T/W*(1-Gamma/6)-3/(2*Gamma*E_m^2*T/W)));
end
```

Cruise Path Angle

```
function [OUT] = gamma_cr(x,u,aircraft)
% Best range path angle for graded cruise
OUT=8.38*aircraft(6)/v_cr(x,u,aircraft)/aircraft(7);
end
```

Descending Path Angle

```
function [OUT] = gamma_des(~,~,aircraft)
OUT=-1/aircraft(7);
end
```

Mass Change Function

```
function [OUT] = masschange(x,u,aircraft)
C=fuelcons_jet(x,u,aircraft);         % Fuel consumption constant
W=x(6);                               % Momentarily Weight
OUT=-W*C/9.81;                        % Fuel Consumption
end
```

Atmosphere Model

```
function [T,P,ro,a] = non_ISA(dT,h)
%% Parameter Definitions
T_0=288.15; % [K] Sea level standard atmospheric temperature
P_0=101325; % [Pa] Sea level standard atmospheric pressure
ro_0=1.225; % [kg/m3] Sea level standard atmospheric density
a0_0=340.294; % [m/s] Sea level speed of sound

kappa=1.4; % [] Adiabatic index of air
R=287.05287; % [m2/(Ks2)] Real gas constant of air
g_0=9.80665; % [m/s2] Gravitational acceleartion

grad=-0.0065; % [K/m] Temperature gradient below tropopause
```

```
h_tp=11000; % [m] Tropopause altitude

%% Temperature
if h < h_tp
    T=T_0+dT+grad*h;
else % Else cond. for temperature above troposphere
    T=T_0+dT+grad*h_tp;
end

%% Pressure
if h <= h_tp
    P=P_0*((T-dT)/T_0)^(-g_0/(grad*R)); % Below Troposphere
else
    P_trop=P_0*((T-dT)/T_0)^(-g_0/(grad*R));
    P=P_trop*exp((h_tp-h)*(g_0/(R*(T-dT)))); % Above Troposhpere
end

%% Density
ro=P/R/T; % From perfect gas law

%% Speed of sound
a=sqrt(kappa*R*T); % From the definition

end
```

Thrust Function

```
function [OUT] = thrust(x,~,aircraft)
% Assume in stratosphere
[~,~,rho_sl,~]=non_ISA(0,0);
[~,~,rho,~]=non_ISA(0,x(3));
T_max=aircraft(5)*rho/rho_sl;
% Thrust at altitude
OUT=T_max;
end
```

Turn Rate Function

```
function [OUT] = turnrate(x,u,aircraft)
S=aircraft(2);                    % m^2

[~,~,rho,~]=non_ISA(0,x(3));              % kg/m^3
c_L=x(6)/(1/2*rho*x(4)^2*S);              % -

```

```
OUT=-c_L*S*rho*x(4)/2/x(6)*sin(u(2));
end
```

Cruise Airspeed Function

```
function [OUT] = v_cr(x,~,aircraft)
g=9.81;
[~,~,rho,~]=non_ISA(0,x(3));
% Best range velocity for graded cruise
OUT=((2*x(6)*g/aircraft(2))/(rho))^.5*(3*aircraft(4)/aircraft(3))
    ^.25;
end
```

X Component of Ground Projected Airspeed

```
function [OUT] = x_vel(x,u)
OUT=x(4)*cos(x(5))*cos(u(3));
end
```

Y Component of Ground Projected Airspeed

```
function [OUT] = y_vel(x,u)
OUT=x(4)*sin(x(5))*cos(u(3));
end
```

Rate of Climb

```
function [OUT] = z_vel(x,u)
OUT=x(4)*sin(u(3));
end
```