Atividade 5

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1) Sejam $F(x,y,z)=(x+2y-z, x-2y+z, -x+z)$, $G(x,y,z)=(3x+y-z, x-y+2z, -x-y)$ e $H(x,y,z)=(z,y,x)$. Determine:

a) $F \circ G$

$$F(G(x,y,z)) = (3x+y-z+2(x-y+2z)-(-x-y),\ 3x+y-z-2(x-y+2z)-x-y,\ -(3x+y-z)-x-y)$$

$$\boxed{F(G(x,y,z)) = (6x+3z, 2y-5z, -4x-2y+z)}$$

b) $G \circ F$

$$G(F(x,y,z)) = (3(x+2y-z)+x-2y+z-(-x+z),\ x+2y-z-(x-2y+z)+2(-x+z),\ -(x+2y-z)-(x-2y+z))$$

$$\boxed{G(F(x,y,z)) = (5x+4y-3z, -2x+4y, -2x)}$$

c) $H \circ F$

$$\boxed{H(F(x,y,z)) = (-x+z, x-2y+z, x+2y-z)}$$

d) $H \circ G$ → $\boxed{H(G(x,y,z)) = (-x-y, x-y+2z, 3x+y-z)}$

2) Determine a lei da transformação linear $G: \mathbb{R}^3 \to \mathbb{R}^2$, cuja matriz em relação às bases $B=\{(1,1,1);(1,1,0);(1,0,0)\}$ e $C=\{(1,3);(2,5)\}$ é $(G)_{B,C}=\begin{pmatrix} -7 & -33 & -12 \\ 4 & 19 & 8 \end{pmatrix}$.

- $F(1,1,1) = -7(1,3)+4(2,5) = (1,-1)$
- $F(1,0,0) = -13(1,3)+8(2,5) = (3,1)$
- $F(1,1,1) = -33(1,3)+19(2,5) = (5,-4)$

→ $U(x,y,z) = a(1,1,1)+b(1,1,0)+c(1,0,0)$ → $\{(1,1,1),(1,1,0),(1,0,0)\}$ é base do $\mathbb{R}^3$.

logo,

$$\begin{cases} a+b+c=x \\ a+b=y \\ a=z \end{cases} \Rightarrow \therefore\ a=z,\ b=y-z,\ c=x-y$$

$$F(x,y,z) = aF(1,1,1)+bF(1,1,0)+cF(1,0,0)$$
$$= z(1,-1)+(y-z)(5,4)+(x-y)(3,1)$$
$$= (z+5y-5z+3x-3y),(-z-4y+4z+x-y)$$
$$= (-4z+2y+3x),(3z-5y+x)$$

$$\therefore\ F(x,y,z)=(-4z+2y+3x,\ 3z-5y+x)$$