Aula

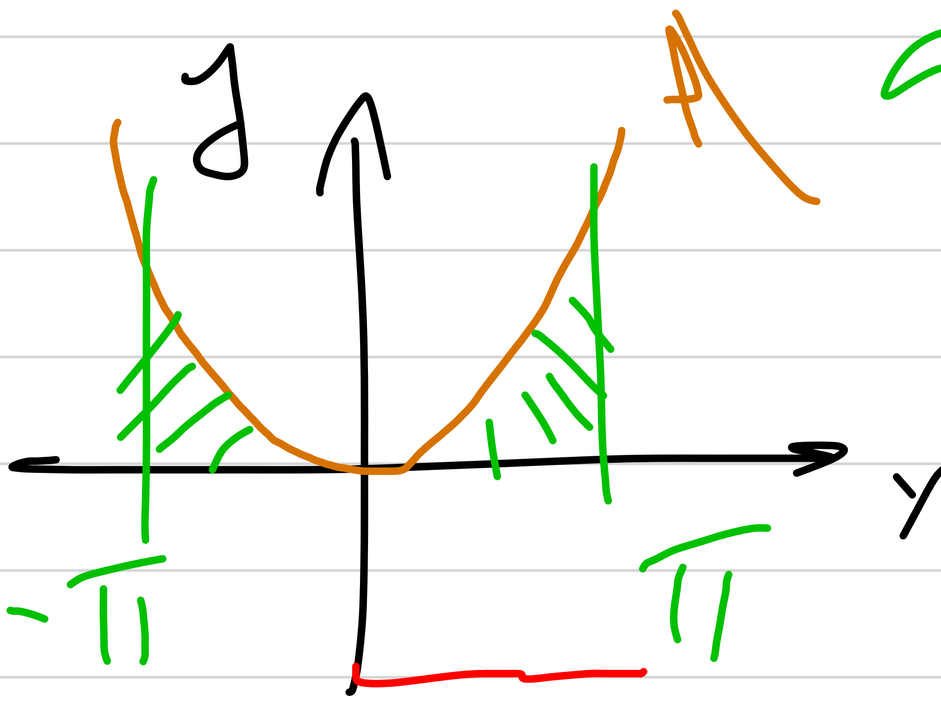
CA4141

12 nov 21

$$f(x) = x^2, \quad -\pi \le x \le \pi$$

$$f \text{ par} \implies b_n = 0 \quad \forall n \ge 1$$

$$a_0 = \frac{1}{\pi}\underbrace{\int_{-\pi}^{\pi} x^2\, dx}_{\text{área}}$$



$$a_0 = \frac{2\pi^2}{3}$$

$$a_0 = \frac{2}{\pi}\int_0^{\pi} x^2\, dx$$

$$a_0 = \frac{2}{\pi}\cdot\frac{x^3}{3}\Big|_0^{\pi} = \frac{2\pi^{\cancel{3}\,2}}{3\cancel{\pi}} - 0$$

$$a_n = \frac{1}{\pi} \boxed{\int_{-\pi}^{\pi} x^2 \cos(nx)\, dx}$$

tabellen (B)

$$a_n = \cancel{\frac{1}{\pi}} (-1)^n \cdot \frac{4\cancel{\pi}}{n^2}, \quad n \geq 1$$

$$\boxed{a_n = (-1)^n \cdot \frac{4}{n^2}, \quad n \geq 1}$$

$$\boxed{a_0 = \frac{2\pi^2}{3}}$$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx)$$

Série de Fourier

de $f(x) = x^2$

$$\frac{1}{2} a_0 + \sum_{n=1}^{\infty} a_n \cos(nx)$$

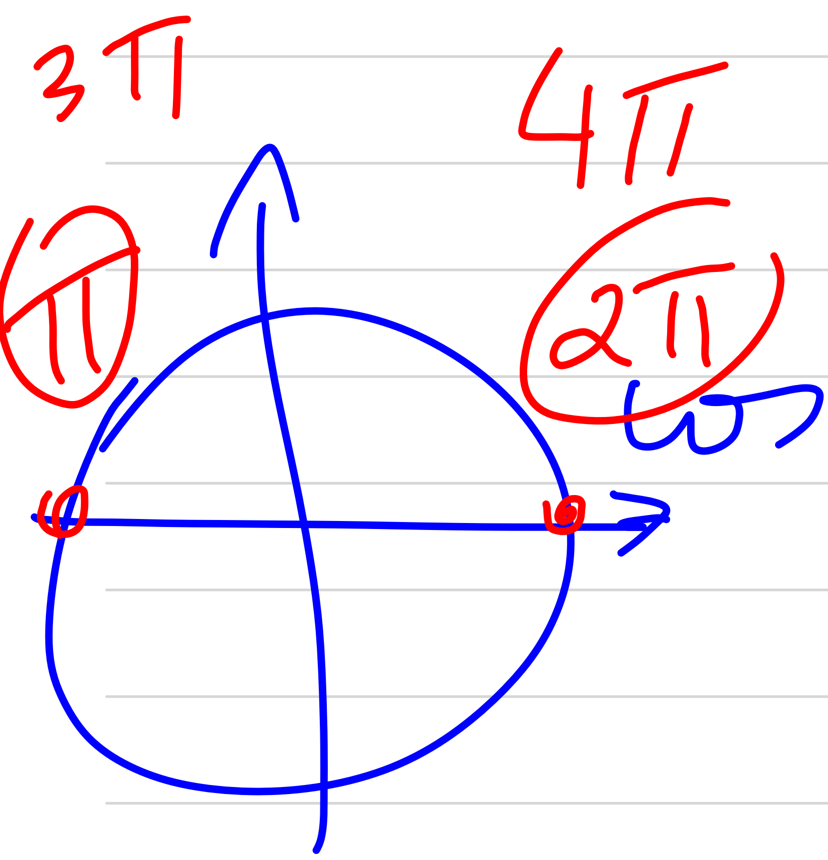
$$\frac{1}{\cancel{2}} \cdot \frac{\cancel{2}\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \frac{4}{n^2} \cos(nx)$$

$$\boxed{\frac{\pi^2}{3} + \sum_{n=1}^{\infty} (-1)^n \frac{4\cos(nx)}{n^2}}$$

$$\frac{\pi^2}{3} + 4\left[\overbrace{-\frac{\cos x}{1^2}}^{n=1} + \overbrace{\frac{\cos(2x)}{2^2}}^{n=2} \overbrace{-\frac{\cos(3x)}{3^2}}^{n=3} + \frac{\cos(4x)}{4^2} - \frac{\cos(5x)}{5^2} + \cdots \right]$$

$$X = \pi$$

$$\pi^2 = \frac{\pi^2}{3} + 4\left[-\overset{-1}{\cos \pi} + \frac{\overset{1}{\cos(2\pi)}}{4} - \frac{\overset{-1}{\cos(3\pi)}}{9} + \frac{\overset{1}{\cos(4\pi)}}{16} - \cdots \right]$$



$$\pi^2 = \frac{\pi^2}{3} + 4\left[\underbrace{\frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \frac{1}{16} + \cdots}\right]$$

$$\pi^2 - \frac{\pi^2}{3} = 4\left[\overbrace{1}^{n=1} + \overbrace{\frac{1}{2^2}}^{n=2} + \overbrace{\frac{1}{3^2}}^{n=3} + \overbrace{\frac{1}{4^2}}^{n=4} + \dots\right]$$

$$\frac{\cancel{2}\pi^2}{\cancel{4}_{2}\,3} = \frac{\cancel{4}}{\cancel{4}} \cdot \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$\boxed{\frac{\pi^2}{6} = \sum_{n=1}^{\infty} \frac{1}{n^{2}}}$$

soma

série-p com $p = 2 > 1$

conv

$$x^2 - \frac{\pi^2}{3} = 4\sum_{n\geq 1} (-1)^n \frac{\cos(nx)}{n^2} \quad (*)$$

Série de Fourier integral

de $y = \frac{x^3}{3} - \frac{\pi^2 x}{3}$?

$$\int \cos(nx)\,dx = \frac{\operatorname{sen}(nx)}{n} + c$$

$u = nx$

$du = n\,dx$

Integrando em x dos dois lados da igualdade $(*)$

$$\int\left(x^2-\frac{\pi^2}{3}\right)dx = 4\int \sum \frac{(-1)^n \cos(nx)}{n^2}\,dx$$

$$\frac{x^3}{3}-\frac{\pi^2}{3}x = 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\int \cos(nx)\,dx$$

Impar

$$= 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cdot\frac{\operatorname{sen}(nx)}{n}$$

$$= 4\sum_{n=1}^{\infty}(-1)^n\frac{\operatorname{sen}(nx)}{n^3}$$

$$+ C$$

$-\pi \qquad 0 \qquad \pi$

$$\frac{x^3}{3}-\frac{\pi^2}{3}x = 4\sum(-1)^n\frac{\operatorname{sen}(nx)}{n^3} + C$$

$x=0 \Rightarrow C=0$

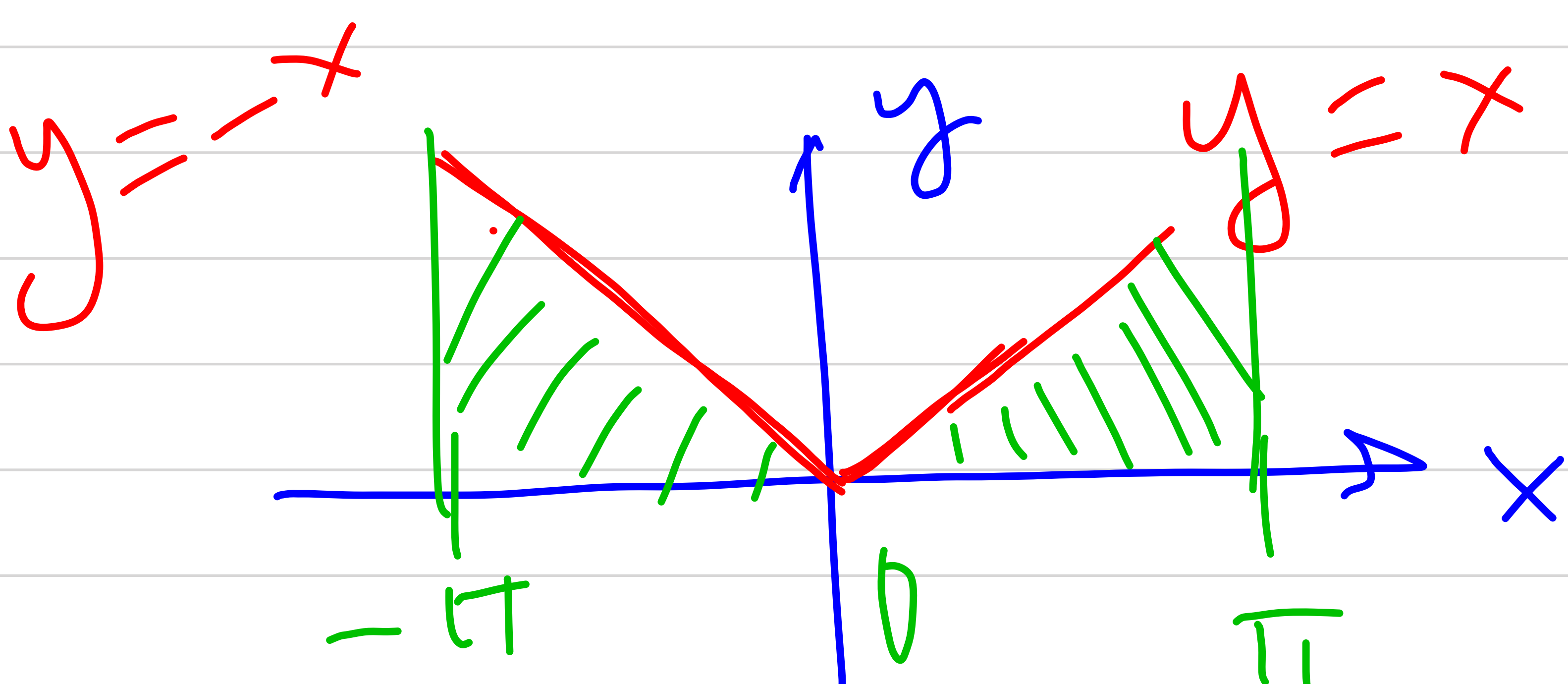
$f(x) = |x|, \quad -\pi \leq x \leq \pi$

f par ou impar?

$f(-x) = |-x| = |x| = f(x)$

f par $\Rightarrow b_n = 0, \forall n \geq 1$

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} |x| \, dx$$



$$|x| = \begin{cases} x, & \text{se } x \geq 0 \\ -x, & \text{se } x < 0 \end{cases}$$

$y = -x$ $\qquad$ $y = x$

$\longrightarrow x$

0

par

$$a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} |x|\, dx$$

$$\boxed{a_0 = \pi}$$

$$a_0 = \frac{2}{\pi} \int_0^{\pi} x\, dx$$

$$a_0 = \frac{\cancel{2}}{\pi} \cdot \frac{x^2}{\cancel{2}} \Big|_0^{\pi} = \frac{\pi^{\cancel{2}}}{\cancel{\pi}} - 0 = \pi$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} \overset{\text{pair}}{|x|}\,\overset{\text{pair}}{\cos(nx)}\,dx$$

$$\int_{-\pi}^{0} (-x)\cos(nx)\,dx$$

$$+\int_{0}^{\pi} (x)\cos(nx)\,dx$$

Ou

$$\int_{-\pi}^{\pi} |x|\cos(nx)\,dx$$

$$= 2\int_{0}^{\pi} x\cos(nx)\,dx$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} x\cos(nx)\,dx$$

A T

$$\int x \cos(nx)\,dx$$

u dv

parts

LIATE

$$\int u\,dv = uv - \int v\,du$$

$u = x$

$du = dx$

$\int dv = \int \cos(nx)\,dx$

$v = \frac{\sin(nx)}{n}$

$$\int x\cos(nx)\,dx = uv - \int v\,du$$

$$= x\frac{\sin(nx)}{n} - \int \frac{\sin(nx)}{n}\,dx$$

w

$w = nx$

$dw = n\,dx$

$$= \frac{x \sin(nx)}{n} - \frac{1}{n}\int \sin(nx)\,dx$$

$$= \frac{x \sin(nx)}{n} - \frac{1}{n}\int \sin w \,\frac{dw}{n}$$

$$= \frac{x \sin(nx)}{n} - \frac{1}{n^2}\left(-\cos w\right)$$

$$= \frac{x \sin(nx)}{n} + \frac{\cos(nx)}{n^2}$$

$$a_n = \frac{2}{\pi}\left[\frac{x \sin(nx)}{n} + \frac{\cos(nx)}{n^2}\right]\Bigg|_{0}^{\pi}$$

$$\Downarrow$$

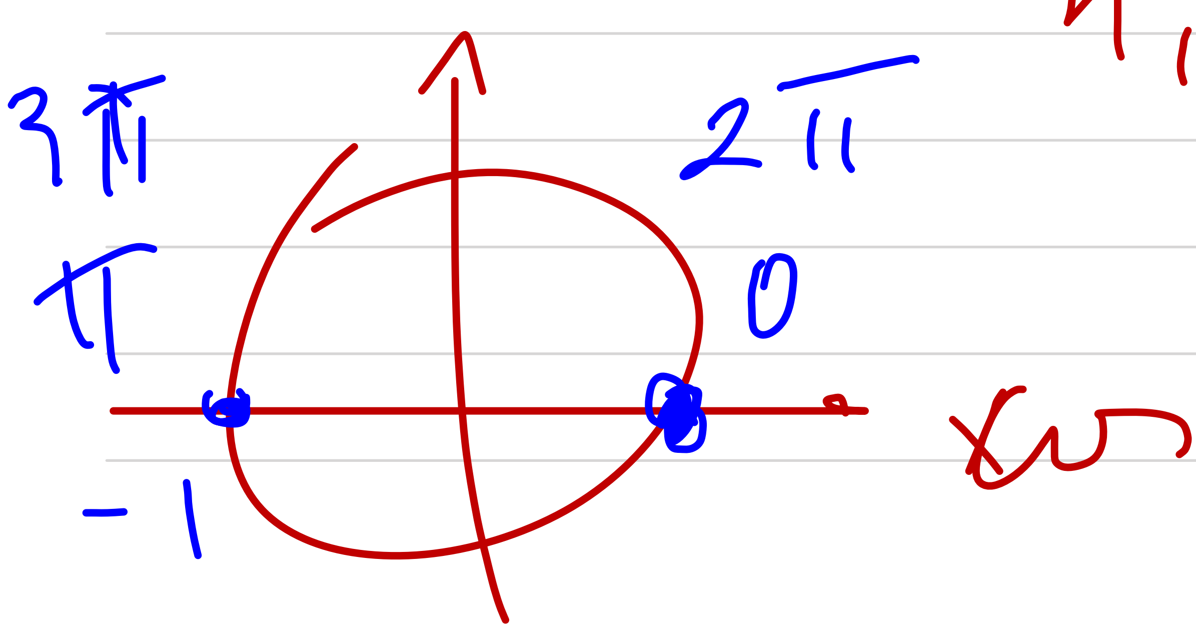
$$a_n = \frac{2}{\pi}\left[\left(\frac{\pi \cancel{\sin(n\pi)}^{\,0}}{n} + \frac{\cos(n\pi)}{n^2}\right) - \left(\cancel{\frac{0 \sin(0)}{n}} + \frac{\overbrace{\cos(0)}^{1}}{n^2}\right)\right]$$

$$\boxed{a_n = \frac{2}{\pi}\left[\frac{\cos(n\pi)}{n^2} - \frac{1}{n^2}\right]}$$

Coeficientes de Fourier de $f=|x|$

$$\begin{cases} a_0 = \pi & \neq 1 \\ a_n = \dfrac{2}{\pi}\left[\dfrac{\cos(n\pi) - 1}{n^2}\right] & n \geq 1 \end{cases}$$

$$\cos(n\pi) \begin{cases} 1 & n \text{ par} \\ -1 & n \text{ impar} \end{cases}$$



$$\cos(n\pi) - 1 = 0$$

(Ne n par)

$$\underbrace{\cos(n\pi)}_{-1} - 1 = -2$$

(Ne n impar)

$$a_n = \frac{2}{\pi}\frac{(-2)}{n^2} = \frac{-4}{\pi n^2}$$

(n impar)

Série de Fourier

de $f(x) = |x|$

$$\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(nx)$$

$$\frac{\pi}{2} + \sum \left(\frac{-4}{\pi n^2} \right) \cos(nx)$$

n ímpar

$$\frac{2n-1}{2n+1}$$

$n \geq 1$

$$\frac{\pi}{2} - \frac{4}{\pi} \sum \frac{\cos(nx)}{n^2}$$

n impar

$$\boxed{\frac{\pi}{2} - \frac{4}{\pi} \sum_{n=1}^{\infty} \frac{\cos((2n-1)x)}{(2n-1)^2}}$$

$$\frac{\pi}{2} - \frac{4}{\pi} \left[\overbrace{\cos x}^{n=1} + \overbrace{\frac{\cos(3x)}{3^2}}^{n=2} + \overbrace{\frac{\cos(5x)}{5^2}}^{n=3} + \cdots \right]$$