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**COMBINED ALGORITHM FOR SIMPLICIAL WEIGHT INTERPOLATION AND EXTRAPOLATION**

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**Disclaimer.** In this article the algorithm for multivariative simplicial weight interpolation and extrapolation is given. The algorithm requires a *n*-dimensional data set, which consists of points and corresponding function values and provides an interpolative function for this set, which is continuous on ***R****n* along with its given number of derivatives.

**Key Words.** Simplex, simplicial complex, weight interpolation, multivariative interpolation.

**Introduction.** The multivariative simplicial weight interpolation was introduced in paper [3]. This scheme provides continuous function with predetermined order of smoothness determined in a convex polyhedron built on a given point set. Out of this region the function remains undefined. This definition is trivial for 1-dimensional case, as the union of basis function, defined in first and last points of an ordered set, is the necessary extrapolating function. For 2-dimensional case such definition is considered in paper [2] along with interpolating function given in [1].

Defining *n*-variative function, that stays continuous along with its’ given number of derivatives on the whole ***R****n*, is a matter of this current work.

**The algorithm.** Consider point data set of N points ***x****i*=*{xi1, xi2, …, xin}*, *i=1..N, n –* is the dimension of its’ space. For each point ***x****i* a corresponding value *yi* is set. Also consider proper basis functions set for each point: *fi(****x****i)*, continuous on ***R****n*, and that *fi(****x****i) = yi.* For more detailed information on basis functions see [3].

Our goal is to define a function: *F(x1, x2, …, xn),* meeting a condition *F(xi1, xi2, …, xin) = yi* and continuous on ***R****n* along with its’ derivatives up to determined level, which is set by basis functions and weight coefficients.

Set of *n+1* points determines a *n*-simplex in *n*-dimensional space. If these point lie in a single *n*-dimensional hyperplane, the simplex considered degenerative, and such cases are not to be considered in this particular paper. On a set of *n+2* and more points building an n-dimensional simplicial complex is possible. Its’ margin will be a convex polyhedron. Therefore each point in *n*-dimensional space lies in one of *n*-simplexes, or in one of *n*-simplexes edges, which are *0..(n-1)-*simplexes themselves, or out of the inner space of simplicial complex.

Combined algorithm for defining the simplicial weight interpolation and extrapolation function *F(****x****)* value in a point***x*,** considering proper complex built,will be:

1. Check if ***x*** belongs to a data set. If *x=xi*, *F(****x****) = F(****x****i) = yi* by definition.

2. Determine if ***x*** lies in one of *m-*simplexes - ***S****m*, m = 1..n. For this following should be done:

Consider [***x****1*, ***x****2*, …, ***x****m+1*] – a set of *m-*simplex vertices. A set of vectors [***x****1-****x****m+1*, ***x****2-****x****m+1*, …, ***x****m-****x****m+1*] provides a m-dimensional basis. Let point ***x*** be lying in a hyperplane set by simplex vertices. A vector ***x****-****x****m+1*, has its coordinates in that basis *(a1;a2;…;am).* This coordinates can be found by solving a system of linear equations:

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|  | (1) |

Here *n* – is the simplex vertices dimension, *xij*– the *j*-th coordinate of point ***x****i*.

If system (1) has a solution and*ai: 0<ai<1,* along with *0<ai<1,* then point ***x*** lies in simplex ***S****m.* If these conditions are not met strictly, yet *ai: 0≤ai<1* with *0<ai≤1* then point ***x*** lies on ***S****m* margin, therefore it belongs to one of ***S****m-1* simplexes or, recursively, lies on their margin.

2.1. If point x lies in one of given complex’ *m*-simplexes – ***S****m*, *m=1..n* – interpolating function is:

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|  | (2) |

Here***x****k* – are projection of point ***x*** on a hyperplane set by *(m-1)*-simplexes’ vertices; *k(h)* – are weight functions, which are basically continuous scalar functions, continuously decreasing for *h>0*, and having vertical asymptote in *h=0*.

3. If point x lies out of simplicial complex:

1. 3.1. Define an empty set of (simplex, scalar) tupples - ***SS***.
2. 3.2. For each of *m*-simplices ***S****m* – *m*-faces *m=0..(n-1)* of each n-simpex of given complex
   * 1. 3.2.1. If m = 0: add tupple *(****S****m, d(x,* ***S****m0))* to ***SS***. Here ***S****m0* is the one and only vertex of 0-simplex.
     2. 3.2.2. if m > 0: find ***xk*** – a projection of ***x*** on ***S****m*; add tupple *(****S****m, d(x, xk))* to ***SS.***
3. 3.3. Find a tupple (***S****i*, *d*) in ***SS***, which has the smallest *d*.
4. 3.4. For found ***S****i* determine extrapolating function as:

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|  | (3) |

Formula (3) is a modification of (2): *k(h)* – should be the same function as in interpolation. ***x****si* – is a projection of ***x*** on ***S****i*

Point ***x****k* – the orthogonal projection of point ***x*** on a hyperplane set by ***S****m-1* vertices – can be found in the following way. As ***x****k-****x****1* lies in *(m-1)*-coordinate basis [***x****2-****x****1*, ***x****3-****x****1*, …, ***x****m-****x****1*], set by 1-edges of ***S****m-1*, and this edges are orthogonal to ***x****k-****x***, we can state:

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|  | (4) |

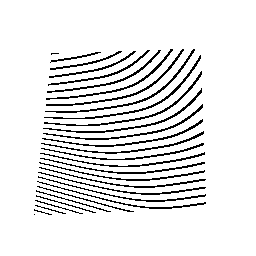
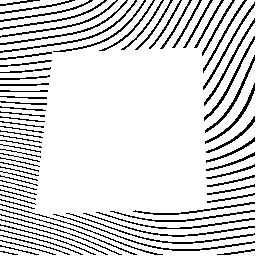
System (4) can be transformed into linear equations system:

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|  | (5) |

This system has the rank of *(m-1)* and has a solution for nondegenerate simplexes. This solution is the coordinates of ***x****k* in [***x****2-****x****1*, ***x****3-****x****1*, …, ***x****m-****x****1*] basis. Point coordinates in its’ original basis can be found by this formula:

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|  | (6) |

Here on fig. 1 an example of working algorithm product is shown. It is the contour lines of *3*-variative function interpolation and extrapolation functions cross section.

*а) interpolation; б) extrapolation; в) combined algorithm*

*Fig. 1. An example of algorithm work*

Derivative continuousness of extrapolating and interpolating functions are dependent on the types of basis and weight functions as described in [1, 3]. The functions are also dependent on the way simplicial complex is built.

Simplicial weight interpolation and extrapolation allows using different types of basis and weight functions in different points of an input data set. Basis functions can be constant, linear, polynomial, radial etc. Weight functions are quite arbitrary, but this type is the most frequently used:

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|  | (7) |

Here *p* is the constant, which helps to regulate derivative continuousness. E. g. constant basis functions used with weight function of such type provide continuous interpolating function with continuous *1*-st derivative when *p=2*, and when *p=1* the 1-st derivative is discontinuous.

Due to recursive nature of simplicial weight interpolation and extrapolation, basis functions may not be defined in points only, but on simplexes edges and faces of any order up to *n*. Therefore given algorithm can be used together with other interpolating and extrapolating schemes yet providing continuous function.

**Conclusion.** The combined algorithm for multivariative simplicial weight interpolation and extrapolation provides continuous function on ***R****n* along with its’ derivatives up to the predetermined order. This function depends on the basis and weight functions, and also on the way simplicial complex is built.

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