MMSE detector for 2x2 MIMO receiver

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***Abstract*—Minimum mean square error (MMSE) linear detector is able to achieve the near-optimal bit error rate (BER) performance for uplink multi-user massive MultipleInput Multiple-Output (MIMO) systems due to the increase in the number of base station (BS) antennas.For MIMO-OFDM systems, this work considers a field programmable gate array (FPGA) implementation of a linear minimum mean square error (LMMSE) detector[1]. For the implementation of the LMMSE detector, two square root free techniques based on QR decomposition (QRD) are presented**

***Index Terms*—MMSE,QR decomposition,Gram Schmidt**

I. Introduction

Multiple-Input and Multiple-Output (MIMO), is a method for multiplying the capacity of a radio link using multiple transmission and receiving antennas to exploit multipath propagation.Large bandwidths are required by the ever-increasing data rates in wireless communication systems. Orthogonal frequency division multiplexing (OFDM) is a frequently used technology in broadband wireless networks to minimize receiver complexity. When compared to single antenna channels, multiple-input multiple-output (MIMO)[2] channels provide increased capacity and great promise for better dependability. Layered space-time (LST) architectures paired with channel coding are pragmatic yet powerful ways to boost the user data rate in systems with multi-element antenna arrays in rich scattering environments (MEAs). MIMO combined with OFDM (MIMO-OFDM) has been highlighted as a viable solution for high spectral efficiency wideband systems.
In the literature, systolic array architectures with communicating processing units are frequently used to create matrix computations (PEs)[3]. Detector designs for 2X2 antenna systems are discussed and compared in this research. For lower-dimensional systems, a fast and parallel design is examined, whereas for bigger systems, a less complicated architecture with simple scaling and time sharing PEs is considered. The computational complexity of each implementation is examined and contrasted using an FPGA hardware implementation

II. System Model

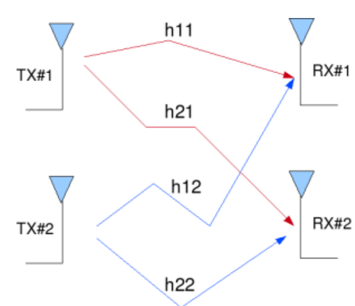
The transmission channel between the transmitter antennas (Tx) and the receiver antenna (Rx) as depicted in the figure below is called a Multiple Input-Multiple Output (MIMO) channel. A transmission system on a MIMO channel is called a MIMO transmission system[4]
In the first time slot, the received signal on the first receive antenna is

$$y1 = h1,1 * x1 + h1,2 * x2 + n1 = \begin{bmatrix} h1,1 & h1,2 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} + n1$$

The received signal on the second receive antenna is,

$$y2 = h2,1 * x1 + h2,2 * x2 + n2 = \begin{bmatrix} h2,1 & h2,2 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} + n2$$

where
y1, y2 are the received symbol on the first and second antenna respectively,
h1,1 is the channel 1st from 2nd transmit antenna to receive antenna,
h1,2 is the channel 1st from 2nd transmit antenna to receive antenna,
h2,1 is the channel 1st from 2nd transmit antenna to receive antenna,
h2,2 is the channel 1st from 2nd transmit antenna to receive antenna,
x1,x2 are the transmitted symbols and
n1,n2 is the noise on 1st,2nd receive antennas.



(a) 2x2 MIMO

Figure 1. 2 Transmit 2 Receive (2×2) MIMO channel.

We assume that the receiver knows h1,1; h1,2; h2,1 and h2,2 . The receiver also knows y1 and y2. For convenience, the above equation can be represented in matrix notation as follows:

$$\begin{bmatrix} y1 \\ y2 \end{bmatrix} = \begin{bmatrix} h1,1 & h1,2 \\ h2,1 & h2,2 \end{bmatrix} \begin{bmatrix} x1 \\ x2 \end{bmatrix} + \begin{bmatrix} n1 \\ n2 \end{bmatrix}$$

Equivalently,

$$y = Hx + n \tag{1}$$

The Minimum Mean Square Error (MMSE) approach tries to find a coefficient which minimizes the criterion,

$$W = [H^H H + NoI]^{-1} * H^H \tag{2}$$

where No is Gaussian noise

$$x = [H^H H + NoI]^{-1} * H^H * y \tag{3}$$

The MMSE (Minimum Mean Square Error) signal splitter takes into consideration the noise characteristics at the receiving antenna branches in addition to the statistical features of the signal from the sending antennas[5].

Because of its adaptive algorithms, the MMSE signal splitter has the benefit of simplicity and ease of application in reality. Furthermore, because the MMSE splitter considers the noise characteristics, the ZF splitter's noise amplification disadvantage may be addressed. As a result, an MMSE splitter's BER or SINR quality is often superior to that of a ZF splitter. The MMSE splitter, like the ZF splitter, has a low computational complexity

III. Related Work

A significant amount of work has been done lately regarding calculate MMSE detector by CORDIC algorithm [6]. For example, the authors present a sequential implementation of the CORDIC algorithm on that results in a design that is well suited for applications.

Two FPGA implementations of a MMSE detector were considered based on the CORDIC and SGR algorithms for MIMO-OFDM systems, where the detector complexity and the number of required operations depend mainly of the number of subcarriers and the number of antennas. The detector architecture solutions were presented and compared for 2 × 2 and 4×4 antenna systems.

The FPGA hardware implementations for both detectors were presented and the computational complexity of each implementation was evaluated and compared. The CORDIC [7] based implementation was found to require more slices and less block multipliers compared to SGR based design. This is due to the normal arithmetic applied in the SGR algorithm and rotation based arithmetic applied in CORDIC algorithm.

IV. QR DECOMPOSITION

A. *QR Decomposition*

There are two algorithms which have been used widely to process QR decomposition: Gram-Schmidt and HouseHolder. Since many analyses and implementations have been done for both of them, Gram-Schmidt is proven to be better to implement in FPGA for two reasons: 1) high speed multipliers is now available in FPGA, and 2) Gram-Schmidt is widely used in software using floating point

The QR signal splitter is based on the QR factorization method of the channel matrix H. According to the QR factorization method, any channel matrix H can all be decomposed into [8]

$$H^H H + NoI = QR \tag{4}$$

Where R is an upper triangular matrix of the form as follows:

$$R = \begin{bmatrix} h1,1 & h1,2 & h1,3 & h1,4 \\ 0 & h2,2 & h2,3 & h2,4 \\ 0 & 0 & h3,3 & h3,4 \\ 0 & 0 & 0 & h4,4 \end{bmatrix}$$

And Q is a unitary matrix with property:

$$Q^H \times Q = Q^{-1} \times Q = 1 \tag{5}$$

From the system equation y = Hx + n,[9] using the QR method and the simple matrix property first, multiply both sides of one, multiply both sides of the equation by the equation we have:

$$Rx = Q^T \times H^H \times y$$

B. *Gram-Schmidt*

The Gram–Schmidt process is a method for orthonormalizing a set of vectors in an inner product space, most commonly the Euclidean space Rn equipped with the standard inner product. The Gram–Schmidt process takes a finite, linearly independent set of vectors S = v1, ..., vk for k n and generates an orthogonal set S = u1, ..., uk that spans the same k-dimensional subspace of Rn as S.[10]

The method is named after Jørgen Pedersen Gram and Erhard Schmidt, but Pierre-Simon Laplace had been familiar with it before Gram and Schmidt. In the theory of Lie group decompositions it is generalized by the Iwasawa decomposition.

The application of the Gram–Schmidt process to the column vectors of a full column rank matrix yields the QR decomposition (it is decomposed into an orthogonal and a triangular matrix).[11]

QR-GramSchmidt Algorithm
Function [Q, R] = QR-Gram-Schmidt(H);

1: [m,n] = size(H);
2: R(1,1) = norm(H(:, 1));
3: Q(:,1) =H(:, 1)/R(1, 1);
4: for k = 2:n
. 1: R(1:k-1, k) = Q(1:m, 1:k-1)' $\times H(1:m,k)$;
. $2: z = H(1:m,k) - Q(1:m,1:k-1) * R(1:k-1,k)$;
. $3: R(k,k) = norm(z)$;
. $4: Q(1:m,k) = z/R(k,k)$;
$5: end$

V. CALCULATION AND SIMULATION

Input and output is a complex matrix 2x2 and 2x1 , because it is a complex number we need to perform data format to be able to perform calculations on ModelSim. From the 2x2 matrix we transform into 4x4 and 2x1 transform into a 4x2 matrix. Each complex number in the matrix becomes a 2x2 matrix containing the real and imaginary parts of that complex number as shown below:

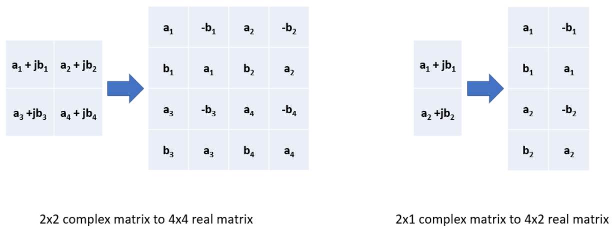


Figure 2. Data format

Input H is a 4x4 complex matrix transformed from matrix 2x2 with value: 0.2+1.2i ; i ; 0.4+1.6i ; -1.8i ;

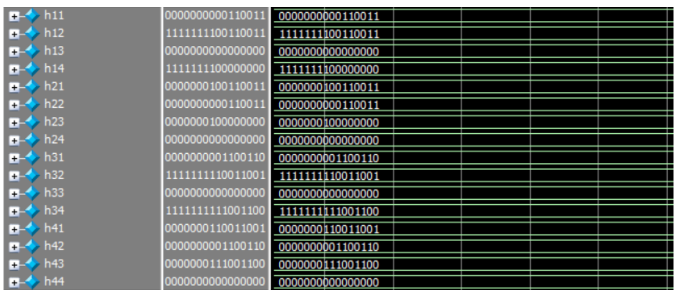


Figure 3. Input H

Input Y is a 2x1 complex matrix transformed from 4x2 matrix with value: 1.2+i ; 1.6+1.8i ;

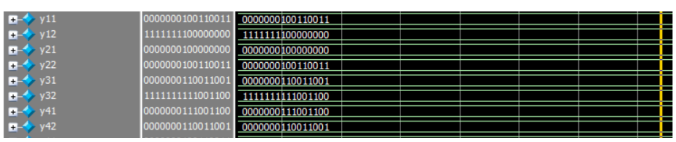


Figure 4. Input Y

Input N is a complex number(1.2+1.8i) multiple an unit matrix transformed from 2x1 matrix
After multiple an unit matrix, we transform 4x2 matrix like data format above

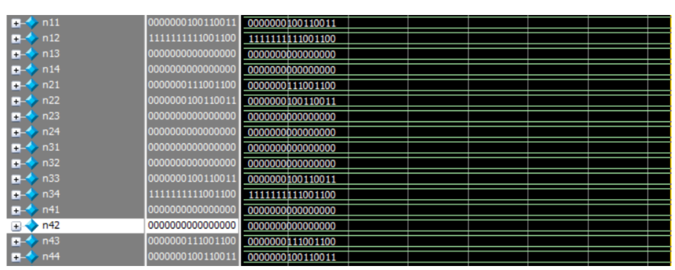


Figure 5. Input N

After simulation finish, we will receive Output X is transmission signal

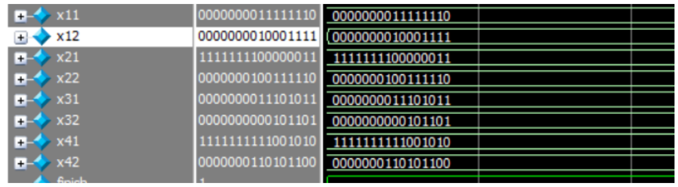


Figure 6. The simulation result is value of transmission signal

VI. Discussion

This paper was successfully completed with the implementation of QR algorithm to calculate for MMSE for 2x2 MIMO.The decrypted text are analyzed and proved to be correct. The MMSE suppresses both the interference and noise components

VII. Conclusion

In this article, we have shown how to perform a search for the transmission signal of MMSE detector for 2x2 MIMO. The article has been analyzed and proven to be accurate The proposed simulation results have been studied and met with satisfactory results

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