

# A Domain Specific Language (DSL) for Legal Rules

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## Introduction

### Overview

This notebook defines a package of Wolfram Language functions that facilitates the modeling of legal rules. This is a domain specific language (DSL) - a high-level programming vocabulary tailored to a specific purpose. The larger context of why one would want to do this and numerous other aspects of policy automation are explained elsewhere.

A DSL for legal rules has to do four basic things:

1. **Information-seeking:** Dynamically seek out missing information in order to make a determination;
2. **Handling uncertainty:** Keep track of various kinds of uncertainty;
3. **Time series operators:** Handle certain time series computations; and
4. **Proof trees:** Provide justifications for its determinations.

These features are defined below, along with unit tests that verify their correctness and demonstrate how to use them.

The functions in this library fall into two categories. Some of the functions generalize ordinary Wolfram Language functions to time series operators. For example, the arithmetic operations are generalized to time series objects, so you can add, subtract, multiply, and divide entire time series together, just as you would ordinarily add two scalar quantities (like  $2+2$ ) together. Other functions, while also applying to time series objects, have no counterpart in the Wolfram Language. For example, a group of functions determine how much time has elapsed during which a Boolean time series has been true. So this DSL generalizes a small subset of the Wolfram Language, and also extends it to include certain time-related operations. This DSL also provides functions for easily generalizing a Wolfram Language function to a time series, so that it can be extended even further when necessary, with very little effort.

### Development Conventions

Where a function in this DSL generalizes an existing Wolfram Language function, the naming and syntax generally follow the Wolfram Language conventions. However, this is not always the case. Some function signatures have not been generalized, and occasionally it was not possible for the names to be used consistently. All of this is explained in a separate syntax reference document.

Typically, user-defined Wolfram Language functions start with lower case letters, to avoid conflicts with functions that are part of the core language. However, in this library, function names start with capital letters, for two reasons. First, this ensures that they will stand out from functions defined by users of

this DSL. And second, functions that start with upper case letters are omitted from proof trees (see below). If the function names in this DSL end up conflicting with functions released in future versions of the Wolfram Language, adjustments will have to be made.

## Interface to this Package

This defines the package's interface to the outside world. In other words, these are the functions that are expected to be used to model the legal/policy rules. (Note that overloaded Wolfram Language functions are tentatively not included in this list. Also, many functions that have not yet been defined are not currently included in this list.)

```
BeginPackage["DSL`"];

AlwaysQ::usage = "";
AnnualTimeLine::usage = "";
ApplyRules::usage = "";
Ask::usage = "";
AsOf::usage = "";
Date::usage = "";
DawnOfTime::usage = "";
EndOfTime::usage = "";
EverQ::usage = "";
IfThen::usage = "asdfasdf";
InterviewPopup::usage = "";
InterviewAPI::usage = "";
InterviewWidget::usage = "";
MissingData::usage = "";
Numeric::usage = "";
ProofTree::usage = "";
RuleStub::usage = "";
SwitchThen::usage = "";
TimeLine::usage = "";
TimeLineMap::usage = "";
TimeLinePlot::usage = "";
TrueBefore::usage = "";
TrueBetween::usage = "";
TrueOnOrAfter::usage = "";
Uncertain::usage = "";

Begin["`Private`"];
DSL`Private`
```

## Background: Modeling Legal Rules

### Facts

Facts are represented as relations (symbolic expressions), with associated values. For example:

```
gender["Mary"] → "Female"
married["Mary", "Tom"] → True
datePurchased["Joan", "Whiteacre"] → DateObject[{2016, 4, 1}]
```

### Rules

Rules derive new facts from existing ones, using Ask to mark inputs. Base-level facts are undefined symbols. The following functions are used to demonstrate the syntax and for testing purposes.

```
qualifyingRelativeOf[a_, b_] :=
  age[a] < 18 &&
  age[b] ≥ 18 &&
  Ask[gender[b]] == "Female" &&
  Ask[pregnantQ[b]] &&
  Ask[familyRelationship[a, b]] == "Child";

(* age[p_] := QuantityMagnitude[DateDifference[Ask[doB[p]], Now, "Year"]]; *)
qualifyingRelationship[a_, b_] := Ask[age[a]] > 18 && Ask[gender[b]] == "Female";
```

---

## Information-Seeking

### Identifying Needed Inputs

#### Ask

The Ask function doesn't take any action on its own; it's just a wrapper used to mark inputs, for example Ask[doB[p]]. It may later be expanded to include question definitions: Ask[fact\_, q\_QuestionObject].

#### ApplyRules

This function resolves goals by applying the rules that we defined to a set of facts:

```
In[220]:= ApplyRules[goals_List, facts_Association] :=
  Quiet[goals /. Normal[KeyMap[Ask[#] &, facts]]];
SetAttributes[ApplyRules, HoldAll];
```

## MissingData

The MissingData function harvests the Ask expressions from the expression that results from the application of rules to facts. TODO: Overload the definition of this function to give it a signature like ApplyRules?

```
In[222]:= MissingData[rs_] :=
  Quiet[DeleteDuplicates[Cases[rs, Ask[_], All]] /. Ask[f_] -> f];
SetAttributes[MissingData, HoldAll];
```

## Interactive Interviews

### InterviewPopup

This function creates a pop up interview (in a Wolfram Language notebook) that can be used to test rules interactively. It is recursively driven by the list of missing facts.

```
In[224]:= InterviewPopup[goals_List, facts_Association] :=
  Quiet[Module[{re, remd, newFactValue},
    re = ApplyRules[goals, facts];
    remd = MissingData[re];

    (* If there's no more missing data, return the evaluated goal. Otherwise,
    ask the next question and resubmit. *)
    If[remd === {}, Return[re],
      newFactValue = Input[ToString[remd[[1]]] <> "?"];
      If[newFactValue === quit, Return[re],
        Return[InterviewPopup[goals, Append[facts, remd[[1]] -> newFactValue]]]]
  ]];
SetAttributes[InterviewPopup, HoldFirst];
```

There are no unit tests because this is an interactive feature. Evaluating the code below will trigger an interview. Typing quit will abort it.

```
InterviewPopup[{qualifyingRelationship["Lucy", "Sam"]}, <| |>]
{$Canceled > 18 && $Canceled == Female}
```

If you seed PopupInterview with facts, those will not be asked:

```
InterviewPopup[{qualifyingRelationship["Lucy", "Sam"]}, <|age["Lucy"] → 26|>]
{True}
```

## Interview API

TODO: Given goals and facts, this function returns information about what is known and what needs to be known in order to reach a determination.

## InterviewWidget

TODO: The following code creates a notebook widget that interactively investigates a given goal.

---

# Handling Uncertainty

## Introduction

States of uncertainty allow the system to keep track of facts the user doesn't know and legal rules that have not been modeled. They are used to create a system of multivalued logic with the following additional states:

RuleStub - Indicates where the rule logic is incomplete (e.g. RuleStub["42 U.S.C. 1983"])

Uncertain - Indicates what is unknown by the user (e.g. a fact she doesn't know the value of, such as Uncertain[doB[Mary]])

The following are used to simplify outputs:

In[226]:=

```
RuleStub[{}] = RuleStub[];
Uncertain[{}] = Uncertain[];
```

## Tests for States of Uncertainty

The TimeLine object is described below.

In[228]:=

```
RuleStubQ[x_] := Head[x] === RuleStub;
UncertainQ[x_] := Head[x] === Uncertain;
DSLTypeQ[x_] := TimeLineQ[x] || UncertainQ[x] || RuleStubQ[x];
```

## Merging Uncertain States Together

If there are multiple states of the same kind of uncertainty, they need to be merged together.

```
In[231]:= mergeStubs[list_List] :=
  RuleStub[DeleteDuplicates[Flatten[list /. RuleStub[x___] -> x]]];

mergeUncertains[list_List] :=
  Uncertain[DeleteDuplicates[Flatten[list /. Uncertain[x___] -> x]]];
```

## Short-Circuit Operations

Some operations - And, Or, and Times - short-circuit in their evaluation. For example, False and True short-circuits to False;  $x * 0$  short-circuits to 0. The following functions prevent uncertainty from propagating up the function tree when a short-circuit evaluation is possible. The prefix “static” is intended to distinguish these functions from their time series counterparts.

```
In[233]:= staticAnd[False, q_] = False;
staticAnd[p_, False] = False;
staticAnd[True, True] = True;
staticAnd[p_, q_] := Which[
  MemberQ[{p, q}, _Uncertain],
  mergeUncertains[Select[{p, q}, UncertainQ]],
  MemberQ[{p, q}, _RuleStub], mergeStubs[Select[{p, q}, RuleStubQ]],
  True, p && q];

staticOr[True, q_] = True;
staticOr[p_, True] = True;
staticOr[False, False] = False;
staticOr[p_, q_] := Which[
  MemberQ[{p, q}, _Uncertain],
  mergeUncertains[Select[{p, q}, UncertainQ]],
  MemberQ[{p, q}, _RuleStub], mergeStubs[Select[{p, q}, RuleStubQ]],
  True, p || q];

staticTimes[0, q_] = 0;
staticTimes[p_, 0] = 0;
staticTimes[p_, q_] := handleUncertainty[p * q, {p, q}];
```

## HandleUncertainty

These are wrapper rules that return the highest-precedence type of uncertainty or, if all items in the input list are certain, the specified expression.

```
In[244]:=
handleUncertainty[expr_, And, {a_, b_}] := staticAnd[a, b];
handleUncertainty[expr_, Or, {a_, b_}] := staticOr[a, b];
handleUncertainty[expr_, Times, {a_, b_}] := staticTimes[a, b];
handleUncertainty[expr_, f_, {a_, b_}] := handleUncertainty[expr, {a, b}];
handleUncertainty[expr_ : Except[_Ask], list_?listMayPassQ] := Which[
  MemberQ[list, _RuleStub], mergeStubs[Select[list, RuleStubQ]],
  MemberQ[list, _Uncertain], mergeUncertains[Select[list, UncertainQ]],
  True, expr]
SetAttributes[handleUncertainty, HoldFirst];
```

Helper function: A list should be trapped (not evaluated further) if it contains Ask and not (RuleStub or Uncertain). In other words, if it has RuleStub or Uncertain, it can be evaluated; if it has Ask it cannot; otherwise it can.

```
In[250]:=
listMayPassQ[list_List] := Which[
  MemberQ[list, _RuleStub] || MemberQ[list, _Uncertain], True,
  MemberQ[list, _Ask], False,
  True, True];
```

## Time Series Operations

### Overview

#### Introduction

As explained elsewhere in great detail, modeling time is a particular challenge in building rule-based systems. The following functions create time series objects and operations that are used extensively in legal rules. A time series is a list of date-value pairs (not necessarily regularly spaced), and time series can be added, subtracted, etc., just like scalar values. So as to avoid conflicts with the `TimeSeries` object as defined in the Wolfram Language, we'll create a new object called `TimeLine` and define a variety of operations on them. The core data structure within a `TimeLine` will be compatible with that of a `TimeSeries`, so we can take advantage of the Wolfram Language's existing `TimeSeries` functions when necessary.

#### Beginning and End of Time

There's a need for values that represent the outer limits of a time series. These are not represented as `DateObjects`, even though they are dates, because complex time series operations perform faster this way.

```
In[251]:= DawnOfTime = {2000, 1, 1};
          EndOfTime = {2050, 12, 31};
```

## TimeLine

### TimeLine

Here's an example of a Boolean TimeLine:

```
TimeLine[{{2000, 1, 1}, False}, {{2010, 1, 1}, True}]
```

An internal operation to get data out of a TimeLine object:

```
In[253]:= TimeLineUnbox[t_TimeLine] := t /. TimeLine[x_] -> x;
```

Convert a scalar to a TimeLine (internal):

```
In[254]:= ToTimeLine[v_TimeLine] := v;
          ToTimeLine[v_] := TimeLine[{{DawnOfTime, v}}];
```

Extract the dates from one or more time series:

```
In[700]:= TimeLineDates[ts_TimeLine] := TimeLineUnbox[ts][[All, 1]];
          TimeLineDates[ts1_TimeLine, ts2_TimeLine] :=
            Union[TimeLineDates[ts1], TimeLineDates[ts2]];
```

Extract the values from a time series:

```
In[699]:= TimeLineValues[ts_TimeLine] := TimeLineUnbox[ts][[All, 2]];
```

Delete redundant time intervals from a time series. If there's only one interval remains, it returns a scalar. Despite the name, the input is assumed to be a List, not a timeSeries. (Avg. execution time = 0.00023s.)

```
In[259]:= TimeLineTrim[t_List] := Block[{trim},
          trim = Map[#[[1]] &, Split[t, #1[[2]] === #2[[2]] &]];
          If[Length[trim] === 1, trim[[1, 2]], TimeLine[trim]]
        ];
```

Determine whether something is a TimeLine:

```
In[260]:= TimeLineQ[x_] := Head[x] === TimeLine;
```

Retrieves the value of the first time-value pair in the time series.

```
In[261]:= TimeLineFirstValue[ts_] := handleUncertainty[TimeLineUnbox[ts][[1]][[2]], {ts}];
```



## AsOf

Get the value of a TimeLine at a particular point in time:

```
In[262]:= AsOf[date_, ts_TimeLine] := handleUncertainty[QuickAsOf[date, ts], {date, ts}];
AsOf[date_, ts_stub] := handleUncertainty[ts, {date, ts}];
AsOf[date_, scalar_] := scalar;
```

Wondering how to make it accept temporal inputs:

```
asOf[date_TimeLine, val_] := TimeLineMap[QuickAsOf[#1, #2] &, date, val]
```

```
In[1221]:= asOf[lastDayOfMonth, TrueOnOrAfter[{2017, 5, 5}]]
Out[1221]= TimeLine[{{2000, 1, 1}, False}, {{2017, 5, 5}, True}]]
```

An internal method for doing the above operation quickly:

```
In[1023]:= QuickAsOf[date_, ts_TimeLine] :=
  Block[{i, t = TimeLineUnbox[ts], len, abTime = AbsoluteTime[date]},
    len = Length[t];
    For[i = 1, i <= len, i++,
      (* If the date is before the index date,
       return the value of the prior interval *)
      If[abTime < AbsoluteTime[t[[i, 1]]], Return[t[[i - 1]][[2]]]];
      (* Else, return the value of the last interval *)
      Return[t[[len]][[2]]];
    ];
  QuickAsOf[date_, scalar_] := scalar;
```

An attempt to implement this in a functional style, but it's much slower, presumably due to the SelectFirst[Reverse[\_]]:

```
In[1009]:= QuickAsOf[date_, ts_TimeLine] :=
  Block[{data = TimeLineUnbox[ts], absDate, abses, time},
    absDate = AbsoluteTime[date];
    abses = AbsoluteTime /@ data[[All, 1]];

    time = SelectFirst[Reverse[abses], # <= absDate &];

    Extract[data, Position[abses, time]]
  ];
  QuickAsOf[date_, scalar_] := scalar;
```

## TimeLineMap

This function maps a scalar function (or functions) onto a time series. For performance reasons, the

merging/mapping algorithm varies depending on the nature of the arguments. The order of the following definitions may affect the function's behavior.

```
In[267]:= TimeLineMap[f_, ts_TimeLine] :=

  TimeLineTrim[Map[{#[[1]], handleUncertainty[f[#[[2]]], toList[#[[2]]]]} &,
    TimeLineUnbox[ts]]];

TimeLineMap[f_, scalar_ : Except[_Ask]] :=
  handleUncertainty[f[scalar], {scalar}];

(* Merges two time series like a zipper and
  applies a function to the values in each interval. *)
TimeLineMap[f_, ts1_TimeLine, ts2_TimeLine] :=
  TimeLineMapZip[handleUncertainty[f[#1, #2], f, {#1, #2}] &, ts1, ts2];

(* If the time series doesn't have any stub intervals,
  return the scalar stub. *)
TimeLineMap[f_, ts_TimeLine, s_stub] :=
  If[TimeLineContainsStubInterval[ts],
    TimeLineTrim[
      Map[{#[[1]], handleUncertainty[f[#[[2]], s], f, {#[[2]], s}]} &,
        TimeLineUnbox[ts]]
    ],
    s];
TimeLineMap[f_, s_stub, ts_TimeLine] :=
  If[TimeLineContainsStubInterval[ts],
    TimeLineTrim[
      Map[{#[[1]], handleUncertainty[f[s, #[[2]]], f, {s, #[[2]]}]} &,
        TimeLineUnbox[ts]]
    ],
    s];

(* Map the scalar to the time series and apply the function. *)
TimeLineMap[f_, ts_TimeLine, scalar_ : Except[_ask]] := TimeLineTrim[
  Map[{#[[1]],
    handleUncertainty[f[#[[2]], scalar], f, {#[[2]], scalar}]
  } &, TimeLineUnbox[ts]]
];
TimeLineMap[f_, scalar_ : Except[_Ask], ts_TimeLine] := TimeLineTrim[
  Map[{#[[1]],
    handleUncertainty[f[scalar, #[[2]]], f, {scalar, #[[2]]}]
  } &, TimeLineUnbox[ts]]
];
```

```
(* TODO: Handle stub arguments *)
TimeLineMap[f_, scalar1_ : Except[_Ask], scalar2_ : Except[_ask]] :=
  handleUncertainty[f[scalar1, scalar2], f, {scalar1, scalar2}];

(* Indicates whether any interval in the time series is a stub *)
TimeLineContainsStubInterval[ts_TimeLine] :=
  MemberQ[TimeLineUnbox[ts], {_, _stub}]
```

Utility function

```
In[276]:= toList[val_List] := val;
          toList[val_] := {val};
```

## TimeLineMapZip

Combines two time series like a zipper, which is more efficient than using `Union[TimeLineDates[]]` and then applying `f` to each interval. This function is the same as the core `TimeLineMap`, but doesn't handle uncertainty when applying the function `f` at each interval. This is needed for when the default value in an `IfThen` statement is `Uncertain`.

```
In[278]:= TimeLineMapZip[f_, ts1_TimeLine, ts2_TimeLine] :=
  Block[{result, t1 = TimeLineUnbox[ts1],
    t2 = TimeLineUnbox[ts2], idx1 = 1, idx2 = 1, val1,
    val2, t1Count, t2Count, nextIdx1, nextIdx2,
    nextDate1, nextDate2, pair, lastPair},

    result = Reap[

      val1 = t1[[1, 2]];
      val2 = t2[[1, 2]];
      t1Count = Length[t1];
      t2Count = Length[t2];

      (* Add first time-value pair. Assumes all time series start at dawn! *)
      lastPair = f[val1, val2];
      (* Sow[{DawnOfTime, lastPair}]; *)
      (* Time series starts at earlier of input time series *)
      Sow[
        {Min[{DateObject[t1[[1, 1]]], DateObject[t2[[1, 1]]]}][[1]], lastPair}];

      (* Walk along the two time series,
        index by index, until reaching the end of both. *)
      While[idx1 < t1Count || idx2 < t2Count,
```

```

(* Don't exceed the length of either list *)
nextIdx1 = Min[idx1 + 1, t1Count];
nextIdx2 = Min[idx2 + 1, t2Count];

(* Get the next change date of each time series *)
nextDate1 = t1[[nextIdx1, 1]];
nextDate2 = t2[[nextIdx2, 1]];

Which[
  (* Case 1: If the next dates on each series are the same,
    advance along both *)
  nextDate1 === nextDate2,
  idx1 = nextIdx1;
  val1 = t1[[idx1, 2]];
  idx2 = nextIdx2;
  val2 = t2[[idx2, 2]];
  pair = f[val1, val2];
  (* Only add a value if it differs from the prior value *)
  If[pair != lastPair, Sow[{nextDate1, pair}]; lastPair = pair;],,

  (* Otherwise, advance along the series that's farther behind,
    except when the one that's behind is at its end *)
  (* Case 2: Advance time series 1 *)
  idx2 === t2Count || (idx1 != t1Count &&
    AbsoluteTime[nextDate2] > AbsoluteTime[nextDate1]),
  idx1 = nextIdx1;
  val1 = t1[[idx1, 2]];
  pair = f[val1, val2];

  If[pair != lastPair, Sow[{nextDate1, pair}]; lastPair = pair;],,

  (* Case3: Advance time series 2 *)
  True,
  idx2 = nextIdx2;
  val2 = t2[[idx2, 2]];
  pair = f[val1, val2];

  If[pair != lastPair, Sow[{nextDate2, pair}]; lastPair = pair;];
]; (* End Which *)
]; (* End While loop *)

][[2, 1]]; (* End Reap *)

```

```
Return[If[Length[result] === 1, result[[1, 2]], TimeLine[result]]];
];
```

This version is necessary in order to ensure that the if function doesn't return a time series that starts at the DawnOfTime:

```
In[279]:= TimeLineMapZip[f_, ts1_TimeLine, scalar_] :=
TimeLineMapZip[f, ts1, TimeLine[{{TimeLineUnbox[ts1][[1, 1]], scalar}}]]
```

## Visualizing TimeLines

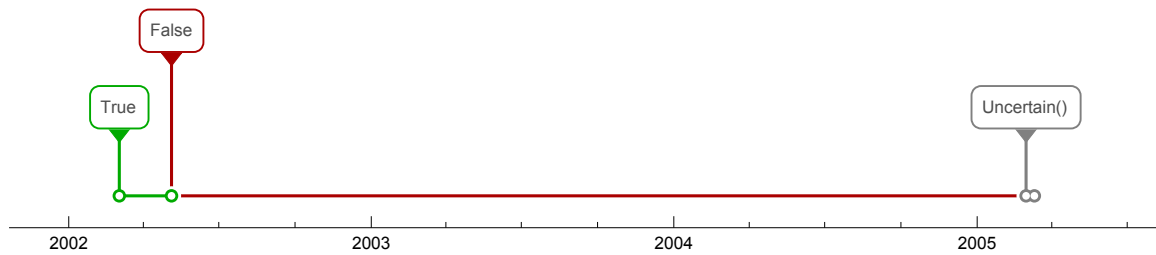
The following functions visually display TimeLines. Note that TimeLinePlot, defined here, is different from TimeLinePlot, defined in the core Wolfram Language.

```
In[280]:= TimeLinePlot[ts_TimeLine] :=
TimelinePlot[plotData[ts], PlotLayout → "Overlapped"];
TimeLinePlot[list_List] := TimelinePlot[
plotData[#] & /@ list, PlotLayout → "Overlapped"];

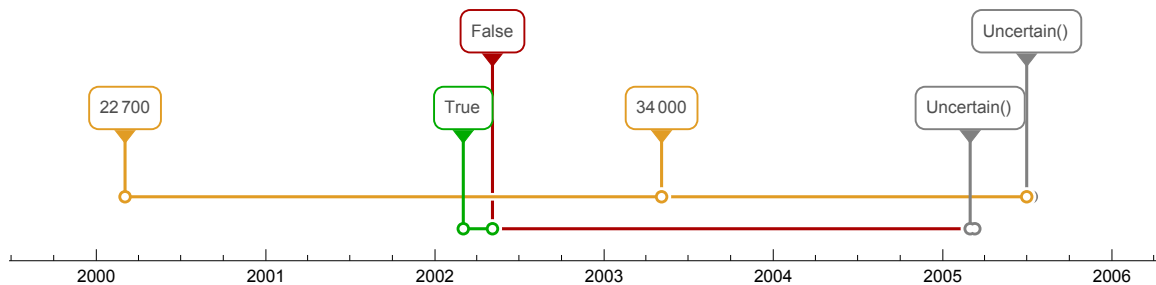
plotData[ts_TimeLine] := intervalStyling[#[[1]], #[[3]], #[[2]]] & /@
Partition[Flatten[TimeLineUnbox[ts], 1], 3, 2, 1, x]

intervalStyling[start_, x, value_] :=
intervalStyling[start, DatePlus[start, {10, "Day"}], value];
intervalStyling[start_, end_, True] := Style[Labeled[
Interval[{DateObject[start], DateObject[end]}], True], Darker[Green]];
intervalStyling[start_, end_, False] := Style[
Labeled[Interval[{DateObject[start], DateObject[end]}], False], Darker[Red]];
intervalStyling[start_, end_, val_Uncertain] :=
Style[Labeled[Interval[{DateObject[start], DateObject[end]}], val], Gray];
intervalStyling[start_, end_, val_RuleStub] :=
Style[Labeled[Interval[{DateObject[start], DateObject[end]}], val], Purple];
intervalStyling[start_, end_, val_] :=
Labeled[Interval[{DateObject[start], DateObject[end]}], val];
```

```
TimeLinePlot[
  TimeLine[{{2002, 3, 3}, True}, {{2002, 5, 5}, False}, {{2005, 3, 1}, Uncertain[]}]
```



```
TimeLinePlot[
  {TimeLine[{{2002, 3, 3}, True}, {{2002, 5, 5}, False}, {{2005, 3, 1}, Uncertain[]}]},
  TimeLine[{{2000, 3, 3}, 22700},
    {{2003, 5, 5}, 34000}, {{2005, 7, 1}, Uncertain[]}]}
```



## TimeLineWindow

Extracts a subset of a TimeLine. The input dates must be in the {y,m,f} format.

```
In[1096]:= TimeLineWindow[t_, start_, end_] := Block[{s, e},
  s = AbsoluteTime[start];
  e = AbsoluteTime[end];
  TimeLine[Select[TimeLineUnbox[t], s ≤ AbsoluteTime[#[[1]]] < e &]]
]
```

```
In[1104]:= TimeLineWindow[theMonth, {2000, 1, 1}, {2001, 1, 1}] // AbsoluteTiming
```

```
Out[1104]= {0.013619,
  TimeLine[{{2000, 1, 1}, 1}, {{2000, 2, 1}, 2}, {{2000, 3, 1}, 3}, {{2000, 4, 1}, 4},
    {{2000, 5, 1}, 5}, {{2000, 6, 1}, 6}, {{2000, 7, 1}, 7}, {{2000, 8, 1}, 8},
    {{2000, 9, 1}, 9}, {{2000, 10, 1}, 10}, {{2000, 11, 1}, 11}, {{2000, 12, 1}, 12}]}
```

## TimeLineGenerator

Using the date values of an existing TimeLine, this function applies a generating function to establish

the new values for those dates.

```
In[290]:= TimeLineGenerator[timeLine_, fcn_] := Block[{dates},
  dates = TimeLineDates[timeLine];
  TimeLine[Transpose[{dates, Array[fcn, Length[dates]]}]]
];
SetAttributes[TimeLineGenerator, HoldAll];
```

```
TimeLineGenerator[theYear, #^2 &]
```

## Helper Functions

Helper function to keep dates in a format that maximizes performance:

```
In[292]:= toDateList[date_DateObject] := DateList[date][[1 ;; 3]]
```

Zip up a list of dates and a list of values to form a new timeline:

```
In[293]:= zipDatesValues[dates_, vals_] := TimeLine[Transpose[{dates, vals}]]
```

Determines whether the first interval in a time series represents the dawn of time:

```
In[294]:= timelineStartsAtDawn[ts_] := handleUncertainty[
  DateList[TimeLineUnbox[ts][[1]][[1]]][[1 ;; 3]] ==
  DateList[DawnOfTime][[1 ;; 3]], {ts}];
```

## AnnualTimeLine

Keep?:

(\* Creates a time series starting at the beginning of a calendar year,  
with values that change annually \*)

```
AnnualTimeLine[startYear_, list_List] := TimeLine[
  Insert[Transpose[{Map[{#, 1, 1} &,
    Range[startYear, startYear + Length[list] - 1]], list}],
    {DawnOfTime, RuleStub["Law"]}, {1}]
];
```

## Boolean TimeLines

### Boolean Operators

Ordinary logical operators, but they apply to time series objects rather than Boolean objects.

```

In[295]:= and[p_, q_] := TimeLineMap[And, p, q];
Unprotect[And];
And[False, a_] = False;
And[a_, False] = False;
And[a_, b_TimeLine] := and[a, b];
And[a_, b_RuleStub] := and[a, b];
And[a_, b_Uncertain] := and[a, b];
Protect[And];

or[p_, q_] := TimeLineMap[Or, p, q];
Unprotect[Or];
Or[True, a_] = True;
Or[a_, True] = True;
Or[a_, b_TimeLine] := or[a, b];
Or[a_, b_RuleStub] := or[a, b];
Or[a_, b_Uncertain] := or[a, b];
Protect[Or];

not[p_] := TimeLineMap[!#&, p];
Unprotect[Not];
Not[a_TimeLine] := not[a];
Not[a_RuleStub] := a;
Not[a_Uncertain] := a;
Protect[Not];

```

## Constructing Boolean TimeLines

An easy way to construct time series whose truth value changes on a particular date (or dates).



```

In[317]:= (* Creates a Boolean time series that's false before a given date,
and true thereafter *)
TrueOnOrAfter[date_] := handleUncertainty[
  If[date === DawnOfTime, True,
    TimeLine[{{DawnOfTime, False}, {date, True}}]],
  {date}];

(* Creates a Boolean time series that's true before a given date,
and false thereafter *)
TrueBefore[date_] := handleUncertainty[
  If[date === DawnOfTime, False,
    TimeLine[{{DawnOfTime, True}, {date, False}}]],
  {date}];

(* Creates a Boolean time series that's true between two dates,
and otherwise false *)
TrueBetween[date1_, date2_] := and[TrueOnOrAfter[date1], TrueBefore[date2]];

```

## Ever or Always True

Does a time series ever have a particular value? Does it always? (TODO: Deal with scalars and temporal uncertainty.)

```

In[320]:= EverQ[ts_, val_] := MemberQ[TimeLineValues[ts], val];
EverQ[ts_] := EverQ[ts, True];

AlwaysQ[ts_, val_] := Block[{vals = TimeLineValues[ts]},
  and[Length[vals] === 1, MemberQ[vals, val]]]; (* TODO: rewrite *)
AlwaysQ[ts_] := AlwaysQ[ts, True];

```

## When Next or Last True

At any point in time, when will a Boolean time series next be true? If, at a given point in time, the TimeLine is true, the value for that interval is theDate. TODO: handle scalars, temporal uncertainty, test, performance, and prevent TimeLines as arguments.

In[324]:=

```

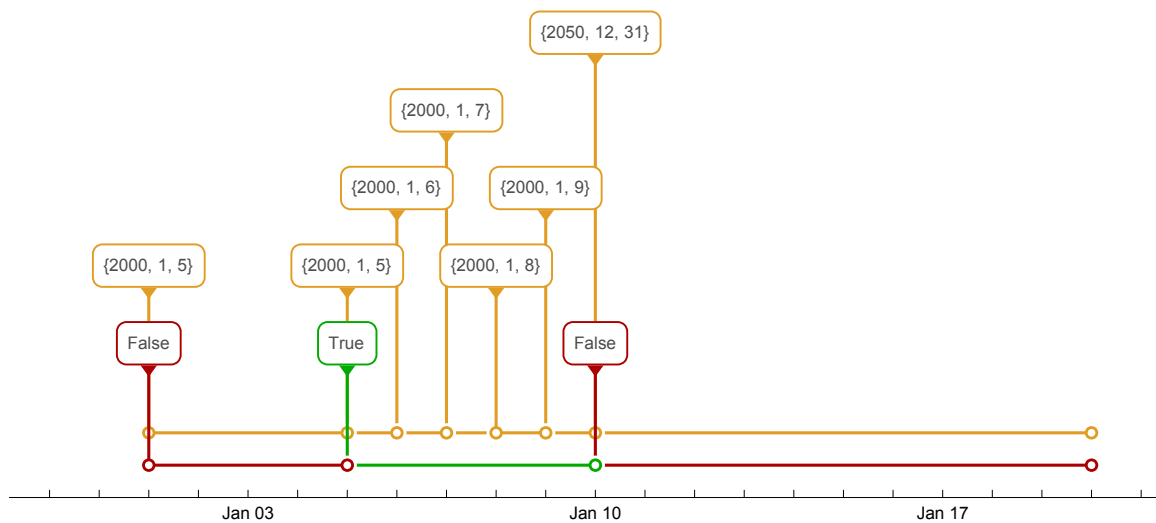
DateNextTrue[timeLine_] := Block[{currentlyTrue, vals, newVals},
  currentlyTrue = IfThen[timeLine, theDate, x];
  (* Assign theDate to True intervals. Slow b/c it uses theDate. *)
  vals = TimeLineValues[currentlyTrue];
  If[Last[vals] == x, vals[[-1]] = EndOfTime]; (* Deal with last interval *)
  newVals = Map[If[#[[1]] == x, #[[2]], #[[1]]] &, Partition[vals, 2, 1, 1, x]];
  (* Get date of next True interval *)
  zipDatesValues[TimeLineDates[currentlyTrue], newVals]
]

```

```

testval = TrueOnOrAfter[{2000, 1, 5}] && TrueBefore[{2000, 1, 10}];
TimeLinePlot[{testval, DateNextTrue[testval]}]

```



At any point in time, when was a Boolean time series last true?

In[325]:=

```

DateLastTrue[timeLine_] := RuleStub[];

```

## Conditionals

### Usage Note

The following conditional functions implement time series versions of `If`, `Which`, and `Switch`. Due to the difficulty of filtering function definitions so as to cleanly separate the time series version from the core Wolfram Language version, the conditional functions have names that diverge from the Wolfram Language. (Conditionals are delicate since, if implemented carelessly, they can easily spin out into infinite loops.)

### Internallf

The if-then-else function for time series, used only within this package. This evaluates lazily to permit

temporal recursion.

```
In[326]:= InternalIf[True, val1_, val2_] := val1;
InternalIf[False, val1_, val2_] := val2;
InternalIf[test_, val1_, val2_] :=
  Block[{first = TimeLineMap[ifTrueMap, test, val1]},
    If[!EverQ[first, False], first, TimeLineMapZip[ifFalseMap, first, val2]]
  ];
SetAttributes[InternalIf, HoldRest]

(* Broken out to make if[] slightly easier to read. *)
ifTrueMap[val1_, val2_] := If[val1, val2, val1];
ifFalseMap[val1_, val2_] := If[val1 === False, val2, val1];
```

## IfThen

This is the externally exposed function that implements the time series version of If and Which. Note that, unlike Which, the last argument is the default value.

```
In[332]:= IfThen[test1_, val1_, args___, def_] :=
  InternalIf[test1, val1, IfThen[args, def]];
S
IfThen[def_] := def;
SetAttributes[IfThen, HoldRest];
```

Out[332]= S

## SwitchThen

This is the time series version of Switch. Note that it's different than Switch because here the last argument is the default value.

```

In[335]:= SwitchThen[test_List, t1_, v1_, args___, def_] :=
  InternalIf[listEqual[t1, test], v1, SwitchThen[test, args, def]];
SwitchThen[test_, t1_, v1_, args___, def_] :=
  InternalIf[t1 == test, v1, SwitchThen[test, args, def]];
SwitchThen[irrelevantTest_, def_] := def;
SetAttributes[SwitchThen, HoldRest];

(* Tests for the equality of two lists *)
listEqual[{a1_, a2_}, {b1_, b2_}] :=
  handleUncertainty[a1 == b1 && a2 == b2, {a1, a2, b1, b2}];
listEqual[a_List, b_List] := handleUncertainty[a == b, Join[a, b]];

```

## Arithmetic

### Addition & Subtraction

Note: The order of these overloaded definitions seems to affect performance (or did in a previous implementation from which this was adapted).

Addition & subtraction:

```

In[341]:= plus[ts1_, ts2_] := TimeLineMap[Plus, ts1, ts2]
Unprotect[Plus];
Plus[a_?DSLTypeQ, b_?DSLTypeQ] := plus[a, b];
Plus[a_?DSLTypeQ, b_] := plus[a, b];
Plus[a_, b_?DSLTypeQ] := plus[a, b];
Protect[Plus];

```

Possibly to add to the definition of Plus: Without this, interview question order sometimes gets rearranged. However, Orderless only remains cleared in Akkadian.nb upon first initialization. Upon the second evaluation of Attributes[Plus], Orderless is mysteriously back.

```
ClearAttributes[Plus, Orderless];
```

### Multiplication

Multiplication:

```
In[347]:= times[ts1_, ts2_] := TimeLineMap[Times, ts1, ts2]
Unprotect[Times];
Times[a_ ? DSLTypeQ, b_ ? DSLTypeQ] := times[a, b];
Times[a_ ? DSLTypeQ, b_] := times[a, b];
Times[a_, b_ ? DSLTypeQ] := times[a, b];
Protect[Times];
```

## Division

Division. Because the Wolfram Language represents  $a/b$  as `Times[a, Power[b, -1]]`, the `Power` function has to be overloaded as well.

```
In[353]:= Unprotect[Power];
Power[t_ ? DSLTypeQ, -1] := TimeLineMap[1 / # &, t];
Protect[Power];
```

## Numeric

Numerical form. Like `N[]`, this converts fractions to decimal form.

```
In[356]:= Numeric[ts_] := TimeLineMap[N, ts];
```

## Modulo

Gives the remainder on division of  $m$  by  $n$ :

```
In[357]:= Modulo[m_, n_] := TimeLineMap[Mod, m, n]
```

## Rounding

Round to the nearest multiple (ties broken by rounding toward the even alternative):

```
In[440]:= Unprotect[Round];
Round[a_ ? DSLTypeQ, b_ ? DSLTypeQ] := TimeLineMap[Round, a, b];
Round[a_ ? DSLTypeQ, b_] := TimeLineMap[Round, a, b];
Round[a_, b_ ? DSLTypeQ] := TimeLineMap[Round, a, b];
Round[a_ ? DSLTypeQ] := TimeLineMap[Round, a, 1];
Protect[Round];
```

Ceiling (round up to the nearest multiple):

In[423]:=

```
Unprotect[Ceiling];
Ceiling[a_ ? DSLTypeQ, b_ ? DSLTypeQ] := TimeLineMap[Ceiling, a, b];
Ceiling[a_ ? DSLTypeQ, b_] := TimeLineMap[Ceiling, a, b];
Ceiling[a_, b_ ? DSLTypeQ] := TimeLineMap[Ceiling, a, b];
Ceiling[a_ ? DSLTypeQ] := TimeLineMap[Ceiling, a, 1];
Protect[Ceiling];
```

Floor (round down to the nearest multiple):

In[430]:=

```
Unprotect[Floor];
Floor[a_ ? DSLTypeQ, b_ ? DSLTypeQ] := TimeLineMap[Floor, a, b];
Floor[a_ ? DSLTypeQ, b_] := TimeLineMap[Floor, a, b];
Floor[a_, b_ ? DSLTypeQ] := TimeLineMap[Floor, a, b];
Floor[a_ ? DSLTypeQ] := TimeLineMap[Floor, a, 1];
Floor[Ceiling];
```

## Comparison

### Comparison

Comparison operators:

In[358]:=

```

greater[ts1_, ts2_] := TimeLineMap[Greater, ts1, ts2]
Unprotect[Greater];
Greater[a_ ? DSLTypeQ, b_ ? DSLTypeQ] := greater[a, b];
Greater[a_ ? DSLTypeQ, b_] := greater[a, b];
Greater[a_, b_ ? DSLTypeQ] := greater[a, b];
Protect[Greater];

greaterEqual[ts1_, ts2_] := TimeLineMap[GreaterEqual, ts1, ts2]
Unprotect[GreaterEqual];
GreaterEqual[a_ ? DSLTypeQ, b_ ? DSLTypeQ] := greaterEqual[a, b];
GreaterEqual[a_ ? DSLTypeQ, b_] := greaterEqual[a, b];
GreaterEqual[a_, b_ ? DSLTypeQ] := greaterEqual[a, b];
Protect[GreaterEqual];

less[ts1_, ts2_] := TimeLineMap[Less, ts1, ts2]
Unprotect[Less];
Less[a_ ? DSLTypeQ, b_ ? DSLTypeQ] := less[a, b];
Less[a_ ? DSLTypeQ, b_] := less[a, b];
Less[a_, b_ ? DSLTypeQ] := less[a, b];
Protect[Less];

lessEqual[ts1_, ts2_] := TimeLineMap[LessEqual, ts1, ts2]
Unprotect[LessEqual];
LessEqual[a_ ? DSLTypeQ, b_ ? DSLTypeQ] := lessEqual[a, b];
LessEqual[a_ ? DSLTypeQ, b_] := lessEqual[a, b];
LessEqual[a_, b_ ? DSLTypeQ] := lessEqual[a, b];
Protect[LessEqual];

equal[ts1_, ts2_] := TimeLineMap[Equal, ts1, ts2]
Unprotect[Equal];
Equal[a_ ? DSLTypeQ, b_ ? DSLTypeQ] := equal[a, b];
Equal[a_ ? DSLTypeQ, b_] := equal[a, b];
Equal[a_, b_ ? DSLTypeQ] := equal[a, b];
Protect[Equal];

unequal[ts1_, ts2_] := TimeLineMap[Unequal, ts1, ts2]
Unprotect[Unequal];
Unequal[a_ ? DSLTypeQ, b_ ? DSLTypeQ] := unequal[a, b];
Unequal[a_ ? DSLTypeQ, b_] := unequal[a, b];
Unequal[a_, b_ ? DSLTypeQ] := unequal[a, b];
Protect[Unequal];

```

## Date Comparisons

AbsoluteTime

```
{2000, 2, 2} > {2000, 2, 1}
```

```
{2000, 2, 2} > {2000, 2, 1}
```

## Date Operations

### Composition

At least one of the inputs must be a scalar. (Need TimeLineMap to handle three inputs in order to implement this fully.)

```
In[1200]:= date[y_? DSLTypeQ, m_ , d_] := TimeLineMap[{#, m, d} &, y];
date[y_? DSLTypeQ, m_?DSLTypeQ, d_] := TimeLineMap[{#1, #2, d} &, y, m];
date[y_? DSLTypeQ, m_ , d_?DSLTypeQ] := TimeLineMap[{#1, m, #2} &, y, d];
date[y_ , m_? DSLTypeQ, d_] := TimeLineMap[{y, #, d} &, m];
date[y_ , m_? DSLTypeQ, d_?DSLTypeQ] := TimeLineMap[{y, #1, #2} &, m, d];
date[y_ , m_ , d_? DSLTypeQ] := TimeLineMap[{y, m, #} &, d];
date[y_ , m_ , d_] := {y, m, d};
```

```
In[1199]:= Clear[date]
```

```
In[1198]:= date[2016, 3, theMonth]
```

### Decomposition

Extract the components of a (possibly temporal) date:

```
In[394]:= GetYear[date_] := TimeLineMap[#[[1]] &, date];
GetMonth[date_] := TimeLineMap[#[[2]] &, date];
GetDay[date_] := TimeLineMap[#[[3]] &, date];
```

## Differences Between Dates

These functions do not follow the naming pattern of their counterparts in the Wolfram Language.

Gives the number of days between two dates (without units of measure):

```
In[453]:= DayDiff[a_ , b_] :=
  TimeLineMap[QuantityMagnitude[DateDifference[#1, #2]] &, a, b];
```

Gives the number of weeks between two dates (without units of measure):

```
In[466]:= WeekDiff[a_ , b_] :=
  TimeLineMap[QuantityMagnitude[DateDifference[#1, #2, "Week"]] &, a, b];
```



Gives the number of months between two dates (without units of measure):

```
In[467]:= MonthDiff[a_, b_] :=  
    TimeLineMap[QuantityMagnitude[DateDifference[#1, #2, "Month"]] &, a, b];
```

Gives the number of years between two dates (without units of measure):

```
In[463]:= YearDiff[a_, b_] :=  
    TimeLineMap[QuantityMagnitude[DateDifference[#1, #2, "Year"]] &, a, b];
```

## A Date Plus Some Interval

These functions do not follow the naming pattern of their counterparts in the Wolfram Language.

```
In[479]:= AddDays[date_, n_] := TimeLineMap[DatePlus[#1, #2] &, date, n];
```

```
In[482]:= AddWeeks[date_, n_] := TimeLineMap[DatePlus[#1, {#2, "Week"}] &, date, n];
```

```
In[487]:= AddMonths[date_, n_] := TimeLineMap[DatePlus[#1, {#2, "Month"}] &, date, n];
```

```
In[489]:= AddYears[date_, n_] := TimeLineMap[DatePlus[#1, {#2, "Year"}] &, date, n];
```

## List Operations

### Length

```
In[1118]:= Len[ts_] := TimeLineMap[Length, ts];
```

### Other

Min

Max

Map

MemberQ

Total

ContainsAny

ContainsAll

## Interval Count Relative to a Point in Time

### YearsSince

Creates a timeline that counts years until/from a given date. Essentially, this returns a number line that has the interval of a year, with the origin located at

the input date. Date input is assumed not to be a time series. Avg. execution time = 0.0002s when time range is 70 years.

```
In[397]:= YearsSince[date_] := handleUncertainty[
  Block[{ts = TimeLine[Table[{{i, date[[2]], date[[3]]}, i - date[[1]]},
    {i, DawnOfTime[[1]], EndOfTime[[1]]}]]},
  If[timeLineStartsAtDawn[ts], ts,
    TimeLine[Insert[TimeLineUnbox[ts],
      {DawnOfTime, TimeLineFirstValue[ts] - 1}, {1}] ]],
  ],
  {date}];
```

## DaysSince

Creates a timeline that counts days until/from a given date. Date input is assumed not to be a time series. Avg. execution time = 0.007s when time range is 70 years.

```
In[398]:= DaysSince[date_] := handleUncertainty[
  Block[{start = QuantityMagnitude[DateDifference[DawnOfTime, date]] * -1,
    diff = Length[absoluteTimeDates] - 1},
  TimeLine[Transpose[{absoluteTimeDates, Range[start, start + diff]}]]],
  ],
  {date}];
```

## MonthsSince

## WeeksSince

Counts weeks until/from a given date:

```
In[496]:= WeeksSince[date_] := Floor[DaysSince[date] / 7];
```

## IntervalsSince

IN PROGRESS. This creates a number line with the origin at a particular point in time:

```
In[400]:= IntervalsSince[origin_, interval_] := TimeLineGenerator[interval, #^2 &]
```

```
IntervalsSince[{2017, 1, 1}, theYear]
```

```
Array[# - 20 &, 50]
```

```
{-19, -18, -17, -16, -15, -14, -13, -12, -11, -10, -9, -8,
 -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12,
 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30}
```

TimeLineDates[theYear]

```
{ {2000, 1, 1}, {2001, 1, 1}, {2002, 1, 1}, {2003, 1, 1}, {2004, 1, 1},
  {2005, 1, 1}, {2006, 1, 1}, {2007, 1, 1}, {2008, 1, 1}, {2009, 1, 1},
  {2010, 1, 1}, {2011, 1, 1}, {2012, 1, 1}, {2013, 1, 1}, {2014, 1, 1},
  {2015, 1, 1}, {2016, 1, 1}, {2017, 1, 1}, {2018, 1, 1}, {2019, 1, 1}, {2020, 1, 1},
  {2021, 1, 1}, {2022, 1, 1}, {2023, 1, 1}, {2024, 1, 1}, {2025, 1, 1}, {2026, 1, 1},
  {2027, 1, 1}, {2028, 1, 1}, {2029, 1, 1}, {2030, 1, 1}, {2031, 1, 1}, {2032, 1, 1},
  {2033, 1, 1}, {2034, 1, 1}, {2035, 1, 1}, {2036, 1, 1}, {2037, 1, 1}, {2038, 1, 1},
  {2039, 1, 1}, {2040, 1, 1}, {2041, 1, 1}, {2042, 1, 1}, {2043, 1, 1}, {2044, 1, 1},
  {2045, 1, 1}, {2046, 1, 1}, {2047, 1, 1}, {2048, 1, 1}, {2049, 1, 1}, {2050, 1, 1} }
```

## Absolute Time

### Internal Machinery

List of dates from the dawn of time to the end of time. This exists for performance reasons, as an internal representation of absolute time; it is not expected to be referenced externally.

```
In[401]:= absoluteTimeDates = DateRange[DawnOfTime, EndOfTime];
```

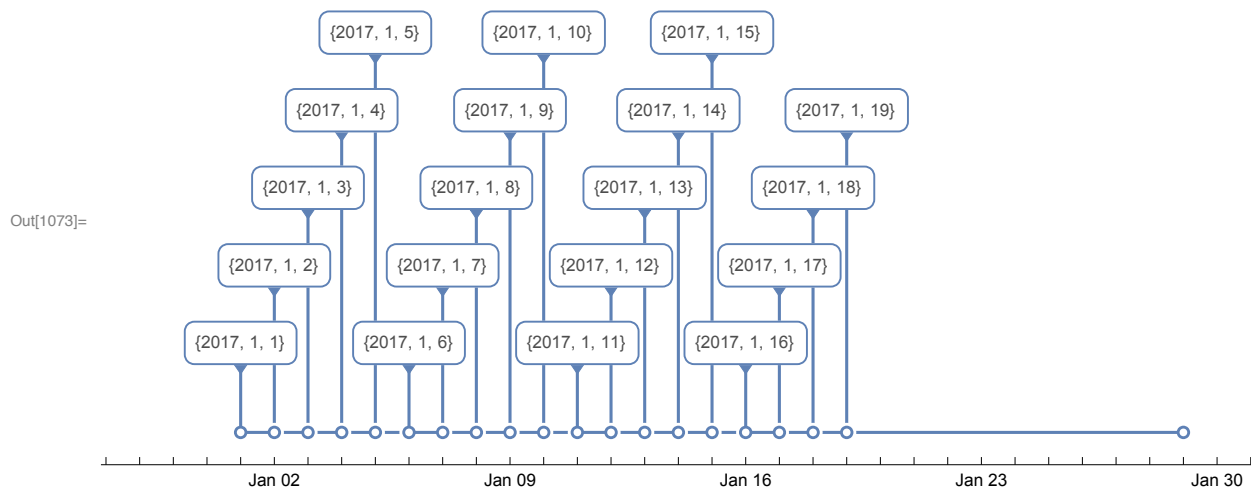
Because the following variables are set using = instead of :=, they must be ordered in the notebook so dependencies are evaluated first (i.e. be further up the page).

### Calendar-Based TimeLines

A time series in which each day has the value of the date itself. For example, on July 11, 2034, the time series value is {2034,7,11}.

```
In[402]:= theDate = TimeLine[Transpose[{absoluteTimeDates, absoluteTimeDates}]];
```

```
In[1073]:= TimeLinePlot[TimeLineWindow[theDate, {2017, 1, 1}, {2017, 1, 19}]]
```



```
In[403]:= theMonth = GetMonth[theDate];
```

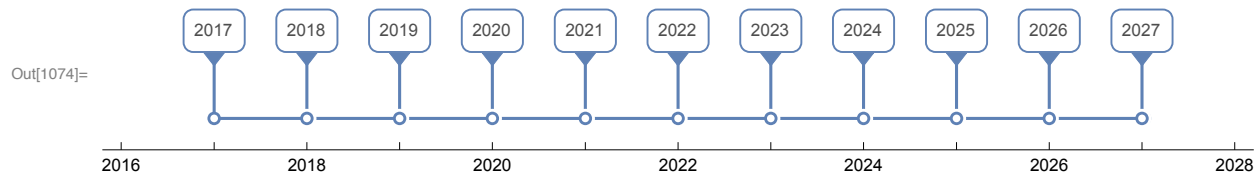
```
In[404]:= theDayOfTheMonth = GetDay[theDate];
```

```
In[405]:= theCalendarQuarter = TimeLineMap[Which[
  # < 4, 1,
  # < 7, 2,
  # < 10, 3,
  True, 4] &,
  theMonth];
```

A time series representing the calendar year.

```
In[406]:= theYear = GetYear[theDate];
```

```
In[1074]:= TimeLinePlot[TimeLineWindow[theYear, {2017, 1, 1}, {2027, 1, 19}]]
```



```
In[407]:= leapYearQ = TimeLineMap[LeapYearQ[{#}] &, theYear];
```

```
In[408]:= theCalendarWeek = RuleStub[];
```

## Interval Ratios

```
In[409]:= daysPerYear = TimeLineMap[If[LeapYearQ[{#}], 366, 365] &, theYear];
```

```
daysPerMonth = SubintervalsPer[theDate, theMonth];
```

```
In[411]:= daysPerCalendarQuarter = RuleStub[];
```

```
In[1214]:= weeksPerMonth = daysPerMonth / 7;
```

```
In[1215]:= weeksPerMonth
```

```
Out[1215]= RuleStub[]
```

```
daysPerQuarter = SubintervalsPerInterval[theDate, theCalendarQuarter]
```

## First/Last Day of Interval

Time series representing the first and last day of every month. It may be the case in practice that a function like `firstDayOfMonth[date]` does the trick and that these are unnecessary.

```
In[1209]:= firstDayOfMonth = date[theYear, theMonth, 1];
```

```
In[1213]:= lastDayOfMonth = AddDays[AddMonths[firstDayOfMonth, 1], -1];
```

## Weekdays

This is an internal helper function. Even though ISO 8601 considers Monday as the first day of the week, the U.S. follows a different convention:

```
In[414]:= dayNameToNumber = <|Sunday → 1, Monday → 2,
    Tuesday → 3, Wednesday → 4, Thursday → 5, Friday → 6, Saturday → 7|>;
```

Day of the week (as a temporal number):

```
In[415]:= dayOfWeek = TimeLine[Transpose[{TimeLineDates[theDate],
    Array[Mod[#, 7] + 1 &, Length[absoluteTimeDates],
        dayNameToNumber[DateValue[DawnOfTime, "DayName"]] - 1]
    }]];
```

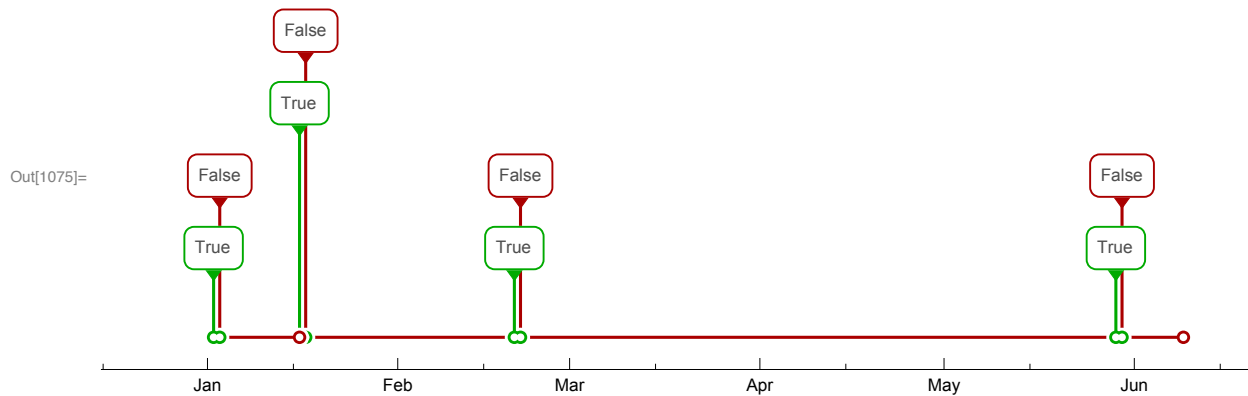
Is the day a weekday?

```
In[416]:= weekdayQ = ! (dayOfWeek == 1 || dayOfWeek == 7);
```

Is the day a U.S. Federal holiday? Note: The legal authority for this is 5 U.S.C. 6103, but we're relying on the Wolfram Language `HolidayCalendar` implementation. This implementation currently (Sept. 2016) has a bug that fails to recognize Veterans Days that fall on a Saturday but that are observed on the preceding Friday. But has been reported and assuming it will eventually get fixed.

```
In[417]:= usFederalHolidayQ = Block[{holidays, firstVal},
    holidays = DayRange[DawnOfTime, EndOfTime, "Holiday"];
    firstVal = {{DatePlus[DawnOfTime, -1], False}};
    TimeLine[Union[firstVal, Map[{toDateList[#], True} &, holidays],
        Map[{toDateList[#], False} &, DatePlus[holidays, 1]]]]
];
```

```
In[1075]:= TimeLinePlot[TimeLineWindow[usFederalHolidayQ, {2017, 1, 1}, {2017, 6, 1}]]
```



```
In[418]:= businessDayQ = weekdayQ && !usFederalHolidayQ;
```

```
In[419]:= nextBusinessDay = DateNextTrue[businessDayQ];
```

## Per Interval

### All Values in a Subinterval

For each interval in a larger time period, gather all of the time series values of a TimeLine with smaller intervals. This is pretty slow.

```
In[1115]:= GatherValuesPer[ts_, interval_] := Block[{dateSpans},
  dateSpans = Partition[TimeLineDates[interval], 2, 1, 1, x] /. x -> EndOfTime;
  TimeLine[Map[
    {#[[1]], TimeLineValues[TimeLineWindow[ts, #[[1]], #[[2]]]]} &, dateSpans]]
]
```

```
In[1119]:= GatherValuesPer[theMonth, theYear]
```

### Number of Subintervals in a Larger Interval

Count how many subintervals there are in each larger interval:

```
In[1124]:= SubintervalsPer[smaller_, larger_] := Len[GatherValuesPer[smaller, larger]];
```

```
In[1125]:= SubintervalsPer[theMonth, theYear]
```

```
Out[1125]= 12
```

## Value at End of Interval

```
IntervalEndValue[ts_, interval_] :=
```

## Other

```
EverPer[T1, T2]
```

```
AlwaysPer[T1, T2]
```

```
SubintervalCountPer
```

```
RunningCountPer
```

For each interval in T2, gather all of the values in T1:

```
GatherPer[T1, T2]
```

## Elapsed Time

## Miscellaneous

```
Regularize[T, interval]
```

Shift the values in a time series to the right or left:

```
Shift[T, n]
```

Maybe more general than Shift (applies fcn to all times):

```
TimeSeriesMapTimes[f, T]
```

---

# Proof Trees

## Proof Trees

A proof tree is a hierarchical report of the reasons for a particular determination. This needs work.

```
ProofTree[{expr_, steps___}] :=
```

```
  OpenerView[{Tooltip[expr, ReleaseHold[expr]], Column[ProofTree /@ {steps}]]}]
```

```
ProofTree[x_] := x
```

```

Trace[standardDeduction["Pat"], _["Pat"]] // ProofTree
▼ standardDeduction[Pat]
  ▼ ineligibleForStandardDeductionQ[Pat]
    ► {mfsQ[Pat], {marriedQ[Pat], {maritalStatus[Pat]}}}
  ▼ shortTaxYearQ[Pat]
    ► taxYearType[Pat]
    ► nonresidentAlienQ[Pat]
    ► dualStatusAlienQ[Pat]
  ► table20Dash3Applies[Pat]
  ► table20Dash2Applies[Pat]
  ► standardDeductionMostPeople[Pat]

```

## EndPackage

```

End[];
EndPackage[];

```