Introduction

Filters are essential components in signal processing and communications systems. They are used to selectively allow signals within certain frequency bands to pass while attenuating others. Among classical filter designs, Chebyshev filtersare popular because they provide a sharper transition between passband and stopband compared to Butterworth filters of the same order, at the expense of ripples in either the passband or the stopband.

Chebyshev filters exist in two variants:

- **Chebyshev Type I**: Equiripple behavior in the passband, monotonic stopband.
- **Chebyshev Type II**: Monotonic passband, equiripple stopband.

Chebyshev Type I Filters

Chebyshev Type I filters are defined by a passband ripple (Rp in dB). Within the passband, the magnitude response oscillates between a maximum and minimum level, producing ripples. The stopband response decreases monotonically.

Key Properties:

- Sharper roll-off compared to Butterworth filters.
- Controlled by filter order (N) and passband ripple (Rp).
- Transfer function poles are derived from Chebyshev polynomials.

Chebyshev Type II Filters

Chebyshev Type II filters are defined by a stopband attenuation (Rs in dB). Unlike Type I, they have a flat (monotonic) passband but equiripple behavior in the stopband.

Key Properties:

- No ripple in the passband (smoother than Type I).
- Controlled by filter order (N) and stopband attenuation (Rs).
- Transfer function includes zeros placed on the unit circle to enforce stopband ripples.

Mathematical Background

- Chebyshev Polynomials

The basis of Chebyshev filters is the Chebyshev polynomial of the first kind:

$$T_n(x) = \begin{cases} cos\big(narccos(x)\big), |x| \leq 1 \\ cosh\big(narccoh(x)\big), |x| \geq 1 \end{cases}$$

- Chebyshev Type I Filter

Transfer function magnitude: $|H(i\omega)|^2 = \frac{1}{1+\varepsilon^2 {T_n}^2(\frac{\omega}{\omega_c})}$ where n is the order of filter and ε is the ripple factor, related to passband ripple A_{max} like so: $\varepsilon = \sqrt{A^{\frac{A_{max}}{10}} - 1}$

Attenuation in dB: $A(\omega) = 10log_{10}(1 + \varepsilon^2 {T_n}^2(\frac{\omega}{\omega_c}))$

Order selection: $N \geq \frac{arccosh(\sqrt{\frac{A^{\frac{A_{min}}{10}}-1}{\varepsilon^2}})}{arccosh(\frac{\omega_s}{\omega_p})}$ where A_{min} is the stopband attenuation in dB, ω_s is the passband edge and ω_p is the stopband edge

- Chebyshev Type II Filter

Transfer function magnitude: $|H(i\omega)|^2 = \frac{1}{1+\frac{1}{\varepsilon^2 {T_n}^2(\frac{\omega}{\omega_c})}}$

Attenuation in dB: $A(\omega) = 10log_{10}(1 + \frac{1}{\varepsilon^2 {T_n}^2(\frac{\omega}{\omega_c})})$

- Zeros of Type-I Chebyshev Filters

All zeros are at infinity. The frequency response is shaped entirely by the pole locations.

- Zeros of Type-II Chebyshev Filters

Unlike Type-I, Type II has finite transmission zeros on the imaginary axis, given by the following equation:

$$ {s_k}^{(z)} = i\omega_c sec(\frac{(2k-1)\pi}{2n}) $$

Where k = 1, 2, 3, …, n ∈ ℕ

- Pole Locations for Type-I Chebyshev Filters

$$ s_k = \omega_c(-sinh(\frac{\beta}{n})sin(\frac{(2k-1)\pi}{2n}) + icosh(\frac{\beta}{n})cos(\frac{(2k-1)\pi}{2n})) $$

Where $\beta = arcsinh(\frac{1}{\varepsilon})$, for k = 1, 2, 3, …, n ∈ ℕ

- Pole Locations for Type-II Chebyshev Filters

$$ s_k = \omega_c(-cosh(\frac{\beta}{n})sin(\frac{(2k-1)\pi}{2n}) + sinh(\frac{\beta}{n})cos(\frac{(2k-1)\pi}{2n})) $$

Where $\beta = arcsinh(\frac{1}{\varepsilon})$, for k = 1, 2, 3, …, n ∈ ℕ

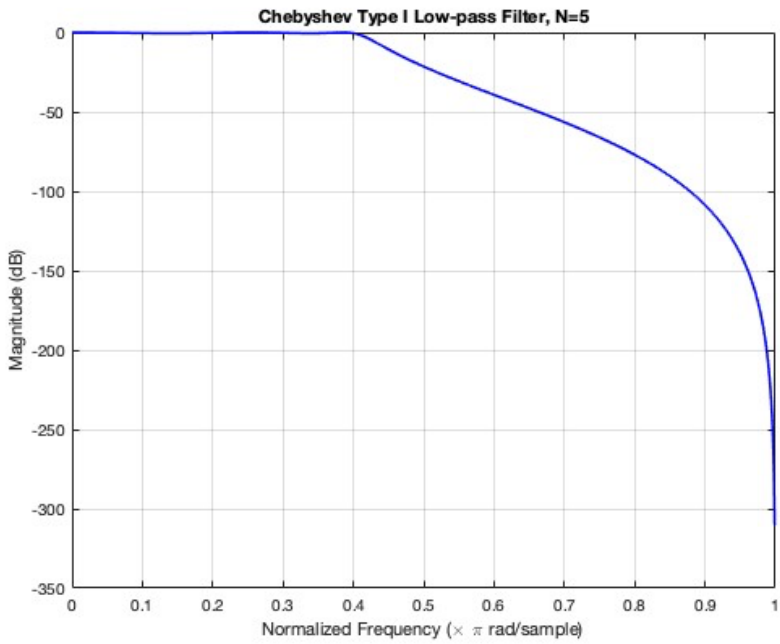
Conclusion

Chebyshev filters provide efficient designs for applications requiring sharper frequency selectivity than Butterworth filters.

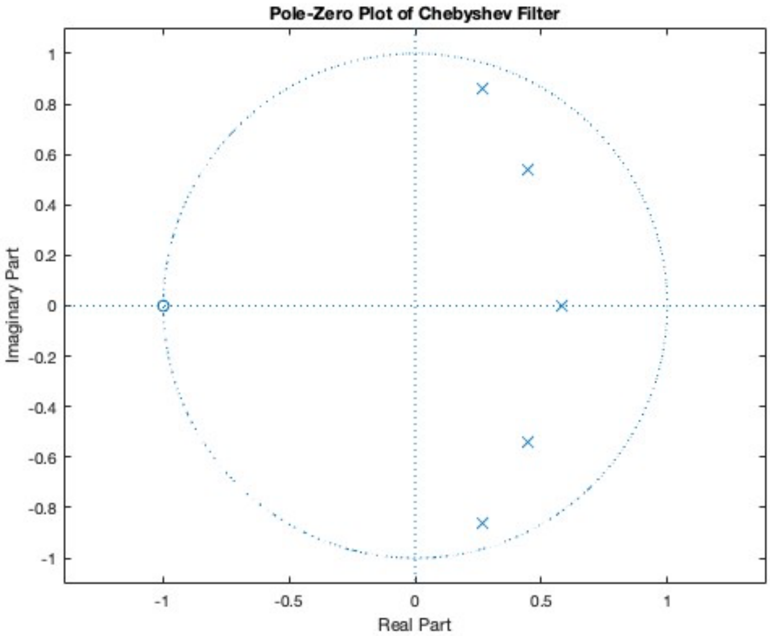
- Type I filters are used when passband ripple is tolerable and sharp transition is desired.
- Type II filters are used when a smooth passband is critical, even though the stopband may contain ripples.

MATLAB Plots

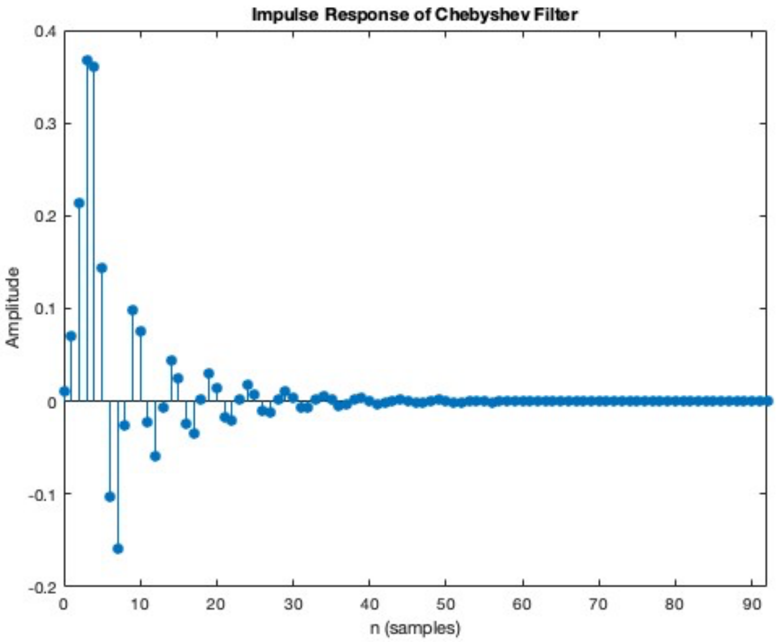
- Type-I Chebyshev Filter



The image displays a frequency response plot for a Chebyshev Type I low-pass filter with a filter order (N) of 5. The plot shows the magnitude of the filter's response, measured in decibels (dB), on the y-axis, and the normalized frequency in radians per sample on the x-axis. The filter exhibits passband ripple, which is a characteristic of Chebyshev filters, and the plot shows that the magnitude is approximately 0 dB across the passband, indicating minimal signal attenuation. Beyond the cutoff frequency, which appears to be around 0.4 on the normalized frequency axis, the magnitude drops off sharply, demonstrating a steep roll-off into the stopband where the signal is heavily attenuated. This rapid decrease in magnitude highlights the filter's effectiveness in suppressing high-frequency components while allowing low-frequency components to pass through.

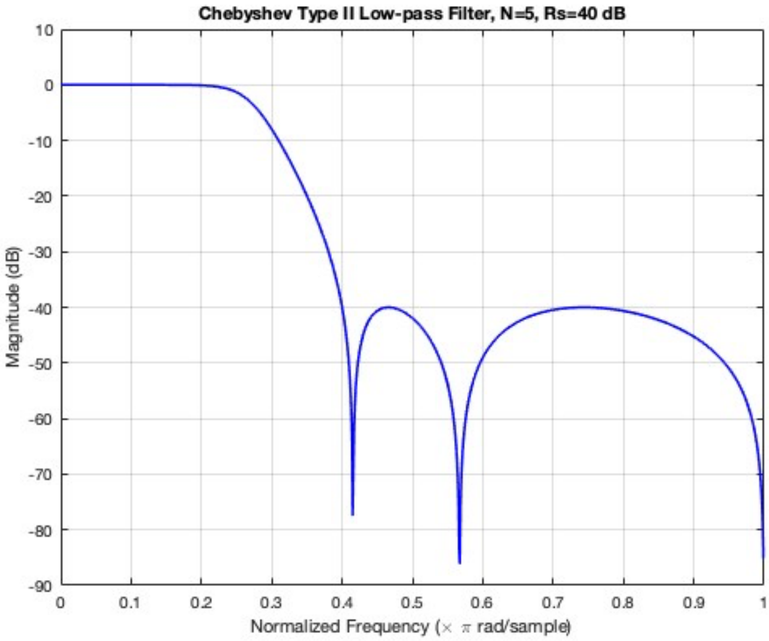


The image shows a pole-zero plot of a Chebyshev filter. The plot is drawn on the complex z-plane, where the horizontal axis represents the real part and the vertical axis represents the imaginary part. A dashed circle, known as the unit circle, is centered at the origin. The filter's poles are marked by 'x's and are located inside the unit circle, which is a condition for stability in a digital filter. The zeros are marked by 'o's, and in this specific plot, one zero is located on the unit circle at z = -1. This particular arrangement of poles and zeros is characteristic of a low-pass filter, where the poles near the unit circle at a given frequency create a large magnitude response (the passband), and the zeros on the unit circle at other frequencies create a zero-magnitude response (the stopband).

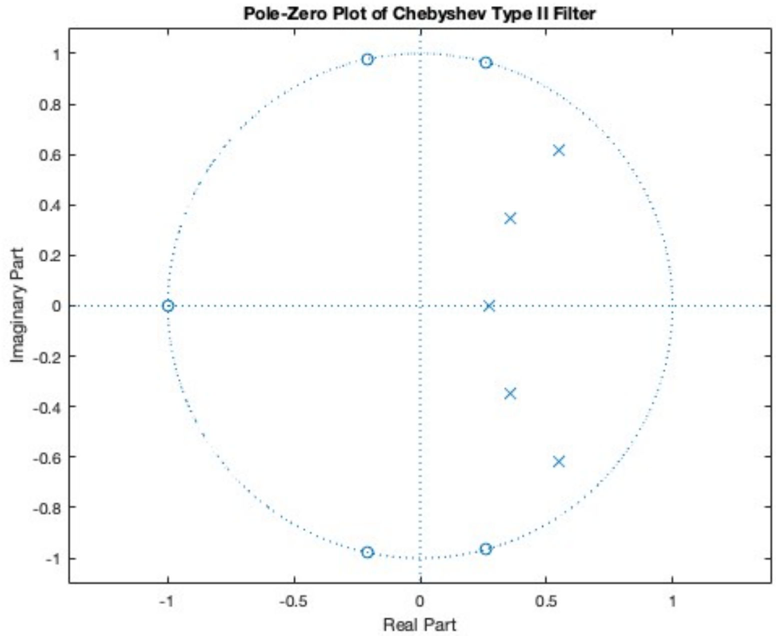


The image displays the impulse response of a Chebyshev filter, a graph showing the filter's output when the input is a single impulse. The x-axis, labeled n (samples), represents discrete time, while the y-axis, Amplitude, shows the filter's output at each sample. The plot shows the output oscillating and decaying over time, which is characteristic of an infinite impulse response (IIR) filter. The response is not zero after a finite number of samples, meaning the filter's output depends not only on the current input but also on all previous inputs and outputs. The oscillations are due to the poles of the filter, and the decay ensures the filter is stable.

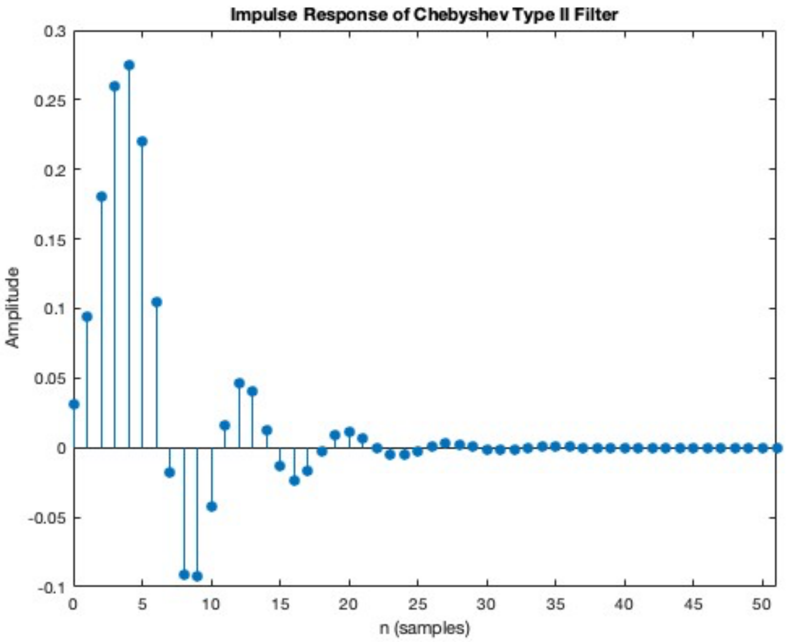
- Type-II Chebyshev Filter



This image is a frequency response plot for a Chebyshev Type II low-pass filter with a filter order (N) of 5 and a stopband attenuation (Rs) of 40 dB. The plot shows the magnitude in decibels (dB) on the y-axis versus normalized frequency on the x-axis. The key characteristics of this filter type are evident in the plot: the passband is maximally flat, with no ripple, and the stopband contains ripples, which are constant after the first drop-off. The plot also shows that the magnitude drops to at least -40 dB in the stopband, confirming the specified Rs value. This design provides a sharp transition from the passband to the stopband, with the trade-off of having ripples in the stopband instead of the passband.



This image is a pole-zero plot for a Chebyshev Type II filter displayed on the complex z-plane. The plot shows the filter's poles (marked by 'x') located inside the unit circle, a characteristic that ensures the filter is stable. The filter's zeros (marked by 'o') are located on the unit circle, specifically in the stopband region, which is a key feature of the Chebyshev Type II filter. These zeros on the unit circle are responsible for the constant ripple in the stopband and the sharp attenuation of frequencies in that region, as seen in the corresponding frequency response. The specific placement of the zeros, especially those clustered near the high-frequency end (approaching z = -1), helps to create the deep notches in the stopband, which is a key difference from a Type I filter.



This image depicts the impulse response of a Chebyshev Type II filter. The plot shows the output amplitude on the y-axis over time, represented by the sample number (n) on the x-axis, after the filter receives a single impulse input. Like the Type I filter, this is an Infinite Impulse Response (IIR) filter, as its output continues to oscillate and decay over an infinite number of

samples rather than becoming zero after a finite time. The oscillations and subsequent decay are characteristic of the filter's poles, which are located inside the unit circle,
ensuring stability. This response shows how the filter's memory of the input signal gradually fades over time.