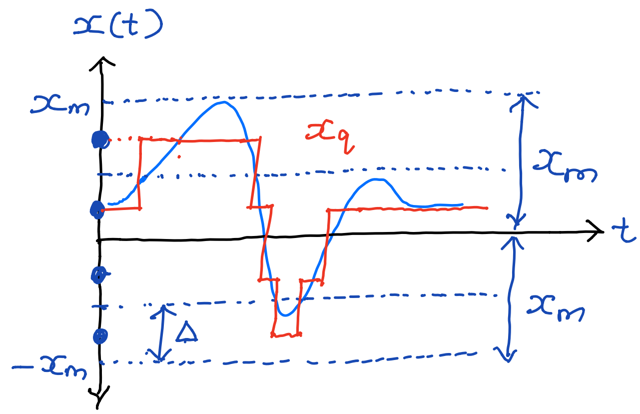
SQNR of a uniform quantizer



$B_x = 2$

(2b quantizer)

$$SQNR_x = \frac{\sigma_x^2}{\sigma_{q_x}^2} \, ; \qquad x_q = x + q_x = Q(x)$$

$q_x \sim \text{Unif}\left[-\frac{\Delta}{2}, \frac{\Delta}{2}\right]$ where Δ is the quantizer step-size

if $Q(x)$ is a uniform quantizer then

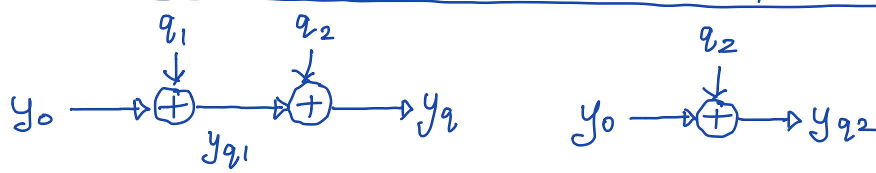
$$\Delta = \frac{2x_m}{2^{B_x}} = x_m \, 2^{-(B_x-1)}$$

$$\sigma_{q_x}^2 = \frac{\Delta^2}{12} = \frac{x_m^2 \times 2^{-2B_x}}{3}$$

$$\therefore \; SQNR_x = \frac{\sigma_x^2}{\sigma_{q_x}^2} = \frac{\sigma_x^2}{x_m^2} \times 3 \times 2^{2B_x} = \frac{3 \times 2^{2B_x}}{\zeta_x^2} \quad \text{—(1)}$$

$$\zeta_x = PAR_x = \frac{x_m}{\sigma_x} \, ; \; \therefore \; SQNR_x(\text{dB}) = 6B_x + 4.8 - \zeta_x$$

Derivation of "Parallel" combination SQNR expression



$$SQNR_T = \frac{\sigma_{y_0}^2}{\sigma_{y_0 - y_q}^2} = \frac{\sigma_{y_0}^2}{\sigma_q^2} = \left[\frac{1}{SQNR_{q_{y_1}}} + \frac{1}{SQNR_{q_{y_2}}}\right]^{-1}$$

$$SQNR_{q_{y_1}} = \frac{\sigma_{y_0}^2}{\sigma_{q_{y_1}}^2} \quad ; \quad SQNR_{q_{y_2}} = \frac{\sigma_{y_0}^2}{\sigma_{q_{y_2}}^2}$$

Proof: Assume q_1 & q_2 are uncorrelated

$$SQNR_T = \frac{\sigma_{y_0}^2}{\sigma_q^2} \quad ; \quad q = q_1 + q_2$$

$$\sigma_q^2 = Var(q_{y_1} + q_{y_2}) = Cov(q_{y_1} + q_{y_2},\ q_{y_1} + q_{y_2})$$

$$= Cov(q_{y_1}, q_{y_1}) + 2\,Cov(q_{y_1}, q_{y_2}) + Cov(q_{y_2}, q_{y_2})$$

$$= \sigma_{q_{y_1}}^2 + 0 + \sigma_{q_{y_2}}^2$$

$$\therefore SQNR_T = \frac{\sigma_{y_0}^2}{\sigma_{q_{y_1}}^2 + \sigma_{q_{y_2}}^2} = \frac{1}{\frac{\sigma_{q_{y_1}}^2}{\sigma_{y_0}^2} + \frac{\sigma_{q_{y_2}}^2}{\sigma_{y_0}^2}}$$

$$= \left[\frac{1}{SQNR_{q_{y_1}}} + \frac{1}{SQNR_{q_{y_2}}}\right]^{-1}$$