(1) Consider an input signal

$$x = x_0 + \eta$$

x → actual, x_0 → ideal, η → noise

Define $SNR_{fl} = \dfrac{\sigma_{x_0}^2}{\sigma_{\eta}^2}$ (floating-point SNR)

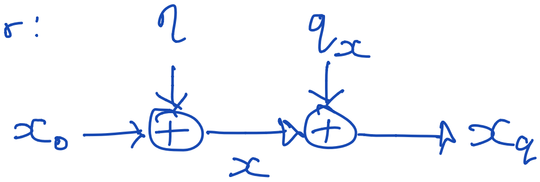
x is quantized to B_x bits to generate $x_q = x + q_x$. Assume x_0, q_x & η are uncorrelated.

$$SNR_{fx} = \frac{\sigma_{x_0}^2}{Var(x_0 - x_q)} \quad \text{.... (fixed-point SNR)}$$

a) What is the minimum value of B_x such that SNR_{fx} is within 0.5 dB of SNR_{fl}?

b) Sketch a generic plot of SNR_{fx} vs. B_x.

Answer:



$$\therefore \; SNR_{fx} = \frac{\sigma_{x_0}^2}{\sigma_\eta^2 + \sigma_{qx}^2} = \left[\frac{1}{SNR_{fe}} + \frac{1}{SQNR_{x_0}}\right]^{-1} \quad -(1)$$

$$SQNR_x = \frac{\sigma_x^2}{\sigma_{qx}^2} = \frac{\sigma_{x_0}^2 + \sigma_\eta^2}{\sigma_{qx}^2} = \frac{\sigma_{x_0}^2}{\sigma_{qx}^2}\left[1 + \frac{1}{SNR_{fe}}\right]$$

$$= SQNR_{x_0}\left[1 + \frac{1}{SNR_{fe}}\right] \quad -(2)$$

if $SNR_{fe} \gg 1 \Rightarrow SQNR_x \approx SQNR_{x_0}$

(high SNR case)

$\therefore$ From (1): $SQNR_{x_0} \geq SNR_{fe} + 9\,dB$ for

$$SNR_{fe} - 0.5\,dB \leq SNR_{fx} \leq SNR_{fe}$$

$$\therefore \; B_x \geq \frac{\overbrace{SNR_{fe} + 9}^{SQNR_x = SQNR_{x_0}} - 4.8 + PAR_{x_0}}{6}$$

if x_0 is a sinusoid & $SNR_{fe} = 20\,dB$ then

$$B_x \geq \frac{20 + 9 - 4.8 + 3}{6} = 4.5 \longrightarrow 5 \text{ bits}$$

(b)