CS 205 - Problem Set 2 - Propositions and Proofs

Sections 01-03 and 07-09

Due Date, Sunday October 02, 2022 in gradescope

Instructions

- Read Rosen 1.4, 1.5 and 1.7 prior to completing this problem set AND/OR
- Watch CUbits videos in Module 1 on Predicates, Quantifiers and proofs -
- This is an individual worksheet. NO group work is permitted.
- You are authorized to seek help from course staff ONLY
- This handout is available in canvas-files.
- If you have any questions about a specific problem, please post to Piazza or email TA.
- Submit solutions to gradescope as instructed

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Your recitation section and/or day of the week : 7 (Tuesday)
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Problem 1

Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives.
(1) First, let the domain consist of all people in your city
(2) Second, let it consist of all people in your country.
First define propositional variables p that can capture what is stated in the statements below such as p = "x has a cellular phone". Each of the statements below may have two interpretations based on the domain. You can use a symbol to define a domain such as, Let A be the set of all people in your city.

- Someone has a cellular phone.

p = x is in your city , q = x has a cellular phone

(1) $\exists x : p(x)$

(2) $\exists x : p(x) \rightarrow q(x)$

- Not Everyone has seen a foreign movie.

p = x is in your city, q = x has seen a foreign movie

(1) $\neg \forall x : p(x)$

(2) $\neg \forall (x) : p(x) \rightarrow q(x)$

- Everyone can sing.

p = x is in your city , q = x can sing

(1) $\forall x : p(x)$

(2) $\forall x : p(x) \rightarrow q(x)$

- There is a someone who can neither sing nor speak Spanish.

p = x is in your city, q = x can neither sing nor speak spanish

(1) $\exists x : \neg p(x)$

(2) $\exists x : \neg p(x) \rightarrow \neg q(x)$

- Some people do not want to go to college.

p = x is in your city , q = x wants to go to college

(1) $\neg \forall x : \neg p(x)$

(2) $\neg \forall x : \neg p(x) \rightarrow \neg q(x)$

Problem 2

Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. In each case, identify the domain and specify the predicates.

- Someone has lost a lot of money playing the lottery.

  $\exists x : p(x)$

  Negation: Nobody has lost a lot of money playing in the lottery
  The domain is the set of all people playing the lottery
  $p =$ The predicate is "has lost a lot of money playing in the lottery"
- There is a someone in your dorm who is related to two other students.

  $\exists x : p(x)$

  Negation: There is no one in your dorm who is related to two other students
  The domain is the set of all people in your dorm
  $p =$ The predicate is "is related to two other students"
- Some student in this class has sent e-mail all other students in this class.

  $\exists x : p(x)$

  Negation: No student in this class has sent e-mail to all other students in this class
  The domain is the set of all students in this class
  $p =$ The predicate is "has sent e-mail to all other students in this class"
- No student solved every exercise in the book.

  $\neg \forall x : p(x)$

  Negation: Every student solved every exercise in the book
  The domain is the set of all students
  $p =$ The predicate is "solved every exercise in the book"
- All students have solved at least one exercise in every section of the book.

  $\forall x : p(x)$

  Negation: No student has solved at least one exercise in every section of the book
  The domain is the set of all students
  $p =$ The predicate is "has solved at least one exercise in every section of the book"

Problem 3

Prove or disprove these universally quantified statements. If proving, you must justify your answer. If disproving you must provide a counterexample. The domain for all variables consists of all real numbers. For all questions assume that $x \neq y$

(a) $\forall$x$\exists$y (x.y = 1)

False. If x=0, there is no real number value of y that will make the product of x and y = 1.

(b) $\exists$x$\forall$y ($y^2 - x < 10$)

True. There will always exist an x which will reduce y^2 to <10 as long as x>y^2-10.

(c) $\forall$x$\forall$y ($x^2 \neq y^3$)

False.

$x = 8$, $y = 4$

8^2 4^3

$64 = 64$ ✓

(c) $\exists$x$\exists$y ($x^2 = y^2$)

True. There will exist the equality of x^2=y^2 when x=-y or when y=-x.

Problem 4

Let n be a natural number.

1. Prove that if $n + 2$ is a perfect square, then n cannot be a perfect square.

$n + 2 = x^2$

Take x to be a natural number as well: as x increases, the difference between x^2 and (x+1)^2 will also increase. Starting at the smallest consecutive natural numbers 1 and 2, 1^2=1 and 2^2=4, their difference being 3, hence leaving it impossible for n and n+2 both to be perfect squares.

2. Prove that if $17n + 2$ is odd, then n is odd.

Assume n is an even number. It is a fact that all multiples of an even number are also even. It is also a fact that the addition of any 2 even numbers results in an even number. Therefore, n is even -> 17n+2 is even. And by contraposition, 17n+2 is odd -> n is odd.

contraposition: $p \to q$

$\Updownarrow$

$\neg q \to \neg p$

p = n is even

q = 17n+2 is even

Problem 5

1. Show that these statements about the real numbers a, b are equivalent:

   - $a < b$
   - $(a + b)/2 > a$
   - $(a + b)/2 < b$.

$$(a+b)/2 > a$$
$$a+b > 2a$$
$$b > a$$
$$\boxed{a < b}$$

$$(a+b)/2 < b$$
$$a+b < 2b$$
$$\boxed{a < b}$$

$\boxed{\text{all 3 inequalities are equivalent}}$

2. Critique the following proof, where x, y are real numbers.

   suppose $x > y$

   this implies $x^2 > y^2$

   we can write above as $x^2 - y^2 > 0$

   factor $(x - y)(x + y) > 0$

   divide $x + y > 0$

   we conclude $x > -y$

   Therefore $x > y \implies x > -y$ ∀x,y

Cannot be implied because x & y may be negative. renders everything below false.

has no basis to divide by x-y. This would cause a loss of solutions. Also since both x & y are variables don't know whether to flip the inequality or not.

ex. $x = -1$
$y = -5$

$-1 > -5$

$(-1)^2 > (-5)^2$

$1 \not> 25$