# Midterm - F22 - Sections 07-09

● Graded

**Student**

AKSHAJ KAMMARI

**Total Points**

66 / 75 pts

**Question 1**

## Cover Page

**0** / 0 pts

- ✔ **– 0 pts** Correct

**Question 2**

## Question 1

9 / 10 pts

- **– 0 pts** Correct
- **– 1 pt** Question 1 variables are wrong
- **– 1 pt** Question 1 statements are wrong
- **– 1 pt** Question 2 variables are wrong
- **– 2 pts** Question 2 statements are wrong
- **– 1 pt** Question 2 statements partial credit
- **– 1 pt** Question 3 variables are wrong
- ✔ **– 1 pt** Question 3 statements are wrong
- **– 1 pt** Question 4 variables are wrong
- **– 2 pts** Question 4 statements are wrong
- **– 1 pt** Question 4 statements partial credit

**Question 3**

## Question 2

**15** / 15 pts

- ✔ **– 0 pts** Correct
- **– 5 pts** 2.1
- **– 5 pts** 2.2a
- **– 5 pts** 2.2b

Question 4

## Question 3 & 4

18.5 / 20 pts

**+ 0 pts** Correct

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Q3

- ✔ **+ 5 pts** 3.1
- ✔ **+ 5 pts** 3.2

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Q4

- ✔ **+ 5 pts** 4.1
- ✔ **+ 5 pts** 4.2

**+ 0 pts** Incorrect

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**− 1.5 pts** 3.1 - Final answer should not have any term in 'i' instead, it should be 'n'. Last term should be n instead of n-1.

Question 5

## Question 5

**15** / 15 pts

- ✔ **− 0 pts** Correct

**− 7.5 pts** Part 1 wrong

**− 7.5 pts** Part 2 wrong

**− 2 pts** Partial credit part 1

**− 2 pts** Partial credit part 2

**− 4 pts** Partial credit part 1

**− 4 pts** Partial credit part 2

**Question 6**

## Question 6

**8.5 / 15 pts**

1. The sum is $1^2 + 1^2 + 2^2 + 3^2 + 5^2 + 8^2 = 104$.

   **– 1 pt** Sum not correct.

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2. Which is equal to $F_6 \cdot F_7$.

   **– 0.5 pts** I assume you meant to write $F_6 \times F_7$.

   **– 1 pt** Did you forget to plug in $n = 6$?

   **– 3 pts** The product given does not match your answer to the first part or the correct answer.

   **– 3 pts** No attempt.

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3. $\sum_{i=1}^{n} F_i^2 = F_n \cdot F_{n+1}$.

   **– 0.5 pts** (typo) I assume you meant to write $F_n \times F_{n+1}$.

   ✔ **– 1.5 pts** (off by 1) I assume you meant to write $F_n \times F_{n+1}$.

   **– 3 pts** We want to relate $\sum_{i=1}^{n} F_i^2$ to something, not $\sum_{i=0}^{5} F_{n+i}^2$.

   **– 3 pts** Not correct.

   **– 3 pts** No attempt.

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$F_n < 2^n$.

   **– 3 pts** Missing base case.

   **– 1 pt** Minor error in inductive step.

   **– 2 pts** Not so minor error in inductive step.

   **– 3 pts** The inductive step is not clear.

   **– 3 pts** Inductive step incomplete.

   **– 4 pts** The intuition is good, but this is not a proof. You need to prove $P(n+1)$ (assuming either just $P(n)$, or $P(1), P(2), \ldots, P(n)$).

   **– 4 pts** Significant error in the induction step.

   **– 5 pts** Inductive step: Why is $F_{k+1} < 2^k \cdot 2$?

   ✔ **– 5 pts** Inductive step missing / not actually proven at all.

   **– 8 pts** No attempt.

   **– 8 pts** You had to prove $F_n < 2^n$ --- not the relation you found in part 3.

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   **– 0 pts** Correct

**− 15 pts** No attempt.

# CS 205 - Midterm Exam

Sections 07-09

Administer Date: October 26, 2022 in class

Instructions

- This test is based on propositions, sets, sequences, proofs and induction
- Please write the answers ONLY in the space provided
- You have 80 minutes to complete this test (no grace period)

your work may not be graded without you signing below
I certify that the answers to this test represents my own work and I have read RU academic integrity policies https://www.cs.rutgers.edu/academic-integrity/introduction.

**SIGN your name :** [signature]
**PRINT your name :** Akshaj Kammari
**netID :** ak1990
**Recitation (1-5) :** 7

| Question | Points | Score | grader |
|---|---|---|---|
| 1 | 10 | | |
| 2 | 15 | | |
| 3 | 10 | | |
| 4 | 10 | | |
| 5 | 15 | | |
| 6 | 15 | | |
| Total | 75 | | |

## Question 1 - Propositions (10 pts)

For each of the following, introduce proposition symbols for each simple proposition in the argument (for example, P = "I will ace this homework"). Then write out the logical form of the argument(s) using one or more compound propositional statements. If the argument form corresponds to a known inference rule, say which it is. If the proposition can be simplified, please leave the answer in the most simplified form. For example, $p \wedge q \rightarrow p$

1. I will ace this homework if I have the time to study.

   P = I will ace this hw

   Q = I have the time to study

   $Q \rightarrow P$

2. I either will go to lecture or watch the videos but will not do both. Also it is sufficient to watch videos in order to pass the exam.

   A = I will go to lecture

   B = I will watch the videos

   C = I will pass the exam

   $(A \vee B) \wedge \neg(A \wedge B) \wedge (B \rightarrow C)$

3. I have not taken Discrete Structures. I can't take Machine Learning unless I have taken Discrete Structures. Therefore, I can't take ML.

   x = I have taken Discrete Structures

   y = I have taken Machine Learning

   $(\neg x \wedge x \Leftrightarrow y) \rightarrow \neg y$

4. If I work all night on this homework, I will answer all the exercises. If I answer all the exercises, I will understand the material. Therefore, if I work all night on this homework, I will understand the material.

   m = I work all night

   n = I answer all the exercises

   o = I understand the material

   $((m \rightarrow n) \wedge (n \rightarrow o)) \rightarrow (m \rightarrow o)$

## Question 2 - Sets and Functions (15 pts)

1. Prove or disprove: Let A and B be two sets. If $A = B$ then powerset(A) = powerset(B). To disprove you need to give a counter example. To prove it, you need to provide a formal proof by showing if x is an element of powerset(A), then x is an element of powerset(B) and vice versa.

$\exists x : x \in A$ given

$x \in B$ because $A = B$

$A \subseteq$ powerset(A) def of powerset

powerset(A) = powerset(B) given

$x \in$ powerset(B) implies

$\therefore$ if $A = B \rightarrow P(A) = P(B)$

2. Show that $f : N \rightarrow N+$ and $f(n) = 2^n$ is not a bijection by showing that it is 1-1 but not onto.

   (a) Prove that f is 1-1 using the definition of 1-1

$f(a) = f(b) \Rightarrow 2^a = 2^b \Rightarrow a = b$

$\therefore f(n)$ is 1-1

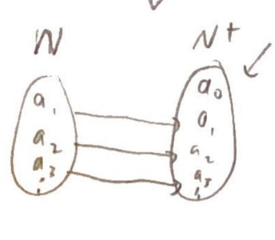
   (b) Prove that f is not onto by showing not every element in N+ has a pre-image in N

given $y = f(n)$

$y = 2^n$

$\log_2 y = n$

$\Downarrow$




N+ has elements that are not matched at least once

$\therefore f(n)$ is not onto

## Question 3 - Summations (10 pts)

1. Find the following sums using known sums. DO NOT simplify. Final answer will not have any summation symbols

   (a) $\sum_{i=1}^{n}(2i^2 + i + 1)$

   $$2\sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} i + \sum_{i=1}^{n} 1$$

   $$2\left(\frac{i(i+1)(2i+1)}{6}\right) + \frac{i(i+1)}{2} + (n-1)$$

   $$\boxed{\frac{i(i+1)(2i+1)}{3} + \frac{i(i+1)}{2} + (n-1)}$$

   (b) $\sum_{i=n}^{2n}(2^i + i/2)$

   $$\sum_{i=n}^{2n} 2^i + \sum_{i=n}^{2n} \frac{i}{2} \rightarrow \left(\sum_{0}^{2n} 2^i - \sum_{0}^{n-1} 2^i\right) + \left(\sum_{0}^{2n} \frac{i}{2} - \sum_{0}^{n-1} \frac{i}{2}\right)$$

   $$\left(\frac{2^{2n+1}-1}{2-1} - \frac{2^{(n-1)+1}-1}{2-1}\right) + \left(\frac{2n(2n+1)}{4} - \frac{(n-1)(n-1+1)}{4}\right)$$

   $$\boxed{\left((2^{2n+1}-1) - (2^n-1)\right) + \left(\left(\frac{n(2n+1)}{2}\right) - \left(\frac{n(n-1)}{4}\right)\right)}$$

## Question 4 - Proving Inferences (10 pts)

1. Prove using known inference rules that $\neg P \wedge (P \vee Q) \Rightarrow Q$ is a tautology. Do not use truth tables.

   ~~$\neg P \wedge (P \vee Q) \Rightarrow Q$~~

   ~~$\neg P \wedge \neg(P \vee Q) \vee Q$~~

   ~~$\neg P \wedge \neg P \wedge \neg Q \vee Q$~~

   ~~idempotent law~~

   ~~$\neg P \wedge T$~~

   $\neg(\neg P \wedge (P \vee Q)) \vee Q$

   $P \vee \neg(P \vee Q) \vee Q$

   $P \vee \neg P \wedge \neg Q \vee Q$

   $T \wedge T = \boxed{\text{True}}$

2. Prove that $(Q \Rightarrow (P \Rightarrow (Q \wedge P))) \Rightarrow \neg Q \wedge (P \vee Q) \Rightarrow P$ is always true. (hint: Part I can help)

   $Q \Rightarrow (\neg P \vee (Q \wedge P))$

   $\neg(\neg Q \vee (\neg P \vee (Q \wedge P))) \Rightarrow \neg Q \wedge (P \vee Q)$

   $\neg((Q \wedge P \wedge \neg Q \vee \neg P) \vee (\neg Q \wedge (P \vee Q))) \Rightarrow P$

   $\neg Q \vee \neg P \vee (Q \wedge P \wedge Q) \vee \neg P \wedge \neg Q \vee P$

   $T \quad T$

   $T \vee T \vee (Q \wedge P \wedge Q) \vee (\neg P \wedge \neg Q)$

   $= \boxed{\text{True}}$

   because logical operators are $\vee$

## Question 5 - Proofs (15 pts)

For each claim below, prove or disprove the claim. To prove a claim, you need to provide a complete proof. To disprove a claim, you need to provide at least one counter example.

1. Every non-negative integer $n \geq 0$ , the expression $n^2 + n + 1$ is always odd.

direct

case 1: (odd $\mathbb{Z}^+$)

$\exists n ; n = 2k+1$ (odd) & $n \in \mathbb{Z}^+$

$n^2+n+1 \rightarrow (2k+1)^2 + (2k+1)+1$

$\rightarrow 4k^2+4k+1+2k+1+1$

$\rightarrow 4k^2+4k+2k+2+1$

$\rightarrow \underbrace{2(2k^2+2k+k+1)}_{\text{even part}}+1$ $\boxed{\text{odd}}$

case 2: (even $\mathbb{Z}^+$)

$\exists n : n = 2k$ (even) & $n \in \mathbb{Z}^+$

$n^2+n+1 \rightarrow (2k)^2+2k+1$

$\rightarrow 4k^2+2k+1$

$\underbrace{2(2k^2+k)}_{\text{even part}}+1$

$\boxed{\text{odd}}$

$\therefore$ for every non-negative integer $n \geq 0 \rightarrow n^2+n+1$ always odd $\boxed{\text{TRUE}}$

2. $\forall x \in R$, $\sqrt[6]{2x-1} \leq \sqrt[3]{x}$

$(\sqrt[6]{2x-1})^6 \leq (\sqrt[3]{x})^6$ $\boxed{\text{TRUE}}$

$2x-1 \leq x^2$

$0 \leq x^2-2x+1$

$0 \leq (x-1)^2$

## Question 6 - Fibonacci (15 pts)

The Fibonacci numbers are defined by

$F_1 = 1$

$F_2 = 1$

$F_n = F_{n-2} + F_{n-1}$ for $n > 2$

1. write down the sum of the squares of the first 6 Fibonacci numbers $F_1, F_2, ...F_6$ for $n = 6$.
   $F_1^2 + F_1^2 + .. + F_6^2$

   $F_1 = 1$ $F_4 = 3$

   $F_2 = 1$ $F_5 = 5$

   $F_3 = 2$ $F_6 = 8$

   $1 + 1 + 4 + 9 + 25 + 64$

   $= 104$

2. The sum in the previous part is equal to the product of two adjacent Fibonacci numbers. What are those two Fibonacci numbers? (hint: you might want to look beyond $n = 6$ find a relation between sum of the first 6 and those two numbers).

   $F_7 = 13$

   $F_8 = 21$

   8 & 13

3. Write a general statement (without proof) that indicates a relation between sum of the squares of first n Fibonacci numbers and two other Fibonacci numbers. (try to generalize the answer to the previous part for any n)

   The sum of the squares of the first n Fibonacci numbers is equal to the product of the next 2 Fibonacci numbers in the sequence, $(n+1)(n+2)$.

4. Prove by induction that $\forall n \geq 1$, $F_n < 2^n$ You must identify and prove the base case and inductive hypothesis, say $P(n)$, and prove that $P(n) \implies P(n+1)$. Show all work to receive full credit

   Base Case: $F_n < 2^1 \rightarrow 1 < 2$ ✓

   inductive hypothesis: For all $n \geq 1$, the term $F_n$ will always be less than $2^n$.

   conclusion: $F_{n+1} < 2^{n+1}$

   $F_{1+1} < 2^{1+1}$

   $F_2 < 2^2$

   $1 < 4$

   $\therefore F_{n+1} < 2^{n+1}$