CS 205 - Problem Set 3 - Proofs and Sets

Sections 01-03, 07-09

Due Date, Sunday October 16, 2022 @ 11:59 PM

Instructions

- Read Rosen 1.6-1.8 and 2.1-2.2 and/OR
- Watch cubits videos in Modules 1 and 2
- This is an individual worksheet. NO group work is permitted.
- You are authorized to seek help from course staff ONLY
- This handout is available in canvas-files.
- If you have any questions about a specific problem, please contact TA or instructor
- Submit solutions to gradescope as instructed. Failure to submit specific pages to assigned problems will result in loss of points.

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Sign and PRINT your name : Akshaj Kammari
Your recitation section and/or day of the week : 7 (Tuesday)
NetID : AK1990

Problem 1 - Proofs

Prove the following, where n is an integer. Clearly state the proof technique you are using (direct, contraposition, contradiction)

1. if $n^3 + 1$ is odd then n is even

$\boxed{\text{Contraposition}}$

assume n is odd

and $n = 2k+1$ $\Rightarrow$

n^3+1

$= (2k+1)^3+1$

$= 8k^3+12k^2+6k+2$

$= 2(4k^3+6k^2+3k+1)$

makes n^3+1 even

if n is odd

$\rightarrow n^3+1$ is even, $\Leftarrow$

hence, if n^3+1 is odd $\rightarrow$ n is even

2. If n is even, then $n^3 + 1$ is odd .

$\boxed{\text{Direct}}$

$n = 2k$

makes n^3 even

n^3+1

$= (2k)^3+1 = 8k^3+1 = \underbrace{\underbrace{2(4k)^3}_{\text{even}}+1}_{\text{odd}}$

hence, n is even $\rightarrow n^3+1$ is odd

3. $\sqrt[3]{2}$ is irrational.

$\boxed{\text{Contradiction}}$

assume $\sqrt[3]{2}$ is rational

$\sqrt[3]{2} = \frac{p}{q}$ where $q \neq 0$ & $\frac{p}{q}$ is simplified

$2 = \left(\frac{p}{q}\right)^3 = \frac{p^3}{q^3}$

$\underline{2}q^3 = p^3$ is even integer: p is even too, $p = (2k)$

$2q^3 = (2k)^3$

$2q^3 = 8k^3$

$q^3 = 4k^3$

$q^3 = 2(2k)^3$

makes q^3 even also

both p & q are even.
they are not simplified
hence making them
irrational

Problem 2 - Proofs of Rational and Irrational Number Properties

1. Prove that the sum two rational numbers is rational. You need to give a proof starting with the definition of a rational number. You will not get any credit for providing examples.

a rational number is one that can be represented as a quotient of 2 integers $\frac{p}{q}$ such that $q \neq 0$.

assume a & b are rational numbers. so $a = \frac{x}{y}$ & $b = \frac{m}{n}$ $y \neq 0$ & $n \neq 0$. therefore $a+b = \frac{x}{y} + \frac{m}{n} = \frac{xn+ym}{yn}$

$xn+ym$ must be an integer because x, y, m, & n are all integers therefore $\frac{xn+ym}{yn}$ is also rational because it is of form $\frac{\text{int}}{\text{int}}$. this means that $a+b$ must be a rational number. concluding, the sum of rational numbers is rational.

2. Prove that the sum of an irrational number and a rational number must be irrational. You need to give a complete proof to get credit. You will not get any credit for providing examples.

an irrational number is one that cannot be expressed as a quotient of 2 integers.

proof by contradiction: lets assume a rational number + an irrational number = a rational number

take $\frac{x}{y}$ & $\frac{m}{n}$ to be rational. take a to be irrational.

$\frac{x}{y} + a = \frac{m}{n}$ so $a = \frac{m}{n} - \frac{x}{y} = \frac{my - xn}{ny}$

since m, n, x, y are all integers, $\frac{my-xn}{ny}$ must take the form $\frac{\text{int}}{\text{int}}$. this contradicts our statement of rational + irrational = rational. hence, by contradiction, rational + irrational = irrational.

Problem 3 - Sets

Provide answers to the following questions

1. Given $R = \{1, 2\}$ and $S = \{1, 2, 3\}$

   - find $R \times S$ and $S \times R$.

   $R \times S = \{(1,1),(1,2),(1,3),(2,1),(2,2),(2,3)\}$

   $S \times R = \{(1,1),(1,2),(2,1),(2,2),(3,1),(3,2)\}$

   - Is the Cartesian product relation commutative? That is, is it true that $R \times S = S \times R$ for any two sets R and S?. If the answer is YES, then give a proof. If the answer is NO give a counter example

   No, the cartesian product relation is not commutative.

   $R = \{1,2\}$, $S = \{3\}$ $\quad (1,3) \neq (3,1)$ & $(2,3) \neq (3,2)$

   $R \times S = \{(1,3),(2,3)\}$ & $S \times R = \{(3,1),(3,2)\}$

2. Given the predicate $P(x) : 0 < x^3 - x < 100$, where x is an integer, find the truth set for the predicate. The truth set of the predicate is all values of x that satisfies the predicate. Express the answer in set notation

$$\{x \mid x \in \mathbb{Z} \wedge 2 \leq x \leq 4\}$$

3. Claim: If powerset(A) = powerset(B) then $A = B$. To disprove you only need to give a counter example. To prove it, you need to show detailed work.

Assume $A \neq B$. specifically let x be an element where $\{x\} \in A \wedge \{x\} \notin B$. From this we can conclude that $\{x\} \in$ powerset$(A) \wedge \{x\} \notin$ powerset(B). in simpler terms, $\{x\}$ is not in both powersets A & B. Hence, by contraposition,

powerset(A) = powerset$(B) \longrightarrow A = B$

Problem 4 - Proofs with Sets

1. Suppose A, B, and C are sets. If $B \subseteq C$, then $A \times B \subseteq A \times C$

Assume $A \times B \nsubseteq A \times C$. $\{(x,y)\} \in A \times B \wedge \{(x,y)\} \notin A \times C$

$x \in A \wedge y \in B$ but $x \in A \wedge y \notin C$

C doesn't contain element y, so $B \nsubseteq C$.

hence, by contraposition,

if $B \subseteq C \longrightarrow A \times B \subseteq A \times C$

2. If A, B, and C are sets, then $A - (B \cup C) = (A - B) \cap (A - C)$
   Do not prove using Venn Diagrams.

let x be an element in sets such that

$x \in A$ so $x \in A - (B \cup C)$ but $x \notin B \wedge x \notin C$.

that means $x \notin B \cup C$. the element x would still exist such that $x \in (A - B) \cap (A - C)$.

hence, by definition of a set:

$$A - (B \cup C) = (A - B) \cap (A - C).$$

Problem 5 - Proofs

1. Prove by cases, that for all real numbers r, s we can show that, $max(r,s) + min(r,s) = r + s$

Case 1: $r > s$

$max(r,s) = r$

$min(r,s) = s$

$max(r,s) + min(r,s) = \underline{r+s}$

Case 2: $s > r$

$max(r,s) = s$

$min(r,s) = r$

$max(r,s) + min(r,s) = \underline{r+s}$

Case 3: $r = s$

$max(r,s) = min(r,s)$

$max(r,s) + min(r,s) =$

$2r = 2s = s+r = \underline{r+s}$

therefore $r \wedge s \in \mathbb{R}$,

$max(r,s) + min(r,s) = r+s$

2. Prove that if $a\dot{b} = n$, then either a or b must be $\leq \sqrt(n)$ where a, b, and n are nonnegative real numbers.

$ab = n$, $a \wedge b \leq \sqrt{n}$

proof by contradiction:

$ab > (\sqrt{n})^2 \Rightarrow ab > n$ which is false

hence, if $ab = n \longrightarrow a \wedge b \leq \sqrt{n}$ by contradiction