Analysis of Torque Loaded Members

ME 210: Mechanics of Materials

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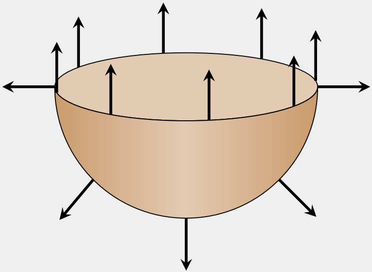
Outline



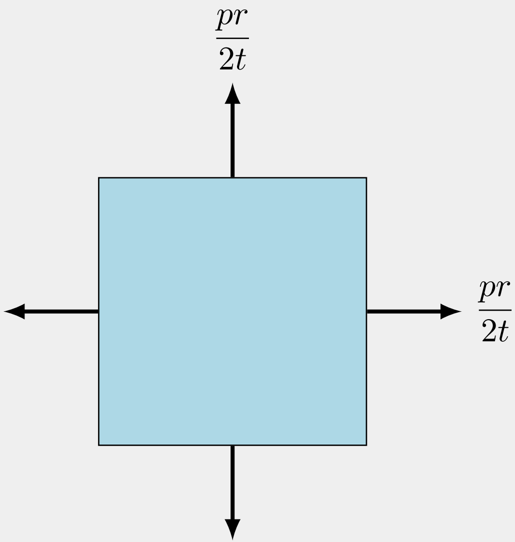
$$F_\sigma = F_p$$

$$\sigma_2\,(2\pi r t) = p\pi r^2$$

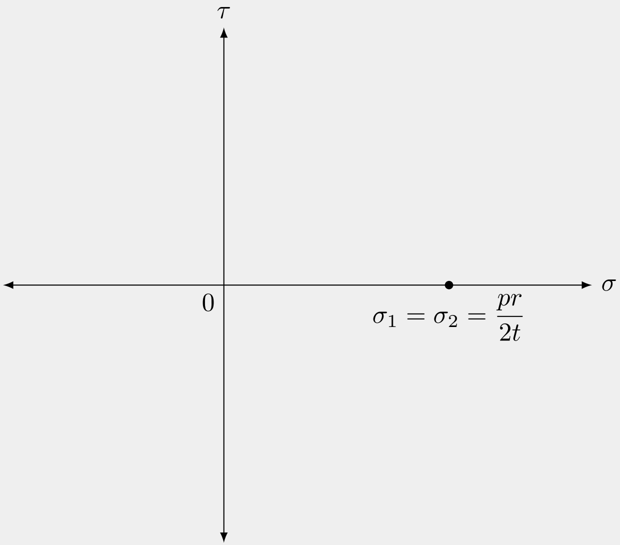
$$\sigma_2 = \frac{pr}{2t}$$

- What about σ_1? How do you draw a Mohr's circle to represent state of stress?

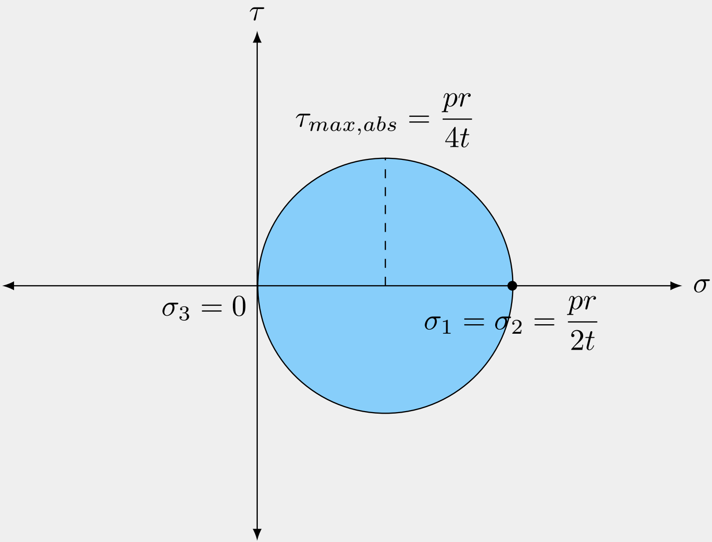
State of Stress of of Vessel Wall



Mohr's Circle of State of Stress



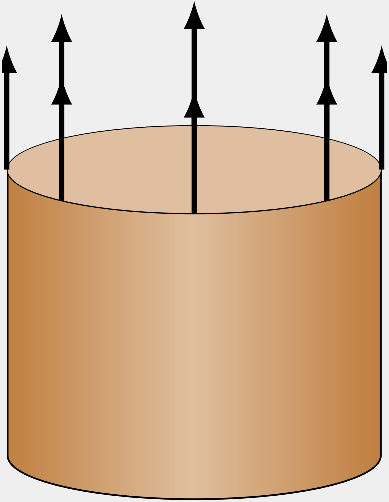
Absolute Maximum Shear Stress of Spherical Pressure Vessel



Spherical Pressure Vessel: Summary

- $\sigma_1 = \sigma_2 = \dfrac{pr}{2t}$
- $\tau_{xy} = \tau_{max} = 0$
- $\tau_{max,abs} = \dfrac{pr}{4t}$

Cylindrical Pressure Vessels



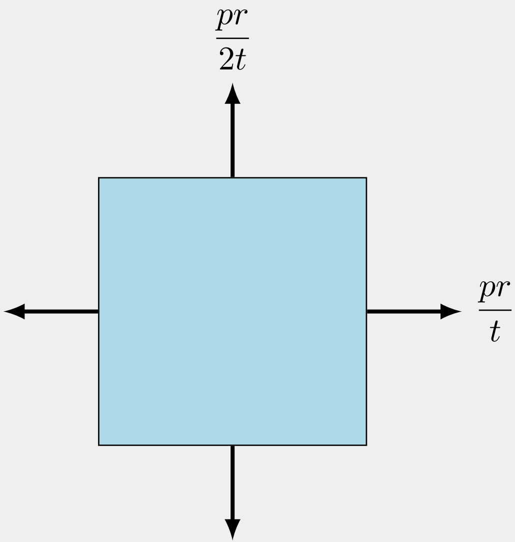
$$F_\sigma = F_p$$

$$\sigma_1 \left(2tdy\right) = p\left(2rdy\right)$$

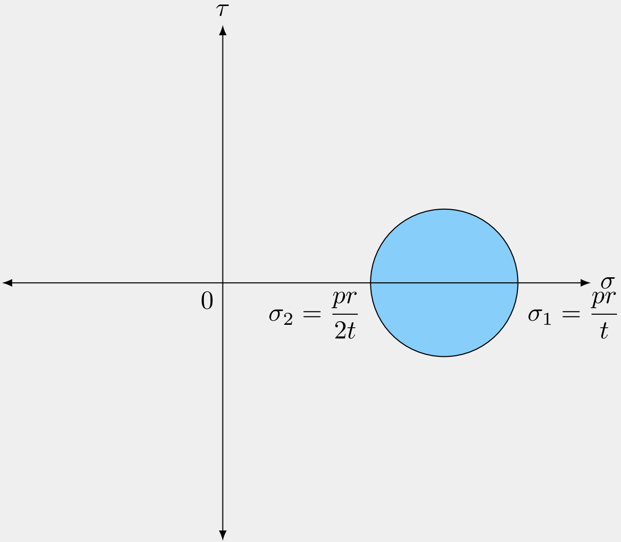
$$\sigma_1 = \frac{pr}{t}$$

$$\sigma_2 \left(2\pi rt\right) = p\pi r^2$$

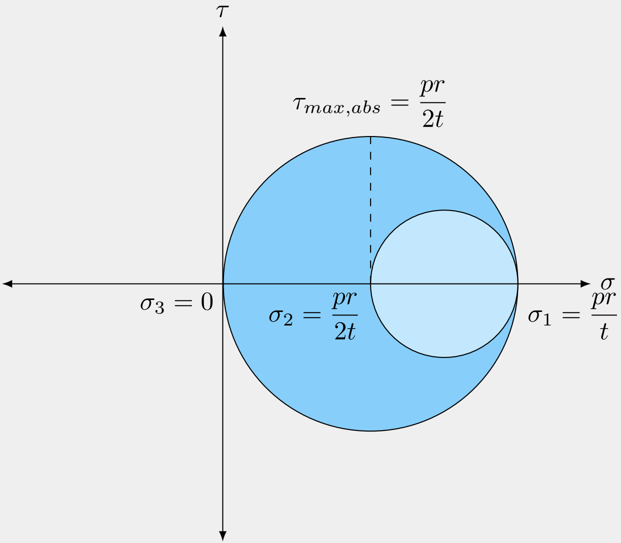
$$\sigma_2 = \frac{pr}{2t}$$


$\frac{pr}{2t}$
$\frac{pr}{t}$

Mohr's Circle of Cylindrical Vessel



Absolute Maximum Shear Stress of Cylindrical Vessel



Cylindrical Pressure Vessel: Summary

- $\sigma_1 = \sigma_c = \dfrac{pr}{t}$
- $\sigma_2 = \sigma_l = \dfrac{pr}{2t}$
- $\tau_{xy} = 0, \tau_{max} = \dfrac{pr}{4t}$
- $\tau_{max,abs} = \dfrac{pr}{2t}$

- Failure of a shotgun barrel

Outline

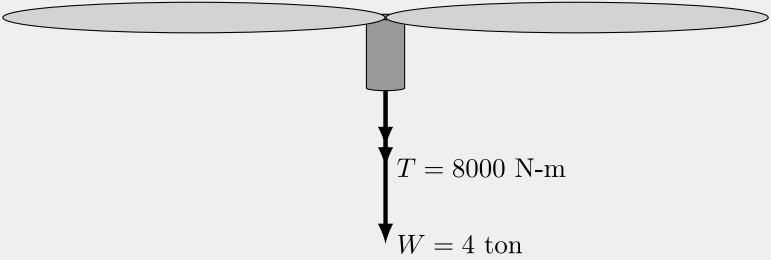
- Multiaxial stress conditions come from
  - Simultaneous application of loads
  - Complex geometry of component
- Superposition is always the key
  - Find stress(es) from each load
  - combine resultant stresses using multiaxial stress analysis

- We need to know where failure starts
- For a single-material component, failure starts where *combined* stress is the highest
- This is called the "critical point"

How to Identify the Critical Point

- Identify each type of load (axial, bending, or torsion)
- Mark locations of maximum stress for each load
- Locate location(s) with multiple maximum stresses

We want to determine the proper diameter of a rotor shaft for a 4-ton helicopter. The shaft is connected to the engine that provides the maximum torque of 8000 N-m. The shaft is made of AISI1023 steel with σ_{allow} = 400 MPa.



Helicopter Rotor Shaft: Solution

- Determine the critical point
- Determine state of stress
- Find proper radius r

Helicopter Rotor Shaft: Solution

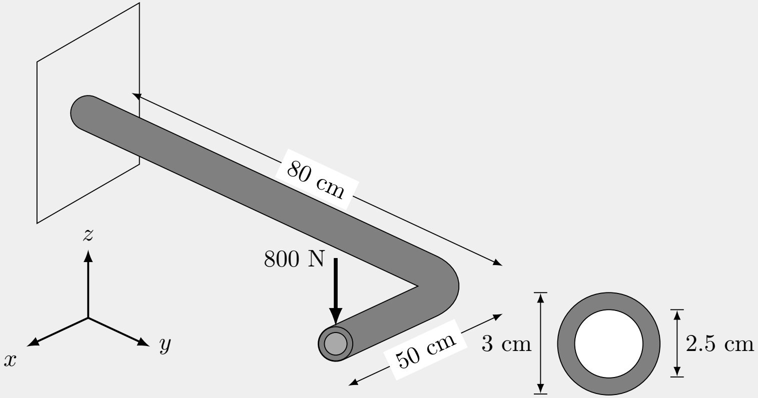
$$
\begin{aligned}
\sigma &= \frac{F}{A} = \frac{4(1000\text{ kg/ton})(10\text{ N/kg})}{\pi r^2} \\
&= \frac{12732}{r^2} \\
\tau &= \frac{Tr}{J} = \frac{8000(r)}{\pi r^4/2} \\
&= \frac{5093}{r^3}
\end{aligned}
$$

So the state of stress at the critical surface is a combination of normal stress and shear stress. Since the given material is limited by its normal stress, we need to determine the maximum principal stress.

$$\sigma_1 = \sigma_{allow} = 400 \times 10^6 = \frac{12732}{2r^2} + \sqrt{\left(\frac{12732}{2r^2}\right)^2 + \left(\frac{5093}{r^3}\right)^2}$$

This equation can be solved numerically to obtain r = 2.38 cm.

L-pipe



Questions

- Find the critical point. Elaborate your reasoning.
- Determine the state of stress of the critical point.
- Draw a Mohr's circle representing the state of stress.