Intro to Material Behaviors

ME 210: Mechanics of Materials

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A Little Intro

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Childhood

- Born and raised in Khon Kaen, Thailand

Bachelor's Degree

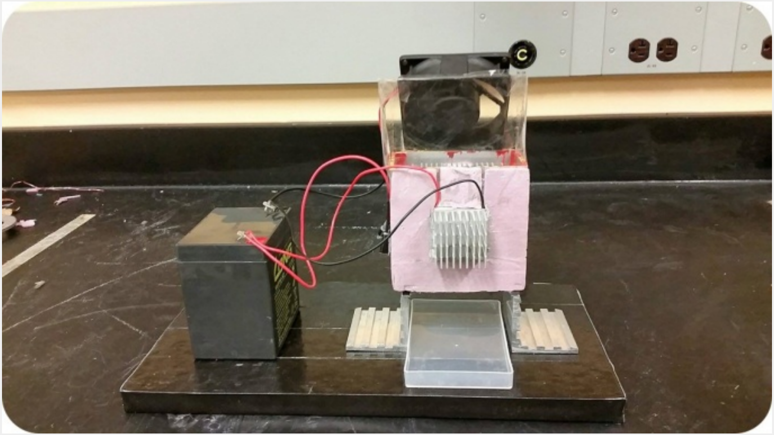
- Sc.B. Brown University, 2002

Master and Doctoral

- M.S. 2005, Ph.D. 2008, MIT

Research Interests

- Solar energy utilization
- Water generation



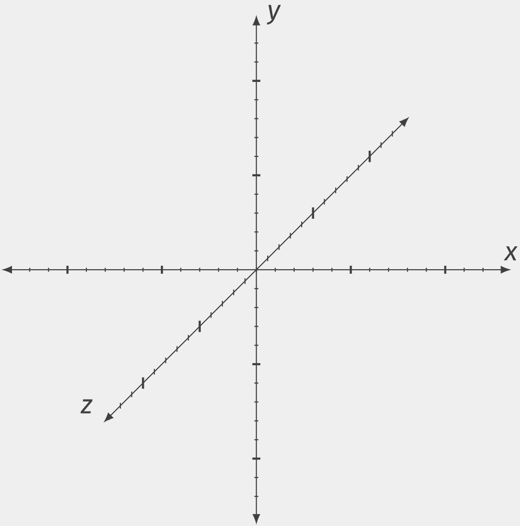
Material Behaviors

- The study of material response to
  - External load(s)
  - Thermal change(s)

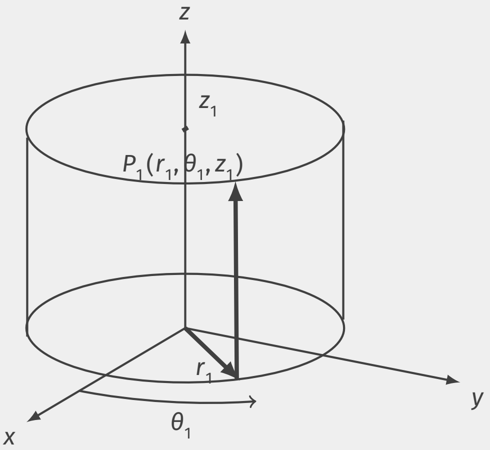
Coordinate System

- Most quanitities in this class are vectors
- Coordinate systems make it clear on orientation and direction of anything
- Three main systems
  - Cartesian
  - Cylindrical
  - Spherical

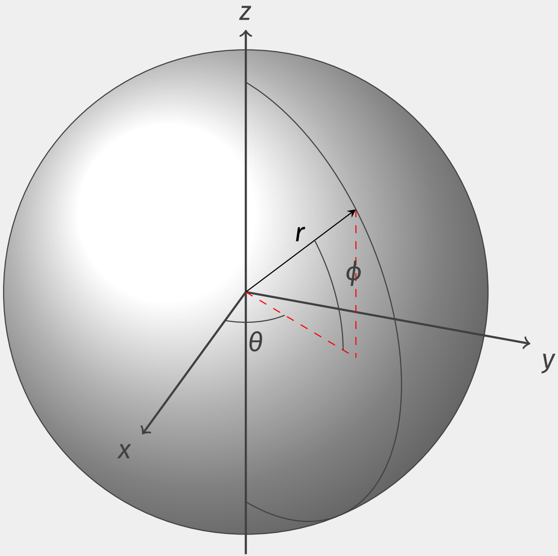
Cartesian



Cylindrical



Spherical Coordinates

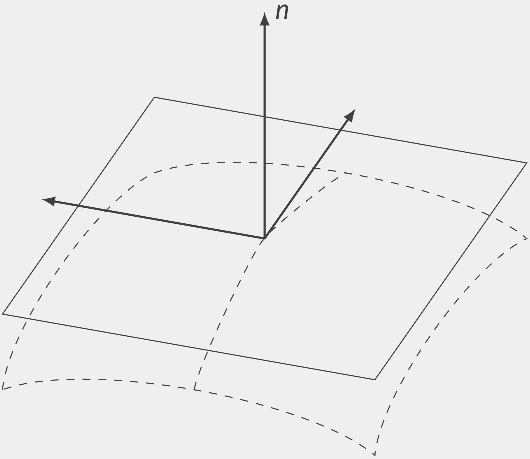


External Loads

- Force(s) or Moment(s)
- Results in deformation, depending on *direction* and *surface*
- How do we define *direction* and *surface*

Surface Direction

- Direction of vector *normal* to the surface



- Normal forces → same direction as surface
- Shear forces → perpendicular to surface direction

Moments

- Follow right-hand rule.
- Torsional moments: same direction as surface
- Bending moments: perpendicular to surface direction

The Singularity Equation (for this class)

- Equilibrium equation

$$\sum F = 0$$
$$\sum M = 0$$

- We will be trying to determine stresses and deformation of things
- Need to find internal load at any point/surface
- Method of sections

Method of sections

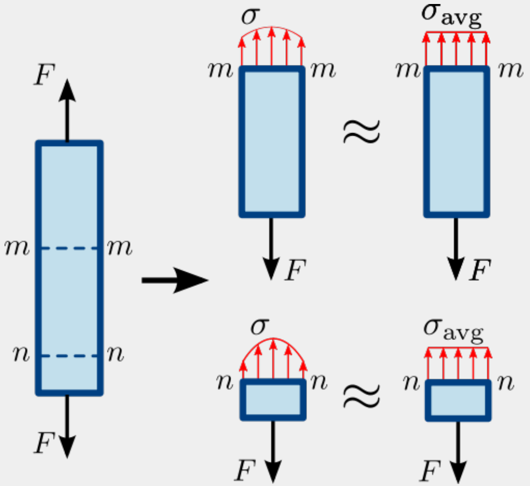
- Use free body diagram to determine internal forces/moments on surface at any point
- What if there are too loads/chages all at once
- Principle of Superposition

- Split the loads/changes
- Determine individual response
- Add the responses up

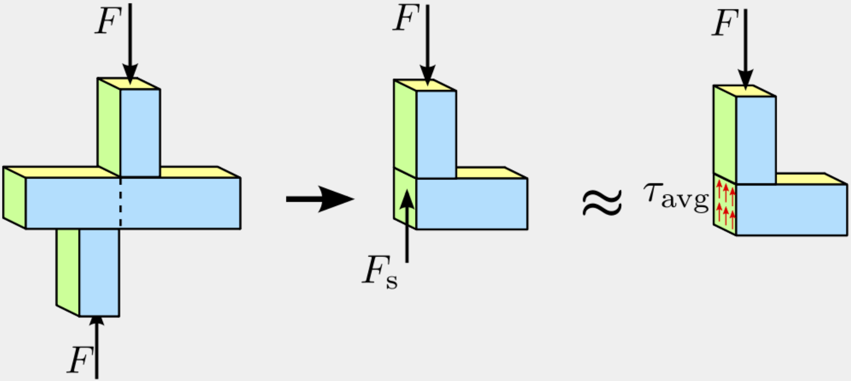
Review of High School Physics

- Normal Stresses: same direction as surface
- Shear stresses: perpendicular to surface direction

$$\sigma = \frac{F_{normal}}{A}$$



$$\tau = \frac{F_{shear}}{A}$$



Allowable Stresses

- Real design needs to take care of uncertainties: materials, conditions, loads, ...
- Use σ_{allow} and τ_{allow} instead

$$\sigma_{allow} = \frac{\sigma_f}{N_s}$$

$$\tau_{allow} = \frac{\tau_f}{N_s}$$

Safety Factors, N_s

- N_s is called the *safety factor*
- $N_s \geqslant 1$ always
- Why? Is there an upper limit to N_s?

Example: Design with Safety Factor

- We need a steel rod that will take the load of 20 kN with a safety factor of 2. The steel rod has the maximum yield strength of 300 MPa. Determine the required diameter of the rod.

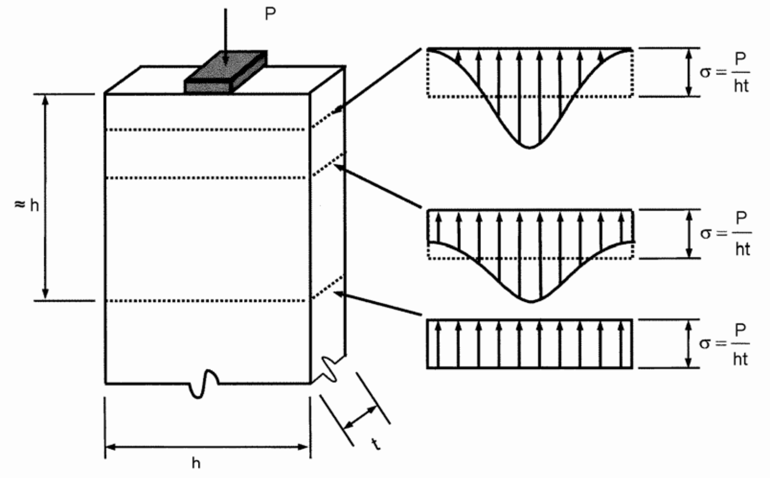
$$\sigma_{allow} = \frac{F}{\pi r^2} = \frac{\sigma_f}{N_s}$$

$$\frac{20000}{\pi r^2} = \frac{300 \times 10^6}{2}$$

$$r^2 = 4.24 \times 10^{-5}$$

$$r = 7.98 \times 10^{-3} \text{ m}$$

St. Venant's Principle



- Far enough away from load, stresses follow theoretical values

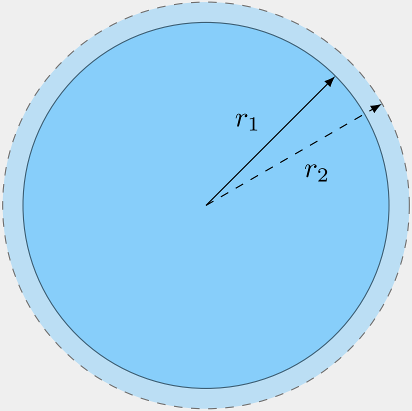
Normal Strain

- Strain from lengthening or shortening of material

$$\varepsilon = \frac{\delta}{L}$$

Example: Ballooon

- Air filled balloon with original radius r_1 is pressurized until its radius becomes r_2. What is its strain?



Shear Strain

- Change in angular orientation of material

$$\gamma = \frac{\pi}{2} - \theta_f$$

Hooke's Law

- How are stresses and strains related?
- Normal stress-strain

$$\sigma = E\varepsilon$$

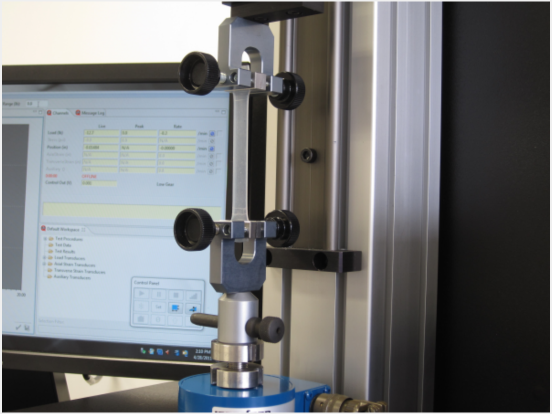
- Shear stress-strain

$$\tau = G\gamma$$

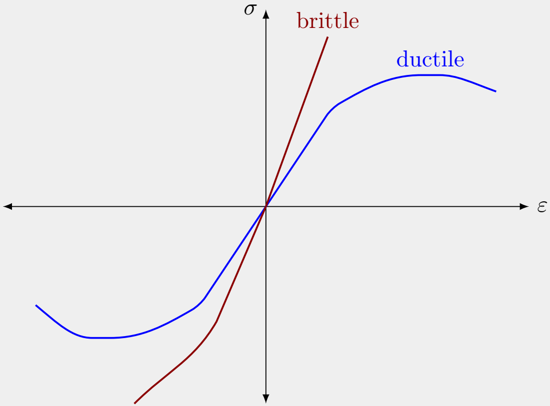
- E is Young's modulus or modulus of elasticity
- G is shear modulus

- Most engineering materials have two regions
    - Elastic behavior: deformation is reversible
    - Plastic behavior: deformation is permanent

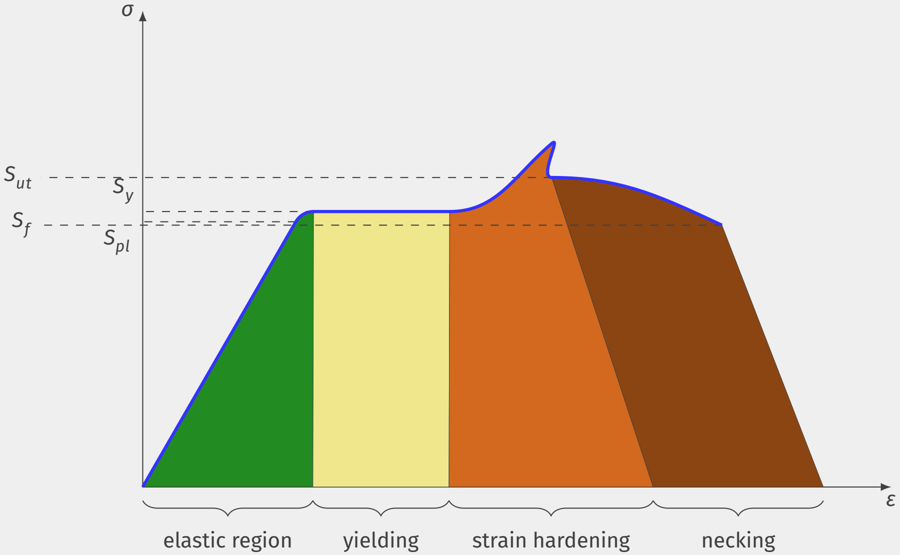
- Tensile Test: testing for material response



Material Types



Stress - Strain Diagram: Ductile



Elastic Region

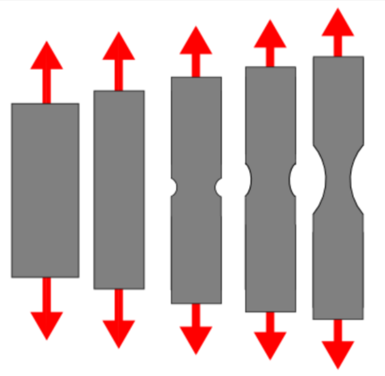
- Deformation is reversible → object returns to original shape once load is removed
- Most designed parts are meant to operate in this region

Yield

- Transition from elastic to plastic deformation ~ material failure
- Difficult to specify exact location
- Definition can vary
    1. proportionality limit
    2. elastic limit
    3. offset yield point (0.2% rule)

Strain Hardening

- Deformation in materials cause temperary hardness increases
- Material can take additional stress because of this



- Final phase of plastic deformation before failure
- Cross-sectional area decreases → increased stress

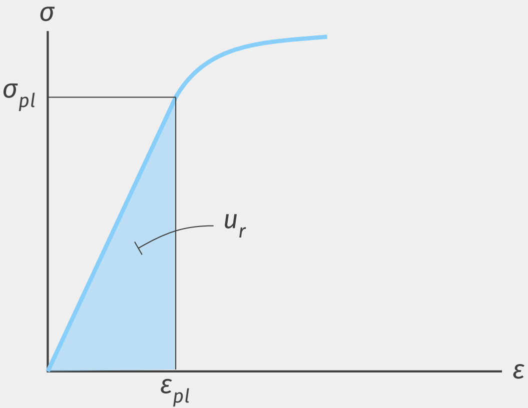
Strain Energy

- Energy stored in deformed body
- Assumed equal to work done by external loadings

$$
\begin{aligned}
u &= \frac{1}{2}\sigma\varepsilon \\
&= \frac{1}{2}\frac{\sigma^2}{E}
\end{aligned}
$$

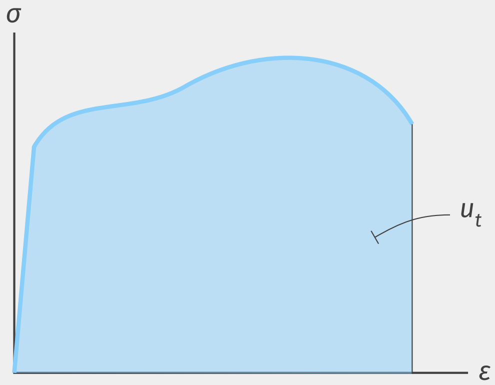
- Area under $\sigma - \varepsilon$ curve
- Important to material strength under impact loading

Modulus of Resilience



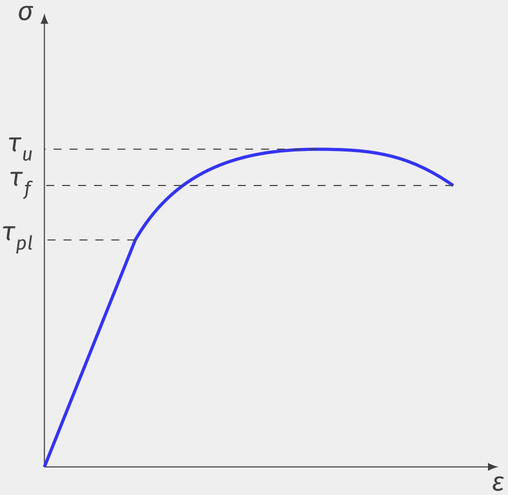
- Amount of energy to permanently deform body

Modulus of Toughness



- Amount of energy to fracture body

Shear Stress-Strain Relationship



- Change in temperature causes material deformation
  - material normally expands when heated and contracts when cooled
- Definition in 1-D

$$\alpha = \frac{1}{L}\frac{dL}{dT}$$

- α is called the coefficient of thermal expansion (CTE)

Properties of α

- α is typically a function of T
- For many engineering materials (solids), α ~ constant

$$\delta = \int_0^L \alpha \Delta T dx$$

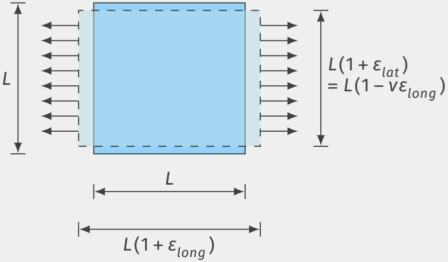
- For uniform temperature change

$$\delta = \alpha \Delta T L$$

Example: Heated Bar

- If a beam has an original length of 2 m and initial T = 20 C
- The beam is heated, after which the temperature along the beam is $T(x) = 20x^2 + 10x + 30$ C. Beam has $\alpha = 2.5\ 10^{-6}$
  - What is the deformation of the middle point of the beam?
  - What is the final length of the beam?

Poisson's Effect



- Material's lateral contraction (extension) under longitudinal tensile (compressive) load

$$\nu = -\frac{\varepsilon_{lat}}{\varepsilon_{long}}$$

- ν is called *Poisson's ratio*

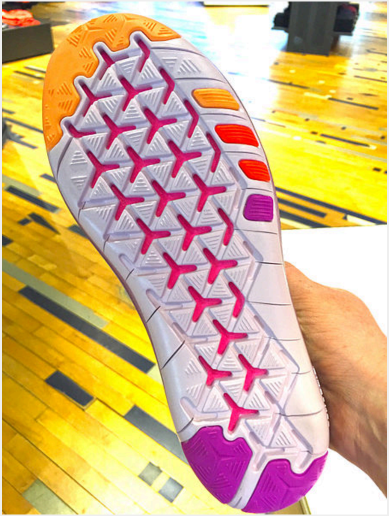
Poisson's Ratio Range

Material	Poisson's ratio
rubber	0.4999
gold	0.42–0.44
saturated clay	0.40–0.49
magnesium	0.252-0.289
titanium	0.265-0.34
copper	0.33
aluminium-alloy	0.32
clay	0.30–0.45
stainless steel	0.30–0.31
steel	0.27–0.30
cast iron	0.21–0.26
sand	0.20–0.45
concrete	0.1-0.2
glass	0.18–0.3
foam	0.10–0.50
cork	0

- Usual engineering materials have $0 \leqslant \nu \leqslant 0.5$

Auxetic Material

- Materials that exhibit negative Poisson's ratios
- How is that possible?
- Rely on material microstructure
- Useful in many design situations



Mechanical Strains vs Thermal Strains

- Strains caused by load vs temperature change
- Mechanical strains: normal strains + lateral strains from Poisson's effect
- Thermal strains: strains in all direction, *no* Poisson's effect.
- $\varepsilon_{\text{total}} = \varepsilon_{\text{mech}} + \varepsilon_{\text{therm}}$ (mind the signs)

Mech vs Therm Strains Example

A circular cross-sectioned steel bar with radius $r = 1$ cm and length $L = 2$ m is stretched along its length by a stress of 100 MPa. If the steel has $E = 210$ GPa, $v = 0.3$, and $\alpha = 16 \times 10^{-6} / ^{\circ}C$, how much temperature change does it need to return to its original volume?

Solution: Mech vs Therm Strains

Match before and after volumes
before:

$$V_0 = \pi r^2 l$$

after:

$$l_1 = l\left(1 + \frac{\sigma}{E} + \alpha \Delta T\right)$$
$$r_1 = r\left(1 - v\frac{\sigma}{E} + \alpha \Delta T\right)$$

Set $V_0 = V_1$

$$\pi r^2 l = \pi r^2 \left(1 - v\frac{\sigma}{E} + \alpha \Delta T\right)^2 l \left(1 + \frac{\sigma}{E} + \alpha \Delta T\right)$$

Keep only first-order terms:

$$\begin{aligned} 0 &= \frac{\sigma}{E} - v\frac{\sigma}{E} - v\frac{\sigma}{E} + 2\alpha \Delta T + \alpha \Delta T \\ \Delta T &= \frac{-\sigma(1 - 2v)}{3\alpha E} \\ &= -3.97 \text{ C} \end{aligned}$$

Thermal Stress

- Stress in a cooled/heated material constrained from deforming freely

Example: Hot Bar / Cool Bar

- A metal bar is constrained between two walls the same distance as the beam's length, how would you change the temperature so that the beam is ...
  - in tension?
  - in compression?
- We can intuit the *direction* of temperature change (up or down), *but* not yet its magnitude (how much)
- We will learn that soon enough ...