

ANALYSIS OF MEMBERS UNDER COMBINED LOADINGS

ME 210: MECHANICS OF MATERIALS

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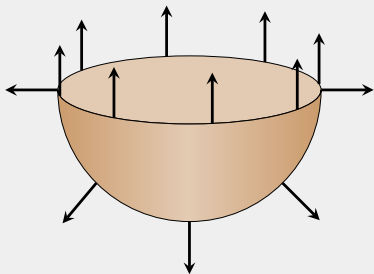
DEPARTMENT OF MECHANICAL ENGINEERING, TSE

OUTLINE

1 Pressure Vessels

2 Combined Loadings

SPHERICAL PRESSURE VESSELS



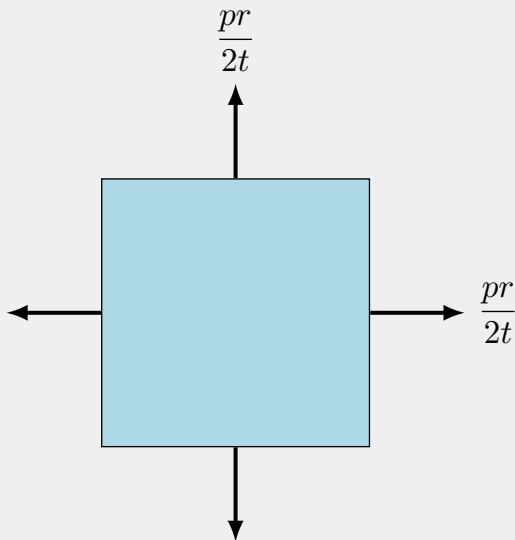
$$F_{\sigma} = F_p$$

$$\sigma_2 (2\pi r t) = p \pi r^2$$

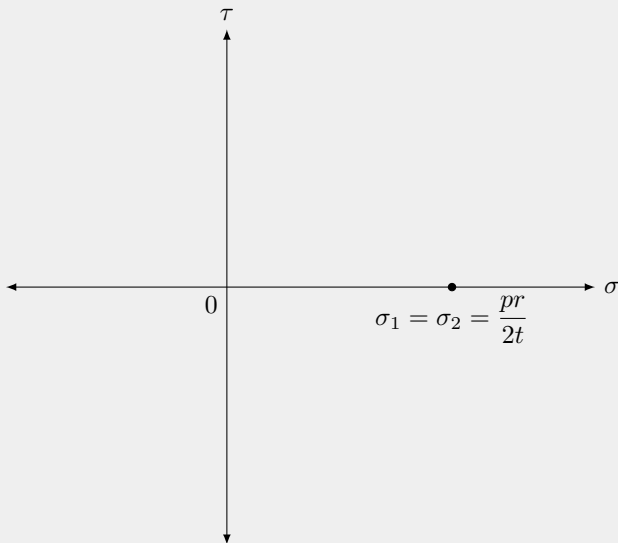
$$\sigma_2 = \frac{pr}{2t}$$

- What about σ_1 ? How do you draw a Mohr's circle to represent state of stress?

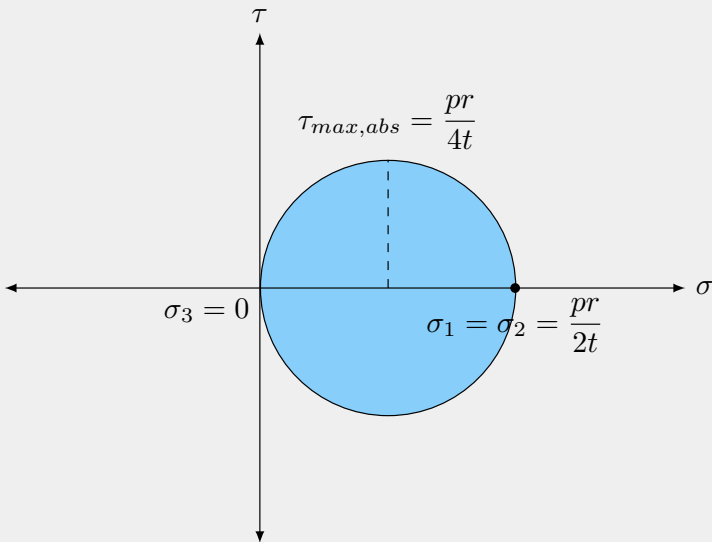
STATE OF STRESS OF OF VESSEL WALL



MOHR'S CIRCLE OF STATE OF STRESS



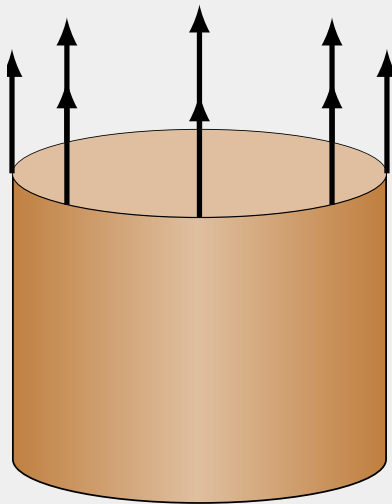
ABSOLUTE MAXIMUM SHEAR STRESS OF SPHERICAL PRESSURE VESSEL



SPHERICAL PRESSURE VESSEL: SUMMARY

- $\sigma_1 = \sigma_2 = \frac{pr}{2t}$
- $\tau_{xy} = \tau_{max} = 0$
- $\tau_{max,abs} = \frac{pr}{4t}$

CYLINDRICAL PRESSURE VESSELS



$$F_{\sigma} = F_p$$

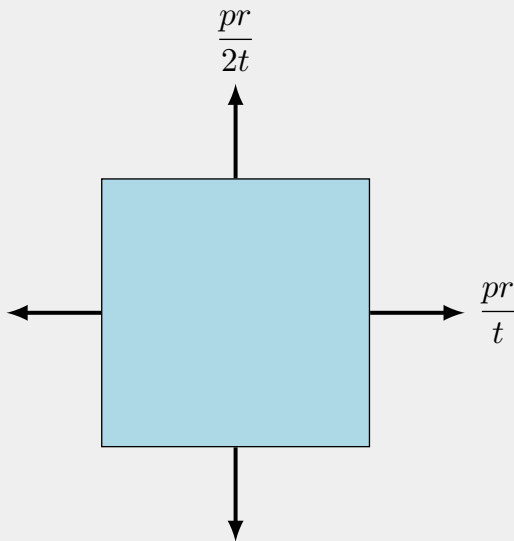
$$\sigma_1 (2t dy) = p (2r dy)$$

$$\sigma_1 = \frac{pr}{t}$$

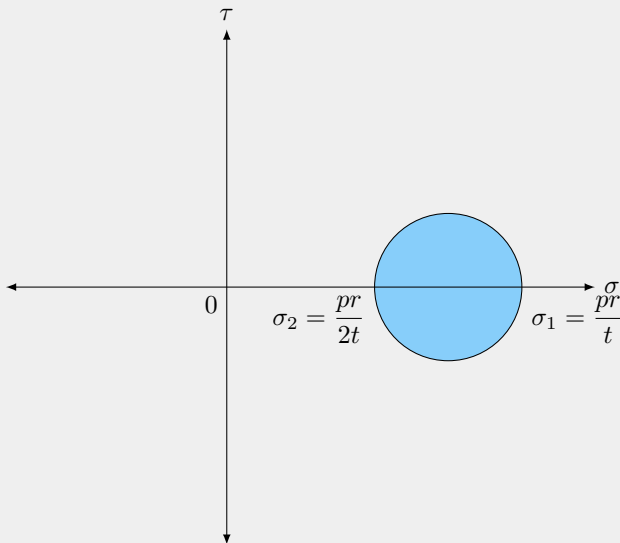
$$\sigma_2 (2\pi r t) = p \pi r^2$$

$$\sigma_2 = \frac{pr}{2t}$$

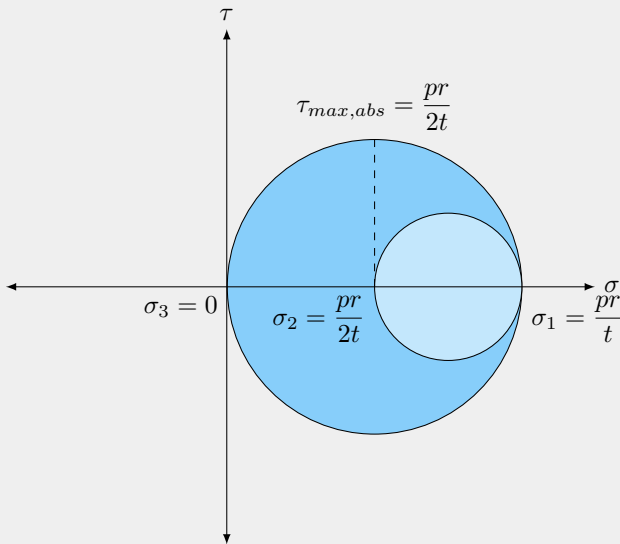
STATE OF STRESS OF OF VESSEL WALL



MOHR'S CIRCLE OF CYLINDRICAL VESSEL



ABSOLUTE MAXIMUM SHEAR STRESS OF CYLINDRICAL VESSEL



CYLINDRICAL PRESSURE VESSEL: SUMMARY

$$\blacksquare \sigma_1 = \sigma_c = \frac{pr}{t}$$

$$\blacksquare \sigma_2 = \sigma_l = \frac{pr}{2t}$$

$$\blacksquare \tau_{xy} = 0, \tau_{max} = \frac{pr}{4t}$$

$$\blacksquare \tau_{max,abs} = \frac{pr}{2t}$$

CYLINDRICAL PRESSURE VESSELS

- Failure of a shotgun barrel



OUTLINE

1 Pressure Vessels

2 Combined Loadings

COMBINED LOADINGS

- Multiaxial stress conditions come from
 - ▶ Simultaneous application of loads
 - ▶ Complex geometry of component
- Superposition is always the key
 - ▶ Find stress(es) from each load
 - ▶ combine resultant stresses using multiaxial stress analysis

DESIGN OF MEMBER UNDER COMBINED LOADINGS

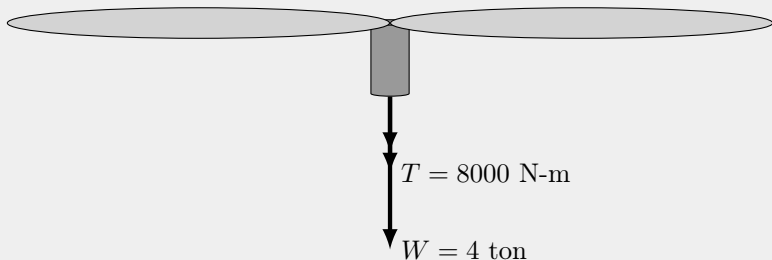
- We need to know where failure starts
- For a single-material component, failure starts where *combined* stress is the highest
- This is called the “critical point”

HOW TO IDENTIFY THE CRITICAL POINT

- Identify each type of load (axial, bending, or torsion)
- Mark locations of maximum stress for each load
- Locate location(s) with multiple maximum stresses

EXAMPLE: HELICOPTOR ROTOR SHAFT

We want to determine the proper diameter of a rotor shaft for a 4-ton helicopter. The shaft is connected to the engine that provides the maximum torque of 8000 N-m. The shaft is made of AISI1023 steel with $\sigma_{allow} = 400 \text{ MPa}$.



HELICOPTER ROTOR SHAFT: SOLUTION

- Determine the critical point
- Determine state of stress
- Find proper radius r

HELICOPTER ROTOR SHAFT: SOLUTION

$$\begin{aligned}\sigma &= \frac{F}{A} = \frac{4(1000 \text{ kg/ton})(10 \text{ N/kg})}{\pi r^2} \\ &= \frac{12732}{r^2} \\ \tau &= \frac{Tr}{J} = \frac{8000(r)}{\pi r^4/2} \\ &= \frac{5093}{r^3}\end{aligned}$$

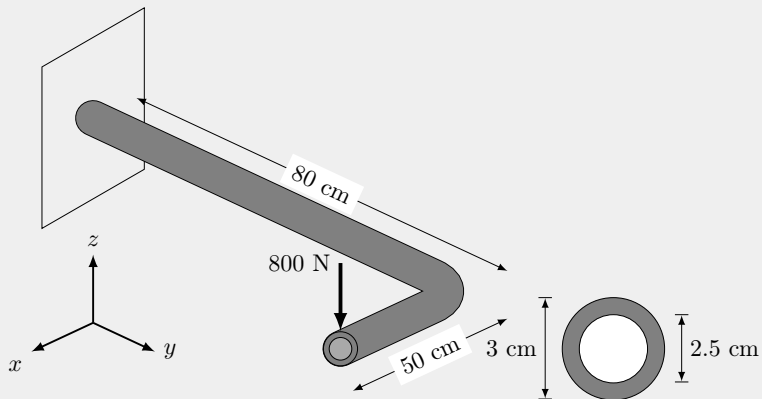
HELICOPTER ROTOR SHAFT: SOLUTION

So the state of stress at the critical surface is a combination of normal stress and shear stress. Since the given material is limited by its normal stress, we need to determine the maximum principal stress.

$$\sigma_1 = \sigma_{allow} = 400 \times 10^6 = \frac{12732}{2r^2} + \sqrt{\left(\frac{12732}{2r^2}\right)^2 + \left(\frac{5093}{r^3}\right)^2}$$

This equation can be solved numerically to obtain $r = 2.38$ cm.

L-PIPE



QUESTIONS

- Find the critical point. Elaborate your reasoning.
- Determine the state of stress of the critical point.
- Draw a Mohr's circle representing the state of stress.