

INTRO TO MATERIAL BEHAVIORS

ME 210: MECHANICS OF MATERIALS

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A LITTLE INTRO

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CHILDHOOD

- Born and raised in Khon Kaen, Thailand



BACHELOR'S DEGREE

■ Sc.B. Brown University, 2002



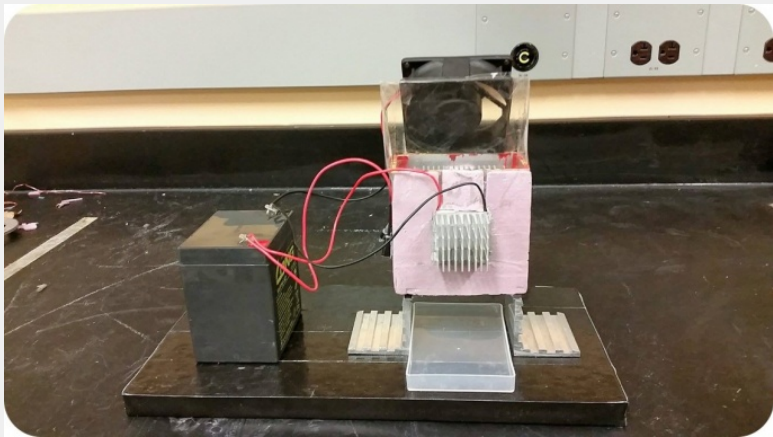
MASTER AND DOCTORAL

■ M.S. 2005, Ph.D. 2008, MIT



RESEARCH INTERESTS

- Solar energy utilization
- Water generation

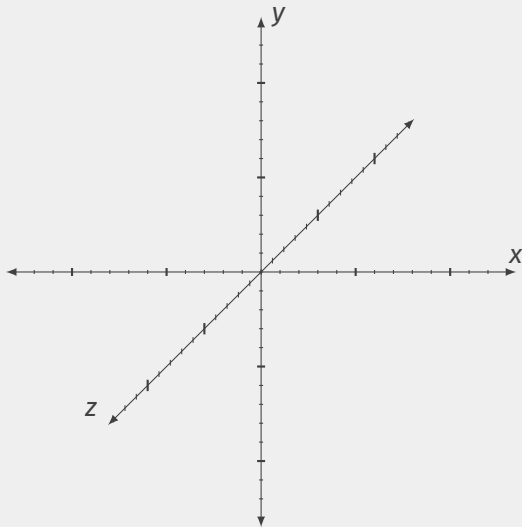


- The study of material response to
 - ▶ Mechanical Changes: external loads
 - ▶ Thermal changes: temperature

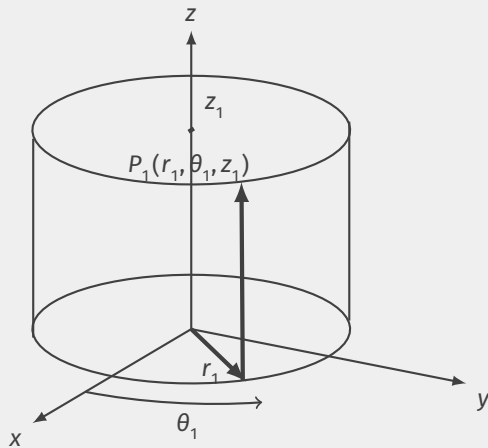
COORDINATE SYSTEM

- Most quantities in this class are vectors
- Coordinate systems make it clear on orientation and direction of anything
- Three main systems
 - ▶ Cartesian
 - ▶ Cylindrical
 - ▶ Spherical

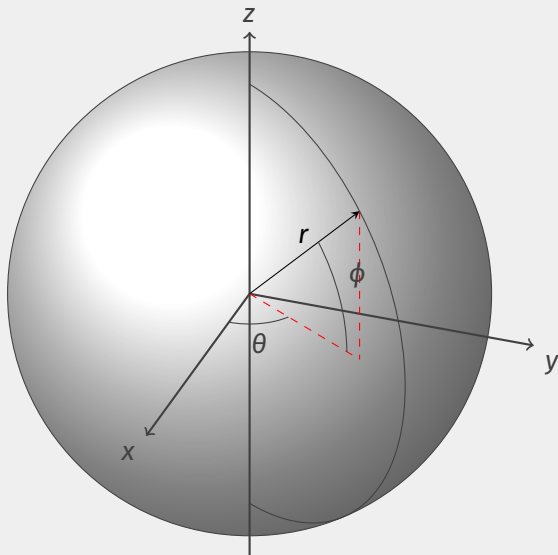
CARTESIAN



CYLINDRICAL



SPHERICAL COORDINATES

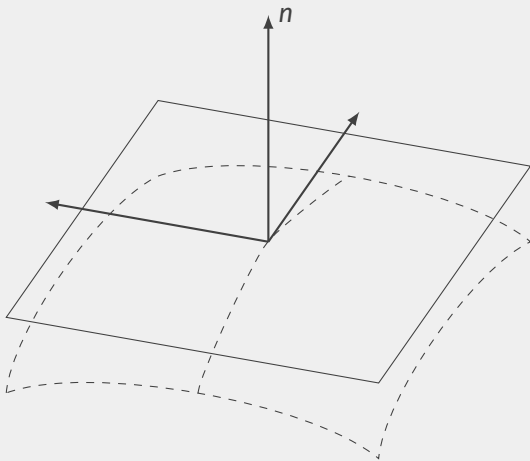


EXTERNAL LOADS

- Force(s) or Moment(s)
- Results in deformation, depending on *direction* and *surface*
- How do we define *direction* and *surface*

SURFACE DIRECTION

- Direction of vector *normal* to the surface



- Normal forces → same direction as surface
- Shear forces → perpendicular to surface direction

- Follow right-hand rule.
- Torsional moments: same direction as surface
- Bending moments: perpendicular to surface direction

THE SINGULARITY EQUATION (FOR THIS CLASS)

- Equilibrium equation

$$\sum F = 0$$

$$\sum M = 0$$

- We will be trying to determine stresses and deformation of things
- Need to find internal load at any point/surface
- Method of sections

METHOD OF SECTIONS

- Use free body diagram to determine internal forces/moments on surface at any point
- What if there are too loads/chages all at once
- Principle of Superposition

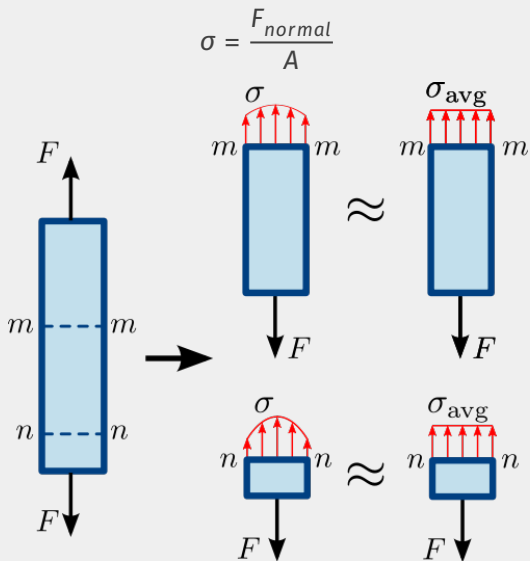
PRINCIPLE OF SUPERPOSITION

- Split the loads/changes
- Determine individual response
- Add the responses up

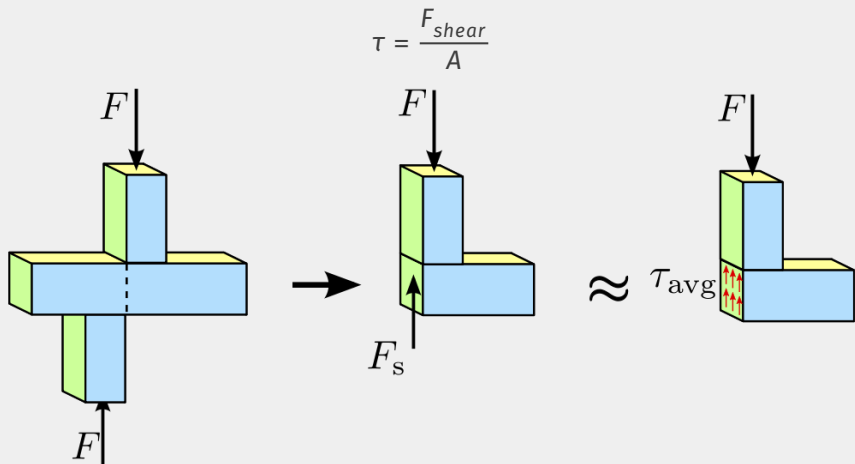
REVIEW OF HIGH SCHOOL PHYSICS

- Normal Stresses: same direction as surface
- Shear stresses: perpendicular to surface direction

NORMAL STRESSES



SHEAR STRESSES



ALLOWABLE STRESSES

- Real design needs to take care of uncertainties: materials, conditions, loads, ...
- Use σ_{allow} and τ_{allow} instead

$$\sigma_{allow} = \frac{\sigma_f}{N_s}$$

$$\tau_{allow} = \frac{\tau_f}{N_s}$$

SAFETY FACTORS, N_s

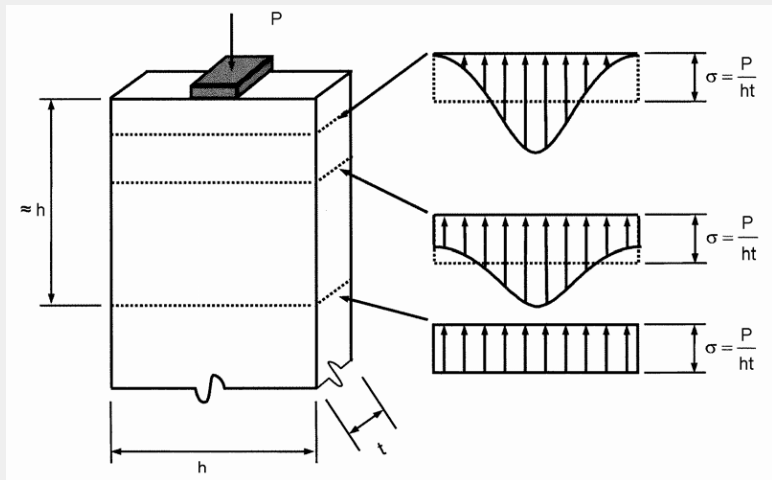
- N_s is called the *safety factor*
- $N_s \geq 1$ always
- Why? Is there an upper limit to N_s ?

EXAMPLE: DESIGN WITH SAFETY FACTOR

- We need a steel rod that will take the load of 20 kN with a safety factor of 2. The steel rod has the maximum yield strength of 300 MPa. Determine the required diameter of the rod.

$$\begin{aligned}\sigma_{allow} &= \frac{F}{\pi r^2} = \frac{\sigma_f}{N_s} \\ \frac{20000}{\pi r^2} &= \frac{300 \times 10^6}{2} \\ r^2 &= 4.24 \times 10^{-5} \\ r &= 7.98 \times 10^{-3} \text{ m}\end{aligned}$$

ST. VENANT'S PRINCIPLE

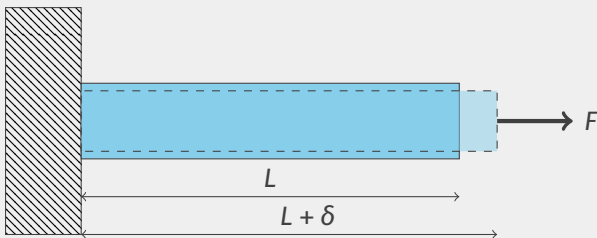


■ Far enough away from load, stresses follow theoretical values

NORMAL STRAIN

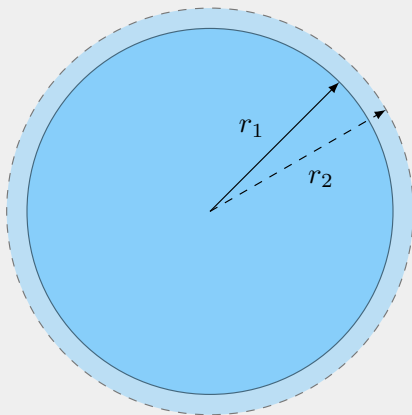
- Strain from lengthening or shortening of material

$$\varepsilon = \frac{\delta}{L}$$



EXAMPLE: BALLOON

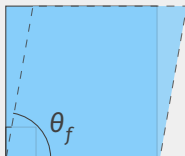
- Air filled balloon with original radius r_1 is pressurized until its radius becomes r_2 . What is its strain?



SHEAR STRAIN

- Change in angular orientation of material

$$\gamma = \frac{\pi}{2} - \theta_f$$



HOOKE'S LAW

- How are stresses and strains related?
- Normal stress-strain

$$\sigma = E \varepsilon$$

- Shear stress-strain

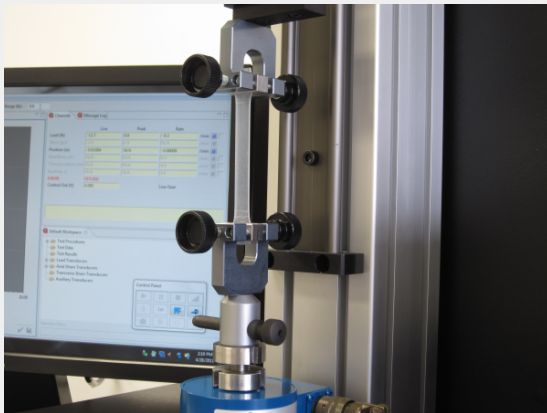
$$\tau = G \gamma$$

- E is Young's modulus or modulus of elasticity
- G is shear modulus

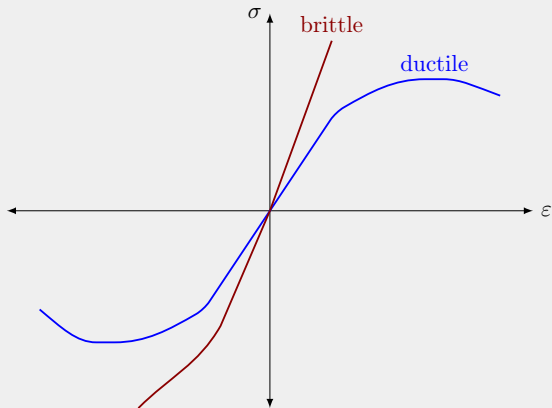
- Most engineering materials have two regions
 - ▶ Elastic behavior: deformation is reversible
 - ▶ Plastic behavior: deformation is permanent

MATERIAL PROPERTY TESTING

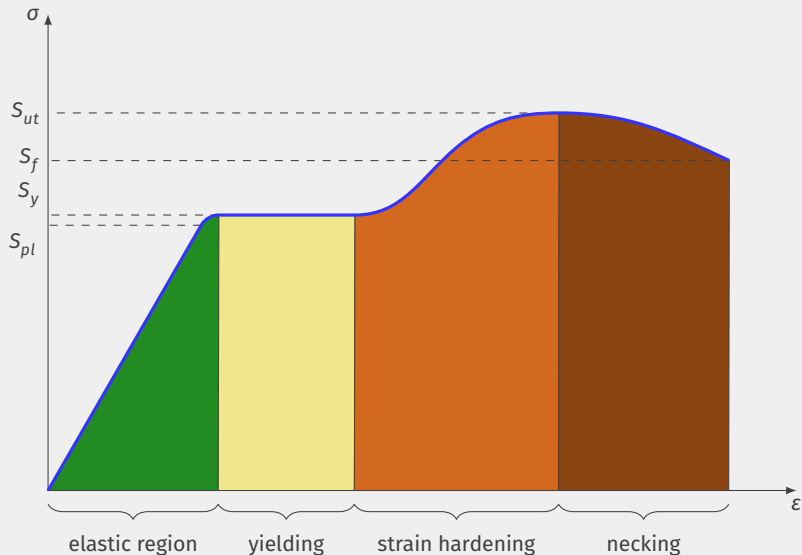
■ Tensile Test: testing for material response



MATERIAL TYPES



STRESS - STRAIN DIAGRAM: DUCTILE



ELASTIC REGION

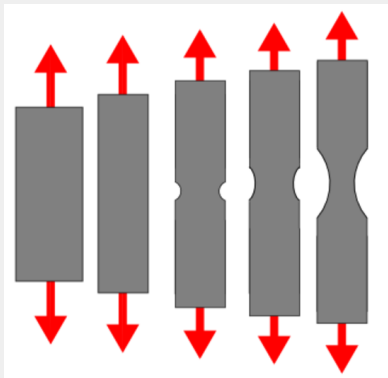
- Deformation is reversible → object returns to original shape once load is removed
- Most designed parts are meant to operate in this region

- Transition from elastic to plastic deformation ~ material failure
- Difficult to specify exact location
- Definition can vary
 1. proportionality limit
 2. elastic limit
 3. offset yield point (0.2% rule)

STRAIN HARDENING

- Deformation in materials cause temporary hardness increases
- Material can take additional stress because of this

NECKING



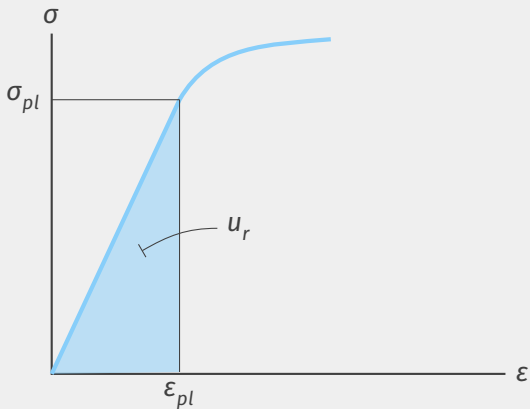
- Final phase of plastic deformation before failure
- Cross-sectional area decreases
→ increased stress

- Energy stored in deformed body
- Assumed equal to work done by external loadings

$$\begin{aligned}u &= \frac{1}{2} \sigma \varepsilon \\&= \frac{1}{2} \frac{\sigma^2}{E}\end{aligned}$$

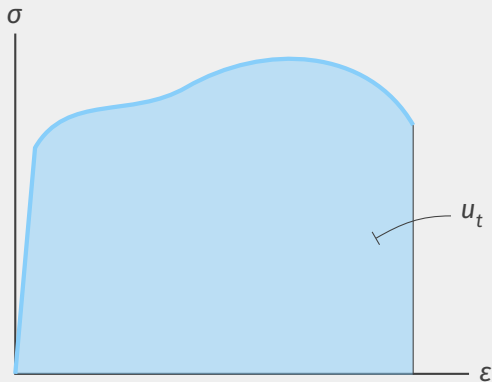
- Area under $\sigma - \varepsilon$ curve
- Important to material strength under impact loading

MODULUS OF RESILIENCE



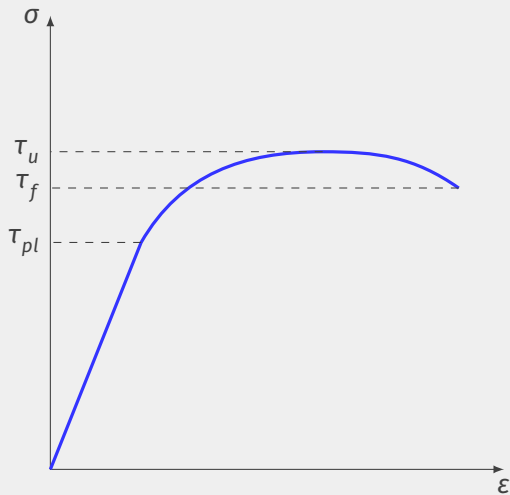
- Amount of energy to permanently deform body

MODULUS OF TOUGHNESS



- Amount of energy to fracture body

SHEAR STRESS-STRAIN RELATIONSHIP



THERMAL STRAIN

- Change in temperature causes material deformation
 - ▶ material normally expands when heated and contracts when cooled
- Definition in 1-D

$$\alpha = \frac{1}{L} \frac{dL}{dT}$$

- α is called the coefficient of thermal expansion (CTE)

PROPERTIES OF α

- α is typically a function of T
- For many engineering materials (solids), $\alpha \sim \text{constant}$

$$\delta = \int_0^L \alpha \Delta T dx$$

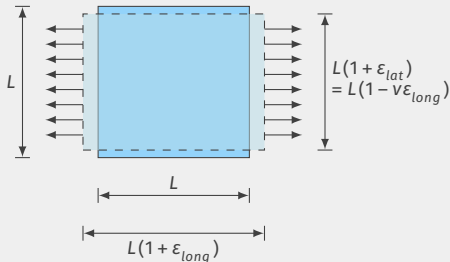
- For uniform temperature change

$$\delta = \alpha \Delta T L$$

EXAMPLE: HEATED BAR

- If a beam has an original length of 2 m and initial $T = 20$ C
- The beam is heated, after which the temperature along the beam is $T(x) = 20x^2 + 10x + 30$ C. Beam has $\alpha = 2.5 \cdot 10^{-6}$
 - ▶ What is the deformation of the middle point of the beam?
 - ▶ What is the final length of the beam?

POISSON'S EFFECT



- Material's lateral contraction (extension) under longitudinal tensile (compressive) load

$$\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}}$$

- ν is called *Poisson's ratio*

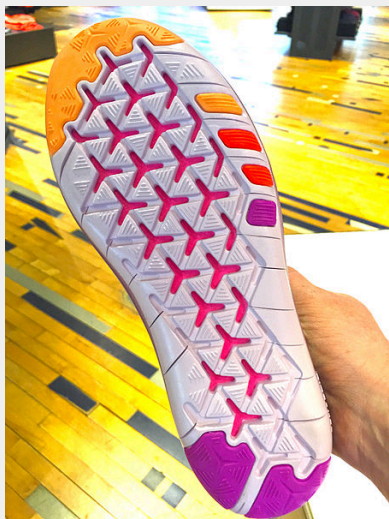
POISSON'S RATIO RANGE

Material	Poisson's ratio
rubber	0.4999
gold	0.42–0.44
saturated clay	0.40–0.49
magnesium	0.252–0.289
titanium	0.265–0.34
copper	0.33
aluminium-alloy	0.32
clay	0.30–0.45
stainless steel	0.30–0.31
steel	0.27–0.30
cast iron	0.21–0.26
sand	0.20–0.45
concrete	0.1–0.2
glass	0.18–0.3
foam	0.10–0.50
cork	0

- Usual engineering materials have $0 \leq \nu \leq 0.5$

AUXETIC MATERIAL

- Materials that exhibit negative Poisson's ratios
- How is that possible?
- Rely on material microstructure
- Useful in many design situations



MECHANICAL STRAINS VS THERMAL STRAINS

- Strains caused by load vs temperature change
- Mechanical strains: normal strains + lateral strains from Poisson's effect
- Thermal strains: strains in all direction, *no* Poisson's effect.
- $\epsilon_{\text{total}} = \epsilon_{\text{mech}} + \epsilon_{\text{therm}}$ (mind the signs)

MECH VS THERM STRAINS EXAMPLE

A circular cross-sectioned steel bar with radius $r = 1$ cm and length $L = 2$ m is stretched along its length by a stress of 100 MPa. If the steel has $E = 210$ GPa, $\nu = 0.3$, and $\alpha = 16 \times 10^{-6} / ^\circ\text{C}$, how much temperature change does it need to return to its original volume?

SOLUTION: MECH VS THERM STRAINS

Match before and after volumes
before:

$$V_0 = \pi r^2 l$$

after:

$$l_1 = l \left(1 + \frac{\sigma}{E} + \alpha \Delta T \right)$$
$$r_1 = r \left(1 - \nu \frac{\sigma}{E} + \alpha \Delta T \right)$$

SOLUTION: MECH VS THERM STRAINS

Set $V_0 = V_1$

$$\pi r^2 l = \pi r^2 \left(1 - \nu \frac{\sigma}{E} + \alpha \Delta T\right)^2 l \left(1 + \frac{\sigma}{E} + \alpha \Delta T\right)$$

Keep only first-order terms:

$$\begin{aligned} 0 &= \frac{\sigma}{E} - \nu \frac{\sigma}{E} - \nu \frac{\sigma}{E} + 2\alpha \Delta T + \alpha \Delta T \\ \Delta T &= \frac{-\sigma(1 - 2\nu)}{3\alpha E} \\ &= -3.97 \text{ C} \end{aligned}$$

- Stress in a cooled/heated material constrained from deforming freely

EXAMPLE: HOT BAR / COOL BAR

- A metal bar is constrained between two walls the same distance as the beam's length, how would you change the temperature so that the beam is ...
 - ▶ in tension?
 - ▶ in compression?
- We can intuit the *direction* of temperature change (up or down), *but* not yet its magnitude (how much)
- We will learn that soon enough ...