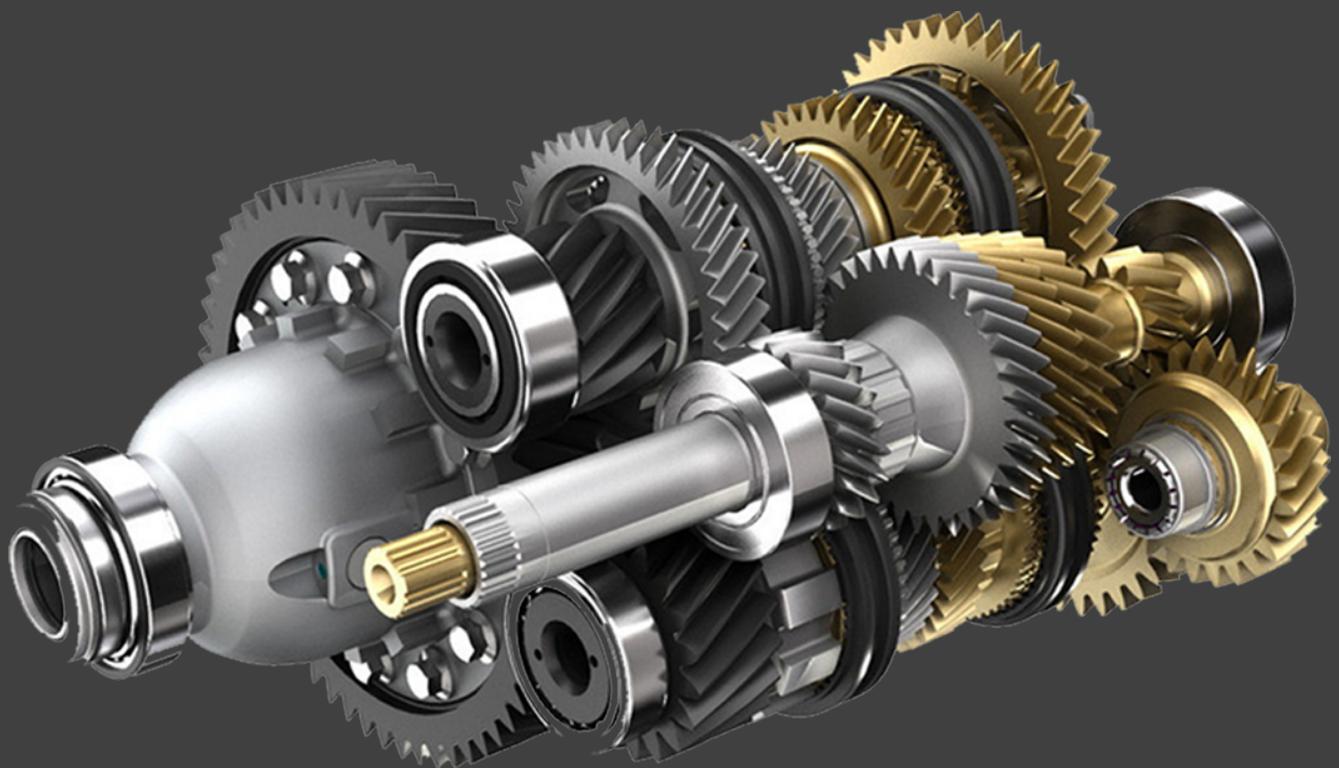




Thammasat University Faculty of Engineering



# ME 310: Mechanical Design

## Power Transmission

Sappinandana Akamphon



## *Preface*

This book is the second part of ME 310: Mechanical Design. In this book, power transmission components are covered.

Sappinandana Akamphon

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## *Revisions*

*September 2019*

Start of Mechanical Design: Power Transmission

*January 2020*

Began working on the chapters on shaft components, gears, brakes and clutches

*October 2021*

Completed chapters on gears, shafts,

# *Shaft and Shaft Components*

Most engineering systems are powered by rotational machinery such as internal combustion engines and electrical motors. It is, thus, extremely important that we understand and properly design power transmission mechanism from the rotational machinery power source to intended components. Shafts remains one of the most prevalent methods of transmitting rotational power, and so it is our first chapter into the world of power transmission design.

By itself, shaft design is no more complex than other components under static or cyclic loadings already covered in the first book of this series. However, as shafts lie at the heart of most machine design applications, its final design will also depend on the design of other components that are to be mounted or connected to the shaft.

## *Shaft Materials*

There are two main concerns with designing a shaft: deflection and strength. Deflection is not affected by material strength. In fact, it depends only on the stiffness, represented by the Young's modulus. Since the Young's modulus is essentially the same for all steels, material choice matters little for deflection.

Strength, on the other hand, constitute a major concern for shaft design. Strength is necessary to resist loading stresses and fatigue stresses from the constant rotation. Many shafts are made from low carbon, cold-drawn or hot-rolled steel such as ANSI 1020-1050 steels. Significant strengthening from heat treatment or high alloy is often not needed. And fatigue failure is only reduced moderately by increase in strength, after a certain level notch sensitivity begins to counteract its benefit.

A good way to select a proper shaft material is to start with an inexpensive low or medium carbon steel for the first round of calculations. If strength considerations turn out to be the limiting factor

over deflection, select a higher strength material and reduce the shaft size accordingly until deflection becomes an issue. The cost of the material and its manufacturing processes must be weighed against the need for smaller shaft diameter.

When additional strengthening is needed, typical alloy steels for heat treatment include ANSI 340-50, 3140-50, 4140, 4340, 5140, and 8650. Shafts don't usually need surface hardening unless there is significant risk of wear from journal bearing, in which case surface hardening include carburizing grades of ANSI 1020, 4320, 4820, and 8620. [1]

### *Shaft Layout*

Generally a shaft is a cylinder with segments of varying cross sections to accommodate components like gears, bearings, pulleys, etc. Shaft shoulders are normally used to axially locate elements and to take any axial thrust loads. illustrates the cross-section of a vertical worm-gear speed reducer. In this figure, the shaft steps are used to axially locate the two tapered bearings and the spur gear.

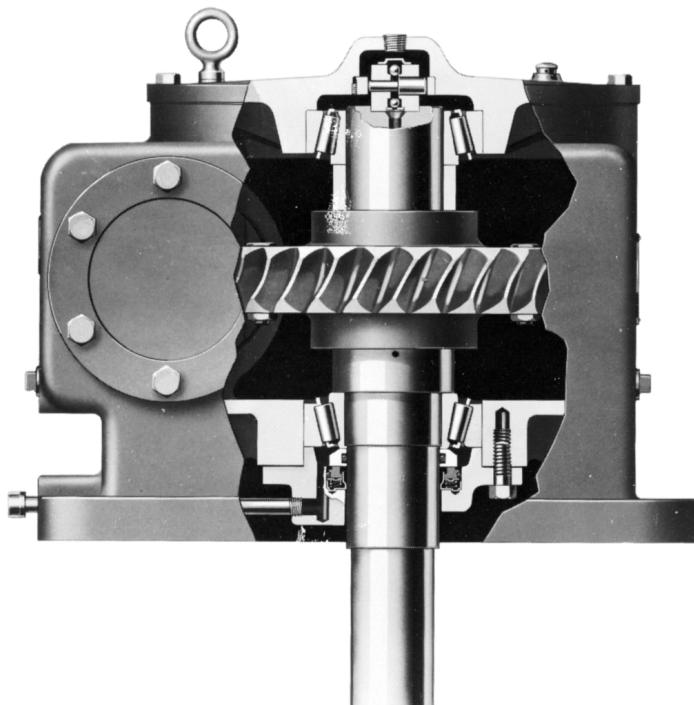


Figure 1: A vertical worm-gear speed reducer. (Courtesy of the Cleveland Gear Company.)

### *Axial Layout of Components*

The axial positioning of components is often preset by the layout of the housing and other meshing components. Generally it is advisable to support any loading-carrying components between bearings. (TODO: NEED FIGURE) However, pulleys and sprockets tend to be mounted outside of the bearing pairs for ease of installation of the belt or chain. The length of the shaft should be kept as short as possible to minimize deflection and resultant load on the bearings.

In most cases, a shaft will only require two bearings, but in cases with long shafts carrying multiple load-carrying components, it may be necessary to use more than two bearings for additional support. In such a case, particular care must be given to the alignment of the bearings.

### *Supporting Axial Loads*

There are use cases where axial loads in shaft may be significant, in which case it is important to provide a means to transfer the axial loads into the shaft. The shaft would then transfer the loads to a bearing and to the ground. This is necessary for shafts mounted with components that generate axial loads like bevel and helical gears, or tapered roller bearings.

It is usually sufficient to have only one bearing carry the axial load. This allows for greater tolerances on shaft length dimensions and prevents binding from shaft thermal expansions, which is especially significant in long shafts. TODO: figures showing shaft that carries axial loads.

### *Support for Torque Transmission*

Most shafts serve to transfer torque from an input (gear, pulley, engine, motor, etc.) to an output gear or pulley. The shaft must be sized to support torsional stress and its resultant angle of twist. It is also important to provide a way to transmit the torque between the gear (or pulley) and the shaft itself. Common torque transfer methods are:

- Keys
- Splines

- Setscrews
- Pins
- Press or shrink fits
- Tapered fits

There are also shafts that are designed to fail if excessive torque is applied, to prevent failure of more expensive components. Details of the components and their design process is covered in .

### *Shaft Design for Stress*

It is not always necessary to evaluate stresses at every point; the same goes for shafts as well. Only a few potentially critical locations should be more than enough. Since the main types of load on shafts are torsion and bending, it follows that most critical locations on the shafts are on the outer surface—typically where the bending moment is large, the torque is large, and where stress concentrations exist.

In order to determine the bending moments, torques, and shear forces on a shaft, it is usually a good idea to draw shear and bending moment diagrams. Since most shafts are loaded by gears and pulleys, introducing forces in two planes, two diagrams are needed to determine the loads. Resultant moments can be obtained simply by adding the moments as vectors at points of interest. The normal stress due to bending will be highest on the outer surfaces and will contribute to fatigue on a rotating shaft.

Axial stresses on shafts from axial loads caused by helical gears or tapered roller bearings are typically negligible compared to the bending stress. The axial stresses are also usually constant, meaning that they rarely contribute significantly to fatigue. However, axial stresses resulting from axial loads applied through other means should be explicitly considered.

Let us now consider the shaft stresses, which are usually the combination of normal stresses from bending and axial stresses, and shear stress from torsion.

$$\begin{aligned}\sigma_a &= K_f \frac{M_a y}{I} & \sigma_m &= K_f \frac{M_m y}{I} \\ \tau_a &= K_{fs} \frac{T_a r}{J} & \tau_m &= K_{fs} \frac{T_m r}{J}\end{aligned}\tag{1}$$

If we assume a solid shaft with circular cross section, we can further simplify the expression to

$$\begin{aligned}\sigma_a &= K_f \frac{32M_a}{\pi d^3} & \sigma_m &= K_f \frac{32M_m}{\pi d^3} \\ \tau_a &= K_{fs} \frac{16T_a}{\pi d^3} & \tau_m &= K_{fs} \frac{16T_m}{\pi d^3}\end{aligned}\quad (2)$$

The stresses can be combined into stress amplitude and average stress using maximum distortion energy theory (MDET or von Mises) as

$$\sigma_{ae} = \left( \sigma_a^2 + 3\tau_a^2 \right)^{1/2} = \left[ \left( \frac{32K_f M_a}{\pi d^3} \right)^2 + 3 \left( \frac{16K_{fs} T_a}{\pi d^3} \right)^2 \right]^{1/2} \quad (3)$$

$$\sigma_{me} = \left( \sigma_m^2 + 3\tau_m^2 \right)^{1/2} = \left[ \left( \frac{32K_f M_m}{\pi d^3} \right)^2 + 3 \left( \frac{16K_{fs} T_m}{\pi d^3} \right)^2 \right]^{1/2} \quad (4)$$

The stress concentration factors for the average stress component in ductile materials can sometimes be ignored since the materials can yield locally at the discontinuity.

These equivalent stresses can be evaluated in design equations to determine the safety factor  $N_s$  or the required diameter  $d$  using the modified Goodman diagram as

$$\frac{1}{N_s} = \frac{\sigma_{ae}}{S_e} + \frac{\sigma_{me}}{S_{ut}} \quad (5)$$

Substituting for  $\sigma_{ae}$  and  $\sigma_{me}$  results in

$$\frac{1}{N_s} = \frac{16}{\pi d^3} \left\{ \frac{1}{S_e} \left[ 4 \left( K_f M_a \right)^2 + 3 \left( K_{fs} T_a \right)^2 \right]^{1/2} + \frac{1}{S_{ut}} \left[ 4 \left( K_f M_m \right)^2 + 3 \left( K_{fs} T_m \right)^2 \right]^{1/2} \right\} \quad (6)$$

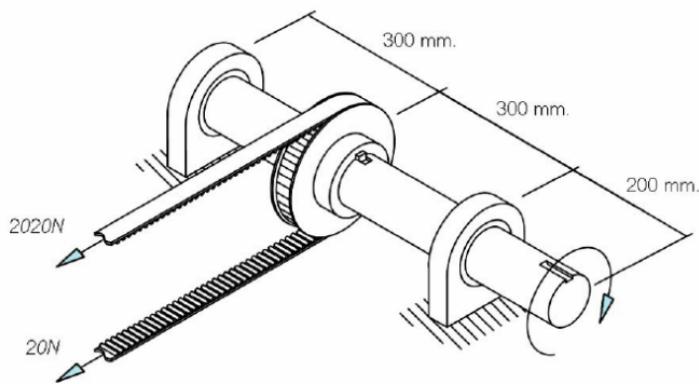
The required diameter  $d$  can be solved from the previous equation as

$$d = \left( \frac{16N_s}{\pi} \left\{ \frac{1}{S_e} \left[ 4 \left( K_f M_a \right)^2 + 3 \left( K_{fs} T_a \right)^2 \right]^{1/2} + \frac{1}{S_{ut}} \left[ 4 \left( K_f M_m \right)^2 + 3 \left( K_{fs} T_m \right)^2 \right]^{1/2} \right\} \right)^{1/3} \quad (7)$$

In many applications, rotating shafts will be under constant bending and torsion, resulting in completely reverse bending stress ( $M_m = 0$ ) and constant torsional shear stress ( $T_a = 0$ ). This means the required diameter becomes

$$d = \left( \frac{16N_s}{\pi} \left\{ \frac{2K_f M_a}{S_e} + \frac{\sqrt{3}K_{fs} T_m}{S_{ut}} \right\} \right)^{1/3} \quad (8)$$

*Example: Shaft sizing*



Size the shaft (AISI 1040,  $S_y = 400$  MPa,  $S_{ut} = 600$  MPa) using

1. MDET
2. Soderberg theory

so that the safety factor  $N_s = 3$ .

*Solution*

1. MDET: The torque loaded on the pulley by the belt is

$$\begin{aligned} T &= (2020 - 20)(0.1) \\ &= 200.0 \text{ N-m} \end{aligned}$$

There is also 2040 N of force pulling at the pulley due to the combined belt tension. The torque generates shear stress throughout the shaft, with the maximum value at the surface. The belt

tension creates bending stresses, whose maximum values are at the top and bottom of the shaft at the middle. This means that the critical points on the shaft (without considering stress concentration from the key/keyseat) are at the top and bottom of the shaft at the middle. In this problem, we will take the bottom of the shaft at the middle. The stress concentration of the keyseat is taken to be  $K_f = 2.14$  in bending and  $K_{fs} = 3.0$  in torsion.

$$\begin{aligned}\sigma &= K_f \frac{My}{I} = 2.14 \frac{2040(0.6)(r)}{4\pi r^4/4} \\ &= \frac{834}{r^3} \text{ Pa} \\ \tau &= K_{fs} \frac{Tr}{J} = 3 \frac{(200.0)(r)}{\pi r^4/2} \\ &= \frac{382}{r^3} \text{ Pa}\end{aligned}$$

2. Soderberg: using the criteria, we must calculate the minimum and maximum bending moments and torques, which will then be used to determine the stress amplitudes and average stresses. We already determine the maximum bending moment and torque, which we used to determine the corresponding stresses for MDET. We now only need to find out the minimum bending moment and torque. The minimum bending moment occurs when the shaft rotates by half a revolution, for which the beam will be under a compressive stress of the same magnitude.

$$\begin{aligned}\sigma_{\min} &= -\frac{834}{r^3} \\ \sigma_a &= \frac{\sigma_{\max} - \sigma_{\min}}{2} \\ &= \frac{834}{r^3}\end{aligned}$$

If the shaft is under continuous operation, the applied torque is constant, which means that the torque amplitude  $T_a = 0$  and the average torque  $T_m = T$ . We can plug this into the equation to determine equivalent amplitude and average stresses.

$$\begin{aligned}\sigma_{ae} &= \sqrt{\sigma_a^2 + 3\tau_a^2} = \frac{834}{r^3} \\ \sigma_{me} &= \sqrt{\sigma_m^2 + 3\tau_m^2} = \frac{662}{r^3} \\ \frac{1}{3} &= \frac{834}{0.5(6.00 \times 10^8)r^3} + \frac{662}{4.00 \times 10^8(r^3)} \\ r &= 0.0237 \text{ m}\end{aligned}$$

Using the settings from the previous example, redetermine the shaft size if the maximum operating speed is 10000 rpm.

For a simply supported shaft, the first natural frequency that can cause shaft whirling is

$$\omega_1 = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{EI}{A\rho}} \quad (9)$$

We must first convert the angular velocity into rad/s: 10000 rpm =  $10000(2\pi/60)$  = rad/s. To achieve the safety factor of 3, the first natural frequency of the shaft must be

$$\omega_1 = 3(1047) = \left(\frac{\pi}{0.6}\right)^2 \sqrt{\frac{2.10 \times 10^{11} \pi r^4 / 4}{\pi r^2 \rho}}$$

$r = 0.044 \text{ m}$

The designed shaft has to follow the largest shaft that satisfy each of the condition, therefore the required radius is 4.4 cm.

### *Torque Transmission Components*

For a designed shaft to properly transfer torque to its intended target, not only must the shaft be able to withstand the stresses resulting from the torque, but it must also be assembled with torque transmission components that are capable of transferring such torque. Some of the mechanisms currently in use can be categorized into:

1. Mechanical drive assembly
2. Interference fit assembly
3. Welded assembly

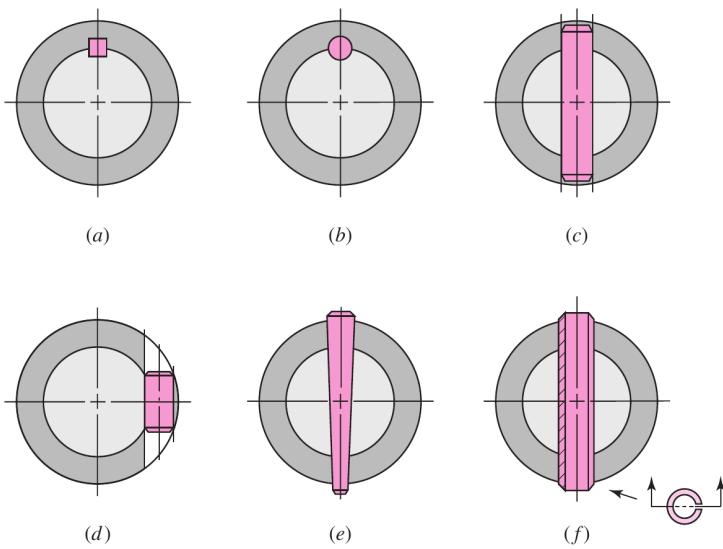
### *Mechanical Drive Assembly*

Mechanical drive assembly typically requires adjustments to the shaft and/or hub of the component to be assembled. The most common mechanical-drive assembly is the conventional key and keyway. Other assemblies are set screws, pins, and spline shafts.

### Keys and Pins

Keys are axially inserted metal that provides interference between the shaft and hub, allowing torque transmission between the two. Aligning keyways need to be cut into both the shaft and the hub.

Pins, on the other hand, are transversely inserted perpendicular to the axis of the shaft. Both the shaft and hub still needs to be drilled.



While pins allow for torque transmission and axial positioning, keys only allow torque transmission. However, because of the typical long length of keys inserted axially which provides larger cross-sectional area, they typically allow larger torque transmission than pins of the same size.

Torque capacity of keys and pins can be calculated by

$$P_{\max} = \frac{S_y}{\sqrt{3}} A = 0.577 S_y A$$

$$T_{\max} = P_{\max} r_{\text{shaft}}$$

### *Limitation of Mechanical Drive Assembly*

Mechanical drive assembly can provide relatively high torque transmission with relatively easy assembly. However, they have a few shortcomings to consider.

1. Stress concentration: shafts and hubs need to be machined to

provide holes/keyways/splines, which will obviously introduce increased stress concentration and thus reduced shaft/hub strength.

2. Backlash: even with very strict tolerances, pins and keys that do not perfectly fit in the holes or keyways will allow relative motion between the shaft and the hub, leading to backlashes. However, this can be eliminated or mitigated with the use of tapered keys or pins.  
Tapered keys or pins have a slowly increasing cross section from one end to the other, therefore allowing a snug fit.
3. Machining costs: as keyways/splines/holes require precise machining, this will incur additional costs on the shaft assembly.
4. Uneven distribution of mass: uniform distribution of mass about the shaft is extremely important for shaft stability especially in high speed application. Usually components such as gears, pulleys, sprockets, etc. are designed to be axisymmetric for this reason. However, the required machining and insertion of keys or pins will introduce asymmetry to the shaft assembly.

### *Example: Key Sizing*

A steel shaft whose  $S_y = 450$  MPa has a diameter of 5 cm. The shaft rotates at 600 rpm and transmits 40 hp through a gear. Select an appropriate key for the gear. Use safety factor = 3.

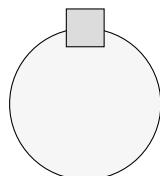


Figure 2: Key and shaft cross section for example ...

### *Solution: Key Sizing*

To keep things simple, pick a square key and pick key length = 2 cm.

$$T = \frac{H}{\omega} = \frac{40(746)}{600(2\pi/60)} \\ = 475 \text{ N-m}$$

For the width (and height) of the key section,

$$N_s T_{\max} = 0.577 S_y b l r_{\text{shaft}}$$

$$b = \frac{3(475)}{0.577(450 \times 10^6)(0.02)(0.05)}$$

$$b = 0.00549 \text{ m}$$

### *Interference Fit Assemblies*

Interference fit refers to a type of joint that relies on friction between the hub and shaft to transfer torque. The friction results from the compression of the hub on the shaft, which means the diameter of the hole on the hub must be slightly smaller than that of the shaft. These type of assemblies are divided into 3 categories based on method of assembly.

1. Press fit:

The word 'press' here refers to the assembly method of pressing the shaft into a hole that is slightly smaller. This process relies on the elasticity of both materials to allow the shaft to slide in without any permanent deformation. This requires extremely strict tolerances on both the shaft and the hub.

2. Tapered fit:

In case of tapered fits, an additional collet, which is a collar that is tapered on the outside along the length, and has a constant cross section on the inside. The collet essentially acts like a wedge between the shaft and hub, allowing the assembler to control the magnitude of compressive (and resulting friction) force on the shaft by the axial load exerted on the collet.

3. Shrink fit:

Similar to press fit in that there is no additional 'collet', but instead of 'pressing' the shaft onto the hub, either the shaft is cooled or the hub is heated (or both) before assembly. This eliminates the difficulty of assembly and allows for greater difference in the diameters of the shaft and hub.

These interference fits do not require any machining on the shaft or hub, thus do not increase the stress concentration nor incur additional machine costs. However, they do have the following limitations to take into consideration.

1. Materials, surface, and design restrictions: interference fits rely on friction, so the material, surface finishing, and shaft dimensions (diameter mostly) affect the magnitude of friction that can be generated. This limits your available choices.

2. Close tolerance: interference fits require that the hubs and shafts have extremely close and accurate tolerances, requiring precise machining. This leads to high machining costs.
3. Fretting: high stress on surfaces that undergo repeated motions are prone to fretting corrosion.
4. Surface galling: high compressive load on mating surfaces can cause them bind. This complicates disassembly, which usually results in surface failure.
5. High stress in components: interference fit requires high compressive force from the hub on the shaft to generate friction. This leads to circumferential stresses on the hub and the shaft as well.

### *Stresses in Interference Fits*

Assumed uniform pressure on shaft (i) and hub (o)

$$p = \frac{d_{\text{shaft}} - d_{\text{hub}}}{\frac{d}{E_o} \left( \frac{d_o^2 + d^2}{d_o^2 - d^2} + \nu_o \right) + \frac{d}{E_i} \left( \frac{d^2 + d_i^2}{d^2 - d_i^2} - \nu_i \right)}$$

When both are of the same material

$$p = \frac{E(d_{\text{shaft}} - d_{\text{hub}})}{2d^3} \left[ \frac{(d_o^2 - d^2)(d^2 - d_i^2)}{d_o^2 - d_i^2} \right]$$

Tangential and radial stresses in shaft and hub are

$$\sigma_{t,\text{shaft}} = -p \frac{d^2 + d_i^2}{d^2 - d_i^2} \quad (10)$$

$$\sigma_{t,\text{hub}} = p \frac{d_o^2 + d^2}{d_o^2 - d^2} \quad (11)$$

$$\sigma_{r,\text{shaft}} = -p \quad (12)$$

$$\sigma_{r,\text{hub}} = -p \quad (13)$$

Combine  $\sigma_t$  and  $\sigma_r$  using MDET to determine failure

### Torque Capacity of Interference Fits

Depends on friction generated between shaft and hub → pressure from interference fits

$$\begin{aligned} f &= \mu N = \mu(pA) \\ &= \pi\mu pld \end{aligned} \quad (14)$$

$$\begin{aligned} T &= fd/2 = \pi\mu pld(d/2) \\ &= \frac{\pi}{2}\mu pld^2 \end{aligned} \quad (15)$$

### Example: Torque Capacity of an Interference Fit

A solid shaft whose diameter is 5 cm is pressed onto a gear whose hub inner diameter is 4.99 cm and outer diameter is 6 cm. If both are made of the same steel whose  $E = 210$  GPa and  $\nu = 0.3$ , determine the radial and tangential stresses, along with the torque capacity of the fit. Assume steel-on-steel  $\mu = 0.3$ , and the hub is 7 cm long.

### Solution: Torque Capacity of an Interference Fit

$$\begin{aligned} p &= \frac{E(d_{\text{shaft}} - d_{\text{hub}})}{2d^3} \left[ \frac{(d_o^2 - d^2)(d^2 - d_i^2)}{d_o^2 - d_i^2} \right] \\ &= \frac{210 \times 10^9 (0.05 - 0.0499)}{2(0.05)^3} \left[ \frac{(0.06^2 - 0.05^2)(0.05^2 - 0)}{0.06^2 - 0} \right] \\ &= 64.2 \text{ MPa} \end{aligned}$$

$$\sigma_{r,\text{shaft}} = \sigma_{r,\text{hub}} = -64.2 \text{ MPa}$$

$$\sigma_{t,\text{shaft}} = -64.2 \frac{0.05^2}{0.05^2} = -64.2 \text{ MPa}$$

$$\sigma_{t,\text{hub}} = 64.2 \frac{0.06^2 + 0.05^2}{0.06^2 - 0.05^2} = 356 \text{ MPa}$$

$$T = \frac{\pi}{2}\mu pld^2 = \frac{\pi}{2}(0.3)64.2 \times 10^6 (0.07)(0.05^2) = 5294 \text{ N-m}$$

### Weld Assembly

In weld assembly, the shaft and hub of intended components are welded together. This provides a relatively quick and permanent

connection between the components. However, weld assemblies also inherit the same disadvantages from welded joints as mentioned in chapter ....

1. Welding only works on compatible materials. Plastics on metals is a no-no. Woods cannot be welded. Ceramics cannot be welded. You get the idea.
2. Heating can cause warpage. Welding introduces uneven heating of the workpiece, which can result in warpage especially in parts with complex geometries.
3. Disassembly. Welding is permanent... mostly. Welding can be undone, although the process is not straightforward and can cause permanent surface damages.
4. Skilled personnel. Welding requires skilled craftsmanship, which means it is usually expensive and its quality is hard to control.
5. Cleaning and machining. Welding typically needs cleaning and machining afterwards to obtain desirable surface finish.

### *Torque Capacity in Weld Assembly*

The strength of weld can be applied to determine the torque capacity of shafts and their components in weld assembly. In most cases, weld joints in weld assembly are fillet welds and the corresponding equations apply.

# *Journal Bearings and Lubrication*

## *Overview of Bearings*

### *What are bearings?*

- A feature that allows relative motions between components
  - Linear motions
  - Rotary motions

### *Two types of bearings*

- Contact: sliding or rolling
- Non-contact: fluid film or magnetic

# *Contact Bearings*

Contact bearings are features that allow relative movements between two surfaces wherein the surfaces remain in contact with each other either directly or indirectly through some solid common medium.

## *Sliding Contact Bearings*



Figure 3: Brass sliding contact bearing

- Commonly used in low- to medium-speed applications
- Lubrication is used to reduce wear and friction

## *Materials for Sliding Contact Bearing*

- Typically hard materials (shaft) on soft (bearing)
- Materials:

- Polymers: nylon is king!
- Brass
- Ceramics
- Check on bearing stress
- Aluminum-on-aluminum is a no-no

### Bearing Contact Pressure

$$P = \frac{F}{DL} \quad (16)$$

$$P_{\max} = \frac{4}{\pi} \frac{F}{DL} \quad (17)$$

### PV Factor

In selecting materials for sliding-contact bearings.

- pressure  $\times$  velocity
- tradeoff in choosing bearing materials
- higher pressure  $\rightarrow$  low speed, and vice versa

Material	Static <i>P</i>		Dynamic <i>P</i>		<i>V</i> m/s (fpm)	<i>PV</i>	
	MPa	(ksi)	MPa	(ksi)		MPa $\cdot$ m/s (ksi $\cdot$ fpm)	
Bronze	55	(8)	14	(2)	6.1 (1200)	1.8	(50)
Lead-bronze	24	(3.5)	5.5	(0.8)	7.6 (1500)	2.1	(60)
Copper-iron	138	(20)	28	(4)	1.1 (225)	1.2	(35)
Hardenable copper-iron	345	(50)	55	(8)	0.2 (35)	2.6	(75)
Iron	69	(10)	21	(3)	2.0 (400)	1.0	(30)
Bronze-iron	72	(10.5)	17	(2.5)	4.1 (800)	1.2	(35)
Lead-iron	28	(4)	7	(1)	4.1 (800)	1.8	(50)
Aluminum	28	(4)	14	(2)	6.1 (1200)	1.8	(50)

Figure 4: PV Table for Metals

Material	<i>P</i>		Temperature		<i>V</i> m/s (fpm)	<i>PV</i>	
	MPa	(ksi)	°C	(°F)		MPa $\cdot$ m/s (ksi $\cdot$ fpm)	
Phenolics	41	(6)	93	(200)	13 (2500)	0.53	(15)
Nylon	14	(2)	93	(200)	3.0 (600)	0.11	(3)
TFE	3.5	(0.5)	260	(500)	0.25 (50)	0.035	(1)
Filled TFE	17	(2.5)	260	(500)	5.1 (1000)	0.35	(10)
TFE fabric	414	(60)	260	(500)	0.76 (150)	0.88	(25)
Polycarbonate	7	(1)	104	(220)	5.1 (1000)	0.11	(3)
Acetal	14	(2)	93	(200)	3.0 (600)	0.11	(3)
Carbon (graphite)	4	(0.6)	400	(750)	13 (2500)	0.53	(15)
Rubber	0.35	(.05)	66	(150)	20 (4000)	—	—
Wood	14	(2)	71	(160)	10 (2000)	0.42	(12)

Figure 5: PV Table for Non-metals

*Example: Sleeve Bearing for a Low-speed Shaft*

A 30-cm long shaft whose diameter  $D$  is 3 cm is operated at 1000 rpm. The shaft has a spur gear whose  $R_{\text{pitch}} = 10$  cm mounted in the middle with a bearing at each end. The gear is transferring the power of 1.5 kW. The gear has pressure vessel  $\theta = 20^\circ$ . Determine the minimum bearing length  $L$  using nylon.

*Solution*

First, let us determine the force on the bearing. Since spur gears don't generate any axial load, the forces will simply be the radial + tangential load, perpendicular to the shaft.

$$T = \frac{P}{\omega} \quad (18)$$

$$= \frac{1500}{1000(2\pi/60)} = 14.3 \text{ N-m} \quad (19)$$

$$F = \frac{T}{R_{\text{pitch}} \cos \theta} \quad (20)$$

$$= \frac{14.3}{0.1 \cos 20^\circ} = 152 \text{ N} \quad (21)$$

Since the gear is mounted in the middle, the force on each bearing is half of the force.

$$F_{\text{bearing}} = \frac{152}{2} = 76 \text{ N} \quad (22)$$

We can't determine the bearing pressure yet since we don't know the bearing length. We can determine the surface velocity, however.

$$v = \omega(D/2) = 1000(2\pi/60)(0.03/2) = 1.57 \text{ m/s} \quad (23)$$

We double-check that  $v < V_{\text{nylon}}$  ( $1.57 < 3.0$ ) so nylon is an acceptable choice. The length of bearing, then should be

$$P_{bearing}v < (PV)_{nylon} \quad (24)$$

$$\frac{F_{bearing}}{DL}v < 0.11 \times 10^6 \quad (25)$$

$$\frac{76}{0.03L}1.57 < 1.1 \times 10^5 \quad (26)$$

$$L > 0.036 = 3.6 \text{ cm} \quad (27)$$

# *Rolling Contact Bearings*

## *Rolling Elements*

- suitable for medium- to high-speed applications
- use balls or rollers to avoid friction

## *Rolling Element Types*

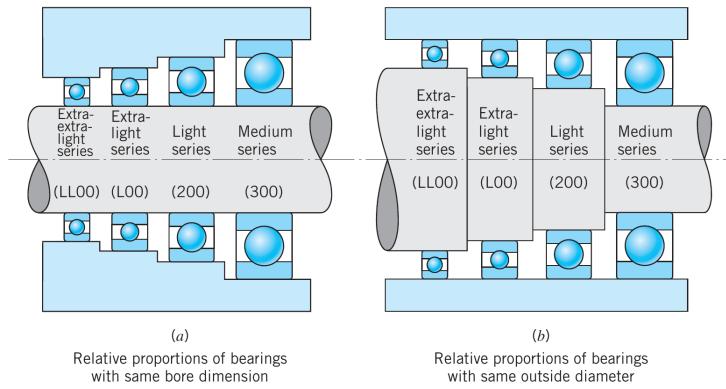


Figure 6: Bearing Series

## *Bearing Life Requirement*

$$L = L_R K_r \left( \frac{C}{F_e} \right)^{10/3} \quad (28)$$

$$C = F_e \left( \frac{L}{K_r L_R} \right)^{0.3} \quad (29)$$

- $L$  life corresponding to equivalent load  $F_e$   
 $L_R$  life corresponding to rated capacity =  $9 \times 10^7$  rev  
 $K_r$  reliability factor  
 $C$  rated capacity  
 $F_e$  equivalent load

Bearing Basic Number	Bore (mm)	Ball Bearings					Roller Bearings				
		OD (mm)	w (mm)	r <sup>a</sup> (mm)	d <sub>S</sub> (mm)	d <sub>H</sub> (mm)	OD (mm)	w (mm)	r <sup>a</sup> (mm)	d <sub>S</sub> (mm)	d <sub>H</sub> (mm)
L03	17	35	10	0.30	19.8	32.3	35	10	0.64	20.8	32.0
203	17	40	12	0.64	22.4	34.8	40	12	0.64	20.8	36.3
303	17	47	14	1.02	23.6	41.1	47	14	1.02	22.9	41.4
L04	20	42	12	0.64	23.9	38.1	42	12	0.64	24.4	36.8
204	20	47	14	1.02	25.9	41.7	47	14	1.02	25.9	42.7
304	20	52	15	1.02	27.7	45.2	52	15	1.02	25.9	46.2
L05	25	47	12	0.64	29.0	42.9	47	12	0.64	29.2	43.4
205	25	52	15	1.02	30.5	46.7	52	15	1.02	30.5	47.0
305	25	62	17	1.02	33.0	54.9	62	17	1.02	31.5	55.9
L06	30	55	13	1.02	34.8	49.3	47	9	0.38	33.3	43.9
206	30	62	16	1.02	36.8	55.4	62	16	1.02	36.1	56.4
306	30	72	19	1.02	38.4	64.8	72	19	1.52	37.8	64.0
L07	35	62	14	1.02	40.1	56.1	55	10	0.64	39.4	50.8
207	35	72	17	1.02	42.4	65.0	72	17	1.02	41.7	65.3
307	35	80	21	1.52	45.2	70.4	80	21	1.52	43.7	71.4
L08	40	68	15	1.02	45.2	62.0	68	15	1.02	45.7	62.7
208	40	80	18	1.02	48.0	72.4	80	18	1.52	47.2	72.9
308	40	90	23	1.52	50.8	80.0	90	23	1.52	49.0	81.3
L09	45	75	16	1.02	50.8	68.6	75	16	1.02	50.8	69.3
209	45	85	19	1.02	52.8	77.5	85	19	1.52	52.8	78.2
309	45	100	25	1.52	57.2	88.9	100	25	2.03	55.9	90.4
L10	50	80	16	1.02	55.6	73.7	72	12	0.64	54.1	68.1
210	50	90	20	1.02	57.7	82.3	90	20	1.52	57.7	82.8
310	50	110	27	2.03	64.3	96.5	110	27	2.03	61.0	99.1
L11	55	90	18	1.02	61.7	83.1	90	18	1.52	62.0	83.6
211	55	100	21	1.52	65.0	90.2	100	21	2.03	64.0	91.4
311	55	120	29	2.03	69.8	106.2	120	29	2.03	66.5	108.7

Figure 7: Bearing table

Bore (mm)	Radial Ball, $\alpha = 0^\circ$			Angular Ball, $\alpha = 25^\circ$			Roller		
	L00 Xlt (kN)	200 It (kN)	300 med (kN)	L00 Xlt (kN)	200 It (kN)	300 med (kN)	1000 Xlt (kN)	1200 It (kN)	1300 med (kN)
10	1.02	1.42	1.90	1.02	1.10	1.88			
12	1.12	1.42	2.46	1.10	1.54	2.05			
15	1.22	1.56	3.05	1.28	1.66	2.85			
17	1.32	2.70	3.75	1.36	2.20	3.55	2.12	3.80	4.90
20	2.25	3.35	5.30	2.20	3.05	5.80	3.30	4.40	6.20
25	2.45	3.65	5.90	2.65	3.25	7.20	3.70	5.50	8.50
30	3.35	5.40	8.80	3.60	6.00	8.80	2.40 <sup>a</sup>	8.30	10.0
35	4.20	8.50	10.6	4.75	8.20	11.0	3.10 <sup>a</sup>	9.30	13.1
40	4.50	9.40	12.6	4.95	9.90	13.2	7.20	11.1	16.5
45	5.80	9.10	14.8	6.30	10.4	16.4	7.40	12.2	20.9
50	6.10	9.70	15.8	6.60	11.0	19.2	5.10 <sup>a</sup>	12.5	24.5
55	8.20	12.0	18.0	9.00	13.6	21.5	11.3	14.9	27.1
60	8.70	13.6	20.0	9.70	16.4	24.0	12.0	18.9	32.5

Figure 8: Bearing rated capacity C

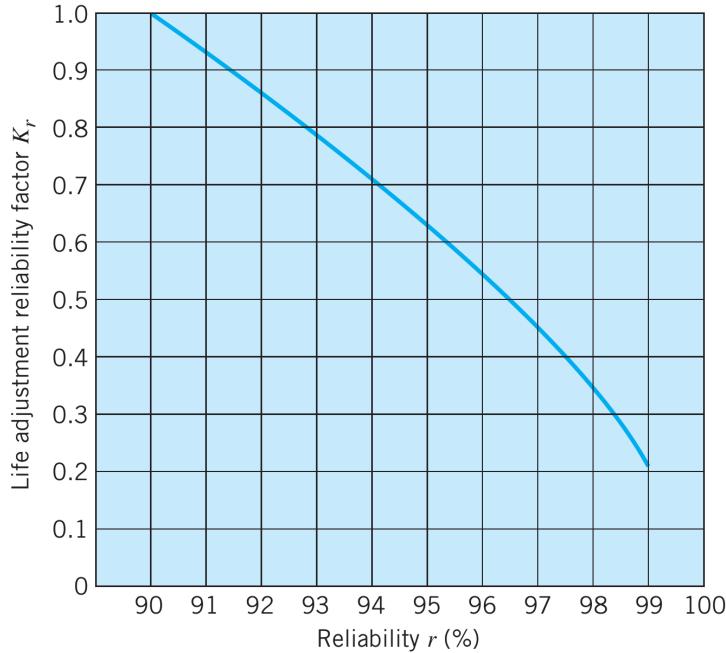


Figure 9: Bearing reliability factor  $K_r$

### Equivalent Load

Let  $e = F_a/F_r$

for radial ball bearings

$$F_e = \begin{cases} F_r & e < 0.35 \\ F_r [1 + 1.115(e - 0.35)] & 0.35 < e < 10 \\ 1.176F_a & e > 10 \end{cases} \quad (30)$$

for angular ball bearings

$$F_e = \begin{cases} F_r & e < 0.68 \\ F_r [1 + 0.87(e - 0.68)] & 0.68 < e < 10 \\ 0.911F_a & e > 10 \end{cases} \quad (31)$$

### Typical Bearing Design Life

#### Radial Ball Bearing Selection

Select a radial ball bearing for a shaft intended for a continuous 8-hr-a-day operation at 1800 rpm with 95% reliability. Axial and radial loads are 1.2 kN and 1.5 kN, respectively.

*Solution*

First, we need to calculate  $F_e$ .

$$e = \frac{F_a}{F_r} = \frac{1.2}{1.5} = 0.8$$

For radial ball bearing,

$$F_e = F_r [1 + 0.87(e - 0.68)] \quad (32)$$

$$= 1500 [1 + 1.115(0.8 - 0.35)] \quad (33)$$

$$= 2276 \text{ N} \quad (34)$$

Required life for 8-hr-a-day service (assumed every day) = 30000 hrs

Life in revolutions

$$L = 1800(30000)(60) = 3.24 \times 10^9 \text{ revolutions}$$

For 95% reliability  $K_r = 0.63$

$$C = 2276 \left( \frac{3.24 \times 10^9}{0.63(9 \times 10^7)} \right)^{0.3} = 7661 \text{ N} = 7.66 \text{ kN} \quad (35)$$

For extra-light, light, and medium series, the required bore are 55, 35, and 30 mm, respectively

The models corresponding to the bore are L11, 207, and 306, respectively.

# *Gears*

## *Gear Overview*

### Why Gears?

- Convert high speed and low torque to that requires low speed and high torque
- Speed: easy to get because voltage is easy
- Torque: hard to get because it requires large current

### Principles of Gears

- Allow positive engagement between teeth
- High forces can be transmitted while in rolling contact
- Do not need friction to operate

### Basic Law of Gearing

- Point of contact between two mating gears is always the same relative distances from the two centers.
- Any gear tooth profiles that follow the law of gearing will result in constant relative speed of rotation

### Gear Geometry

#### Module of a Gear, $m$

- Term used to define gear tooth size

- Defined as ratio of pitch diameter to number of teeth

$$m = \frac{D_{\text{pitch}}}{z} \quad (36)$$

- A pair of meshing gears must have the same modules!

## Gear Types

### Gear Terminology

*Pinion* smaller of two gears, usually driving

*Gear* Larger of the two. Also called *wheel*. Usually driven.

### Gear Materials

- Steel: medium-carbon steel + heat treatment + grinding
- Cast iron: surface fatigue > bending fatigue
- Nonferrous: bronzes → corrosion + wear resistant, low friction
- Nonmetallic: Nylon → low friction and weight + corrosion resistant, but low thermal conductivity

### Gear Efficiency

- With friction, gears are 90 - 95% efficient because of mostly rolling contact

$$T_{\text{out}} = \frac{\eta T_{\text{in}} d_{\text{out}}}{d_{\text{in}}} \quad (37)$$

$$\omega_{\text{out}} = \frac{\omega_{\text{in}} d_{\text{in}}}{d_{\text{out}}} \quad (38)$$

$$P_{\text{out}} = T_{\text{out}} \omega_{\text{out}} \quad (39)$$

## Gear Trains

*Gear Trains* When large reduction is required

- Large gear + small pinion: simple, but large stress and interference
- Multiple pairs of gears and pinions: less simple, low stress, large space

- Planetary gears: complex, low stress, small space

### Normal Gear Trains

$$e_{total} = e_1 e_2 \dots \quad (40)$$

(41)

### Planetary/Epicyclic Gear Train

- Planetary or epicyclic gears enable a high reduction ratio in small spaces

### Planetary Gear Components

#### Planetary Gears: Torque, Forces, and Reduction Ratios

- Symmetry → no net force on shaft
- Multiple planet gears reduce individual torque/force
- Any combination of fixed, input, output gears
- 1 gear box -> multiple gear reduction ratios

Fixed ring:

$$\omega_{carrier} = 9 \quad (42)$$

$$\omega_{planet} = (9) \frac{60/2 + 20/2}{20/2} = 36 \quad (43)$$

$$\omega_{sun} = (36) \frac{20}{30} = 24 \quad (44)$$

$$e = 9/24 = 0.375 \quad (45)$$

Fixed	Input	Planet	Output	RR
Ring	Carrier 9	36	Sun 24	0.375
Sun	Carrier 9	36	Ring 14.4	0.625
Carrier	Sun 9	27	Ring 5.4	1.667

## *Spur Gears*

Spur gears have straight involute teeth. They transfer torque by applying forces perpendicular to their involute face. And because their teeth are perpendicular to their axis, spur gears do not generate axial loads—they generate only tangential and radial forces.

$$F_t = \frac{T}{R_{\text{pitch}}} \quad (46)$$

$$F_r = F_t \tan \phi \quad (47)$$

$$F_a = 0 \quad (48)$$

The tangential force is perpendicular to gear teeth, leading to tooth bending

### *Spur Gear Stress*

The equation for bending stress in spur gears is a modified lewis equation that takes into account the shapes, stress concentrations, and operating conditions of the gear.

- Bending Stress → AGMA stress equation
- Consider tooth as a cantilever beam

$$\sigma = \frac{F_t}{bY_f m} K_O K_m K_v \quad (49)$$

$F_t$  tangential force

$b$  face width

$Y_f$  geometry factor

$m$  module

The geometry factor  $Y_J$  takes into account the involute shape and stress concentration factor of the tooth.

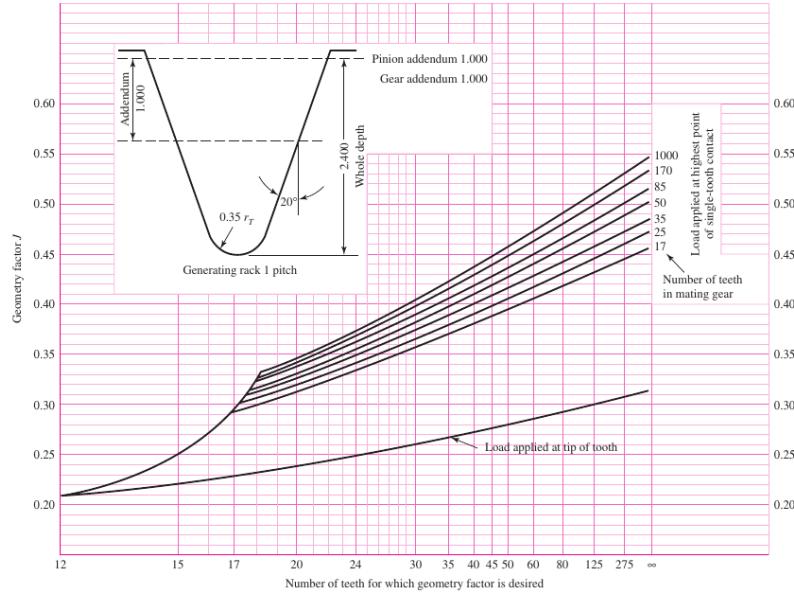


Figure 10: Geometry factor  $Y_J$  for spur gear design

From Figure 10, gears with large number of teeth or that are mating with gears with large number of teeth have higher geometry factors, leading to lower bending stresses. This is because large number of teeth means the teeth are shorter, hence the lower stresses.

Overload Factor:  $K_O$

This factor takes into account the shock and impact loading during operation, which can cause a sharp increase in the stress. We consider the source of shock and impact loading from both the power source (input) and the driven machine (output).

Power source	Driven Machine			
	Uniform	Light shock	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.50	1.75
Light shock	1.20	1.40	1.75	2.25
Moderate shock	1.30	1.70	2.00	2.75

Table 1: Overload factor  $K_O$  for spur gear design

Power sources can be categorized based on their shock/impact loading generated, along with some examples, as follows.

*Uniform* Electric motor, constant-speed turbine

*Light* Water turbine, variable-speed drive

*Moderate* Multicylinder engine

Driven machines are categorized based on operating conditions, which depends mostly on the resisting load on the system.

*Uniform* Continuous generator

*Light* Fans, low-speed pumps, conveyors

*Moderate* high-speed pumps, compressors, heavy conveyors

*Heavy* rock crushers, punch press drivers

Mounting Factor:  $K_m$

The factor accounts for the increase in stress when tooth faces do not align perfectly. This can happen due to inaccurate mountings, the use of normal (rather than precision) gears, or off-center mountings.

Characteristics of Support	Face Width (cm)			
	0 to 5 cm	15	22.5	40
Accurate mountings, small bearing clearances, precision gears	1.3	1.4	1.5	1.8
Less rigid moutings, standard gears, full face contact	1.6	1.7	1.8	2.2
Less than full face contact	Over 2.2			

Velocity Factor:  $K_v$

This factor accounts for the increase in stress from increased pitch line velocity of the gears. This combines with the overloading factor  $K_O$  to describe the effect of shock and impact loading on stress.

$$K_v = \left( \frac{A + \sqrt{200v_t}}{A} \right)^B \quad (50)$$

$$A = 50 + 56(1 - B) \quad (51)$$

$$B = 0.25(12 - Q)^{2/3} \quad (52)$$

$v_t$  pitch line velocity [m/s]

***Q*** AGMA Quality Number

AGMA recommends designers choose gears based on the level of precision they require from the designed mechanisms. Some of the guidelines are listed in Table 2

$v_t$ [m/s]	$Q$	Applications
0 - 4	6 - 8	Paper box making machine, cement, mill drives
4 - 10	8 - 10	Washing machine, printing press, computing mechanism
10 - 20	10 - 12	Automotive transmission, Antenna drive, propulsion drive
$\geq 20$	12 - 14	Gyroscope

***Gear Material Strength  $S'_e$*** 

Because they are used exclusively in rotational machinery, gears are constantly under repeated loadings. Thus, their main mode of failure is fatigue. Hence, whenever we consider the strength of gear material for design, endurance limits should be the first factor on our list.

$$S'_e = S_e C_L C_G C_S k_r k_t k_{ms} \quad (53)$$

where

$S_e$  endurance limit

$C_L$  load factor (= 1 for bending)

$C_G$  gradient surface = 1

$C_S$  surface factor (= 0.75 for machined surface)

$k_r$  reliability factor

$k_t$  temperature factor

$k_{ms}$  median-stress factor (1 for two-way bending (followers), 1.4 for one-way bending (input or output))

Now let us consider each of endurance limit modifier in more details.

Reliability Factor:  $k_r$

Since most material properties—endurance limits included—are reported by their average values, obviously there will be a gear whose

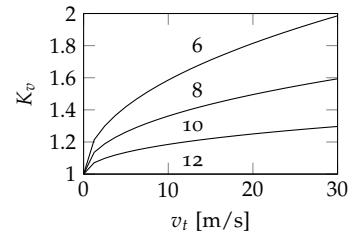


Figure 11: Velocity factor  $K_v$  as a function of pitch line velocity  $v_t$  for various gear quality number  $Q$

Table 2: AGMA recommended quality number for various applications

actual endurance limit is lower than the reported value. This reliability factor represents this probability so that a given design based on the reduced endurance limit will have a higher probability of achieving the designated lifetime.

Reliability (%)	$k_r$
50	1.000
90	0.897
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659

Table 3: Gear reliability factor  
 $k_r$

Temperature Factor:  $k_t$

Temperature directly affects endurance limit as already discussed in "Introduction to Theories of Failure" chapter.

$$k_t = \begin{cases} 1 & T \leq 160 \text{ F} \\ \frac{620}{460 + T} & T > 160 \text{ F} \end{cases} \quad (54)$$

Aside from the strength criteria governed by the given equations. There are more 'guidelines' – a set of recommended rules – that can be used to facilitate the spur gear design process.

1.  $e \leq 1/6$
2. Use multi-stage gears for larger than  $e > 1/6$
3.  $8m \leq b \leq 16m$
4. many small teeth  $\gg$  few large teeth
5. few teeth  $\rightarrow$  small gear, but be careful about interference
6. Avoid exact ratio  $\rightarrow$  hunting tooth

*Example: Spur gear design for a conveyor belt*

A pair of spur gears with face width  $b = 3$  cm is used in a conveyor belt drive. The input motor has  $\omega_{\max}$  of 200 rad/s. The pinion has 18 teeth. The conveyor has moderate shock and should be driven at 100 rad/s. The gears have pressure angles  $\theta$  of  $20^\circ$ . Both pinion and gear has  $m = 1$  cm. Determine the maximum power that the gears can transmit continuously with 1% chance of bending fatigue failure. Steel has  $S_{ut} = 400$  MPa

*Solution: spur gear design for a conveyor belt*

First, the bending fatigue stress is

$$\begin{aligned}\sigma &= \frac{F_t}{bY_Jm} K_O K_m K_v \\ &= \frac{F_t}{(0.03)(0.32)(0.01)} (1.25)(1.6) \left( \frac{65.12 + \sqrt{200(18)}}{65.12} \right)^{0.73} \\ &= 33542 F_t\end{aligned}$$

Next, the material fatigue strength is

$$\begin{aligned}S'_e &= S_e C_L C_G C_S k_r k_t k_{ms} \\ &= (400 \times 10^6 (0.5))(1)(1)(0.75)(0.814)(1)(1.4) \\ &= 1.71 \times 10^8\end{aligned}$$

We can then find the maximum allowable tangential force

$$\begin{aligned}F_t &= \frac{1.71 \times 10^8}{33542} = 5096 \text{ N} \\ P &= T\omega = F_t v_{pitch} = 5096 \times 18 = 9.17 \times 10^4 \text{ W}\end{aligned}$$

### Rack and Pinion

Racks are essentially linear spur gears—the teeth line up on a straight rather than a circular path. When used with gears, the pairs can convert torque and rotational motion to force and linear motion. They are less expensive than power screws, but also less accurate and provide no mechanical advantages.

As they are considered linear spur gears, the forces acting on them are identical to those on spur gears.

$$\begin{aligned}F_t &= \frac{T}{R_{pitch}} \\ F_r &= F_t \tan \phi \\ F_a &= 0\end{aligned} \tag{55}$$

## *Helical Gears*

Another type of gears have teeth that are slanted at a constant angle, forming helices about their axes of rotation. These are called **helical gears**.

### *Helical Gear Analysis*

Geometrically, they can be analyzed similarly to spur gears, the only different being that the teeth are at an angle of  $\psi$  with the gear axis.

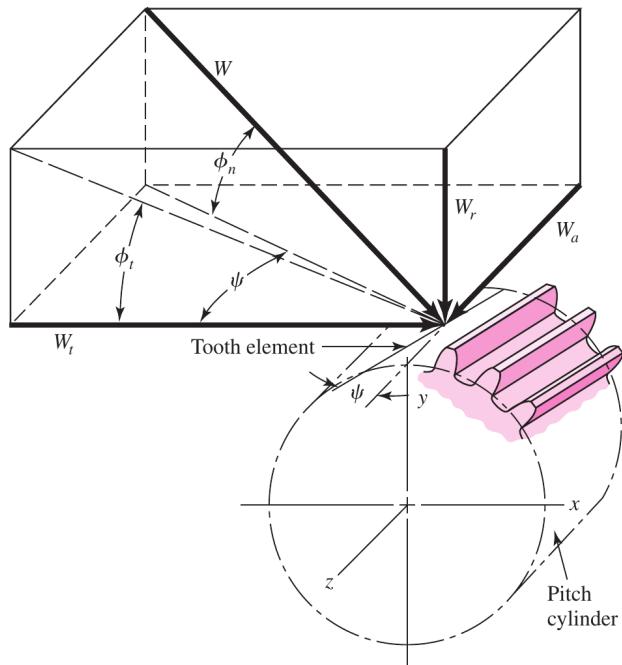


Figure 12: Forces acting on a helical gear

$$F_t = \frac{H}{v_t} \quad (56)$$

$$F_r = F_t \tan \phi_t \quad (57)$$

$$F_a = F_t \tan \psi \quad (58)$$

$$\tan \phi_n = \tan \phi_t \cos \psi \quad (59)$$

$$m_n = m_t \cos \psi \quad (60)$$

Design Equations Same as spur gear equation with small modification

$$\sigma = \frac{F_t}{bY_J m} K_v K_o (0.93K_m) \quad (61)$$

$$S'_e = S_e C_L C_G C_S k_r k_t k_{ms} \quad (62)$$

0.93 indicated helical gears less sensitivity to mounting factor

$Y_J$  needs small modification for helical teeth

Geometry Factor:  $Y_J$

Because of the helix angle  $\psi$ , the geometry factor which accounts for the tooth size and its geometry is slightly modified.

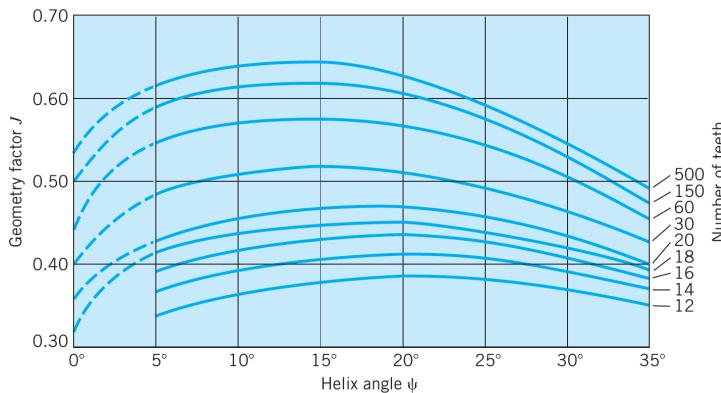


Figure 13: Geometry factor for helical gears

The geometry factor also has to be modified by another multiplier which accounts for the mating gear.

*Example: Meshing helical gears*

A pair of meshing helical gears is connected at the input side to a 0.5-hp motor at 1800 rpm and to an output shaft at 600 rpm. The input gear has 18 teeth,  $\phi_n = 20^\circ$ ,  $m_n = 0.00173$ ,  $\psi = 30^\circ$ ,  $b = 2$  cm. From the given information, determine the pitch line velocity  $v_t$ , gear tooth forces  $F_t$ ,  $F_r$ , and  $F_a$ , and bending stress  $\sigma$ .

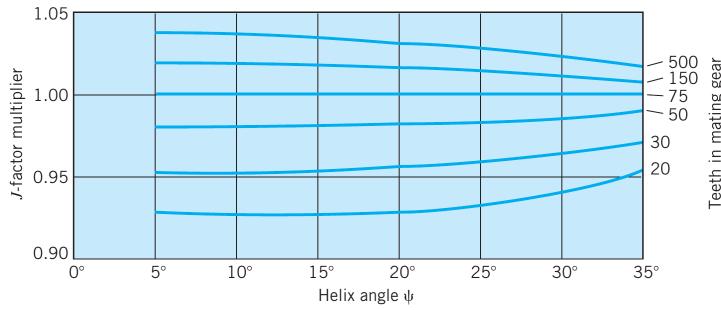


Figure 14: Geometry factor multiplier for helical gears

Calculate tangential module from normal module, then pitch diameter and tangential velocity.

$$m_t = \frac{m_n}{\cos 30^\circ} = \frac{0.00173}{\cos 30^\circ} = 0.002$$

$$d = mz = 0.002(18) = 0.036 \text{ m}$$

$$v_t = \omega \frac{d}{2} = (1800) \frac{2\pi}{60} \frac{0.036}{2} = 3.4 \text{ m/s}$$

Transmitted power only depends on tangential force, after which we can calculate axial and radial forces.

$$F_t = \frac{H}{v_t} = \frac{0.5(746)}{3.4} = 104 \text{ N}$$

$$\tan \phi_t = \frac{\tan \phi_n}{\cos \psi} = \frac{\tan 20^\circ}{\cos 30^\circ} = 0.42$$

$$\phi_t = 22.8^\circ$$

$$F_r = F_t \tan \phi_t = 104 \tan 22.8^\circ = 43.7 \text{ N}$$

$$F_a = F_t \tan \psi = 104 \tan 30^\circ = 60 \text{ N}$$

$$\sigma = \frac{F_t}{bY_Jm} K_v K_o (0.93K_m)$$

$$b = 0.02 \text{ m}$$

$$\text{For 18-teeth to 54-teeth mesh, } Y_J = 0.99(0.42) = 0.416$$

$$\text{Uniform-uniform input-output, } K_o = 1$$

For  $K_v$ , since  $v_t = 3.57$  m/s, let  $Q = 6$ .

$$B = 0.25(12 - 6)^{2/3} = 0.825$$

$$A = 50 + 56(1 - 0.825) = 59.8$$

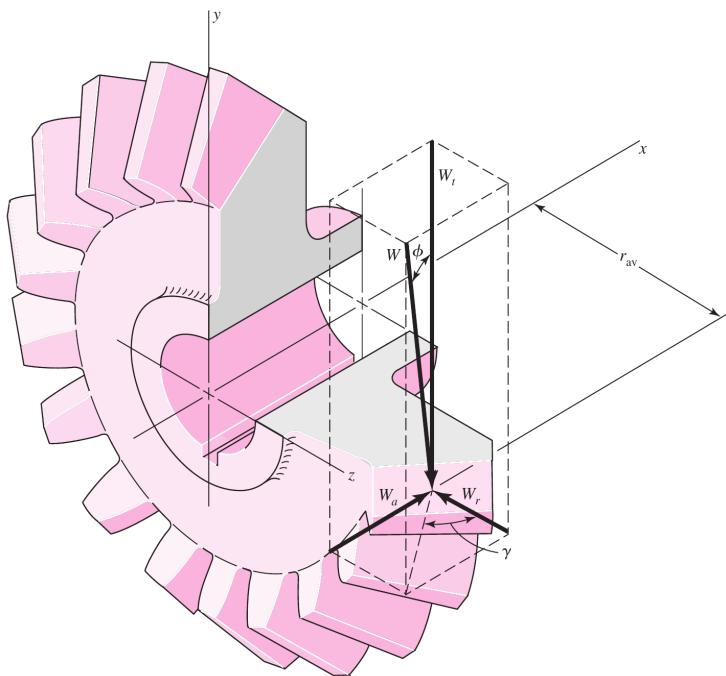
$$K_v = \left( \frac{59.8 + \sqrt{200v_t}}{59.8} \right)^{0.825} = 1.36$$

For  $K_m$ , nothing specific about gears or mounting, let's go with the middle case for  $b = 2$  cm.  $K_m = 1.6$

We can finally calculate  $\sigma$

$$\begin{aligned}\sigma &= \frac{F_t}{bY_f m} K_v K_o (0.93 K_m) \\ &= \frac{104}{(0.02)(0.416)(0.002)} (1.36)(1)((0.93)1.6) \\ &= 1.26 \times 10^7 = 12.6 \text{ MPa}\end{aligned}$$

## Bevel Gears



$$\begin{aligned}
 d_{av} &= d - b \sin \gamma \\
 v_{av} &= \omega \frac{d_{av}}{2} \\
 F_t &= \frac{H}{v_t} \\
 F_a &= F_t \tan \phi \sin \gamma \\
 F_r &= F_t \tan \phi \cos \gamma
 \end{aligned} \tag{63}$$

Design equations for bevel gears are similar to the spur gear equations with small modification.

$$\sigma = \frac{F_t}{bY_Jm} K_v K_o K_m \tag{64}$$

$$S'_e = S_e C_L C_G C_S k_r k_t k_{ms} \tag{65}$$

### Geometry Factor: $Y_J$

Because the tooth bevel gears do not have constant thickness, the geometry factors is modified.

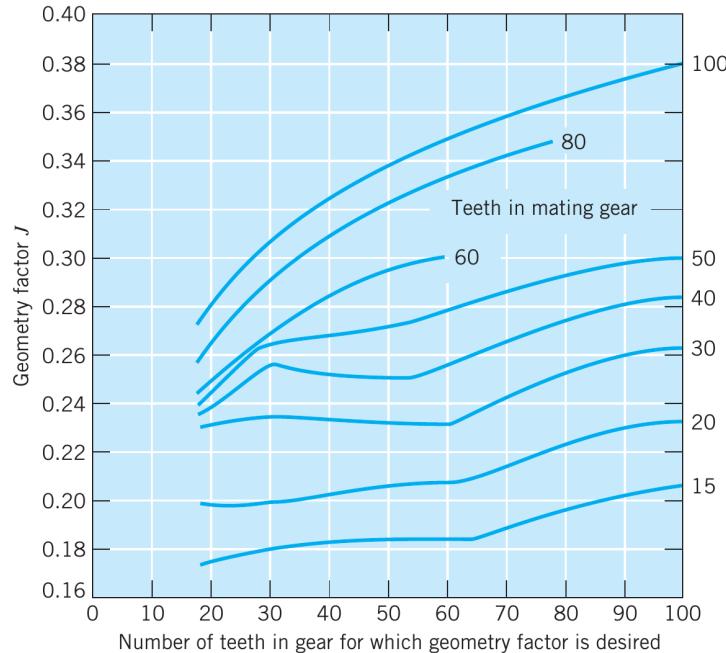


Figure 15: Geometry factors for bevel gears

### Mounting Factor: $K_m$

This factor accounts for increased stress from misalignment in the gears. In the case of bevel gears, this depends mainly on how the gears are supported. In a straddle mounting, a gear is fitted in between two bearings, providing the best support and highest rigidity. On the other hand, in an overhung mounting, a gear is fitted onto a free end of the shaft, which provides minimal rigidity.

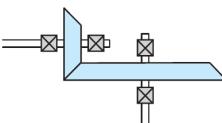
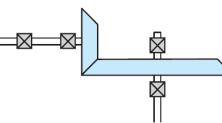
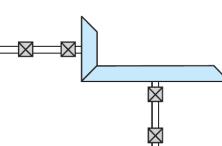
Mounting type	Mounting Rigidity
Both straddle-mounted	 1.0 to 1.25
straddle-overhung	 1.1 to 1.4
Both overhung	 1.25 to 1.5

Table 4: Mounting factor  $K_m$  for bevel gears

*Example: Bevel Gearset Design*

Identical bevel gears has a module of 0.005 m/teeth, 25 teeth, 2-cm face width, and a 20° normal pressure angle. The gear quality is  $Q = 7$ . Both requires overhung mounting. The gears are made of ductile iron whose  $S_e = 95$  MPa. Determine the power rating of the gearset at 600 rpm.

*Solution: Bevel Gearset Design*

For the stress side,

$$d_{av} = mz/1000 = 0.125 \text{ m}$$

$$v_t = \omega \frac{d_{av}}{2} = 600 \frac{2\pi}{60} \frac{0.125}{2} = 3.93 \text{ m/s}$$

For uniform-uniform loading,  $K_o = 1$

$$B = 0.25(12 - 7)^{2/3} = 0.731$$

$$A = 50 + 56(1 - 0.731) = 65$$

$$K_v = \left( \frac{65 + \sqrt{200(3.93)}}{65} \right)^{0.731} = 1.3$$

For both-overhung mounting,  $K_m = 1.5$

For 25-teeth pair,  $Y_J = 0.22$

Now, onto the strength side,

$C_L = 1$  for bending

$C_s = 0.75$  for machined surface

$C_G = 1$

No requirement on the reliability. Let's be generous, give it 90%

For normal operating temperature,  $k_t = 1$ .

For one-way bending,  $k_{ms} = 1.4$ .

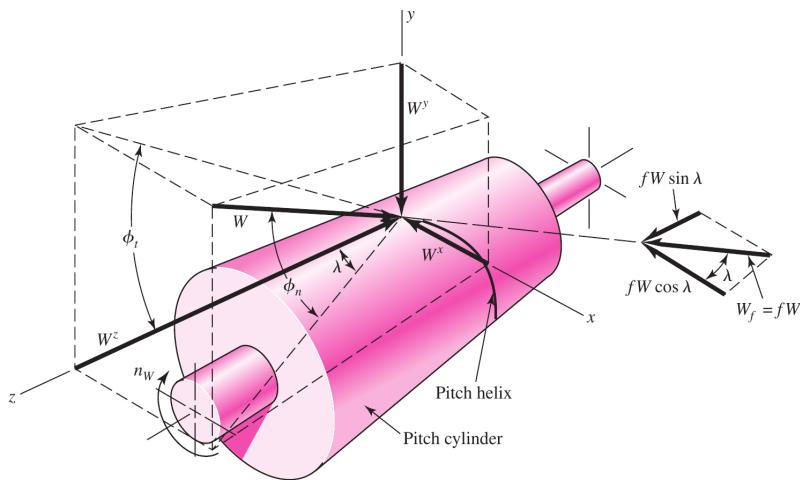
Set the two sides equal ( $N_s = 1$ ), we have

$$\frac{F_t}{(0.02)(0.22)(0.005)} (1.3)(1)(1.5) = 95 \times 10^6 (1)(1)(0.75)(0.897)(1)(1.4)$$

$$F_t = 1009 \text{ N}$$

$$H = F_t v_t = 1009(3.93) = 3965 \text{ W} = 5.31 \text{ hp}$$

## Worms and Worm Gears



- Without friction

$$F_{wt} = F \cos \phi_n \sin \lambda \quad (66)$$

$$F_{wr} = F \sin \phi_n \quad (67)$$

$$F_{wa} = F \cos \phi_n \cos \lambda \quad (68)$$

- With friction  $F_f = \mu F$

$$F_{wt} = F \cos \phi_n \sin \lambda + \mu F \cos \lambda = F_{ga} \quad (69)$$

$$F_{wr} = F \sin \phi_n = F_{gr} \quad (70)$$

$$F_{wa} = F \cos \phi_n \cos \lambda - \mu F \sin \lambda = F_{gt} \quad (71)$$

### Worm Efficiency

- Worm and worm gear velocities can be related by

$$\frac{v_g}{v_w} = \tan \lambda$$

$$v_s = \sqrt{v_w^2 + v_g^2} = v_g \sqrt{1 + \tan^2 \lambda}$$

- Efficiency  $\eta$  is

$$\eta = \frac{F_{gt} v_g}{F_{wt} v_w} \quad (72)$$

$$= \frac{\cos \phi_n \cos \lambda - \mu \sin \lambda}{\cos \phi_n \sin \lambda + \mu \cos \lambda} \tan \lambda \quad (73)$$

$$= \frac{\cos \phi_n - \mu \tan \lambda}{\cos \phi_n + \mu \cot \lambda} \quad (74)$$

### *Self locking*

- Thread will lock itself (not backdrivable) when  $F_{wt} \leq 0$

$$F_{wt} = F \cos \phi_n \sin \lambda - \mu F \cos \lambda \leq 0 \quad (75)$$

$$\mu \geq \cos \phi_n \tan \lambda \quad (76)$$

- Desirable in cases where auto-braking is needed
- In systems with large inertia, sudden stop can break the worm tooth → alternative brake mechanism is needed

### *Design Equation*

Worm gears have higher stresses than worm, so our main concern is designing the gear.

$$F_{gt,allow} = \frac{C_s d^{0.8} b C_m C_v}{75.948}$$

$C_s$  material factor

$d$  gear diameter [mm]

$b$  effective face width (actual width but less than  $0.67d_w$ ) [mm]

$C_m$  ratio correction factor

$C_v$  velocity factor

Worm Gear Material Fatigue Strength,  $S'_e$

Materials	$S'_e$
Manganese Bronze	117 MPa
Phosphor Bronze	165 MPa
Cast Iron	$0.35S_{ut}$

Allowable Load due to Wear For rough estimates:

$$F_w = D_{gear}bK_w \quad (77)$$

$F_w$  maximum allowable dynamic load

$D_{gear}$  pitch diameter of gear

$b$  face width of gear

$K_w$  material and geometry factor

$C_s$ : Material factor

For center distance  $C < 7.62$  cm

$$C_s = 720 + 0.000633C^3$$

For  $C \geq 7.62$  cm

Sand-cast gears:

$$\begin{aligned} C_s &= 1000 & d &\leq 6.35 \text{ cm} \\ C_s &= 1856.104 - 467.5454 \log d & d &> 6.35 \text{ cm} \end{aligned}$$

Chilled-cast gears:

$$\begin{aligned} C_s &= 1000 & d &\leq 20.32 \text{ cm} \\ C_s &= 2052.011 - 455.8259 \log d & d &> 20.32 \text{ cm} \end{aligned}$$

Centrifugally-cast gears:

$$\begin{aligned} C_s &= 1000 & d &\leq 63.5 \text{ cm} \\ C_s &= 1053.811 - 179.7503 \log d & d &> 63.5 \text{ cm} \end{aligned}$$

$C_m$ : Ratio correction factor

Depends on reduction ratio,  $RR = \omega_i/\omega_o$

$$C_m = \begin{cases} 0.02\sqrt{-RR^2 + 40RR - 76} + 0.46 & 3 < RR \leq 20 \\ 0.0107\sqrt{-RR^2 + 56RR + 5145} & 20 < RR \leq 76 \\ 1.1483 - 0.00658RR & RR > 76 \end{cases}$$

$C_v$ : Velocity factor

Depends on sliding velocity at mean worm diameter  $v_s$ :

$$C_v = \begin{cases} 0.659e^{-0.2165v_s} & 0 < v_s \leq 3.556 \text{ m/s} \\ 0.652v_s^{-0.571} & 3.556 < v_s \leq 15.24 \text{ m/s} \\ 1.098v_s^{-0.774} & v_s > 15.24 \text{ m/s} \end{cases}$$

*Example: Worm gear speed reducer*

A 2-hp, 1200-rpm motor drives a 60-rpm mechanism by using a work gear reducer. The gear has  $D_{gear} = 20$  cm. The worm has  $\alpha = 12^\circ$ ,  $\theta = 20^\circ$ , and  $D_{worm} = 5$  cm. Assume  $\mu = 0.03$ , determine

1. all force components according to the rated power
2. power delivered to the driven mechanism
3. whether the drive is self-locking
4. safety factor of worm gear

*Solution: Worm gear speed reducer*

First, determine  $v_w$  to determine  $v_g$

$$\begin{aligned} v_w &= \omega_w(d_w/2) = 3.14 \text{ m/s} \\ v_g &= v_w \tan \lambda = 3.14 \tan 12^\circ \\ &= 0.667 \text{ m/s} \end{aligned}$$

Power output at the worm gear is

$$\eta = \frac{\cos \phi_n - \mu \tan \lambda}{\cos \phi_n + \mu \cot \lambda} = \frac{\cos 20^\circ - 0.1 \tan 12^\circ}{\cos 20^\circ + 0.1 \cot 12^\circ} = 0.65$$

$$H_g = 0.65(2)(746) = 970 \text{ W}$$

$$F_{gt} = \frac{H_g}{v_g} = \frac{970}{0.667} \\ = 1454 \text{ N}$$

The other forces can then be calculated.

$$F_{ga} = F_{wt} = \frac{H_w}{v_w} = \frac{2(746)}{3.14} = 475 \text{ N}$$

To find  $F_{wr} = F_{gr}$ , we need first to find  $F$ , which we can solve from either  $F_{gt}$  or  $F_{ga}$

$$F_{ga} = 475 = F \cos \phi_n \sin \lambda + \mu F \cos \lambda = F (\cos 20^\circ \sin 12^\circ + 0.1 \cos 12^\circ)$$

$$475 = 0.293F$$

$$F = 1620 \text{ N}$$

$$F_{gr} = F \sin \phi_n = 1620 \sin 20^\circ = 554 \text{ N}$$

Self locking

$$\mu \geq \cos 20^\circ \tan 12^\circ$$

$$0.1 \geq 0.20$$

Nope!

Definition of safety factor

$$N_s = \frac{F_{gt,allow}}{F_{gt}}$$

Determine the allowable tangential force on worm gear and material factor

$$F_{gt,allow} = \frac{C_s d^{0.8} b C_m C_v}{75.948}$$

$$C = \frac{d_g}{2} + \frac{d_w}{2} = \frac{0.2 + 0.05}{2} = 0.125 \text{ m}$$

$$C_s = 1000 \quad (\text{assume centrifugally-cast})$$

Ratio correction factor

$$RR = 1200/60 = 20$$

$$C_m = 0.02 \sqrt{-20^2 + 40(20) - 76} + 0.46 = 0.82$$

Velocity factor

$$v_s = v_g \sqrt{1 + \tan^2 \lambda} = 0.667 \sqrt{1 + \tan^2 12^\circ} = 0.682 \text{ m/s}$$

$$C_v = 0.659 e^{-0.2165(0.682)} = 0.569$$

Finally, the safety factor

$$F_{gt,allow} = 1000(200)^{0.8}(0.67(50))(0.82)(0.569)/75.948 = 14265 \text{ N}$$

$$N_s = \frac{14265}{1454} = 9.81$$

# *Clutches and Brakes*

- Rely on friction to transfer torque
- Easy to engage/disengage

## *Clutches vs Brakes*

when engaged

*Clutches*  $\omega_{in} = \omega_{out} \neq 0$

*Brakes*  $\omega_{in} = \omega_{out} = 0$

## *Considerations for Clutch and Brake*

*Actuating force* force to engage clutch/brake

*Transmitted torque* torque through mechanism

*Energy loss* energy dissipated before mechanism is fully engaged

*Temperature rise* temperature increase from energy loss

## *Types of Clutches and Brakes*

### *Drum Brakes*

### *Disc Brakes*

### *Band Brakes*

### *Brake Linings*

#### *Materials*

*Molded* thermosetting polymer or rubber + heat resistant fibers

*Woven* fibers + brass or zinc woven into fabric + resin

*Sintered metal* metal powder + inorganic fillers molded and sintered

### *Dry Linings*

#### *Wet Linings*

#### *Drum Brake*

##### *Internal Drum Brake*

$$p = \frac{p_{\max}}{(\sin \theta)_{\max}} \sin \theta \quad (78)$$

$$M_n = \int_{\theta_1}^{\theta_2} dN(a \sin \theta) \quad (79)$$

$$dN = p(rd\theta)b \quad (80)$$

#### *Moment Generated on Drum by Normal Force*

$$dN = \frac{p_{\max}br \sin \theta d\theta}{(\sin \theta)_{\max}} \quad (81)$$

$$M_n = \int_{\theta_1}^{\theta_2} \frac{p_{\max}bra \sin^2 \theta}{(\sin \theta)_{\max}} d\theta \quad (82)$$

$$= \frac{p_{\max}bra}{(\sin \theta)_{\max}} \int_{\theta_1}^{\theta_2} \sin^2 \theta d\theta \quad (83)$$

$$= \frac{p_{\max}bra}{4(\sin \theta)_{\max}} [2(\theta_2 - \theta_1) - \sin 2\theta_2 + \sin 2\theta_1] \quad (84)$$

*Moment Generated on Drum by Friction*

$$M_f = \int_{\theta_1}^{\theta_2} \mu dN(r - a \cos \theta) \quad (85)$$

$$= \int_{\theta_1}^{\theta_2} \frac{\mu p_{\max} \sin \theta r d\theta b}{(\sin \theta)_{\max}} (r - a \cos \theta) \quad (86)$$

$$= \frac{\mu p_{\max} br}{(\sin \theta)_{\max}} \left[ r(\cos \theta_1 - \cos \theta_2) + \frac{a}{4} (\cos 2\theta_2 - \cos 2\theta_1) \right] \quad (87)$$

*Self-energizing Brake*

- if  $M_f \geq M_n$ , the brake is **self-energizing**
- The shoe sticks to the drum without actuating force  $F$

*Torque Generated on the Drum*

$$T = \int_{\theta_1}^{\theta_2} \mu r dN \quad (88)$$

$$= \frac{\mu r^2 b p_{\max}}{(\sin \theta)_{\max}} \int_{\theta_1}^{\theta_2} \sin \theta d\theta \quad (89)$$

$$= \frac{\mu r^2 b p_{\max}}{(\sin \theta)_{\max}} (-\cos \theta) \Big|_{\theta_1}^{\theta_2} \quad (90)$$

$$= \frac{\mu r^2 b p_{\max}}{(\sin \theta)_{\max}} (\cos \theta_1 - \cos \theta_2) \quad (91)$$

(92)

*External Drum Brake*

*Torque Generated on the Drum*

- identical equations to internal drum brake, only need to be careful about the direction of actuating force

*Example: Braking torque of a drum brake*

- $F = 2000 \text{ N}$

- $\mu = 0.3$
- $b = 3 \text{ cm}$

Determine the braking torque.

### Solution

First, we must determine  $p_{\max}$  on the right shoe. In this case,  $M_n$  and  $M_f$  go in opposite directions.

$$Fc = M_n - M_f \quad (93)$$

$$M_n = \frac{p_{\max}bra}{4(\sin \theta)_{\max}} [2(\theta_2 - \theta_1) - \sin 2\theta_2 + \sin 2\theta_1] \quad (94)$$

Let us first find  $M_n$  as a function of  $p_{\max}$

$$a = \sqrt{0.112^2 + 0.05^2} = 0.123 \text{ m} \quad (95)$$

$$M_n = \frac{p_{\max}bra}{4(\sin \theta)_{\max}} [2(\theta_2 - \theta_1) - \sin 2\theta_2 + \sin 2\theta_1] \quad (96)$$

$$= \frac{p_{\max}(0.03)(0.15)(0.123)}{4(\sin 90^\circ)} \left[ 2(126^\circ(\frac{\pi}{180^\circ})) - \sin(2(126^\circ)) \right] \quad (97)$$

$$= 7.38 \times 10^{-4} p_{\max} \quad (98)$$

Now find  $M_f$  as a function of  $p_{\max}$

$$M_f = \frac{\mu p_{\max}br}{(\sin \theta)_{\max}} \left[ r(\cos \theta_1 - \cos \theta_2) + \frac{a}{4}(\cos 2\theta_2 - \cos 2\theta_1) \right] \quad (99)$$

$$= \frac{0.3 p_{\max}(0.03)(0.15)}{\sin 90^\circ} [(0.15)(\cos 0 - \cos 126^\circ) + \quad (100)$$

$$\frac{0.123}{4} (\cos 2(126^\circ) - \cos 2(0))] \quad (101)$$

$$= 2.67 \times 10^{-4} p_{\max} \quad (102)$$

$$Fc = M_n - M_f \quad (103)$$

$$2000(0.212) = p_{\max}(7.38 - 2.67) \times 10^{-4} \quad (104)$$

$$p_{\max} = 9.00 \times 10^5 \text{ Pa} \quad (105)$$

Braking torque of the right shoe is

$$T_R = \frac{\mu r^2 b p_{\max}}{(\sin \theta)_{\max}} (\cos \theta_1 - \cos \theta_2) \quad (106)$$

$$= \frac{(0.3)(0.15^2)(0.03)(9.00 \times 10^5)}{1} (\cos 0^\circ - \cos 126^\circ) \quad (107)$$

$$= 289 \text{ N-m} \quad (108)$$

To calculate braking torque in left shoe, we also must calculate  $p_{\max}$ .  $M_n$  and  $M_f$  are now both clockwise.

$$Fc = M_n + M_f \quad (109)$$

$$2000(0.212) = (7.38 + 2.67) \times 10^{-4} p_{\max} \quad (110)$$

$$p_{\max} = 4.22 \times 10^5 \text{ Pa} \quad (111)$$

Braking torque of the left shoe is

$$T_L = \frac{\mu r^2 b p_{\max}}{(\sin \theta)_{\max}} (\cos \theta_1 - \cos \theta_2) \quad (112)$$

$$= \frac{(0.3)(0.15^2)(0.03)(4.22 \times 10^5)}{1} (\cos 0^\circ - \cos 126^\circ) \quad (113)$$

$$= 136 \text{ N-m} \quad (114)$$

Total braking torque is

$$T = T_L + T_R \quad (115)$$

$$= 289 + 136 = 425 \text{ N-m} \quad (116)$$

### *Band Brakes*

#### *Principles of Band Brakes*

- Rely on friction between band and drum
- Similar to pulley-belt system

$$T = (F_1 - F_2)r \quad (117)$$

### Belt Tension

$$dF = \mu dN \quad (118)$$

$$dN = 2(Fd\theta/2) = Fd\theta \quad (119)$$

$$\frac{dF}{F} = \mu d\theta \quad (120)$$

$$\ln \frac{F_1}{F_2} = \mu\theta \quad (121)$$

$$\frac{F_1}{F_2} = e^{\mu\theta} \quad (122)$$

### Example: An Exercise Bike

An exercise bike has an adjustable band brake on the wheel to provide different levels of resistance. What should the slack side belt tension be so that the biker can exercise with  $T = 50 \text{ N-m}$ . Take  $\theta = 150^\circ$  and  $\mu = 0.2$ , the bike wheel  $r = 50 \text{ cm}$ .

### Solution

$$\frac{F_1}{F_2} = e^{\mu\theta} \quad (123)$$

$$T = (F_1 - F_2)r \quad (124)$$

$$T = (e^{\mu\theta} - 1)F_2r \quad (125)$$

$$F_2 = \frac{50}{(e^{0.3(150(\pi/180))} - 1)(0.5)} \quad (126)$$

$$= 83.8 \text{ N} \quad (127)$$

### Disc Clutches and Brakes

#### Working Principles

#### Disc Wear

- new disc is rigid
- uniform pressure at first, but the outer area wears faster because of higher velocity

- after a while, pressure is no longer uniform, but wear becomes uniform

### *Torque Calculation*

1. Uniform pressure: new disc
2. Uniform wear: old disc

### *Uniform Pressure*

$$dF = pdA \quad (128)$$

$$dT = \mu r dF = \mu r p dA \quad (129)$$

$$T = \int_{r_o}^{r_i} \int_0^{2\pi} \mu r p (r dr d\theta) \quad (130)$$

$$= \frac{2}{3} \mu \pi p (r_o^3 - r_i^3) \quad (131)$$

### *Uniform Pressure (cont.)*

- Taking the actuating force  $F = p\pi(r_o^2 - r_i^2)$

$$T = \frac{2\mu F (r_o^3 - r_i^3)}{3 (r_o^2 - r_i^2)}$$

- For  $N$  parallel discs

$$T = \frac{2\mu FN (r_o^3 - r_i^3)}{3 (r_o^2 - r_i^2)}$$

### *Uniform Rate of Wear*

- Rate of Wear  $\propto$  Friction Work Rate

$$pr = C$$

- Max pressure occurs at inside radius, hence the constant is

$$pr = C = p_{\max} r_i$$

### *Braking Torque*

$$dF = pdA \quad (132)$$

$$dT = \mu r dF = \mu r p dA = \mu p_{\max} r_i dA \quad (133)$$

$$T = p_{\max} r_i \int_{r_o}^{r_i} \int_0^{2\pi} r dr d\theta \quad (134)$$

$$= \mu \pi p_{\max} r_i (r_o^2 - r_i^2) \quad (135)$$

### *Braking Torque (cont)*

- Taking into account actuating force  $F$

$$T = \mu F \left( \frac{r_o + r_i}{2} \right)$$

- For  $N$  parallel discs

$$T = \mu FN \left( \frac{r_o + r_i}{2} \right)$$

### *Usual Guideline for Disc Brakes/Clutches*

1.  $0.45r_o < r_i < 0.8r_o$
2. Use uniform wear rate, unless for short-term application

### *Example: Automotive Clutch*

- Design a wet clutch to transfer the torque of 100 N·m using the material with  $\mu = 0.08$  and  $p_{\max} = 1500$  kPa. Space requirements only allow  $r_o \leq 60$  mm. Determine the inner diameter and number of discs.

### *Solution*

- Use  $r_i = 30$  mm,

$$N = \frac{T}{[\mu\pi p_{\max} r_i (r_o^2 - r_i^2)]} \quad (136)$$

$$= \frac{100}{[(0.08)\pi(1500 \times 10^3)(0.03)(0.06^2 - 0.03^2)]} \quad (137)$$

$N = 4$  and  $d_i = 2r_i = 60$  mm

### *Drum Brakes vs Disc Brakes*

Drum	Disc
self-energizing possible	no self-energizing
very sensitive to $\mu$	not sensitive to $\mu$
requires larger force once $\mu$ goes down	well-designed caliper compensate for wear and exert constant pressure

# *Flexible Mechanical Elements*

- belts
- ropes
- chains
- used in transmission of power over long distances

## *Why Flexible?*

- Torque capacity: Gears > Belts, Ropes, Chains
- Flexible elements are better against vibration and shock loads
- Important to check for wear, age, and loss of elasticity

## *Flat Belts*

### *Belt Tension*

- For low speed belt drive

$$T = (F_1 - F_2) r \quad (138)$$

### *Belt Tension II*

- For high speed belt drive

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{\mu\theta} \quad (139)$$

where  $F_c = m\omega^2 r^2$  is the centrifugal force on the belt and  $m$  is the mass per length of belt

*Power Transmitted*

$$P = (F_1 - F_2)v \quad (140)$$

*Maintaining Belt Tension**Belt Design*

$$(F_1)_a = bF_aC_pC_v \quad (140)$$

(141)

$(F_1)_a$  = allowable largest tension (142)

$b$  = belt width (143)

$F_a$  = manufacturer's allowed tension (N/m) (144)

$C_p$  = pulley correction factor (145)

$C_v$  = velocity correction factor = 1 except leather belts (146)

*Manufacturer's Allowed Tension:  $F_a$* *Pulley Correction Factor:  $C_p$* *Velocity Correction Factor:  $C_v$* *V-Belts**Why V Belts?*

- Increased tension forces belt further into groove, providing more friction
- Increased torque capacity at a slightly lower efficiency

### Belt Tension

$$T = (F_1 - F_2) r \quad (147)$$

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{\frac{\mu\theta}{\sin\beta}} \quad (148)$$

### V Belt Design Equation

$$N_s = \frac{P_a N}{P_{nom} K_s} \quad (149)$$

- $N_s$ : safety factor
- $P_a$ : allowable power per belt
- $N$ : number of belts (integer)
- $P_{nom}$ : nominal power =  $(F_1 - F_2)v$
- $K_s$ : service factor

### Allowable Power

#### Service Factor

#### Timing Belts

- No significant stretch or slip → power where speed ratio is important
- efficiency of 97 - 99%
- No need for lubrication
- Quieter than chain drives
- Same design equations as V-belts

#### Example

Design a urethane flat belt to connect two shafts. The driving pulley ( $d = 3$  cm) is connected to a 1-kW motor. The driven pulley has  $D = 8$  cm. The motor rotates at 500 rpm.

### *Solution*

Assuming that the conveyor will utilize full motor power. The belt tension difference is

$$P = (F_1 - F_2) v \quad (150)$$

$$F_1 - F_2 = \frac{2000}{0.1} = 20000N \quad (151)$$

### *Roller Chains*

#### *Why Roller Chain (Chain Drives)?*

- No slip → constant ratio
- Long life
- Can drive multiple shafts from a single source

#### *Equations on Roller Chains*

$$D = \frac{p}{\sin\left(\frac{180^\circ}{N}\right)}$$

$$V = Npn$$

- $N$  = number of sprocket teeth
- $p$  = chain pitch
- $n$  = sprocket speed in revolution/min

### *Design Criteria*

- Roller chains rarely fail because of tensile stress
- Need to worry more about fatigue and wear on rollers
- To minimize chordal speed variation.  $N \geq 17$

### *Power Capacity*

- For link-plate limit

$$H_1 = 0.004N_1^{1.08}n_1^{0.9}p^{3-0.07p} \text{ hp}$$

- For roller limit

$$H_2 = \frac{1000K_t N_1^{1.5} p^{0.8}}{n_1^{1.5}} \text{ hp}$$

- $N_1$  = number of teeth in smaller sprocket
- $n_1$  = sprocket speed [rev/min]
- $p$  = pitch of chain [in]
- $K_t$  = 29 for chain numbers 25,35; 3.4 for 41; and 17 for 40-240

### *Roller Chain Rated Capacity (1)*

### *Roller Chain Rated Capacity (2)*

### *Example*

Pick a roller chain for a motorcycle whose output is 15 hp. The driving sprocket has 20 teeth and the output has 39 teeth.

# *Power Transmission Case Study*

## *Example: Design of a Conveyor Belt System*

Let us consider a design for a conveyor belt system. The conveyor belt needs to be moving at 60 cm/s. Both the driving and driven pulleys have the same diameter of 40 cm. The driving pulley is mounted on the same shaft as spur gear *B*, which is driven by spur gear *A* connected directly to the driving motor.

Spur gear *A* has  $D_{\text{pitch},A} = 5 \text{ cm}$ ,  $z = 10$  teeth, and pressure angle  $\theta = 20^\circ$ . The motor is set to operate constantly at 500 rpm and providing the torque of 2 N-m.

Design the following:

1. the driven spur gear *B* (determine the number of teeth and pitch diameter).
2. width *b* of the conveyor belt. (Belt material is polyamide F-1,  $\mu = 0.5$ , thickness  $t = 1.3 \text{ mm}$ ,  $\gamma = 9.5 \text{ kN/m}^3$ ,  $F_a = 6 \text{ kN/m}$ ,  $C_v = 1$ )
3. the radius of the shaft on which spur gear *B* and the driving pulley are mounted. ( $E = 210 \text{ GPa}$ ,  $S_y = 300 \text{ MPa}$ ,  $N_s = 2.5$ )
4. radial contact bearings for  $9 \times 10^7$  revolutions, 90% reliability (series and bore size).

## *Solution: Design of a Conveyor Belt System*

1. To determine the number of teeth of spur gear *B*, we need to determine the required pitch line velocity. The belt must be moving at 60 cm/s, which means the pulley must be rotating at

$$\begin{aligned}\omega_{\text{pulley}} &= \frac{0.6}{\pi(0.2)} \\ &= 0.95 \text{ rev/s} = 57.3 \text{ rpm}\end{aligned}$$

since spur gear  $B$  is mounted on the same shaft as the pulley, it must have the same  $\omega$ . The pitch line velocities on two meshing gears must be the same. Spur gear  $A$  rotates at 500 rpm, therefore, the pitch radius of spur gear  $B$  is

$$\begin{aligned} r_B &= \frac{\omega_A r_A}{\omega_B} \\ &= 0.218 \text{ m} \end{aligned}$$

Its diameter, of course, is 43.6 cm. Given that meshing gears must also have the same module, the number of teeth on  $B$  is

$$\begin{aligned} m &= \frac{D_{\text{pitch},A}}{z_A} = \frac{0.05}{10} = 0.005 \\ &= \frac{43.6}{z_B} \\ z_B &= 87.3 \text{ teeth} \end{aligned}$$

2. Assuming no power loss, the power the belt-pulley system needs to transfer is  $H = T\omega = 2(500)(2\pi/60) = 104.7 \text{ W}$ .

Required torque transfer of the belt is  $T = P/\omega = 104.7/(57.3(2\pi/60)) = 17.4 \text{ N}\cdot\text{m}$ . The tension difference on the two side needs to be

$$\begin{aligned} T &= (F_1 - F_2)r_{\text{pulley}} \\ F_1 - F_2 &= \frac{T}{r_{\text{pulley}}} = 349 \text{ N} \end{aligned}$$

We can now write an equation to determine the thickness  $b$  of the belt

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{\mu\theta}$$

Set

$$F_1 = (F_1)_a = bF_aC_vC_p = b(6000)(1)(0.7) = 4200b$$

$$F_2 = F_1 - 87 = 4200b - 87$$

$$\mu = 0.5$$

$$\theta = \pi$$

$$\begin{aligned} F_c &= (\gamma/g)bt\omega^2r^2 = (9500/10)b(0.0013)(0.95(2\pi))^2(0.2)^2 \\ &= 1.76b \end{aligned}$$

$$\frac{4200b - 1.76b}{4200b - 87 - 1.76b} = e^{0.5\pi}$$

$$b = 0.0261 = 2.61 \text{ cm}$$

3. To design the shaft, we must first determine the loads on the shaft, which are exerted by spur gear *B* and the pulley. Let us determine the load from each component.

The load from the gear can simply be determined by

$$\begin{aligned} F &= \frac{T}{r \cos \theta} \\ &= \frac{17.4}{(0.2)(\cos 20^\circ)} \\ &= 92.6 \text{ N} \end{aligned}$$

So the force exerted on the shaft by gear *B* is 92.6 N in the direction of  $20^\circ$  downward from the tangential line between the two gears.

The load from the pulley is even simpler

$$\begin{aligned} F_p &= F_1 + F_2 = 4200(0.0261) + 4200(0.0261) - 87 \\ &= 132.24 \text{ N} \end{aligned}$$

The force exerted on the shaft by the pulley is 132 N to the right.

Now all we need is to determine the maximum bending moment on the shaft.

First, bending moment  $M_g$  generated by force in the gear is

$$\begin{aligned} M_g &= \frac{F_g x (L - x)}{L} = \frac{92.6(0.1)(0.4 - 0.1)}{0.4} \\ &= 6.95 \text{ N-m} \end{aligned}$$

Next, bending moment  $M_p$  generated by force on the pulley is

```
M_p = 132*0.1*(0.4-0.1)/0.4
return round(M_p,2)
```

$$\begin{aligned} M_p &= \frac{F_p x (L - x)}{L} = \frac{132(0.3)(0.4 - 0.3)}{0.4} \\ &= 9.9 \text{ N-m} \end{aligned}$$

The maximum bending moment occurs in the middle, where

$$\begin{aligned} M_{\max} &= (6.95 + 9.9)\frac{2}{3} \\ &= 11.2 \text{ N-m} \end{aligned}$$

The maximum torque is just the required torque transfer, which is  $T = 17.4$  N-m. We can now determine the required shaft size.

$$N_s = \frac{S_y}{\sigma_e} \quad (152)$$

$$2 = \frac{300 \times 10^6}{\sqrt{\left(\frac{4(11.2)}{\pi r^3}\right)^2 + 3\left(\frac{2(17.4)}{\pi r^3}\right)^2}} \quad (153)$$

$$r = 0.0054 = 5.4 \text{ mm} \quad (154)$$

4. Since the problem is not symmetrical, we need to determine the bearing under the higher load. This happens to the bearing closer to the higher force, which is the pulley. Taking the other bearing as a fulcrum, the radial load in the bearing on the pulley side is

$$\begin{aligned} F_r &= \frac{132(3)}{4} + \frac{92.6(1)}{4} \\ &= 122.2 \text{ N} \end{aligned}$$

With  $L = 9 \times 10^7$  and  $K_r = 1$ ,  $F_e = F_r = 122.2 = C$ . This is smaller than the smallest bore which is 10mm for all series. We therefore should pick the smallest bore (10 mm) as our choice.

## *Bibliography*

- [1] Joseph Edward Shigley. *Shigley's mechanical engineering design.* Tata McGraw-Hill Education, 2015.