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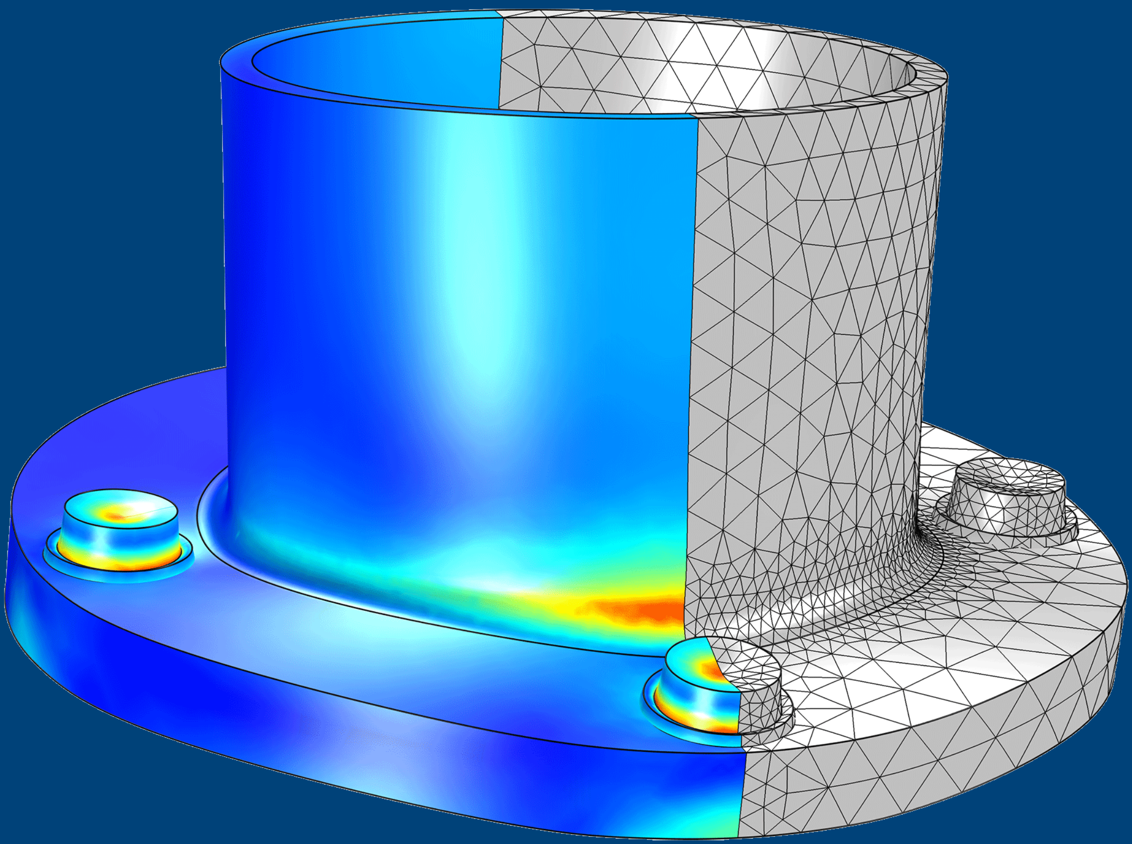

ME 310: Mechanical Design

Part II: Power Transmission

Sappinandana Akamphon

Preface

This book is the second part of ME 310: Mechanical Design. In this book, power transmission components are covered.

Sappinandana Akamphon

Revisions

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1 Shaft and Shaft Components

Most engineering systems are powered by rotational machinery such as internal combustion engines and electrical motors. It is, thus, extremely important that we understand and properly design power tranmission mechanism from the rotational machinery power source to intended components. Shafts remains one of the most prevalent methods of transmitting rotational power, and so it is our first chapter into the world of power transmission design.

By itself, shaft design is no more complex than other components under static or cyclic loadings already covered in the first book of this series. However, as shafts lie at the heart of most machine design applications, its final design will also depend on the design of other components that are to be mounted or connected to the shaft.

1.1 Shaft Materials

There are two main concerns with designing a shaft: deflection and strength. Deflection is not affected by material strength. In fact, it depends only on the stiffness, represented by the Young's modulus. Since the Young's modulus is essentially the same for all steels, material choice matters little for deflection.

Strength, on the other hand, constitute a major concern for shaft design. Strength is necessary to resist loading stresses and fatigue stresses from the constant rotation. Many shafts are made from low carbon, cold-drawn or hot-rolled steel such as ANSI 1020-1050 steels. Significant strengthening from heat treatment or high alloy is often not neeeded. And fatigue failure is only reduced moderately by increase in strength, after a certain level notch sensitivity begins to counteract its benefit.

A good way to select a proper shaft material is to start with an inexpensive low or medium carbon steel for the first round of calculations. If strength considerations turn out to be the limiting factor over deflection, select a higher strength material and reduce the shaft size accordingly until deflection becomes an issue. The cost of the material and its manufacturing processes must be weighed against the need for smaller shaft diameter.

When additional strengthening is needed, typical alloy steels for heat treatment include ANSI 340-50, 3140-50, 4140, 4340, 5140, and 8650. Shafts don't usually need surface hardening unless there is significant risk of wear from journal bearing, in which case surface hardening include carburizing grades of ANSI 1020, 4320, 4820, and 8620. [1]

1.2 Shaft Layout

Generally a shaft is a cylinder with segments of varying cross sections to accommodate components like gears, bearings, pulleys, etc. Shaft shoulders are normally used to axially locate elements and to take any axial thrust loads. Figure 1.1 illustrates the cross-section of a vertical worm-gear speed reducer. In this figure, the shaft steps are used to axially locate the two tapered bearings and the spur gear.

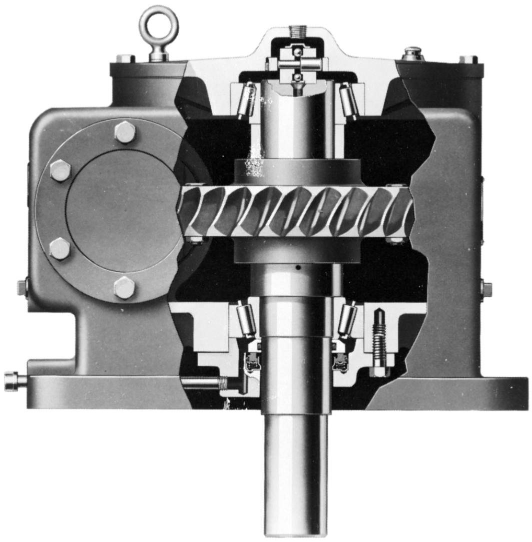


Figure 1.1: A vertical worm-gear speed reducer. *(Courtesy of the Cleveland Gear Company.)*

1.2.1 Axial Layout of Components

The axial positioning of components is often preset by the layout of the housing and other meshing components. Generally it is advisable to support any loading-carrying components between bearings. (TODO: NEED FIGURE) However, pulleys and sprockets tend to be mounted outside of the bearing pairs for ease of installation of the belt or chain. The length of the shaft should be kept as short as possible to minimize deflection and resultant load on the bearings.

In most cases, a shaft will only require two bearings, but in cases with long shafts carrying multiple load-carrying components, it may be necessary to use more than two bearings for additional support. In such a case, particular care must be given to the alignment of the bearings.

1.2.2 Supporting Axial Loads

There are use cases where axial loads in shaft may be significant, in which case it is important to provide a means to transfer the axial loads into the shaft. The shaft would then transfer the loads to a bearing and to the ground. This is necessary for shafts mounted with components that generate axial loads like bevel and helical gears, or tapered roller bearings.

It is usually sufficient to have only one bearing carry the axial load. This allows for greater tolerances on shaft length dimensions and prevents binding from shaft thermal expansions, which is especially significant in long shafts. TODO: figures showing shaft that carries axial loads.

1.2.3 Support for Torque Transmission

Most shafts serve to transfer torque from an input (gear, pulley, engine, motor, etc.) to an output gear or pulley. The shaft must be sized to support torsional stress and its resultant angle of twist. It is also important to provide a way to transmit the torque between the gear (or pulley) and the shaft itself. Common torque transfer methods are:

- Keys
- Splines
- Setscrews
- Pins
- Press or shrink fits
- Tapered fits

There are also shafts that are designed to fail if excessive torque is applied, to prevent failure of more expensive components. Details of the components and their design process is covered in section 1.4.

1.3 Shaft Design for Stress

It is not always necessary to evaluate stresses at every point; the same goes for shafts as well. Only a few potentially critical locations should be more than enough. Since the main types of load on shafts are torsion and bending, it follows that most critical locations on the shafts are on the outer surface–typically where the bending moment is large, the torque is large, and where stress concentrations exist.

In order to determine the bending moments, torques, and shear forces on a shaft, it is usually a good idea to draw shear and bending moment diagrams. Since most shafts are loaded by gears and pulleys, introducing forces in two planes, two diagrams are needed to determine the loads. Resultant moments can be obtained simply by adding the moments as vectors at points of interest. The normal stress due to bending will be highest on the outer surfaces and will contribute to fatigue on a rotating shaft.

Axial stresses on shafts from axial loads caused by helical gears or tapered roller bearings are typically negiligible compared to the bending stress. The axial stresses are also usually constant, meaning that they rarely contribute significantly to fatigue. However, axial stresses resulting from axial loads applied through other means should be explicitly considered.

Let us now consider the shaft stresses, which are usually the combination of normal stresses from bending and axial stresses, and shear stress from torsion.

$$\begin{aligned} \sigma_a &= K_f \frac{M_a y}{I} \quad & \sigma_m &= K_f \frac{M_m y}{I} \\ \tau_a &= K_{fs} \frac{T_a r}{J} \quad & \tau_m &= K_{fs} \frac{T_m r}{J} \end{aligned} \tag{1.1}$$

If we assume a solid shaft with circular cross section, we can further simplify the expression to

$$\begin{aligned} \sigma_a &= K_f \frac{32 M_a}{\pi d^3} \quad & \sigma_m &= K_f \frac{32 M_m}{\pi d^3} \\ \tau_a &= K_{fs} \frac{16 T_a}{\pi d^3} \quad & \tau_m &= K_{fs} \frac{16 T_m}{\pi d^3} \end{aligned} \tag{1.2}$$

The stresses can be combined into stress amplitude and average stress using maximum distortion energy theory (MDET or von Mises) as

$$\sigma_{ae} = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[\left(\frac{32 K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16 K_{fs} T_a}{\pi d^3}\right)^2\right]^{1/2} \tag{1.3}$$

$$\sigma_{me} = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[\left(\frac{32 K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16 K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2} \tag{1.4}$$

The stress concentration factors for the average stress component in ductile materials can sometimes be ignored since the materilas can yield locally at the discontinuity.

These equivalent stresses can be evaluated in design equations to determine the safety factor N_s or the required diameter d using the the modified Goodman diagram as

$$\frac{1}{N_s} = \frac{\sigma_{ae}}{S_e} + \frac{\sigma_{me}}{S_{ut}}$$

Substituting for σ_{ae} and σ_{me} results in

$$\frac{1}{N_s} = \frac{16}{\pi d^3}\left\{\frac{1}{S_e}\left[4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2\right]^{1/2} + \frac{1}{S_{ut}}\left[4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2\right]^{1/2}\right\}$$

The required diameter d can be solved from the previous equation as

$$d = \left(\frac{16N_s}{\pi}\left\{\frac{1}{S_e}\left[4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2\right]^{1/2} + \frac{1}{S_{ut}}\left[4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2\right]^{1/2}\right\}\right)^{1/3}$$

In many applications, rotating shafts will be under constant bending and torsion, resulting in completely reverse bending stress ($M_m = 0$) and constant torsional shear stress ($T_a = 0$). This means the required diameter becomes

$$d = \left(\frac{16N_s}{\pi}\left\{\frac{2K_f M_a}{S_e} + \frac{\sqrt{3}K_{fs} T_m}{S_{ut}}\right\}\right)^{1/3}$$

Example 1.1 Size the shaft (AISI 1040, $S_y = 400$ MPa, $S_{ut} = 600$ MPa) using

1. MDET
2. Soderberg theory

so that the safety factor $N_s = 3$.

Solution

1. MDET: The torque loaded on the pulley by the belt is

$$\begin{aligned} T &= (2020 - 20)(0.1) \\ &= 200.0 \text{ N-m} \end{aligned}$$

There is also 2040 N of force pulling at the pulley due to the combined belt tension. The torque generates shear stress throughout the shaft, with the maximum value at the surface. The belt tension creates bending stresses, whose maximum values are are the top and bottom of the shaft at the middle. This means that the critical points on the shaft (without considering stress concentration from the key/keyseat) are at the top and bottom of the shaft at the middle. In this problem, we will take the bottom of the shaft at the middle. The stress concentration of the keyseat is taken to be $K_f = 2.14$ in bending and $K_{fs} = 3.0$ in torsion.

$$
\begin{aligned}
\sigma &= K_f \frac{My}{I} = (2.14)\frac{2040(0.6)(r)}{4\pi r^4/4} \\
&= \frac{834}{r^3}\ \text{Pa} \\
\tau &= K_{fs}\frac{Tr}{J} = (3)\frac{200.0(r)}{\pi r^4/2} \\
&= \frac{382}{r^3}\ \text{Pa}
\end{aligned}
$$

We can then combine them using MDET to obtain the equivalent stress σ_e.

$$
\begin{aligned}
\sigma_e &= \sqrt{\sigma^2 + 3\tau^2} \\
&= \sqrt{\left(\frac{834}{r^3}\right)^2 + 3\left(\frac{382}{r^3}\right)^2} \\
N_s &= \frac{S_y}{\sigma_e} \\
3 &= \frac{400 \times 10^6 r^3}{1064} \\
r &= 0.02\ \text{m}
\end{aligned}
$$

2. Soderberg: using the criteria, we must calculate the minimum and maximum bending moments and torques, which will then be used to determine the stress amplitudes and average stresses. We already determine the maximum bending moment and torque, which we used to determine the corresponding stresses for MDET. We now only need to find out the minimum bending moment and torque. The minimum bending moment occurs when the shaft rotates by half a revolution, for which the beam willbe under a compressive stress of the same magnitude.

$$
\begin{aligned}
\sigma_{\text{min}} &= -\frac{834}{r^3} \\
\sigma_a &= \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \\
&= \frac{834}{r^3}
\end{aligned}
$$

If the shaft is under continuous operation, the applied torque is constant, which means that the torque amplitude $T_a = 0$ and the average torque $T_m = T$. We can plug this into the sequation to determine equivalent amplitude and average stresses.

$$
\begin{aligned}
\sigma_{ae} &= \sqrt{\sigma_a^2 + 3\tau_a^2} = \frac{834}{r^3} \\
\sigma_{me} &= \sqrt{\sigma_m^2 + 3\tau_m^2} = \frac{662}{r^3} \\
r &= 0.0237\ \text{m}
\end{aligned}
$$

Example 1.2 Using the settings from the previous example, redetermine the shaft size if the maximum operating speed is 10000 rpm.

Solution For a simply supported shaft, the first natural frequency that can cause shaft whirling is

$$\omega_1 = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{EI}{A\rho}}$$

We must first convert the angular velocity into rad/s: 10000 rpm = 10000 * $2\pi/60$ = 1047 rad/s. To achieve the safety factor of 3, the first natural frequency of the shaft must be

$$r = 0.044 \text{ m}$$

The designed shaft has to follow the largest shaft that satisfy each of the condition, therefore the required radius is 4.4 cm.

1.4 Torque Trasmission Components

2 Journal Bearings and Lubrication

Overview of Bearings

What are bearings?

- A feature that allows relative motions between components
  - Linear motions
  - Rotary motions

Two types of bearings

- Contact: sliding or rolling
- Non-contact: fluid film or magnetic

Contact Bearings

Sliding Contact Bearings

- Commonly used in low- to medium-speed applications

- Lubrication is used to reduce wear and friction

Materials for Sliding Contact Bearing

- Typically hard materials (shaft) on soft (bearing)
- Materials:
  - Polymers: nylon is king!
  - Brass
  - Ceramics
- Check on bearing stress
- Aluminum-on-aluminum is a no-no

Bearing Contact Pressure

$$P = \frac{F}{DL}$$
$$P_{\max} = \frac{4}{\pi}\frac{F}{DL}$$

PV Factor

- pressure × velocity
- tradeoff in choosing bearing materials
- higher pressure → low speed, and vice versa

PV Table for Metals

PV Table for Nonmetals

Example: Sleeve Bearing for a Low-speed Shaft

A 30-cm long shaft whose diameter D is 3 cm is operated at 1000 rpm. The shaft has a spur gear whose $R_{\text{pitch}} = 10$ cm mounted in the middle with a bearing at each end. The gear is transferring the power of 1.5 kW. The gear has pressure vessel $\theta = 20°$. Determine the minimum bearing length L using nylon.

Solution

First, let us determine the force on the bearing. Since spur gears don't generate any axial load, the forces will simply be the radial + tangential load, perpendicular to the shaft.

$$\begin{aligned} T &= \frac{P}{\omega} \\ &= \frac{1500}{1000(2\pi/60)} = 14.3 \text{ N-m} \\ F &= \frac{T}{R_{\text{pitch}} \cos\theta} \\ &= \frac{14.3}{0.1 \cos 20°} = 152 \text{ N} \end{aligned}$$

Solution

Since the gear is mounted in the middle, the force on each bearing is half of the force.

$$F_{bearing} = \frac{152}{2} = 76 \text{ N}$$

We can't determine the bearing pressure yet since we don't know the bearing length. We can determine the surface velocity, however.

$$v = \omega(D/2) = 1000(2\pi/60)(0.03/2) = 1.57 \text{ m/s}$$

Solution

We double-check that $v < V_{nylon} (1.57 < 3.0)$ so nylon is an acceptable choice. The length of bearing, then should be

$$
\begin{aligned}
P_{bearing} v &< (PV)_{nylon} \\
\frac{F_{bearing}}{DL} v &< 0.11 \times 10^6 \\
\frac{76}{0.03L} 1.57 &< 1.1 \times 10^5 \\
L &> 0.036 = 3.6 \text{ cm}
\end{aligned}
$$

3 Rolling-Contact Bearings

Rolling Contact Bearings

Rolling Elements

- suitable for medium- to high-speed applications
- use balls or rollers to avoid friction

Rolling Element Types

Bearing Series

Bearing Table

Bearing Life Requirement

$$L = L_R K_r \left(\frac{C}{F_e}\right)^{10/3}$$

$$C = F_e \left(\frac{L}{K_r L_R}\right)^{0.3}$$

L	life corresponding to equivalent load F_e
L_R	life corresponding to rated capacity = 9×10^7 rev
K_r	reliability factor
C	rated capacity
F_e	equivalent load

Bearing Rated Capacity

Reliability Factor

Equivalent Load

Let $e = F_a/F_r$

for radial ball bearings

$$F_e = \begin{cases} F_r & e < 0.35 \\ F_r\left[1 + 1.115(e - 0.35)\right] & 0.35 < e < 10 \\ 1.176F_a & e > 10 \end{cases}$$

for angular ball bearings

$$F_e = \begin{cases} F_r & e < 0.68 \\ F_r\left[1 + 0.87(e - 0.68)\right] & 0.68 < e < 10 \\ 0.911F_a & e > 10 \end{cases}$$

Typical Bearing Design Life

Example 3.1 Radial Ball Bearing Selection

Select a radial ball bearing for a shaft intended for a continuous 8-hr-a-day operation at 1800 rpm with 95% reliability. Axial and radial loads are 1.2 kN and 1.5 kN, respectively.

Solution First, we need to calculated F_e.

$$e = \frac{F_a}{F_r} = \frac{1.2}{1.5} = 0.8$$

For radial ball bearing,

$$\begin{aligned} F_e &= F_r\left[1 + 0.87(e - 0.68)\right] \\ &= 1500\left[1 + 1.115(0.8 - 0.35)\right] \\ &= 2276 \text{ N} \end{aligned}$$

Required life for 8-hr-a-day service (assumed every day) = 30000 hrs

Life in revolutions

$$L = 1800(30000)(60) = 3.24 \times 10^9 \text{ revolutions}$$

For 95% reliability $K_r = 0.63$

$$C = 2276\left(\frac{3.24 \times 10^9}{0.63(9 \times 10^7)}\right)^{0.3} = 7661 \text{ N} = 7.66 \text{ kN}$$

For extra-light, light, and medium series, the required bore are 55, 35, and 30 mm, respectively

The models corresponding to the bore are L11, 207, and 306, respectively.

4 Gears

4.1 Gear Overview

Why Gears?

- Convert high speed and low torque to that requires low speed and high torque
- Speed: easy to get because voltage is easy
- Torque: hard to get because it requires large current

Principles of Gears

- Allow positive engagement between teetch
- High forces can be transmitted while in rolling contact
- Do not need friction to operate

Basic Law of Gearing

- Point of contact between two mating gears is always the same relative distances from the two centers.
- Any gear tooth profiles that follow the law of gearing will result in constant relative speed of rotation

Gear Geometry

Module of a Gear, m

- Term used to define gear tooth size
- Defined as ratio of pitch diameter to number of teeth

$$m = \frac{D_{\text{pitch}}}{z}$$

- A pair of meshing gears must have the same modules!

Gear Types

Gear Terminology

Pinion smaller of two gears, usually driving

Gear Larger of the two. Also called *wheel*. Usually driven.

Gear Materials

- Steel: medium-carbon steel + heat treatment + grinding
- Cast iron: surface fatigue > bending fatigue

- Nonferrous: bronzes → corrosion + wear resistant, low friction
- Nonmetallic: Nylon → low friction and weight + corrosion resistant, but low thermal conductivity

Gear Efficiency

- With friction, gears are 90 - 95% efficient because of mostly rolling contact

$$
\begin{aligned}
T_{\text{out}} &= \frac{\eta T_{\text{in}} d_{\text{out}}}{d_{\text{in}}} \\
\omega_{\text{out}} &= \frac{\omega_{\text{in}} d_{\text{in}}}{d_{\text{out}}} \\
P_{\text{out}} &= T_{\text{out}} \omega_{\text{out}}
\end{aligned}
$$

4.2 Gear Trains

Gear Trains When large reduction is required

- Large gear + small pinion: simple, but large stress and interference
- Multiple pairs of gears and pinions: less simple, low stress, large space
- Planetary gears: complex, low stress, small space

Normal Gear Trains

$$e_{total} = e_1 e_2 \cdots$$

Planetary/Epicyclic Gear Train

- Planetary or epicyclic gears enable a high reduction ratio in small spaces

Planetary Gear Components

Planetary Gears: Torque, Forces, and Reduction Ratios

- Symmetry → no net force on shaft
- Multiple planet gears reduce individual torque/force
- Any combination of fixed, input, output gears
- 1 gear box -> multiple gear reduction ratios

Example 4.1

Solution Fixed ring:

$$\omega_{\text{carrier}} = 9$$
$$\omega_{\text{planet}} = (9)\frac{60/2 + 20/2}{20/2} = 36$$
$$\omega_{\text{sun}} = (36)\frac{20}{30} = 24$$
$$e = 9/24 = 0.375$$

Fixed	Input	Planet	Output	RR
Ring	Carrier 9	36	Sun 24	0.375
Sun	Carrier 9	36	Ring 14.4	0.625
Carrier	Sun 9	27	Ring 5.4	1.667

4.3 Spur Gears

Spur Gears

- Straight involute teeth → most common

$$F = \frac{T}{R_{\text{pitch}} \cos\theta}$$

- Lead to shaft bending

Spur Gear Stress

- Bending Stress → AGMA stress equation
- Consider tooth as a cantilever beam

$$\sigma = \frac{F_t}{bY_J m} K_O K_m K_v$$

F_t tangential force

b face width

Y_J geometry factor

m module

Geometry Factor: Y_J

Overload Factor: K_O Power sources

Power source	Driven Machine			
	Uniform	Light shock	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.50	1.75
Light shock	1.20	1.40	1.75	2.25
Moderate shock	1.30	1.70	2.00	2.75

Uniform Electric motor, constant-speed turbine

Light Water turbine, variable-speed drive

Moderate Multicylinder engine

Driven machine

Uniform Continuous generator

Light Fans, low-speed pumps, conveyors

Moderate high-speed pumps, compressors, heavy conveyers

Heavy rock crushers, punch press drivers

$\boxed{\text{Mounting Factor: } K_m}$

Characteristics of Support	Face Width (cm)			
	0 to 5 cm	15	22.5	40
Accurate mountings, small bearing clearances, precision gears	1.3	1.4	1.5	1.8
Less rigid moutings, standard gears, full face contact	1.6	1.7	1.8	2.2
Less than full face contact	Over 2.2			

$\boxed{\text{Velocity Factor: } K_v}$

- Takes care of shock and impact loading

$$K_v = \left(\frac{A + \sqrt{200 v_t}}{A}\right)^B$$
$$A = 50 + 56(1 - B)$$
$$B = 0.25(12 - Q)^{2/3}$$

v_t pitch line velocity [m/s]

Q AGMA Quality Number

$\boxed{\text{AGMA Recommended Quality Number: } Q}$

v_t [m/s]	Q	Applications
0 - 4	6 - 8	Paper box making machine, cement, mill drives
4 - 10	8 - 10	Washing machine, printing press, computing mechanism
10 - 20	10 - 12	Automotive transmission, Antenna drive, propulsion drive
$\geqslant 20$	12 - 14	Gyroscope

$\boxed{\text{Gear Material Strength}}$

$$S'_e = S_e C_L C_G C_S k_r k_t k_{ms}$$

S_e endurance limit

C_L load factor (= 1 for bending)

C_G gradient surface = 1

C_S surface factor (= 0.75 for machined surface)

k_r reliability factor

k_t temperature factor

k_{ms} median-stress factor (1 for two-way bending (followers), 1.4 for one-way bending (input or output))

$\boxed{\text{Reliability Factor: } k_r}$

Reliability (%)	k_r
50	1.000
90	0.897
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659

$\boxed{\text{Temperature Factor: } k_t}$

$$k_t = \begin{cases} 1 & T \leqslant 160 \text{ F} \\ \dfrac{620}{460 + T} & T > 160 \text{ F} \end{cases}$$

$\boxed{\text{General Guidelines}}$

1. $e \leqslant 1/6$
2. Use multi-stage gears for larger than $e > 1/6$
3. $8m \leqslant b \leq 16m$
4. many small teeth $\gg$ few large teeth
5. few teeth $\longrightarrow$ small gear, but be careful about interference
6. Avoid exact ratio $\longrightarrow$ hunting tooth

$\boxed{\text{Example}}$ A pair of spur gears with face width $b = 3$ cm is used in a conveyor belt drive. The input motor has $\omega_{\max}$ of 200 rad/s. The pinion has 18 teeth. The conveyor has moderate shock and should be driven at 100 rad/s. The gears have pressure angles θ of 20°. Both pinion and gear has $m = 1$ cm. Determine the maximum power that the gears caan transmit continuously with 1% chance of bending fatigue failure. Steel has $S_{ut} = 400$ MPa

$\boxed{\text{Solution}}$ First, the bending fatigue stress is

$$\begin{aligned}\sigma &= \frac{F_t}{bY_J m} K_O K_m K_v \\ &= \frac{F_t}{(0.03)(0.32)(0.01)}(1.25)(1.6)\left(\frac{65.12 + \sqrt{200(18)}}{65.12}\right)^{0.73} \\ &= 33542 F_t\end{aligned}$$

$\boxed{\text{Solution}}$ Next, the material fatigue strength is

$$\begin{aligned}S_e' &= S_e C_L C_G C_S k_r k_t k_{ms} \\ &= (400 \times 10^6 (0.5))(1)(1)(0.75)(0.814)(1)(1.4) \\ &= 1.71 \times 10^8\end{aligned}$$

We can then find the maximum allowable tangential force

$$
\begin{aligned}
F_t &= \frac{1.71 \times 10^8}{33542} = 5096 \text{ N} \\
P &= T\omega = F_t v_{pitch} = 5096 \times 18 = 9.17 \times 10^4 \text{ W}
\end{aligned}
$$

Rack and Pinion

$$
\begin{aligned}
F_t &= \frac{T}{R_{\text{pitch}}} \\
F_r &= \frac{T \tan\theta}{R_{\text{pitch}}}
\end{aligned}
$$

- Rack = linear gear
- Convert torque to force
- Cheaper but less accurate than power screw
- No mechanical advantages

4.4 Other Types of Gears

Helical Gears

$$
\begin{aligned}
F_t &= \frac{T}{R_{\text{pitch}}} \\
F &= \frac{T}{R_{\text{pitch}} \cos\theta \cos\alpha} \\
F_a &= F \cos\theta \sin\alpha = \frac{T \tan\alpha}{R_{\text{pitch}}} \\
F_r &= F \sin\theta
\end{aligned}
$$

Design Equations Same as spur gear equation with small modification

$$
\begin{aligned}
\sigma &= \frac{F_t}{bY_J m} K_v K_o (0.93 K_m) \\
S'_e &= S_e C_L C_G C_S k_r k_t k_{ms}
\end{aligned}
$$

0.93 indicated helical gears less sensitivity to mounting factor

Y_J needs small modification for helical teeth

Geometry Factor: Y_J

Geometry Factor: Y_J

Bevel Gears

$$
\begin{aligned}
F_t &= \frac{T}{R_{\text{av}}} \\
F &= \frac{T}{R_{\text{av}} cos\theta} \\
F_a &= F\sin\theta\sin\gamma = \frac{T}{R_{\text{av}}}\tan\theta\sin\gamma \\
F_r &= F\sin\theta\cos\gamma = \frac{T}{R_{\text{av}}}\tan\theta\cos\gamma
\end{aligned}
$$

Design Equations Same as spur gear equation with small modification

$$
\begin{aligned}
\sigma &= \frac{F_t}{bY_J m} K_v K_o K_m \\
S'_e &= S_e C_L C_G C_S k_r k_t k_{ms}
\end{aligned}
$$

Geometry Factor: Y_J

Mounting Factor: K_m

Worms and Worm Gears

- Worm drives gear
- Sliding contact → η = 30 - 50%

$$
\tan\alpha = \frac{p_{worm}}{\pi D_{worm}}
$$

Worm Force Analysis

- Without friction

$$
\begin{aligned}
F_{wt} &= F\cos\theta\sin\alpha \\
F_{wr} &= F\sin\theta \\
F_{wa} &= F\cos\theta\cos\alpha
\end{aligned}
$$

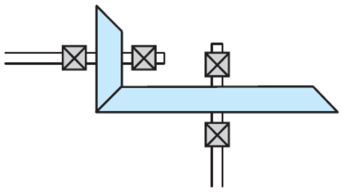
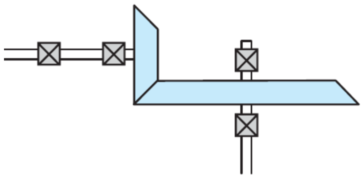
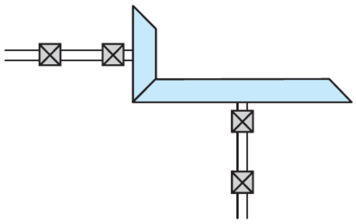
- With friction $F_f = \mu F$

$$
\begin{aligned}
F_{wt} &= F\cos\theta\sin\alpha + \mu F\cos\alpha = F_{ga} \\
F_{wr} &= F\sin\theta = F_{gr} \\
F_{wa} &= F\cos\theta\cos\alpha - \mu F\sin\alpha = F_{gt}
\end{aligned}
$$

Worm Efficiency

- Worm and worm gear velocities can be related by

$$
\frac{v_g}{v_w} = \tan\alpha
$$

Mounting type		Mounting Rigidity
Both straddle-mounted		1.0 to 1.25
straddle-overhung		1.1 to 1.4
Both overhung		1.25 to 1.5

- Efficiency η is

$$\begin{aligned}
\eta &= \frac{F_{gt} v_g}{F_{wt} v_w} \\
&= \frac{\cos\theta\cos\alpha - \mu\sin\alpha}{\cos\theta\sin\alpha + \mu\cos\alpha}\tan\alpha \\
&= \frac{\cos\theta - \mu\tan\alpha}{\cos\theta + \mu\cot\alpha}
\end{aligned}$$

Self-Locking

- Thread will lock itself (not backdrivable) when $F_{wt} \leqslant 0$

$$\begin{aligned}
F_{wt} &= F\cos\theta\sin\alpha - \mu F\cos\alpha \leqslant 0 \\
\mu &\geqslant \cos\theta\tan\alpha
\end{aligned}$$

- Desirable in cases where auto-braking is needed
- In systems with large inertia, sudden stop can break the worm tooth → alternative brake mechanism is needd

Design Equation

$$\begin{aligned}
F_b &\geqslant F_d \\
F_w &\geqslant F_d
\end{aligned}$$

F_b allowable load due to tooth bending

F_w allowable load due to wear

F_d dynamic load $= F_{gt}\dfrac{6.1 + v_g}{6.1}$

Allowable Load due to Tooth Bending: F_s

$$F_s = S_e' y b p$$

y Lewis form factor

θ	y
14.5	0.100
20	0.125
25	0.150
30	0.175

b face width of worm gear (should be $\leqslant 0.67 D_{\text{pitch, worm}}$)

p circular pitch $= \dfrac{\pi D_{\text{pitch}} \cos\alpha}{z}$

Worm Gear Material Fatigue Strength, S_e'

Materials	S_e'
Manganese Bronze	117 MPa
Phosphor Bronze	165 MPa
Cast Iron	$0.35 S_{ut}$

Allowable Load due to Wear For rough estimates:

$$F_w = D_{gear} b K_w$$

F_w maximum allowable dynamic load

D_{gear} pitch diameter of gear

b face width of gear

K_w material and geometry factor

Material and Geometry Factor: K_w

Material (Worm - Gear)	K_w [KPa]		
	$\alpha < 10°$	$\alpha < 25°$	$\alpha > 25°$
Steel - Bronze	414	517	620
Hardened steel - Bronze	551	689	827
Hardened steel - Chill-cast bronze	827	1034	1241
Cast iron - Bronze	1034	1275	1551

Example: Worm gear speed reducer A 2-hp, 1200-rpm motor drives a 60-rpm machanism by using a work gear reducer. The gear has $D_{gear} = 20$ cm. The worm has $\alpha = 12°$, $\theta = 20°$, and $D_{worm} = 5$ cm. Assume $\mu = 0.03$, determine

1. all force components according to the rated power
2. power delivered to the driven mechanism
3. whether the drive is self-locking

Solution

- Pitch line velocity of worm

$$v_w = \omega_w(D_w/2) = 3.14 \text{ m/s}$$
$$F_{wt} = \frac{P}{v_w} = \frac{2(746)}{3.14} = 475 \text{ N}$$

$\boxed{\text{Solution}}$

- Efficiency

$$e = \frac{cos\theta - \mu \tan\alpha}{cos\theta + \mu \cot\alpha} = 0.86$$

Delivered power = $0.86 \times 2(746) = 1283$ W

- Self locking

$$\mu \geqslant \cos 20^\circ \tan 12^\circ$$
$$0.03 \geqslant 0.20$$

Nope!

5 Clutches, Brakes, Couplings, and Flywheels

Clutches + Brakes

- Rely on friction to transfer torque
- Easy to engage/disengage

Clutches vs Brakes

when engaged

Clutches $\omega_{in} = \omega_{out} \neq 0$

Brakes $\omega_{in} = \omega_{out} = 0$

Considerations for Clutch and Brake

Actuating force force to engage clutch/brake

Transmitted torque torque through mechanism

Energy loss energy dissipated before mechanism is fully engaged

Temperature rise temperature increase from energy loss

Types of Clutches and Brakes

Drum Brakes

Disc Brakes

Band Brakes

Brake Linings

Materials

Molded thermosetting polymer or rubber + heat resistant fibers

Woven fibers + brass or zinc woven into fabric + resin

Sintered metal metal powder + inorganic fillers molded and sintered

Dry Linings

Wet Linings

Drum Brake

Internal Drum Brake

$$
\begin{aligned}
p &= \frac{p_{\max}}{(\sin\theta)_{\max}}\sin\theta \\
M_n &= \int_{\theta_1}^{\theta_2} dN(a\sin\theta) \\
dN &= p(r\,d\theta)b
\end{aligned}
$$

Moment Generated on Drum by Normal Force

$$
\begin{aligned}
dN &= \frac{p_{\max} br\sin\theta\, d\theta}{(\sin\theta)_{\max}} \\
M_n &= \int_{\theta_1}^{\theta_2} \frac{p_{\max} bra\sin^2\theta}{(\sin\theta)_{\max}} d\theta \\
&= \frac{p_{\max} bra}{(\sin\theta)_{\max}} \int_{\theta_1}^{\theta_2} \sin^2\theta\, d\theta \\
&= \frac{p_{\max} bra}{4(\sin\theta)_{\max}} [2(\theta_2-\theta_1) - \sin 2\theta_2 + \sin 2\theta_1]
\end{aligned}
$$

Moment Generated on Drum by Friction

$$
\begin{aligned}
M_f &= \int_{\theta_1}^{\theta_2} \mu\, dN(r - a\cos\theta) \\
&= \int_{\theta_1}^{\theta_2} \frac{\mu p_{\max}\sin\theta\, r\,d\theta\, b(r-a\cos\theta)}{(\sin\theta)_{\max}} \\
&= \frac{\mu p_{\max} br}{(\sin\theta)_{\max}} \left[r(\cos\theta_1 - \cos\theta_2) + \frac{a}{4}(\cos 2\theta_2 - \cos 2\theta_1) \right]
\end{aligned}
$$

Self-energizing Brake

- if $M_f \geqslant M_n$, the brake is **self-energizing**
- The shoe sticks to the drum without actuating force F

Torque Generated on the Drum

$$
\begin{aligned}
T &= \int_{\theta_1}^{\theta_2} \mu r dN \\
&= \frac{\mu r^2 b p_{\max}}{(\sin\theta)_{\max}} \int_{\theta_1}^{\theta_2} \sin\theta d\theta \\
&= \frac{\mu r^2 b p_{\max}}{(\sin\theta)_{\max}} (-\cos\theta)|_{\theta_1}^{\theta_2} \\
&= \frac{\mu r^2 b p_{\max}}{(\sin\theta)_{\max}} (\cos\theta_1 - \cos\theta_2)
\end{aligned}
$$

External Drum Brake

Torque Generated on the Drum

- identical equations to internal drum brake, only need to be careful about the direction of actuating force

Example: Braking torque of a drum brake

- $F = 2000$ N
- $\mu = 0.3$
- $b = 3$ cm

  Determine the braking torque.

Solution

First, we must determine $p_{\max}$ on the right shoe. In this case, M_n and M_f go in opposite directions.

$$
\begin{aligned}
Fc &= M_n - M_f \\
M_n &= \frac{p_{\max} b r a}{4(\sin\theta)_{\max}} [2(\theta_2 - \theta_1) - \sin 2\theta_2 + \sin 2\theta_1]
\end{aligned}
$$

Solution

Let us first find M_n as a function of $p_{\max}$

$$
\begin{aligned}
a &= \sqrt{0.112^2 + 0.05^2} = 0.123 \text{ m} \\
M_n &= \frac{p_{\max} b r a}{4(\sin\theta)_{\max}} [2(\theta_2 - \theta_1) - \sin 2\theta_2 + \sin 2\theta_1] \\
&= \frac{p_{\max}(0.03)(0.15)(0.123)}{4(\sin 90^\circ)} \left[2(126^\circ(\frac{\pi}{180^\circ})) - \sin(2(126^\circ))\right] \\
&= 7.38 \times 10^{-4} p_{\max}
\end{aligned}
$$

Solution

Now find M_f as a function of $p_{\max}$

$$\begin{aligned}
M_f &= \frac{\mu p_{\max} b r}{(\sin\theta)_{\max}} \left[r(\cos\theta_1 - \cos\theta_2) + \frac{a}{4}(\cos 2\theta_2 - \cos 2\theta_1) \right] \\
&= \frac{0.3 p_{\max}(0.03)(0.15)}{\sin 90^\circ} \left[(0.15)(\cos 0 - \cos 126^\circ) + \right. \\
&\left. \frac{0.123}{4}(\cos 2(126^\circ) - \cos 2(0)) \right] \\
&= 2.67 \times 10^{-4} p_{\max}
\end{aligned}$$

Solution

$$\begin{aligned}
Fc &= M_n - M_f \\
2000(0.212) &= p_{\max}(7.38 - 2.67) \times 10^{-4} \\
p_{\max} &= 9.00 \times 10^5 \text{ Pa}
\end{aligned}$$

Solution

Braking torque of the right shoe is

$$\begin{aligned}
T_R &= \frac{\mu r^2 b p_{\max}}{(\sin\theta)_{\max}}(\cos\theta_1 - \cos\theta_2) \\
&= \frac{(0.3)(0.15^2)(0.03)(9.00 \times 10^5)}{1}(\cos 0^\circ - \cos 126^\circ) \\
&= 289 \text{ N-m}
\end{aligned}$$

Solution

To calculate braking torque in left shoe, we also must calculate $p_{\max}$. M_n and M_f are now both clockwise.

$$\begin{aligned}
Fc &= M_n + M_f \\
2000(0.212) &= (7.38 + 2.67) \times 10^{-4} p_{\max} \\
p_{\max} &= 4.22 \times 10^5 \text{ Pa}
\end{aligned}$$

Solution

Braking torque of the left shoe is

$$\begin{aligned}
T_L &= \frac{\mu r^2 b p_{\max}}{(\sin\theta)_{\max}}(\cos\theta_1 - \cos\theta_2) \\
&= \frac{(0.3)(0.15^2)(0.03)(4.22 \times 10^5)}{1}(\cos 0^\circ - \cos 126^\circ) \\
&= 136 \text{ N-m}
\end{aligned}$$

Solution

Total braking torque is

$$\begin{aligned} T &= T_L + T_R \\ &= 289 + 136 = 425 \text{ N-m} \end{aligned}$$

Band Brakes

Principles of Band Brakes

- Rely on friction between band and drum
- Similar to pulley-belt system

$$T = (F_1 - F_2)r$$

Belt Tension

$$\begin{aligned} dF &= \mu dN \\ dN &= 2(Fd\theta/2) = Fd\theta \\ \frac{dF}{F} &= \mu d\theta \\ \ln \frac{F_1}{F_2} &= \mu\theta \\ \frac{F_1}{F_2} &= e^{\mu\theta} \end{aligned}$$

Example: An Exercise Bike

An exercise bike has an adjustable band brake on the wheel to provide different levels of resistance. What should the slack side belt tension be so that the biker can exercise with $T = 50$ N-m. Take $\theta = 150°$ and $\mu = 0.2$, the bike wheel $r = 50$ cm.

Solution

$$\begin{aligned} \frac{F_1}{F_2} &= e^{\mu\theta} \\ T &= (F_1 - F_2)r \\ T &= (e^{\mu\theta} - 1)F_2 r \\ F_2 &= \frac{50}{(e^{0.3(150(\pi/180))}) - 1)(0.5)} \\ &= 83.8 \text{ N} \end{aligned}$$

Disc Clutches and Brakes

Working Principles

Disc Wear

- new disc is rigid
- uniform pressure at first, but the outer area wears faster because of higher velocity
- after a while, pressure is no longer uniform, but wear becomes uniform

Torque Calculation

1. Uniform pressure: new disc
2. Uniform wear: old disc

Uniform Pressure

$$\begin{aligned} dF &= pdA \\ dT &= \mu r dF = \mu r p dA \\ T &= \int_{r_o}^{r_i} \int_0^{2\pi} \mu r p (r dr d\theta) \\ &= \frac{2}{3} \mu \pi p \left(r_o^3 - r_i^3 \right) \end{aligned}$$

Uniform Pressure (cont.)

- Taking the actuating force $F = p\pi(r_o^2 - r_i^2)$

$$T = \frac{2\mu F \left(r_o^3 - r_i^3 \right)}{3 \left(r_o^2 - r_i^2 \right)}$$

- For N parallel discs

$$T = \frac{2\mu F N \left(r_o^3 - r_i^3 \right)}{3 \left(r_o^2 - r_i^2 \right)}$$

Uniform Rate of Wear

- Rate of Wear $\propto$ Friction Work Rate

$$pr = C$$

- Max pressure occurs at inside radius, hence the constant is

$$pr = C = p_{\max} r_i$$

Braking Torque

$$
\begin{aligned}
dF &= p dA \\
dT &= \mu r dF = \mu r p dA = \mu p_{\max} r_i dA \\
T &= p_{\max} r_i \int_{r_o}^{r_i} \int_0^{2\pi} r dr d\theta \\
&= \mu \pi p_{\max} r_i \left(r_o^2 - r_i^2\right)
\end{aligned}
$$

Braking Torque (cont)

- Taking into account actuating force F

$$T = \mu F \left(\frac{r_o + r_i}{2}\right)$$

- For N parallel discs

$$T = \mu F N \left(\frac{r_o + r_i}{2}\right)$$

Usual Guideline for Disc Brakes/Clutches

1. $0.45 r_o < r_i < 0.8 r_o$
2. Use uniform wear rate, unless for short-term application

Example: Automotive Clutch

- Design a wet clutch to transfer the torque of 100 N-m using the material with $\mu = 0.08$ and $p_{\max} = 1500$ kPa. Space requirements only allow $r_o \leqslant 60$ mm. Determine the inner diameter and number of discs.

Solution

- Use $r_i = 30$ mm,

$$
\begin{aligned}
N &= \frac{T}{\left[\mu \pi p_{\max} r_i \left(r_o^2 - r_i^2\right)\right]} \\
&= \frac{100}{\left[(0.08)\pi(1500 \times 10^3)(0.03)\left(0.06^2 - 0.03^2\right)\right]}
\end{aligned}
$$

$N = 4$ and $d_i = 2r_i = 60$ mm

Drum Brakes vs Disc Brakes

Drum	Disc
self-energizing possible	no self-energizing
very sensitive to μ	not sensitive to μ
requires larger force once μ goes down	well-designed caliper compensate for wear and exert constant pressure

6 Flexible Mechanical Elements

Flexible Mechanical Elements?

- belts
- ropes
- chains
- used in transmission of power over long distances

Why Flexible?

- Torque capacity: Gears > Belts, Ropes, Chains
- Flexible elements are better against vibration and shock loads
- Important to check for wear, age, and loss of elasticity

Flat Belts

Belt Tension

- For low speed belt drive

$$T = (F_1 - F_2)\, r$$

Belt Tension II

- For high speed belt drive

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{\mu\theta}$$

where $F_c = m\omega^2 r^2$ is the centrifugal force on the belt and m is the mass per length of belt

Power Transmitted

$$P = (F_1 - F_2)v$$

Maintaining Belt Tension

Belt Design

$$(F_1)_a = bF_a C_p C_v$$

$$
\begin{aligned}
(F_1)_a &= \text{allowable largest tension} \\
b &= \text{belt width} \\
F_a &= \text{manufacturer's allowed tension (N/m)} \\
C_p &= \text{pulley correction factor} \\
C_v &= \text{velocity correction factor} = 1 \text{ except leather belts}
\end{aligned}
$$

Manufacturer's Allowed Tension: F_a

Pulley Correction Factor: C_p

Velocity Correction Factor: C_v

V-Belts

Why V Belts?

- Increased tension forces belt further into groove, providing more friction
- Increased torque capacity at a slightly lower efficiency

Belt Tension

$$
\begin{aligned}
T &= (F_1 - F_2)\,r \\
\frac{F_1 - F_c}{F_2 - F_c} &= e^{\frac{\mu\theta}{\sin\beta}}
\end{aligned}
$$

V Belt Design Equation

$$
N_s = \frac{P_a N}{P_{nom} K_s}
$$

- N_s: safety factor
- P_a: allowable power per belt
- N: number of belts (integer)
- P_{nom}: nominal power = $(F_1 - F_2)v$
- K_s: service factor

Allowable Power

Service Factor

Timing Belts

- No significant stretch or slip → power where speed ratio is important
- efficiency of 97 - 99%
- No need for lubrication
- Quieter than chain drives
- Same design equations as V-belts

Example

Design a urethane flat belt to connect two shafts. The driving pulley (d = 3 cm) is connected to a 1-kW motor. The driven pulley has D = 8 cm. The motor rotates at 500 rpm.

Solution

Assuming that the conveyor will utilize full motor power. The belt tension difference is

$$
\begin{aligned}
P &= (F_1 - F_2)\,v \\
F_1 - F_2 &= \frac{2000}{0.1} = 20000N
\end{aligned}
$$

Roller Chains

Why Roller Chain (Chain Drives)?

- No slip → constant ratio
- Long life
- Can drive multiple shafts from a single source

Equations on Roller Chains

$$
D = \frac{p}{\sin\left(\dfrac{180^\circ}{N}\right)}
$$

$$
V = Npn
$$

- N = number of sprocket teeth
- p = chain pitch
- n = sprocket speed in revolution/min

Design Criteria

- Roller chains rarely fail because of tensile stress
- Need to worry more about fatigue and wear on rollers
- To minimize chordal speed variation. $N \geqslant 17$

Power Capacity

- For link-plate limit

$$H_1 = 0.004N_1^{1.08}n_1^{0.9}p^{3-0.07p}\ \text{hp}$$

- For roller limit

$$H_2 = \frac{1000K_tN_1^{1.5}p^{0.8}}{n_1^{1.5}}\ \text{hp}$$

- N_1 = number of teeth in smaller sprocket
- n_1 = sprocker speed [rev/min]
- p = pitch of chain [in]
- K_r = 29 for chain numbers 25,35; 3.4 for 41; and 17 for 40-240

Roller Chain Rated Capacity (1)

Roller Chain Rated Capacity (2)

Example

Pick a roller chain for a motorcycle whose output is 15 hp. The driving sprocket has 20 teeth and the output has 39 teeth.

7 Flexible Machine Elements

8 Power Transmission Case Study

Example 8.1 Let us consider a design for a conveyor belt system. The conveyor belt needs to be moving at 60 cm/s. Both the driving and driven pulleys have the same diameter of 40 cm. The driving pulley is mounted on the same shaft as spur gear B, which is driven by spur gear A connected directly to the driving motor.

Spur gear A has D_{pitch} = 5 cm, z = 10 teeth, and pressure angle $\theta = 20°$. The motor is set to operate constantly at 500 rpm and providing the torque of 2 N-m.

Design the following:

1. the driven spur gear B (determine the number of teeth and pitch diameter).
2. width b of the conveyor belt. (Belt material is polyamide F-1, μ = 0.5, thickness t = 1.3 mm, γ = 9.5 kN/m^3, F_a = 6 kN/m, C_v = 1)
3. the radius of the shaft on which spur gear B and the driving pulley are mounted. (E = 210 GPa, S_y = 300 MPa, N_s = 2.5)
4. radial contact bearings for 9 $\times 10^7$ revolutions, 90% reliability (series and bore size).

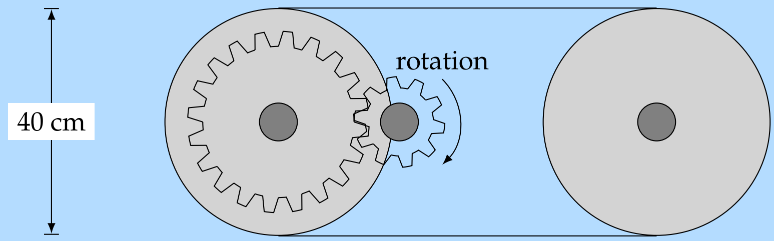


Figure 8.1: side view of conveyor assembly

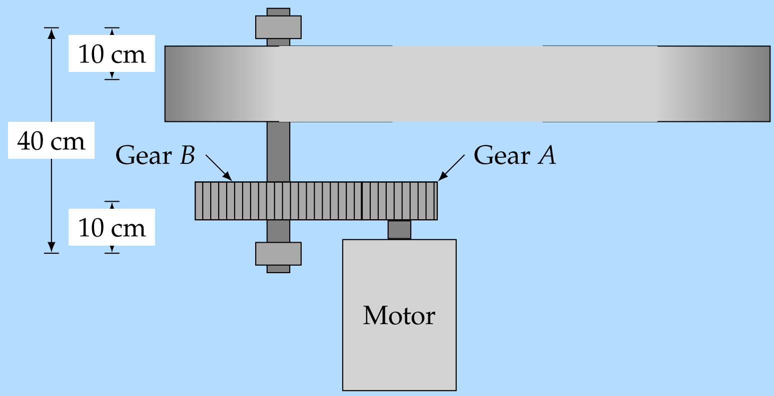


Figure 8.2: top view of conveyor assembly

Solution 1. To determine the number of teeth of spur gear B, we need to determine the required pitch line velocity. The belt must be moving at 60 cm/s, which means the pulley must be rotating at

$$\begin{aligned}\omega_{\text{pulley}} &= \frac{0.6}{\pi(0.2)} \\ &= 0.95 \text{ rev/s} = 57.3 \text{ rpm}\end{aligned}$$

since spur gear B is mounted on the same shaft as the pulley, it must have the same ω. The pitch line velocities on two meshing gears must be the same. Spur gear A rotates at 500 rpm, therefore, the pitch radius of spur gear B is

$$\begin{aligned}r_B &= \frac{\omega_A r_A}{\omega_B} \\ &= 21.8\end{aligned}$$

Its diameter, of course, is 43.6 cm. Given that meshing gears must also have the same module, the number of teeth on B is

$$\begin{aligned}m &= \frac{D_{\text{pitch,A}}}{z_A} = 510 = 0.5 \\ &= \frac{43.6}{z_B} \\ z_B &= 87.2 \text{ teeth}\end{aligned}$$

2. Assuming no power loss, the power the belt-pulley system needs to transfer is $P = T\omega = 2(500)(2\pi/60) = 104.7$ W. Required torque transfer of the belt is $T = P/\omega = 104.7/(57.3(2\pi/60)) = 17.4$ N-m. The tension difference on the two side needs to be

$$\begin{aligned}T &= (F_1 - F_2) r_{\text{pulley}} \\ F_1 - F_2 &= \frac{T}{r_{\text{pulley}}} = 87 \text{ N}\end{aligned}$$

We can now write an equation to determine the thickness b of the belt

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{\mu\theta}$$

Set $F_1 = (F_1)_a = bF_a C_v C_p = b(6000)(1)(0.7) = 4200b$, $F_2 = F_1 - 87 = 4200b - 87$, $\mu = 0.5$, $\theta = \pi$, $F_c = (\gamma/g)bt\omega^2r^2 = (9500/10)b(0.0013)(0.95(2\pi))^2(0.2)^2 = 1.76b$

$$\begin{aligned}\frac{4200b - 1.76b}{4200b - 87 - 1.76b} &= e^{0.5\pi} \\ b &= 0.0261 = 2.61 \text{ cm}\end{aligned}$$

3. To design the shaft, we must first determine the loads on the shaft, which are exerted by spur gear B and the pulley. Let us determine the load from each component.

   The load from the gear can simply be determined by

$$F = \frac{T}{r\cos\theta} = \frac{17.4}{(0.2)(\cos 20^\circ)} = 92.6 \text{ N}$$

So the force exerted on the shaft by gear B is 92.6 N in the direction 20° downward from the tangential line between the two gears.

The load from the pulley is even simpler

$$F_p = F_1 + F_2 = 4200(0.0261) + 4200(0.0261) - 87 = 132.24 \text{ N}$$

The force exerted on the shaft by the pulley is 132 N to the right.

Now all we need is to determine the maximum bending moment on the shaft.

First, bending moment M_g generated by force in the gear is

$$M_g = \frac{F_g x(L-x)}{L} = \frac{92.6(0.1)(0.4-0.1)}{0.4} = 6.95 \text{ N-m}$$

Next, bending moment M_p generaed by force on the pulley is

$$M_p = \frac{F_p x(L-x)}{L} = \frac{132(0.3)(0.4-0.3)}{0.4} = 9.9 \text{ N-m}$$

The maximum bending moment occurs in the middle, where

$$M_{\text{max}} = (6.95 + 9.9)\frac{2}{3} = 11.2 \text{ N-m}$$

The maximum torque is just the required torque transfer, which is $T = 17.4$ N-m. We can now determine the required shaft size.

$$N_s = \frac{S_y}{\sigma_e}$$

$$2 = \frac{300 \times 10^6}{\sqrt{\left(\frac{4(11.2)}{\pi r^3}\right)^2 + 3\left(\frac{2(17.4)}{\pi r^3}\right)^2}}$$

$$r = 0.0054 = 5.4 \text{ mm}$$

4. Since the problem is not symmetrical, we need to determine the bearing under the higher load. This happens to the bearing closer to the higher force, which is the pulley. Taking the other bearing as a fulcrum, the radial load in the bearing on the pulley side is

$$
\begin{aligned}
F_r &= \frac{132(3)}{4} + \frac{92.6(1)}{4} \\
&= 122.2 \text{ N}
\end{aligned}
$$

With $L = 9 \times 10^7$ and $K_r = 1$, $F_e = F_r = 122.2 = C$. This is smaller than the smallest bore which is 10mm for all series. We therefore should pick the smallest bore (10 mm) as our choice.

Bibliography

[1] Joseph Edward Shigley. *Shigley's mechanical engineering design*. Tata McGraw-Hill Education, 2015.