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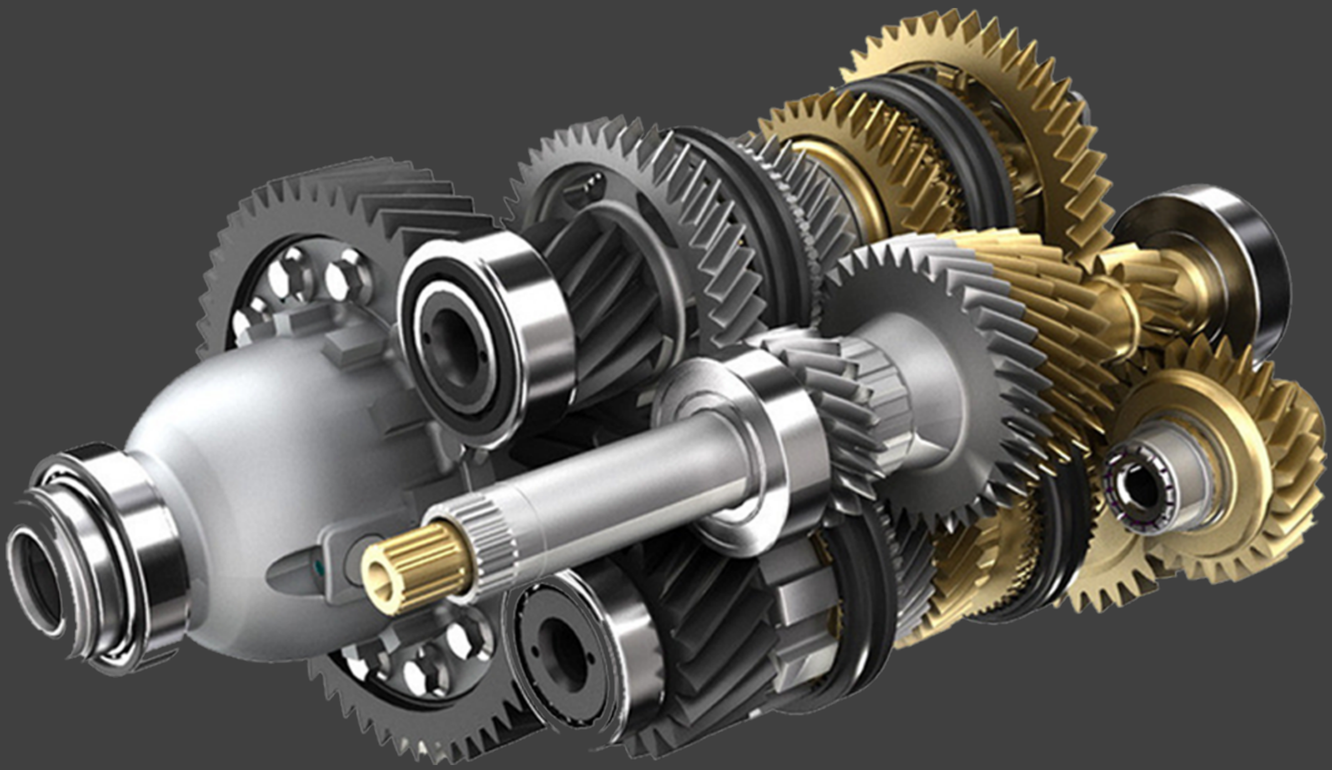

ME 310: Mechanical Design

Power Transmission

Sappinandana Akamphon

Preface

This book is the second part of ME 310: Mechanical Design. In this book, power transmission components are covered.

Sappinandana Akamphon

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Revisions

September 2019

Start of Mechanical Design: Power Transmission

January 2020

Began working on the chapters on shaft components, gears, brakes and clutches

October 2021

Completed chapters on gears, shafts,

Shaft and Shaft Components

Most engineering systems are powered by rotational machinery such as internal combustion engines and electrical motors. It is, thus, extremely important that we understand and properly design power tranmission mechanism from the rotational machinery power source to intended components. Shafts remains one of the most prevalent methods of transmitting rotational power, and so it is our first chapter into the world of power transmission design.

By itself, shaft design is no more complex than other components under static or cyclic loadings already covered in the first book of this series. However, as shafts lie at the heart of most machine design applications, its final design will also depend on the design of other components that are to be mounted or connected to the shaft.

Shaft Materials

There are two main concerns with designing a shaft: deflection and strength. Deflection is not affected by material strength. In fact, it depends only on the stiffness, represented by the Young's modulus. Since the Young's modulus is essentially the same for all steels, material choice matters little for deflection.

Strength, on the other hand, constitute a major concern for shaft design. Strength is necessary to resist loading stresses and fatigue stresses from the constant rotation. Many shafts are made from low carbon, cold-drawn or hot-rolled steel such as ANSI 1020-1050 steels. Significant strengthening from heat treatment or high alloy is often not neeeded. And fatigue failure is only reduced moderately by increase in strength, after a certain level notch sensitivity begins to counteract its benefit.

A good way to select a proper shaft material is to start with an inexpensive low or medium carbon steel for the first round of calculations. If strength considerations turn out to be the limiting factor over deflection, select a higher strength material and reduce the shaft size accordingly until deflection becomes an issue. The cost of the material and its manufacturing processes must be weighed against the need for smaller shaft diameter.

When additional strengthening is needed, typical alloy steels for heat treatment include ANSI 340-50, 3140-50, 4140, 4340, 5140, and 8650. Shafts don't usually need surface hardening unless there is significant risk of wear from journal bearing, in which case surface hardening include carburizing grades of ANSI 1020, 4320, 4820, and 8620. [1]

Shaft Layout

Generally a shaft is a cylinder with segments of varying cross sections to accommodate components like gears, bearings, pulleys, etc. Shaft shoulders are normally used to axially locate elements and to take any axial thrust loads. illustrates the cross-section of a vertical worm-gear speed reducer. In this figure, the shaft steps are used to axially locate the two tapered bearings and the spur gear.

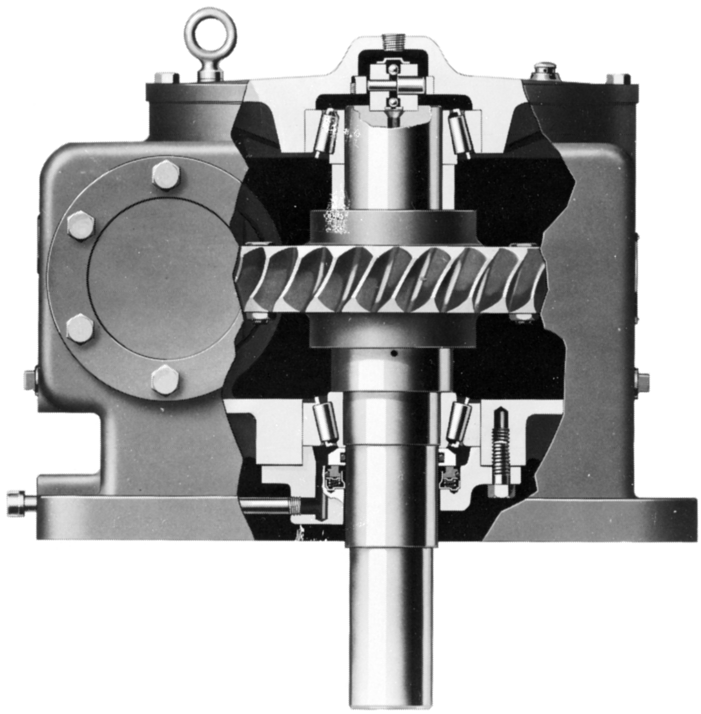


Figure 1: A vertical worm-gear speed reducer. *(Courtesy of the Cleveland Gear Company.)*

Axial Layout of Components

The axial positioning of components is often preset by the layout of the housing and other meshing components. Generally it is advisable to support any loading-carrying components between bearings. However, pulleys and sprockets tend to be mounted outside of the bearing pairs for ease of installation of the belt or chain. The length of the shaft should be kept as short as possible to minimize deflection and resultant load on the bearings.

In most cases, a shaft will only require two bearings, but in cases with long shafts carrying multiple load-carrying components, it may be necessary to use more than two bearings for additional support. In such a case, particular care must be given to the alignment of the bearings.

Supporting Axial Loads

There are use cases where axial loads in shaft may be significant, in which case it is important to provide a means to transfer the axial loads into the shaft. The shaft would then transfer the loads to a bearing and to the ground. This is necessary for shafts mounted with components that generate axial loads like bevel and helical gears, or tapered roller bearings.

Figure 2: Supports in shafts with axial loads.

It is usually sufficient to have only one bearing carry the axial load. This allows for greater tolerances on shaft length dimensions and prevents binding from shaft thermal expansions, which is especially significant in long shafts.

Support for Torque Transmission

Most shafts serve to transfer torque from an input (gear, pulley, engine, motor, etc.) to an output gear or pulley. The shaft must be sized to support torsional stress and its resultant angle of twist. It is also important to provide a way to transmit the torque between the gear (or pulley) and the shaft itself. Common torque transfer methods are:

- Keys
- Splines
- Setscrews

- Pins
- Press or shrink fits
- Tapered fits

There are also shafts that are designed to fail if excessive torque is applied, to prevent failure of more expensive components. Details of the components and their design process is covered in .

Shaft Design for Stress

It is not always necessary to evaluate stresses at every point; the same goes for shafts as well. Only a few potentially critical locations should be more than enough. Since the main types of load on shafts are torsion and bending, it follows that most critical locations on the shafts are on the outer surface–typically where the bending moment is large, the torque is large, and where stress concentrations exist.

In order to determine the bending moments, torques, and shear forces on a shaft, it is usually a good idea to draw shear and bending moment diagrams. Since most shafts are loaded by gears and pulleys, introducing forces in two planes, two diagrams are needed to determine the loads. Resultant moments can be obtained simply by adding the moments as vectors at points of interest. The normal stress due to bending will be highest on the outer surfaces and will contribute to fatigue on a rotating shaft.

Axial stresses on shafts from axial loads caused by helical gears or tapered roller bearings are typically negiligible compared to the bending stress. The axial stresses are also usually constant, meaning that they rarely contribute significantly to fatigue. However, axial stresses resulting from axial loads applied through other means should be explicitly considered.

Let us now consider the shaft stresses, which are usually the combination of normal stresses from bending and axial stresses, and shear stress from torsion.

$$\begin{aligned} \sigma_a &= K_f \frac{M_a y}{I} \quad & \sigma_m &= K_f \frac{M_m y}{I} \\ \tau_a &= K_{fs} \frac{T_a r}{J} \quad & \tau_m &= K_{fs} \frac{T_m r}{J} \end{aligned} \tag{1}$$

If we assume a solid shaft with circular cross section, we can further simplify the expression to

$$\begin{aligned} \sigma_a &= K_f \frac{32M_a}{\pi d^3} \quad \sigma_m = K_f \frac{32M_m}{\pi d^3} \\ \tau_a &= K_{fs} \frac{16T_a}{\pi d^3} \quad \tau_m = K_{fs} \frac{16T_m}{\pi d^3} \end{aligned} \tag{2}$$

The stresses can be combined into stress amplitude and average stress using maximum distortion energy theory (MDET or von Mises) as

$$\sigma_{ae} = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_a}{\pi d^3}\right)^2\right]^{1/2} \tag{3}$$

$$\sigma_{me} = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[\left(\frac{32K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2} \tag{4}$$

The stress concentration factors for the average stress component in ductile materials can sometimes be ignored since the materilas can yield locally at the discontinuity.

These equivalent stresses can be evaluated in design equations to determine the safety factor N_s or the required diameter d using the the modified Goodman diagram as

$$\frac{1}{N_s} = \frac{\sigma_{ae}}{S_e} + \frac{\sigma_{me}}{S_{ut}} \tag{5}$$

Substituting for σ_{ae} and σ_{me} results in

$$\frac{1}{N_s} = \frac{16}{\pi d^3} \left\{ \frac{1}{S_e} \left[4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2\right]^{1/2} + \frac{1}{S_{ut}} \left[4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2\right]^{1/2} \right\} \tag{6}$$

The required diameter d can be solved from the previous equation as

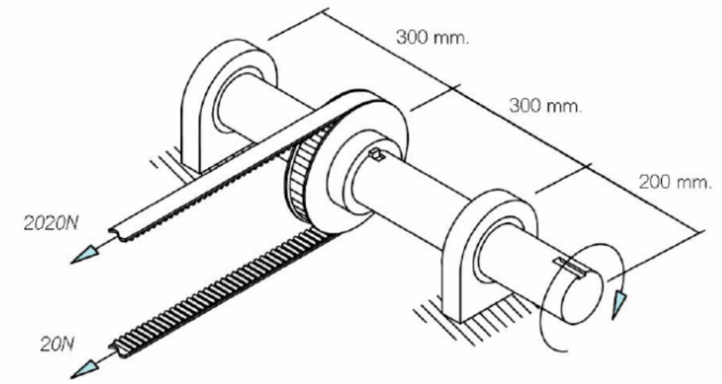
$$d = \left(\frac{16N_s}{\pi} \left\{ \frac{1}{S_e} \left[4\left(K_f M_a\right)^2 + 3\left(K_{fs} T_a\right)^2\right]^{1/2} + \frac{1}{S_{ut}} \left[4\left(K_f M_m\right)^2 + 3\left(K_{fs} T_m\right)^2\right]^{1/2} \right\} \right)^{1/3} \tag{7}$$

In many applications, rotating shafts will be under constant bending and torsion, resulting in completely reverse bending stress

($M_m = 0$) and constant torsional shear stress ($T_a = 0$). This means the required diameter becomes

$$d = \left(\frac{16N_s}{\pi} \left\{ \frac{2K_f M_a}{S_e} + \frac{\sqrt{3}K_{fs}T_m}{S_{ut}} \right\} \right)^{1/3} \tag{8}$$

Example: Shaft sizing



Size the shaft (AISI 1040, S_y = 400 MPa, S_{ut} = 600 MPa) using

1. MDET
2. Soderberg theory

so that the safety factor N_s = 3.

Solution

1. MDET: The torque loaded on the pulley by the belt is

$$\begin{aligned} T &= (2020 - 20)(0.1) \\ &= 200.0 \text{ N-m} \end{aligned}$$

There is also 2040 N of force pulling at the pulley due to the combined belt tension. The torque generates shear stress throughout the shaft, with the maximum value at the surface. The belt tension creates bending stresses, whose maximum values are are the top and bottom of the shaft at the middle. This means that the critical points on the shaft (without considering stress concentration from the key/keyseat) are at the top and bottom of the shaft at the middle. In this problem, we will take the bottom of the shaft at the middle. The stress concentration of the keyseat is taken to be K_f = 2.14 in bending and K_{fs} = 3.0 in torsion.

$$
\begin{aligned}
\sigma &= K_f \frac{My}{I} = 2.14 \frac{2040(0.6)(r)}{4\pi r^4/4} \\
&= \frac{834}{r^3} \text{ Pa} \\
\tau &= K_{fs} \frac{Tr}{J} = 3 \frac{(200.0)(r)}{\pi r^4/2} \\
&= \frac{382}{r^3} \text{ Pa}
\end{aligned}
$$

2. Soderberg: using the criteria, we must calculate the minimum and maximum bending moments and torques, which will then be used to determine the stress amplitudes and average stresses. We already determine the maximum bending moment and torque, which we used to determine the corresponding stresses for MDET. We now only need to find out the minimum bending moment and torque. The minimum bending moment occurs when the shaft rotates by half a revolution, for which the beam willbe under a compressive stress of the same magnitude.

$$
\begin{aligned}
\sigma_{\text{min}} &= -\frac{834}{r^3} \\
\sigma_a &= \frac{\sigma_{\text{max}} - \sigma_{\text{min}}}{2} \\
&= \frac{834}{r^3}
\end{aligned}
$$

If the shaft is under continuous operation, the applied torque is constant, which means that the torque amplitude T_a = 0 and the average torque $T_m = T$. We can plug this into the sequation to determine equivalent amplitude and average stresses.

$$
\begin{aligned}
\sigma_{ae} &= \sqrt{\sigma_a^2 + 3\tau_a^2} = \frac{834}{r^3} \\
\sigma_{me} &= \sqrt{\sigma_m^2 + 3\tau_m^2} = \frac{662}{r^3} \\
\frac{1}{3} &= \frac{834}{0.5(6.00 \times 10^8) r^3} + \frac{662}{4.00 \times 10^8 (r^3)} \\
r &= 0.0237 \text{ m}
\end{aligned}
$$

Using the settings from the previous example, redetermine the shaft size if the maximum operating speed is 10000 rpm.

For a simply supported shaft, the first natural frequency that can cause shaft whirling is

$$\omega_1 = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{EI}{A\rho}} \tag{9}$$

We must first convert the angular velocity into rad/s: 10000 rpm = $10000(2\pi/60)$ = rad/s. To achieve the safety factor of 3, the first natural frequency of the shaft must be

$$\begin{aligned}\omega_1 = 3(1047) &= \left(\frac{\pi}{0.6}\right)^2 \sqrt{\frac{2.10 \times 10^{11} \pi r^4/4}{\pi r^2 \rho}} \\ r &= 0.044 \text{ m}\end{aligned}$$

The designed shaft has to follow the largest shaft that satisfy each of the condition, therefore the required radius is 4.4 cm.

Torque Trasmission Components

For a designed shaft to properly transfer torque to its intended target, not only must the shaft be able to with stand the stresses resulting from the torque, but it must also be assembled with torque transmission components that are capable of transferring such torque. Some of the mechanisms currently in use can be categorized into:

1. Mechanical drive assembly
2. Interference fit assembly
3. Welded assembly

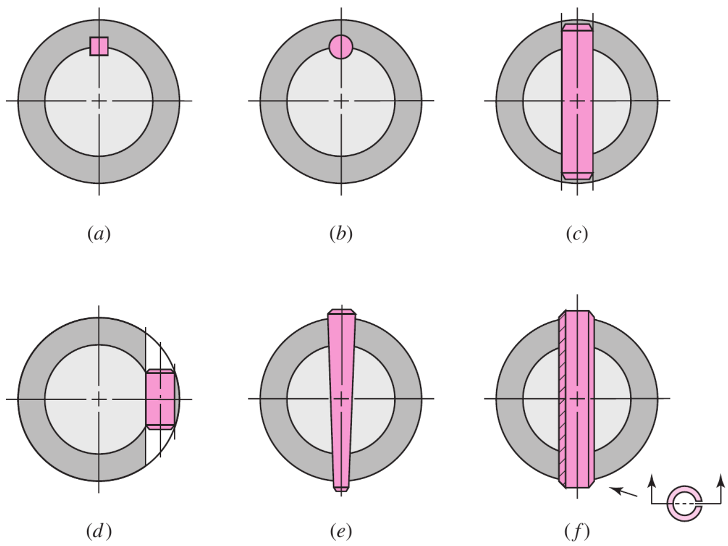
Mechanical Drive Assembly

Mechanical drive assembly typically requires adjustments to the shaft and/or hub of the component to be assembled. The most common mechanical-drive assembly is the conventional key and keyway. Other assemblies are set screws, pins, and spline shafts.

Keys and Pins

Keys are axially inserted metal that provides interference between the shaft and hub, allowing torque trasmission between the two. Aligning keyways need to be cut into both the shaft and the hub.

Pins, on the other hand, are transversely inserted perpendicular to the axis of the shaft. Both the shaft and hub still needs to be drilled.



While pins allow for torque transmission and axial positioning, keys only allow torque transmission. However, because of the typical long length of keys inserted axially which provides larger cross-sectional area, they typically allow larger torque trasmission than pins of the same size.

Torque capacity of keys and pins can be calculated by

$$
\begin{aligned}
P_{\text{max}} &= \frac{S_y}{\sqrt{3}} A = 0.577 S_y A \\
T_{\text{max}} &= P_{\text{max}} r_{\text{shaft}}
\end{aligned}
$$

Limitation of Mechanical Drive Assembly

Mechanical drive assembly can provide relatively high torque transmission with relatively easy assembly. However, they have a few shortcomings to consider.

1. Stress concentration: shafts and hubs need to be machined to provide holes/keyways/splines, which will obviously introduce increased stress concentration and thus reduced shaft/hub strength.
2. Backlash: even with very strict tolerances, pins and keys that do not perfectly fit in the holes or keyways will allow relative

motion between the shaft and the hub, leading to backlashes. However, this can be eliminated or mitigated with the use of tapered keys or pins.

Tapered keys or pins have a slowly increasing cross section from one end to the other, therefore allowing a snug fit.

3. Machining costs: as keyways/splines/holes require precise machining, this will incur additional costs on the shaft assembly.

4. Uneven distribution of mass: uniform distribution of mass about the shaft is extremely important for shaft stability especially in high speed application. Usually components such as gears, pulleys, sprockets, etc. are designed to be axisymmetric for this reason. However, the required machining and insertion of keys or pins will introduce asymmetry to the shaft assembly.

Example: Key Sizing

A steel shaft whose S_y = 450 MPa has a diameter of 5 cm. The shaft rotates at 600 rpm and transmits 40 hp through a gear. Select an appropriate key for the gear. Use safety factor = 3.

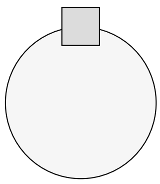


Figure 3: Key and shaft cross section for example ...

Solution: Key Sizing

To keep things simple, pick a square key and pick key length = 2 cm.

$$\begin{aligned} T &= \frac{H}{\omega} = \frac{40(746)}{600(2\pi/60)} \\ &= 475 \text{ N-m} \end{aligned}$$

For the width (and height) of the key section,

$$\begin{aligned} N_s T_{\max} &= 0.577 S_y b l r_{\text{shaft}} \\ b &= \frac{3(475)}{0.577(450 \times 10^6)(0.02)(0.05)} \\ b &= 0.00549 \text{ m} \end{aligned}$$

Interference Fit Assemblies

Interference fit refers to a type of joint that relies on friction between the hub and shaft to transfer torque. The friction results from the compression of the hub on the shaft, which means the diameter of the hole on the hub must be slightly smaller than that of the shaft. These type of assemblies are divided into 3 categories based on method of assembly.

1. Press fit:

   The word 'press' here refers to the assembly method of pressing the shaft into a hole that is slightly smaller. This process relies on the elasticity of both materials to allow the shaft to slide in without any permanent deformation. This requires extremely strict tolerances on both the shaft and the hub.

2. Tapered fit:

   In case of tapered fits, an additional collet, which is a collar that is tapered on the outside along the length, and has a constant cross section on the inside. The collet essentially acts like a wedge between the shaft and hub, allowing the assembler to control the magnitude of compressive (and resulting friction) force on the shaft by the axial load exerted on the collet.

3. Shrink fit:

   Similar to press fit in that there is no additional 'collet', but instead of 'pressing' the shaft onto the hub, either the shaft is cooled or the hub is heated (or both) before assembly. This eliminates the difficulty of assembly and allows for greater difference in the diameters of the shaft and hub.

These interference fits do not require additional machining on the shaft or hub, which would otherwise increase the stress concentration and incur additional machine costs. However, they do have the following limitations to take into consideration.

1. Materials, surface, and design restrictions: interference fits rely on friction, so the material, surface finishing, and shaft dimensions (diameter mostly) affect the magnitude of friction that can be generated. This limits your available choices.

2. Close tolerance: interference fits require that the hubs and shafts have extremely close and accurate tolerances, requiring precise machining. This leads to high machining costs.

3. Fretting: high stress on surfaces that undergo repeated motions are prone to fretting corrosion.

4. Surface galling: high compressive load on mating surfaces can cause them bind. This complicates dissasembly, which usually results in surface failure.

5. High stress in components: interference fit requires high compressive force from the hub on the shaft to generate friction. This leads to circumferential stresses on the hub and the shaft as well.

Stresses in Interference Fits

Assumed uniform pressure on shaft (i) and hub (o)

$$p = \frac{d_{\text{shaft}} - d_{\text{hub}}}{\frac{d}{E_o}\left(\frac{d_o^2 + d^2}{d_o^2 - d^2} + \nu_o\right) + \frac{d}{E_i}\left(\frac{d^2 + d_i^2}{d^2 - d_i^2} - \nu_i\right)}$$

When both are of the same material

$$p = \frac{E(d_{\text{shaft}} - d_{\text{hub}})}{2d^3}\left[\frac{(d_o^2 - d^2)(d^2 - d_i^2)}{d_o^2 - d_i^2}\right]$$

Tangential and radial stresses in shaft and hub are

$$\sigma_{t,\text{shaft}} = -p\frac{d^2 + d_i^2}{d^2 - d_i^2} \tag{10}$$

$$\sigma_{t,\text{hub}} = p\frac{d_o^2 + d^2}{d_o^2 - d^2} \tag{11}$$

$$\sigma_{r,\text{shaft}} = -p \tag{12}$$

$$\sigma_{r,\text{hub}} = -p \tag{13}$$

Combine σ_t and σ_r using MDET to determine failure

Torque Capacity of Interference Fits

Depends on friction generated between shaft and hub $\rightarrow$ pressure from interference fits

$$\begin{aligned} f &= \mu N = \mu(pA) \\ &= \pi\mu p l d \qquad (14) \\ T &= fd/2 = \pi\mu p l d(d/2) \\ &= \frac{\pi}{2}\mu p l d^2 \qquad (15) \end{aligned}$$

Example: Torque Capacity of an Interference Fit

A solid shaft whose diameter is 5 cm is pressed onto a gear whose hub inner diameter is 4.99 cm and outer diameter is 6 cm. If both are made of the same steel whose E = 210 GPa and ν = 0.3, determine the radial and tangential stresses, along with the torque capacity of the fit. Assume steel-on-steel μ = 0.3, and the hub is 7 cm long.

Solution: Torque Capacity of an Interference Fit

$$\begin{aligned} p &= \frac{E(d_{\text{shaft}} - d_{\text{hub}})}{2d^3}\left[\frac{(d_o^2 - d^2)(d^2 - d_i^2)}{d_o^2 - d_i^2}\right] \\ &= \frac{210 \times 10^9(0.05 - 0.0499)}{2(0.05)^3}\left[\frac{(0.06^2 - 0.05^2)(0.05^2 - 0)}{0.06^2 - 0}\right] \\ &= 64.2 \text{ MPa} \\ \sigma_{r,\text{shaft}} &= \sigma_{r,\text{hub}} = -64.2 \text{ MPa} \\ \sigma_{t,\text{shaft}} &= -64.2\frac{0.05^2}{0.05^2} = -64.2 \text{ MPa} \\ \sigma_{t,\text{hub}} &= 64.2\frac{0.06^2 + 0.05^2}{0.06^2 - 0.05^2} = 356 \text{ MPa} \\ T &= \frac{\pi}{2}\mu p l d^2 = \frac{\pi}{2}(0.3)64.2 \times 10^6(0.07)(0.05^2) = 5294 \text{ N-m} \end{aligned}$$

Weld Assembly

In weld assemby, the shaft and hub of intended components are welded together. This provides a relatively quick and permanent connection between the components. However, weld assemblies also inherit the same disadvantages from welded joints as mentioned in chapter ....

1. Welding only works on compatible materials. Plastics on metals is a no-no. Woods cannot be welded. Ceramics cannot be welded. You get the idea.

2. Heating can cause warpage. Welding introduces uneven heating of the workpiece, which can result in warpage especially in parts with complex geometries.

3. Disassembly. Welding is permanent... mostly. Welding can be undone, although the process is not straightforward and can cause permanent surface damages.

4. Skilled personnel. Welding requires skilled craftmanship, which means it is usually expensive and its quality is hard to control.

5. Cleaning and machining. Welding typically needs cleaning and machining afterwards to obtain desirable surface finish.

Torque Capacity in Weld Assembly

The strength of weld can be applied to determine the torque capacity of shafts and their components in weld assembly. In most cases, weld joints in weld assembly are fillet welds and the corresponding equations apply.

Journal Bearings and Lubrication

Overview of Bearings

What are bearings?

- A feature that allows relative motions between components
  - Linear motions
  - Rotary motions

Two types of bearings

- Contact: sliding or rolling
- Non-contact: fluid film or magnetic

Sliding Contact Bearings

Contact bearings are features that allow relative movements between two surfaces wherein the surfaces remain in contact with each other either directly or indirectly through some solid common medium.

Sliding Contact Bearings

In a sliding contact bearing, the shaft is directly in contact with the inner surface of the bearing. As the shaft rotates, this causes relative motions (sliding) between the shaft and the bearing–hence the name. Because of this sliding, friction plays a significant role in this type of bearings. Special care must be taken to select the materials of both the shaft and bearing so that it can withstand the conditions.

Figure 4: Brass sliding contact bearing

Because of friction, sliding contact bearings are typically used in low- to medium-speed applications. The bearings are also used with lubrication to reduce wear and improve its performance.

Materials Selection for Sliding Contact Bearing

Materials that are chosen for sliding contact bearings are, for the most part, softer that the shaft; it is usually easier to switch out worn bearings than worn shafts.

Even though the bearings are designed for wear, we must make sure that it can handle the contact pressure and the heat generation rate.

First, the contact pressure from the shaft depends on the radial force exerted by the shaft and the effective area of contact. For a bearing of length L and inner diameter D, the average contact pressure generated from the shaft with radial force F is

$$P_{\text{avg}} = \frac{F}{DL} \tag{16}$$

The maximum contact pressure is at the bottom of the contact area, where the surface vector is in the same direction as the radial force.

$$P_{\text{max}} = \frac{4}{\pi}\frac{F}{DL} \tag{17}$$

Secondly, the heat generation rate depends on the friction and the surface velocity of the shaft. Friction depends on the radial force (similar to contact pressure), while the surface velocity can be calculated by

$$v = \omega\frac{D}{2} \tag{18}$$

The heat generation rate is friction force multiplied by the surface velocity. Since friction depends on the contact pressure, rate of heat generation is simply a function of contact pressure and surface velocity.

$$\dot{Q} = f(PV) \tag{19}$$

Therefore, in material catalogs that list properties for sliding contact bearings under P, V, and PV, the last one indicates the heat generation rate of the materials.

Table 1: PV Table for Metals

Material	Static P MPa	Dynamic P MPa	V m/s	PV MPa·m/s
Bronze	55	14	6.1	1.8
Lead-bronze	24	5.5	7.6	2.1
Copper-iron	138	28	1.1	1.2
Hardenable copper-iron	345	55	0.2	2.6
Iron	69	21	2.0	1.0
Bronze-iron	72	17	4.1	1.2
Lead-iron	28	7	4.1	1.8
Aluminum	28	14	6.1	1.8

Combining the limit on contact pressure P, surface velocity v, and the product of the two PV, we can represent the **area of safety** using a plot as shown in Figure

Material	P MPa	Temperature °C	V m/s	PV MPa·m/s
Phenolics	41	93	13	0.53
Nylon	14	93	3.0	0.11
TFE	3.5	260	0.25	0.035
Filled TFE	17	260	5.1	0.35
TFE fabric	414	260	0.76	0.88
Polycarbonate	7	104	5.1	0.11
Acetal	14	93	3.0	0.11
Carbon (graphite)	4	400	13	0.53
Rubber	0.35	66	20	–
Wood	14	71	10	0.42

Table 2: PV Table for Non-metals

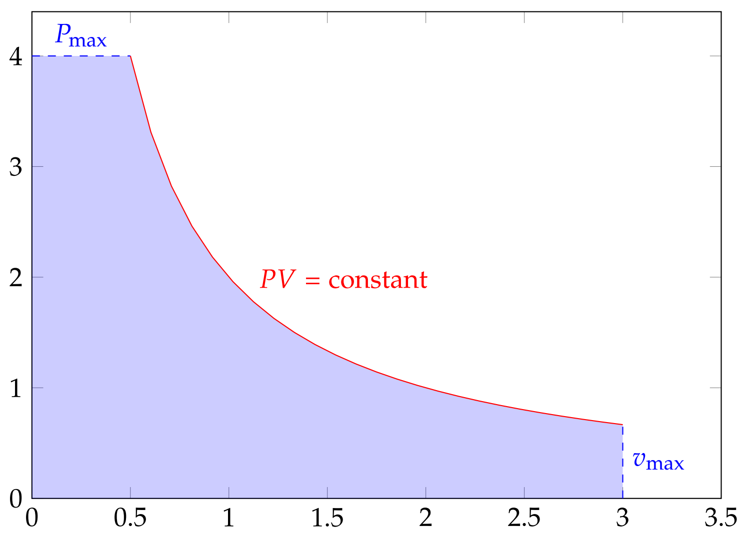


Figure 5: Area of safety for sliding contact bearing materials.

Example: Sleeve Bearing for a Low-speed Shaft

A 30-cm long shaft whose diameter D is 3 cm is operated at 1000 rpm. The shaft has a spur gear whose R_{pitch} = 10 cm mounted in the middle with a bearing at each end. The gear is transferring the power of 1.5 kW. The gear has pressure vessel θ = 20°. Determine the minimum bearing length L using nylon.

Solution

First, let us determine the force on the bearing. Since spur gears don't generate any axial load, the forces will simply be the radial + tangential load, perpendicular to the shaft.

$$\begin{aligned} T &= \frac{P}{\omega} \\ &= \frac{1500}{1000(2\pi/60)} = 14.3 \text{ N-m} \\ F &= \frac{T}{R_{\text{pitch}} \cos\theta} \\ &= \frac{14.3}{0.1 \cos 20^\circ} = 152 \text{ N} \end{aligned}$$

Since the gear is mounted in the middle, the force on each bearing is half of the force.

$$F_{bearing} = \frac{152}{2} = 76 \text{ N}$$

We can't determine the bearing pressure yet since we don't know the bearing length. We can determine the surface velocity, however.

$$v = \omega(D/2) = 1000(2\pi/60)(0.03/2) = 1.57 \text{ m/s}$$

We double-check that $v < V_{nylon}(1.57 < 3.0)$ so nylon is an acceptable choice. The length of bearing, then should be

$$\begin{aligned}
P_{bearing} v &< (PV)_{nylon} \\
\frac{F_{bearing}}{DL} v &< 0.11 \times 10^6 \\
\frac{76}{0.03L} 1.57 &< 1.1 \times 10^5 \\
L &> 0.036 = 3.6 \text{ cm}
\end{aligned}$$

Rolling Contact Bearings

Another type of bearings avoids friction by introducing rolling elements between the inner surface (in contact with shaft) and outer surface (in contact with hub). The rolling elements can be spherical, cylindrical, or conical. Because these bearings eliminate friction almost entirely, they are suitable for medium to high-speed applications.

Types of rolling contact bearings

Ball bearings

Roller bearings

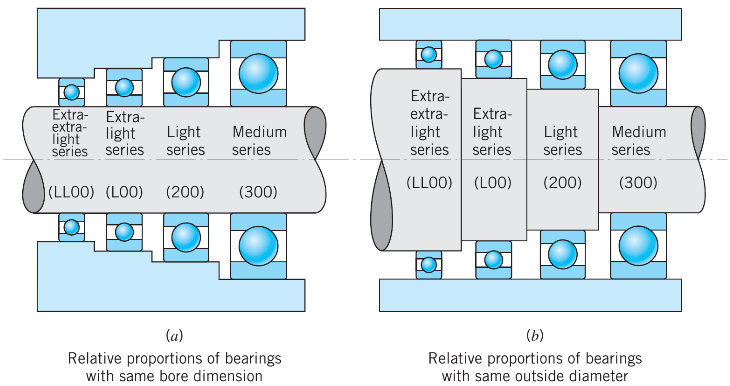


Figure 6: Bearing Series

Bearing Life Requirement

$$L = L_R K_r \left(\frac{C}{F_e}\right)^{10/3} \tag{20}$$

$$C = F_e \left(\frac{L}{K_r L_R}\right)^{0.3} \tag{21}$$

Bearing Basic Number	Bore (mm)	Ball Bearings OD (mm)	w (mm)	r^a (mm)	d_S (mm)	d_H (mm)	Roller Bearings OD (mm)	w (mm)	r^a (mm)	d_S (mm)	d_H (mm)
L03	17	35	10	0.30	19.8	32.3	35	10	0.64	20.8	32.0
203	17	40	12	0.64	22.4	34.8	40	12	0.64	20.8	36.3
303	17	47	14	1.02	23.6	41.1	47	14	1.02	22.9	41.4
L04	20	42	12	0.64	23.9	38.1	42	12	0.64	24.4	36.8
204	20	47	14	1.02	25.9	41.7	47	14	1.02	25.9	42.7
304	20	52	15	1.02	27.7	45.2	52	15	1.02	25.9	46.2
L05	25	47	12	0.64	29.0	42.9	47	12	0.64	29.2	43.4
205	25	52	15	1.02	30.5	46.7	52	15	1.02	30.5	47.0
305	25	62	17	1.02	33.0	54.9	62	17	1.02	31.5	55.9
L06	30	55	13	1.02	34.8	49.3	47	9	0.38	33.3	43.9
206	30	62	16	1.02	36.8	55.4	62	16	1.02	36.1	56.4
306	30	72	19	1.02	38.4	64.8	72	19	1.52	37.8	64.0
L07	35	62	14	1.02	40.1	56.1	55	10	0.64	39.4	50.8
207	35	72	17	1.02	42.4	65.0	72	17	1.02	41.7	65.3
307	35	80	21	1.52	45.2	70.4	80	21	1.52	43.7	71.4
L08	40	68	15	1.02	45.2	62.0	68	15	1.02	45.7	62.7
208	40	80	18	1.02	48.0	72.4	80	18	1.52	47.2	72.9
308	40	90	23	1.52	50.8	80.0	90	23	1.52	49.0	81.3
L09	45	75	16	1.02	50.8	68.6	75	16	1.02	50.8	69.3
209	45	85	19	1.02	52.8	77.5	85	19	1.52	52.8	78.2
309	45	100	25	1.52	57.2	88.9	100	25	2.03	55.9	90.4
L10	50	80	16	1.02	55.6	73.7	72	12	0.64	54.1	68.1
210	50	90	20	1.02	57.7	82.3	90	20	1.52	57.7	82.8
310	50	110	27	2.03	64.3	96.5	110	27	2.03	61.0	99.1
L11	55	90	18	1.02	61.7	83.1	90	18	1.52	62.0	83.6
211	55	100	21	1.52	65.0	90.2	100	21	2.03	64.0	91.4
311	55	120	29	2.03	69.8	106.2	120	29	2.03	66.5	108.7

Figure 7: Bearing table

L	life corresponding to equivalent load F_e
L_R	life corresponding to rated capacity = 9×10^7 rev
K_r	reliability factor
C	rated capacity
F_e	equivalent load

Bore (mm)	Radial Ball, $\alpha = 0°$ L00 Xlt (kN)	200 lt (kN)	300 med (kN)	Angular Ball, $\alpha = 25°$ L00 Xlt (kN)	200 lt (kN)	300 med (kN)	Roller 1000 Xlt (kN)	1200 lt (kN)	1300 med (kN)
10	1.02	1.42	1.90	1.02	1.10	1.88			
12	1.12	1.42	2.46	1.10	1.54	2.05			
15	1.22	1.56	3.05	1.28	1.66	2.85			
17	1.32	2.70	3.75	1.36	2.20	3.55	2.12	3.80	4.90
20	2.25	3.35	5.30	2.20	3.05	5.80	3.30	4.40	6.20
25	2.45	3.65	5.90	2.65	3.25	7.20	3.70	5.50	8.50
30	3.35	5.40	8.80	3.60	6.00	8.80	2.40[a]	8.30	10.0
35	4.20	8.50	10.6	4.75	8.20	11.0	3.10[a]	9.30	13.1
40	4.50	9.40	12.6	4.95	9.90	13.2	7.20	11.1	16.5
45	5.80	9.10	14.8	6.30	10.4	16.4	7.40	12.2	20.9
50	6.10	9.70	15.8	6.60	11.0	19.2	5.10[a]	12.5	24.5
55	8.20	12.0	18.0	9.00	13.6	21.5	11.3	14.9	27.1
60	8.70	13.6	20.0	9.70	16.4	24.0	12.0	18.9	32.5

Figure 8: Bearing rated capacity C

Equivalent Load

Let $e = F_a / F_r$

for radial ball bearings

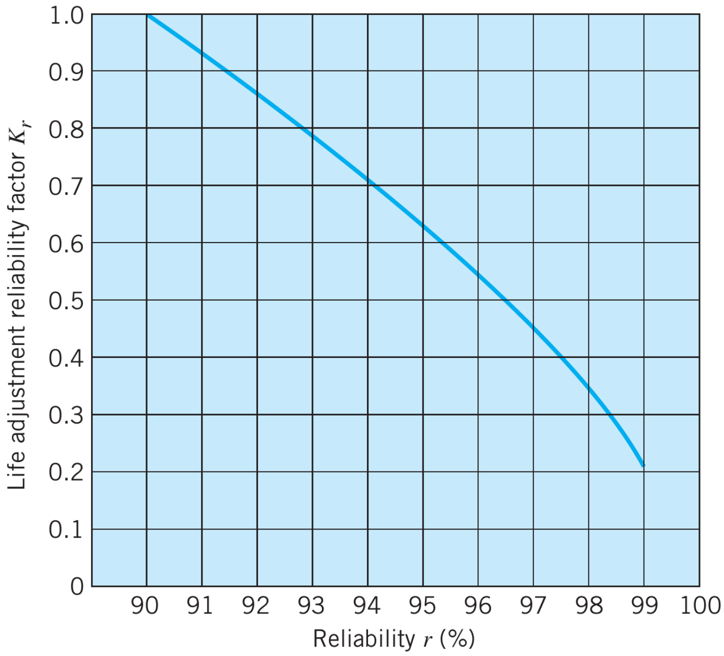


Figure 9: Bearing reliability factor K_r

$$F_e = \begin{cases} F_r & e < 0.35 \\ F_r\left[1 + 1.115(e - 0.35)\right] & 0.35 < e < 10 \\ 1.176F_a & e > 10 \end{cases} \tag{22}$$

for angular ball bearings

$$F_e = \begin{cases} F_r & e < 0.68 \\ F_r\left[1 + 0.87(e - 0.68)\right] & 0.68 < e < 10 \\ 0.911F_a & e > 10 \end{cases} \tag{23}$$

Typical Bearing Design Life

Radial Ball Bearing Selection

Select a radial ball bearing for a shaft intended for a continuous 8-hr-a-day operation at 1800 rpm with 95% reliability. Axial and radial loads are 1.2 kN and 1.5 kN, respectively.

Solution

First, we need to calculated F_e.

$$e = \frac{F_a}{F_r} = \frac{1.2}{1.5} = 0.8$$

For radial ball bearing,

$$\begin{aligned} F_e &= F_r\left[1 + 0.87(e - 0.68)\right] \\ &= 1500\left[1 + 1.115(0.8 - 0.35)\right] \\ &= 2276 \text{ N} \end{aligned}$$

Required life for 8-hr-a-day service (assumed every day) = 30000 hrs

Life in revolutions

$$L = 1800(30000)(60) = 3.24 \times 10^9 \text{ revolutions}$$

For 95% reliability K_r = 0.63

$$C = 2276\left(\frac{3.24 \times 10^9}{0.63(9 \times 10^7)}\right)^{0.3} = 7661 \text{ N} = 7.66 \text{ kN}$$

For extra-light, light, and medium series, the required bore are 55, 35, and 30 mm, respectively

The models corresponding to the bore are L11, 207, and 306, respectively.

Gears

Gear Overview

Why Gears?

- Convert high speed and low torque to that requires low speed and high torque
- Speed: easy to get because voltage is easy
- Torque: hard to get because it requires large current

Principles of Gears

- Allow positive engagement between teeth
- High forces can be transmitted while in rolling contact
- Do not need friction to operate

Basic Law of Gearing

- Point of contact between two mating gears is always the same relative distances from the two centers.
- Any gear tooth profiles that follow the law of gearing will result in constant relative speed of rotation

Gear Geometry

Module of a Gear, m

- Term used to define gear tooth size

- Defined as ratio of pitch diameter to number of teeth

$$m = \frac{D_{\text{pitch}}}{z} \tag{24}$$

- A pair of meshing gears must have the same modules!

Gear Types

Gear Terminology

Pinion smaller of two gears, usually driving

Gear Larger of the two. Also called *wheel*. Usually driven.

Gear Materials

- Steel: medium-carbon steel + heat treatment + grinding
- Cast iron: surface fatigue > bending fatigue
- Nonferrous: bronzes → corrosion + wear resistant, low friction
- Nonmetallic: Nylon → low friction and weight + corrosion resistant, but low thermal conductivity

Gear Efficiency

- With friction, gears are 90 - 95% efficient because of mostly rolling contact

$$T_{\text{out}} = \frac{\eta T_{\text{in}} d_{\text{out}}}{d_{\text{in}}} \tag{25}$$

$$\omega_{\text{out}} = \frac{\omega_{\text{in}} d_{\text{in}}}{d_{\text{out}}} \tag{26}$$

$$P_{\text{out}} = T_{\text{out}} \omega_{\text{out}} \tag{27}$$

Gear Trains

Gear Trains When large reduction is required

- Large gear + small pinion: simple, but large stress and interference
- Multiple pairs of gears and pinions: less simple, low stress, large space

- Planetary gears: complex, low stress, small space

Normal Gear Trains

$$e_{total} = e_1 e_2 \cdots \tag{28}$$

Planetary/Epicyclic Gear Train

- Planetary or epicyclic gears enable a high reduction ratio in small spaces

Planetary Gear Components

Planetary Gears: Torque, Forces, and Reduction Ratios

- Symmetry $\rightarrow$ no net force on shaft
- Multiple planet gears reduce individual torque/force
- Any combination of fixed, input, output gears
- 1 gear box -> multiple gear reduction ratios

Fixed ring:

$$\begin{aligned}
\omega_{\text{carrier}} &= 9 \\
\omega_{\text{planet}} &= (9)\frac{60/2 + 20/2}{20/2} = 36 \\
\omega_{\text{sun}} &= (36)\frac{20}{30} = 24 \\
e &= 9/24 = 0.375
\end{aligned}$$

Fixed	Input	Planet	Output	e
Ring	Carrier 9	36	Sun 24	0.375
Sun	Carrier 9	36	Ring 14.4	0.625
Carrier	Sun 9	27	Ring 5.4	1.667

Spur Gears

Spur gears have straight involute teeth. They transfer torque by applying forces perpendicular to their involute face. And because their teeth are perpendicular to their axis, spur gears do not generate axial loads–they generate only tangential and radial forces.

$$F_t = \frac{T}{R_{\text{pitch}}} \tag{29}$$

$$F_r = F_t \tan\phi \tag{30}$$

$$F_a = 0 \tag{31}$$

The tangential force is perpendicular to gear teeth, leading to tooth bending

Spur Gear Stress

The equation for bending stress in spur gears is a modified lewis equation that takes into account the shapes, stress concentrations, and operating conditions of the gear.

- Bending Stress $\rightarrow$ AGMA stress equation
- Consider tooth as a cantilever beam

$$\sigma = \frac{F_t}{bY_J m} K_O K_m K_v \tag{32}$$

F_t tangential force

b face width

Y_J geometry factor

m module

The geometry factor Y_J takes into account the involute shape and stress concentration factor of the tooth.

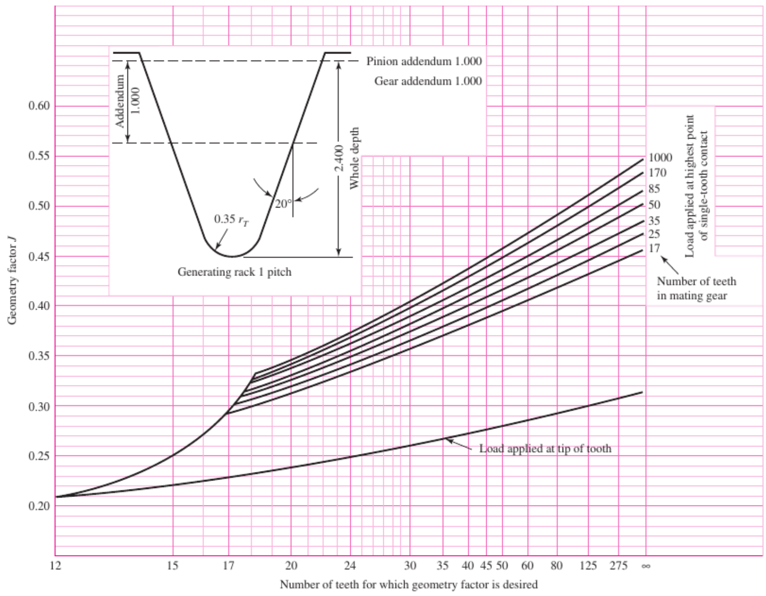


Figure 10: Geometry factor Y_J for spur gear design

From Figure 10, gears with large number of teeth or that are mating with gears with large number of teeth have higher geometry factors, leading to lower bending stresses. This is because large number of teeth means the teeth are shorter, hence the lower stresses.

Overload Factor: K_O

This factor takes into account the shock and impact loading during operation, which can cause a sharp increase in the stress. We consider the source of shock and impact loading from both the power source (input) and the driven machine (output).

Table 3: Overload factor K_O for spur gear design

Power source	Driven Machine			
	Uniform	Light shock	Moderate shock	Heavy shock
Uniform	1.00	1.25	1.50	1.75
Light shock	1.20	1.40	1.75	2.25
Moderate shock	1.30	1.70	2.00	2.75

Power sources can be categorized based on their shock/impact loading generated, along with some examples, as follows.

Uniform Electric motor, constant-speed turbine

Light Water turbine, variable-speed drive

Moderate Multicylinder engine

Driven machines are categorized based on operating conditions, which depends mostly on the resisting load on the system.

Uniform Continuous generator

Light Fans, low-speed pumps, conveyors

Moderate high-speed pumps, compressors, heavy conveyers

Heavy rock crushers, punch press drivers

Mounting Factor: K_m

The factor accounts for the increase in stress when tooth faces do not align perfectly. This can happen due to inaccurate mountings, the use of normal (rather than precision) gears, or off-center mountings.

Characteristics of Support	Face Width (cm)			
	0 to 5 cm	15	22.5	40
Accurate mountings, small bearing clearances, precision gears	1.3	1.4	1.5	1.8
Less rigid moutings, standard gears, full face contact	1.6	1.7	1.8	2.2
Less than full face contact	Over 2.2			

Velocity Factor: K_v

This factor accounts for the increase in stress from increased pitch line velocity of the gears. This combines with the overloading factor K_O to describe the effect of shock and impact loading on stress.

$$K_v = \left(\frac{A + \sqrt{200 v_t}}{A} \right)^B \tag{33}$$

$$A = 50 + 56(1 - B) \tag{34}$$

$$B = 0.25(12 - Q)^{2/3} \tag{35}$$

v_t pitch line velocity [m/s]

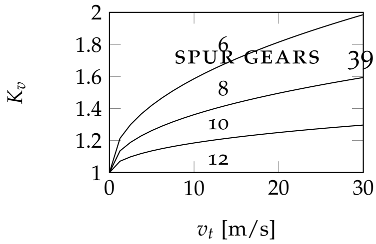


Figure 11: Velocity factor K_v as a function of pitch line velocity v_t for various gear quality number Q

Q AGMA Quality Number

AGMA recommends designers choose gears based on the level of precision they require from the deisgned mechanisms. Some of the guidlines are listed in Table 4

Table 4: AGMA recommended quality number for various applications

v_t [m/s]	Q	Applications
0 - 4	6 - 8	Paper box making machine, cement, mill drives
4 - 10	8 - 10	Washing machine, printing press, computing mechanism
10 - 20	10 - 12	Automotive transmission, Antenna drive, propulsion drive
$\geqslant 20$	12 - 14	Gyroscope

Gear Material Strength S_e'

Because they are used exclusively in rotational machinery, gears are constantly under repeated loadings. Thus, their main mode of failure is fatigue. Hence, whenever we consider the strength of gear material for design, endurance limits should be the first factor on our list.

$$S_e' = S_e C_L C_G C_S k_r k_t k_{ms} \tag{36}$$

where

S_e endurance limit

C_L load factor (= 1 for bending)

C_G gradient surface = 1

C_S surface factor (= 0.75 for machined surface)

k_r reliability factor

k_t temperature factor

k_{ms} median-stress factor (1 for two-way bending (followers), 1.4 for one-way bending (input or output))

Now let us consider each of endurance limit modifier in more details.

Reliability Factor: k_r

Since most material properties–endurance limits included–are reported by their average values, obviously there will be a gear whose actual endurance limit is lower than the reported value. This reliability factor represents this probability so that a given design based on the reduced endurance limit will have a higher probability of achieving the designated lifetime.

Reliability (%)	k_r
50	1.000
90	0.897
99	0.814
99.9	0.753
99.99	0.702
99.999	0.659

Table 5: Gear reliability factor k_r

Temperature Factor: k_t

Temperature directly affects endurance limit as already discussed in "Introduction to Theories of Failure" chapter.

$$k_t = \begin{cases} 1 & T \leqslant 160 \text{ F} \\ \dfrac{620}{460 + T} & T > 160 \text{ F} \end{cases} \tag{37}$$

Aside from the strength criteria governed by the given equations. There are more 'guidelines' – a set of recommended rules – that can be used to facilitate the spur gear design process.

1. $e \leqslant 1/6$
2. Use multi-stage gears for larger than $e > 1/6$
3. $8m \leqslant b \leq 16m$
4. many small teeth $\gg$ few large teeth
5. few teeth $\rightarrow$ small gear, but be careful about interference
6. Avoid exact ratio $\rightarrow$ hunting tooth

Example: Spur gear design for a conveyor belt

A pair of spur gears with face width b = 3 cm is used in a conveyor belt drive. The input motor has $\omega_{\max}$ of 200 rad/s. The pinion has 18 teeth. The conveyor has moderate shock and should be driven at 100 rad/s. The gears have pressure angles θ of 20°. Both pinion and gear has m = 1 cm. Determine the maximum power that the gears caan transmit continuously with 1% chance of bending fatigue failure. Steel has S_{ut} = 400 MPa

Solution: spur gear design for a conveyor belt

First, the bending fatigue stress is

$$
\begin{aligned}
\sigma &= \frac{F_t}{bY_J m} K_O K_m K_v \\
&= \frac{F_t}{(0.03)(0.32)(0.01)}(1.25)(1.6)\left(\frac{65.12+\sqrt{200(18)}}{65.12}\right)^{0.73} \\
&= 33542 F_t
\end{aligned}
$$

Next, the material fatigue strength is

$$
\begin{aligned}
S_e' &= S_e C_L C_G C_S k_r k_t k_{ms} \\
&= (400\times 10^6(0.5))(1)(1)(0.75)(0.814)(1)(1.4) \\
&= 1.71\times 10^8
\end{aligned}
$$

We can then find the maximum allowable tangential force

$$
\begin{aligned}
F_t &= \frac{1.71\times 10^8}{33542} = 5096 \text{ N} \\
P &= T\omega = F_t v_{pitch} = 5096\times 18 = 9.17\times 10^4 \text{ W}
\end{aligned}
$$

Rack and Pinion

Racks are essentially linear spur gears–the teeth line up on a straight rather than a circular path. When used with gears, the pairs can convert torque and rotational motion to force and linear motion. They are less expensive than power screws, but also less accurate and provide no mechanical advantages.

As they are considered linear spur gears, the forces acting on them are identical to those on spur gears.

$$
\begin{aligned}
F_t &= \frac{T}{R_{\text{pitch}}} \\
F_r &= F_t \tan\phi \\
F_a &= 0
\end{aligned}
\tag{38}
$$

Helical Gears

Another type of gears have teeth that are slanted at a constant angle, forming helices about their axes of rotation. These are called **helical gears**.

Helical Gear Analysis

Geometrically, they can be analyze similarly to spur gears, the only different being that the teeth are at an angle of ψ with the gear axis.

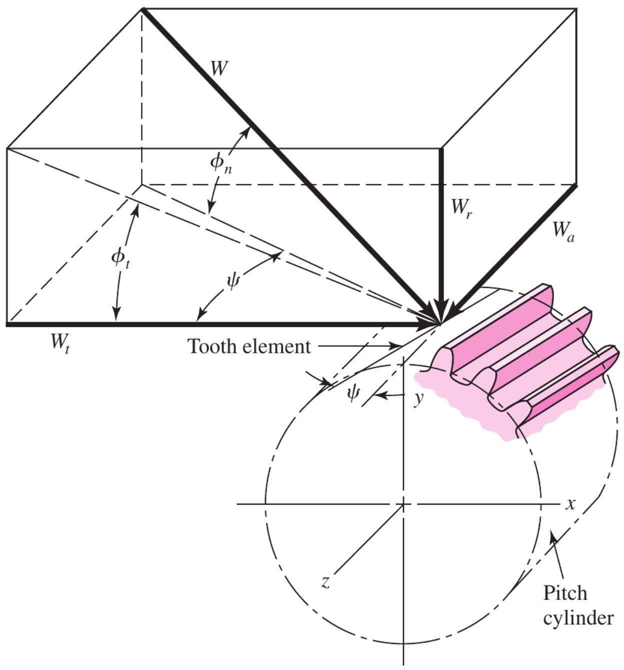


Figure 12: Forces acting on a helical gear

$$F_t = \frac{H}{v_t} \tag{39}$$

$$F_r = F_t \tan\phi_t \tag{40}$$

$$F_a = F_t \tan\psi \tag{41}$$

$$\tan\phi_n = \tan\phi_t \cos\psi \tag{42}$$

$$m_n = m_t \cos\psi \tag{43}$$

Design Equations Same as spur gear equation with small modification

$$\sigma = \frac{F_t}{bY_J m} K_v K_o (0.93 K_m) \tag{44}$$

$$S_e' = S_e C_L C_G C_S k_r k_t k_{ms} \tag{45}$$

0.93 indicated helical gears less sensitivity to mounting factor

Y_J needs small modification for helical teeth

Geometry Factor: Y_J

Because of the helix angle ψ, the geometry factor which accounts for the tooth size and its geometry is slightly modified.

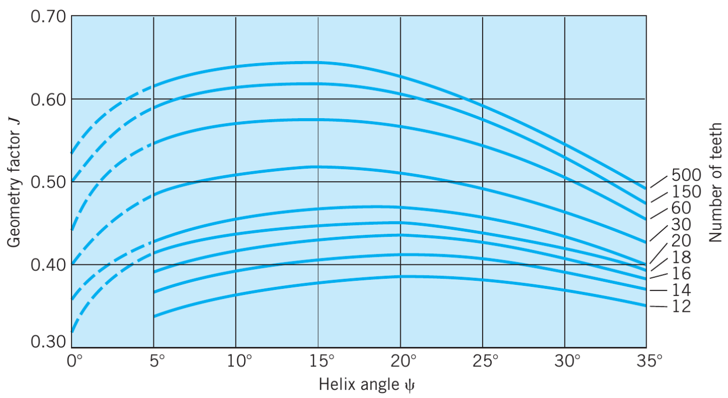


Figure 13: Geometry factor for helical gears

The geometry factor also has to be modified by another multiplier which accounts for the mating gear.

Example: Meshing helical gears

A pair of meshing helical gears is connected at the input side to a 0.5-hp motor at 1800 rpm and to an output shaft at 600 rpm. The input gear has 18 teeth, $\phi_n = 20°$, $m_n = 0.00173$, $\psi = 30°$, $b = 2$ cm. From the given information, determine the pitch line velocity v_t, gear tooth forces $F_t, F_r,$ and F_a, and bending stress σ.

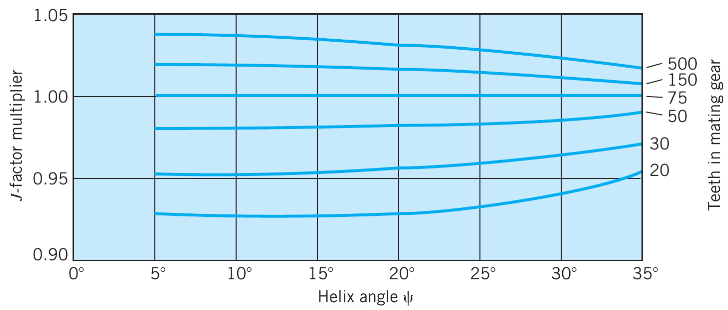


Figure 14: Geometry factor multiplier for helical gears

Calculate tangential module from normal module, then pitch diameter and tangential velocity.

$$m_t = \frac{m_n}{\cos 30^\circ} = \frac{0.00173}{\cos 30^\circ} = 0.002$$
$$d = mz = 0.002(18) = 0.036 \text{ m}$$
$$v_t = \omega\frac{d}{2} = (1800)\frac{2\pi}{60}\frac{0.036}{2} = 3.4 \text{ m/s}$$

Transmitted power only depends on tangential force, after which we can calculate axial and radial forces.

$$F_t = \frac{H}{v_t} = \frac{0.5(746)}{3.4} = 104 \text{ N}$$
$$\tan\phi_t = \frac{\tan\phi_n}{\cos\psi} = \frac{\tan 20^\circ}{\cos 30^\circ} = 0.42$$
$$\phi_t = 22.8^\circ$$
$$F_r = F_t \tan\phi_t = 104 \tan 22.8^\circ = 43.7 \text{ N}$$
$$F_a = F_t \tan\psi = 104 \tan 30^\circ = 60 \text{ N}$$

$$\sigma = \frac{F_t}{bY_J m} K_v K_o (0.93 K_m)$$

b = 0.02 m

For 18-teeth to 54-teeth mesh, Y_J = 0.99(0.42) = 0.416

Uniform-uniform input-output, K_o = 1

For K_v, since v_t = 3.57 m/s, let Q = 6.

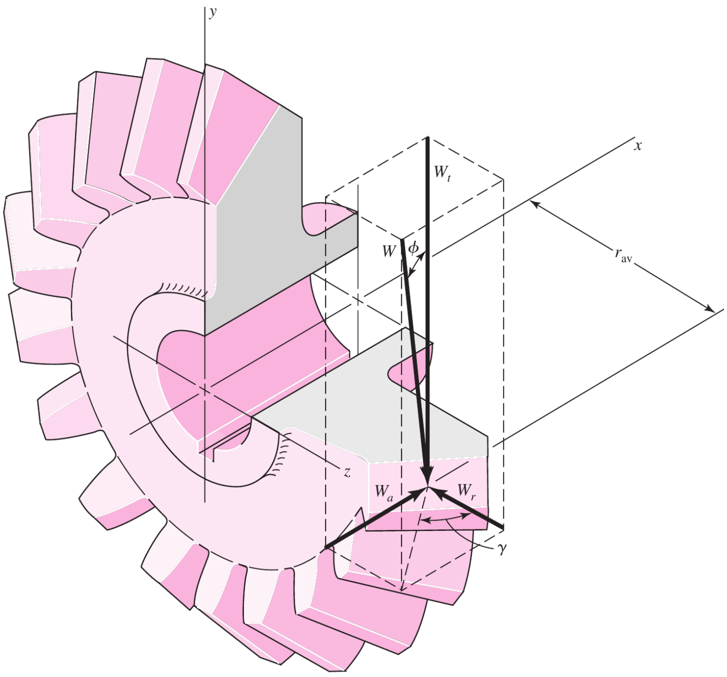
$$
\begin{aligned}
B &= 0.25(12-6)^{2/3} = 0.825 \\
A &= 50 + 56(1-0.825) = 59.8 \\
K_v &= \left(\frac{59.8+\sqrt{200 v_t}}{59.8}\right)^{0.825} = 1.36
\end{aligned}
$$

For K_m, nothing specific about gears or mounting, let's go with the middle case for b = 2 cm. K_m = 1.6

We can finally calculate σ

$$
\begin{aligned}
\sigma &= \frac{F_t}{bY_J m} K_v K_o (0.93 K_m) \\
&= \frac{104}{(0.02)(0.416)(0.002)}(1.36)(1)((0.93)1.6) \\
&= 1.26 \times 10^7 = 12.6 \text{ MPa}
\end{aligned}
$$

Bevel Gears



$$
\begin{aligned}
d_{av} &= d - b\sin\gamma \\
v_{av} &= \omega \frac{d_{av}}{2} \\
F_t &= \frac{H}{v_t} \\
F_a &= F_t \tan\phi \sin\gamma \\
F_r &= F_t \tan\phi \cos\gamma
\end{aligned} \tag{46}
$$

Design equations for bevel gears are similar to the spur gear equations with small modification.

$$\sigma = \frac{F_t}{bY_J m} K_v K_o K_m \tag{47}$$

$$S'_e = S_e C_L C_G C_S k_r k_t k_{ms} \tag{48}$$

Geometry Factor: Y_J

Because the tooth bevel gears do not have constant thickness, the geometry factors is modified.

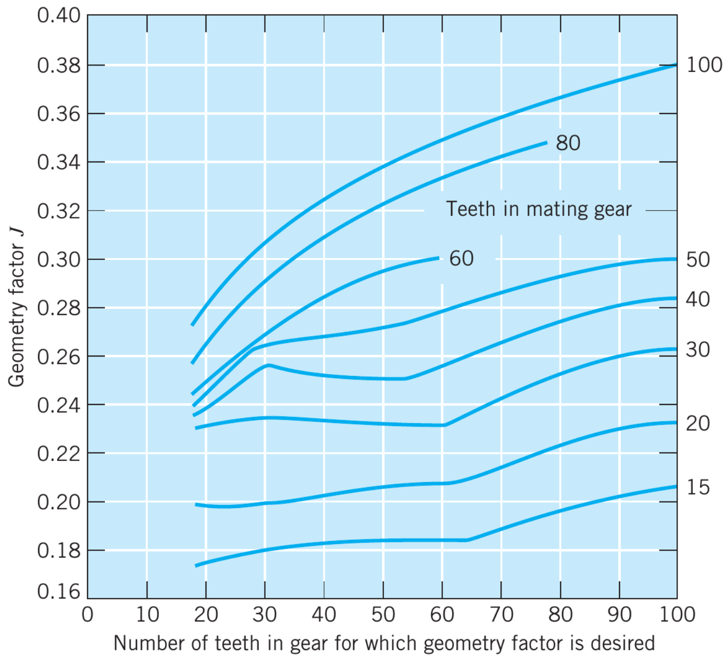


Figure 15: Geometry factors for bevel gears

Mounting Factor: K_m

This factor accounts for increased stress from misalignment in the gears. In the case of bevel gears, this depends mainly on how the gears are supported. In a straddle mounting, a gear is fitted in between two bearings, providing the best support and highest rigidity. On the other hand, in an overhung mounting, a gear is fitted onto a free end of the shaft, which provides minimal rigidity.

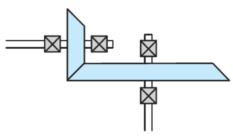
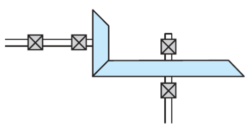
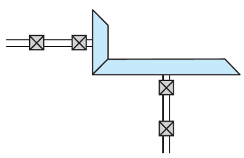
Mounting type		Mounting Rigidity
Both straddle-mounted		1.0 to 1.25
straddle-overhung		1.1 to 1.4
Both overhung		1.25 to 1.5

Table 6: Mounting factor K_m for bevel gears

Example: Bevel Gearset Design

Identical bevel gears has a module of 0.005 m/teeth, 25 teeth, 2-cm face width, and a 20° normal pressure angle. The gear quality is $Q = 7$. Both requires overhung mounting. The gears are made of ductile iron whose S_e = 95 MPa. Determine the power rating of the gearset at 600 rpm.

Solution: Bevel Gearset Design

For the stress side,

$$\begin{aligned} d_{av} &= mz/1000 = 0.125 \text{ m} \\ v_t &= \omega \frac{d_{av}}{2} = 600 \frac{2\pi}{60} \frac{0.125}{2} = 3.93 \text{ m/s} \end{aligned}$$

For uniform-uniform loading, $K_o = 1$

$$\begin{aligned} B &= 0.25(12-7)^{2/3} = 0.731 \\ A &= 50 + 56(1-0.731) = 65 \\ K_v &= \left(\frac{65 + \sqrt{200(3.93)}}{65} \right)^{0.731} = 1.3 \end{aligned}$$

For both-overhung mounting, $K_m = 1.5$

For 25-teeth pair, Y_J = 0.22

Now, onto the strength side,

C_L = 1 for bending

C_s = 0.75 for machined surface

C_G = 1

No requirement on the reliablility. Let's be generous, give it 90\

For normal operating temperature, k_t = 1.

For one-way bending, k_{ms} = 1.4.

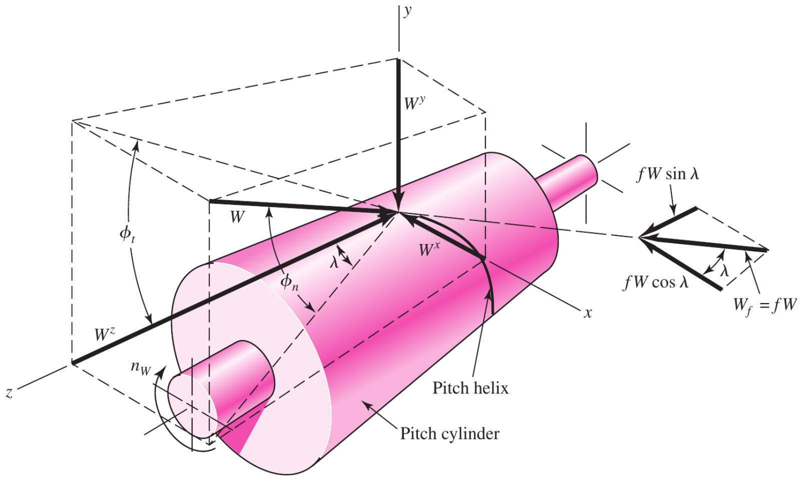
Set the two sides equal (N_s = 1), we have

$$\frac{F_t}{(0.02)(0.22)(0.005)}(1.3)(1)(1.5) = 95 \times 10^6(1)(1)(0.75)(0.897)(1)(1.4)$$

$$F_t = 1009 \text{ N}$$

$$H = F_t v_t = 1009(3.93) = 3965 \text{ W} = 5.31 \text{ hp}$$

Worms and Worm Gears



- Without friction

$$
\begin{aligned}
F_{wt} &= F\cos\phi_n \sin\lambda && (49)\\
F_{wr} &= F\sin\phi_n && (50)\\
F_{wa} &= F\cos\phi_n \cos\lambda && (51)
\end{aligned}
$$

- With friction $F_f = \mu F$

$$
\begin{aligned}
F_{wt} &= F\cos\phi_n \sin\lambda + \mu F\cos\lambda = F_{ga} && (52)\\
F_{wr} &= F\sin\phi_n = F_{gr} && (53)\\
F_{wa} &= F\cos\phi_n \cos\lambda - \mu F\sin\lambda = F_{gt} && (54)
\end{aligned}
$$

Worm Efficiency

- Worm and worm gear velocities can be related by

$$\frac{v_g}{v_w} = \tan\lambda$$
$$v_s = \sqrt{v_w^2 + v_g^2} = v_g\sqrt{1+\tan^2\lambda}$$

- Efficiency η is

$$\begin{aligned}
\eta &= \frac{F_{gt}v_g}{F_{wt}v_w} && (55)\\
&= \frac{\cos\phi_n\cos\lambda - \mu\sin\lambda}{\cos\phi_n\sin\lambda + \mu\cos\lambda}\tan\lambda && (56)\\
&= \frac{\cos\phi_n - \mu\tan\lambda}{\cos\phi_n + \mu\cot\lambda} && (57)
\end{aligned}$$

Self locking

- Thread will lock itself (not backdrivable) when $F_{wt} \leqslant 0$

$$\begin{aligned}
F_{wt} &= F\cos\phi_n\sin\lambda - \mu F\cos\lambda \leqslant 0 && (58)\\
\mu &\geqslant \cos\phi_n\tan\lambda && (59)
\end{aligned}$$

- Desirable in cases where auto-braking is needed
- In systems with large inertia, sudden stop can break the worm tooth $\rightarrow$ alternative brake mechanism is needd

Design Equation

Worm gears have higher stresses than worm, so our main concern is designing the gear.

$$F_{gt,allow} = \frac{C_s d^{0.8} b C_m C_v}{75.948}$$

C_s material factor

d gear diameter [mm]

b effective face width (actual width but less than $0.67d_w$) [mm]

C_m ratio correction factor

C_v velocity factor

Worm Gear Material Fatigue Strength, S'_e

Materials	S'_e
Manganese Bronze	117 MPa
Phosphor Bronze	165 MPa
Cast Iron	$0.35S_{ut}$

Allowable Load due to Wear For rough estimates:

$$F_{\mathrm{w}} = D_{\mathrm{gear}} b K_{\mathrm{w}} \tag{60}$$

F_w maximum allowable dynamic load

D_{gear} pitch diameter of gear

b face width of gear

K_w material and geometry factor

C_s: Material factor

For center distance $C < 7.62$ cm

$$C_s = 720 + 0.000633C^3$$

For $C \geqslant 7.62$ cm

Sand-cast gears:

$$\begin{aligned} C_s &= 1000 & d &\leqslant 6.35 \text{ cm} \\ C_s &= 1856.104 - 467.5454 \log d & d &> 6.35 \text{ cm} \end{aligned}$$

Chilled-cast gears:

$$\begin{aligned} C_s &= 1000 & d &\leqslant 20.32 \text{ cm} \\ C_s &= 2052.011 - 455.8259 \log d & d &> 20.32 \text{ cm} \end{aligned}$$

Centrifugally-cast gears:

$$C_s = 1000 \qquad d \leqslant 63.5 \text{ cm}$$
$$C_s = 1053.811 - 179.7503 \log d \qquad d > 63.5 \text{ cm}$$

C_m: Ratio correction factor

Depends on reduction ratio, $e = \omega_i / \omega_o$

$$C_m = \begin{cases} 0.02\sqrt{-e^2 + 40e - 76} + 0.46 & 3 < e \leqslant 20 \\ 0.0107\sqrt{-e^2 + 56e + 5145} & 20 < e \leqslant 76 \\ 1.1483 - 0.00658e & e > 76 \end{cases}$$

C_v: Velocity factor

Depends on sliding velocity at mean worm diameter v_s:

$$C_v = \begin{cases} 0.659e^{-0.2165v_s} & 0 < v_s \leqslant 3.556 \text{ m/s} \\ 0.652v_s^{-0.571} & 3.556 < v_s \leqslant 15.24 \text{ m/s} \\ 1.098v_s^{-0.774} & v_s > 15.24 \text{ m/s} \end{cases}$$

Example: Worm gear speed reducer

A 2-hp, 1200-rpm motor drives a 60-rpm mechanism by using a work gear reducer. The gear has D_{gear} = 20 cm. The worm has α = 12°, θ = 20°, and D_{worm} = 5 cm. Assume μ = 0.03, determine

1. all force components according to the rated power
2. power delivered to the driven mechanism
3. whether the drive is self-locking
4. safety factor of worm gear

Solution: Worm gear speed reducer

First, determine v_w to determine v_g

$$\begin{aligned} v_w &= \omega_w (d_w/2) = 3.14 \text{ m/s} \\ v_g &= v_w \tan\lambda = 3.14 \tan 12^\circ \\ &= 0.667 \text{ m/s} \end{aligned}$$

Power output at the worm gear is

$$\begin{aligned}
\eta &= \frac{\cos\phi_n - \mu\tan\lambda}{\cos\phi_n + \mu\cot\lambda} = \frac{\cos 20^\circ - 0.1\tan 12^\circ}{\cos 20^\circ + 0.1\cot 12^\circ} = 0.65 \\
H_g &= 0.65(2)(746) = 970 \text{ W} \\
F_{gt} &= \frac{H_g}{v_g} = \frac{970}{0.667} \\
&= 1454 \text{ N}
\end{aligned}$$

The other forces can then be calculated.

$$F_{ga} = F_{wt} = \frac{H_w}{v_w} = \frac{2(746)}{3.14} = 475 \text{ N}$$

To find $F_{wr} = F_{gr}$, we need first to find F, which we can solve from either F_{gt} or F_{ga}

$$\begin{aligned}
F_{ga} = 475 &= F\cos\phi_n\sin\lambda + \mu F\cos\lambda = F\left(\cos 20^\circ \sin 12^\circ + 0.1\cos 12^\circ\right) \\
475 &= 0.293F \\
F &= 1620 \text{ N} \\
F_{gr} &= F\sin\phi_n = 1620\sin 20^\circ = 554 \text{ N}
\end{aligned}$$

Self locking

$$\begin{aligned}
\mu &\geqslant \cos 20^\circ \tan 12^\circ \\
0.1 &\geqslant 0.20
\end{aligned}$$

Nope!

Definition of safety factor

$$N_s = \frac{F_{gt,allow}}{F_{gt}}$$

Determine the allowable tangential force on worm gear and material factor

$$
\begin{aligned}
F_{gt,allow} &= \frac{C_s d^{0.8} b C_m C_v}{75.948} \\
C &= \frac{d_g}{2} + \frac{d_w}{2} = \frac{0.2 + 0.05}{2} = 0.125 \text{ m} \\
C_s &= 1000 \qquad \text{(assume centrifugally-cast)}
\end{aligned}
$$

Ratio correction factor

$$
\begin{aligned}
e &= 1200/60 = 20 \\
C_m &= 0.02\sqrt{-20^2 + 40(20) - 76} + 0.46 = 0.82
\end{aligned}
$$

Velocity factor

$$
\begin{aligned}
v_s &= v_g \sqrt{1 + \tan^2 \lambda} = 0.667\sqrt{1 + \tan^2 12^\circ} = 0.682 \text{ m/s} \\
C_v &= 0.659 e^{-0.2165(0.682)} = 0.569
\end{aligned}
$$

Finally, the safety factor

$$
\begin{aligned}
F_{gt,allow} &= 1000(200)^{0.8}(0.67(50))(0.82)(0.569)/75.948 = 14265 \text{ N} \\
N_s &= \frac{14265}{1454} = 9.81
\end{aligned}
$$

Clutches and Brakes

- Rely on friction to transfer torque
- Easy to engage/disengage

Clutches vs Brakes

when engaged

Clutches $\omega_{in} = \omega_{out} \neq 0$

Brakes $\omega_{in} = \omega_{out} = 0$

Considerations for Clutch and Brake

Actuating force force to engage clutch/brake

Transmitted torque torque through mechanism

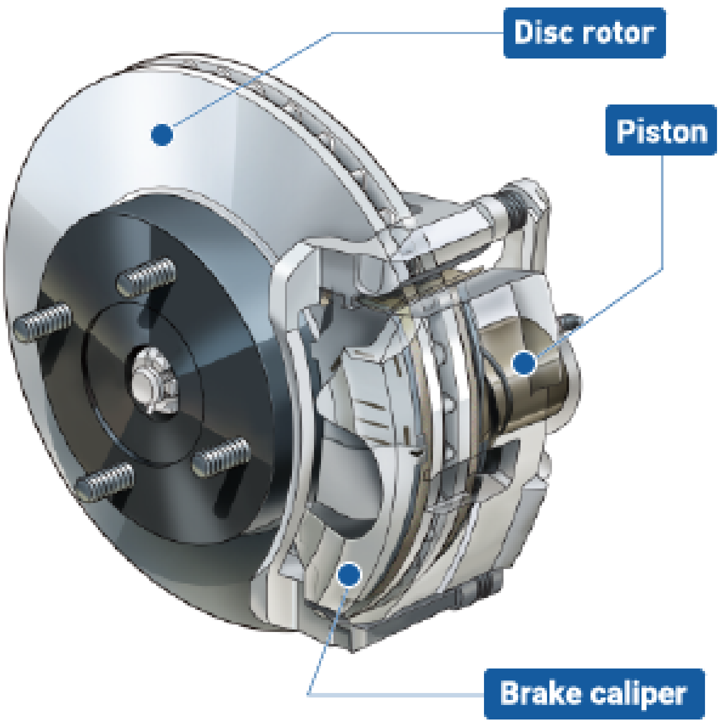
Energy loss energy dissipated before mechanism is fully engaged

Temperature rise temperature increase from energy loss

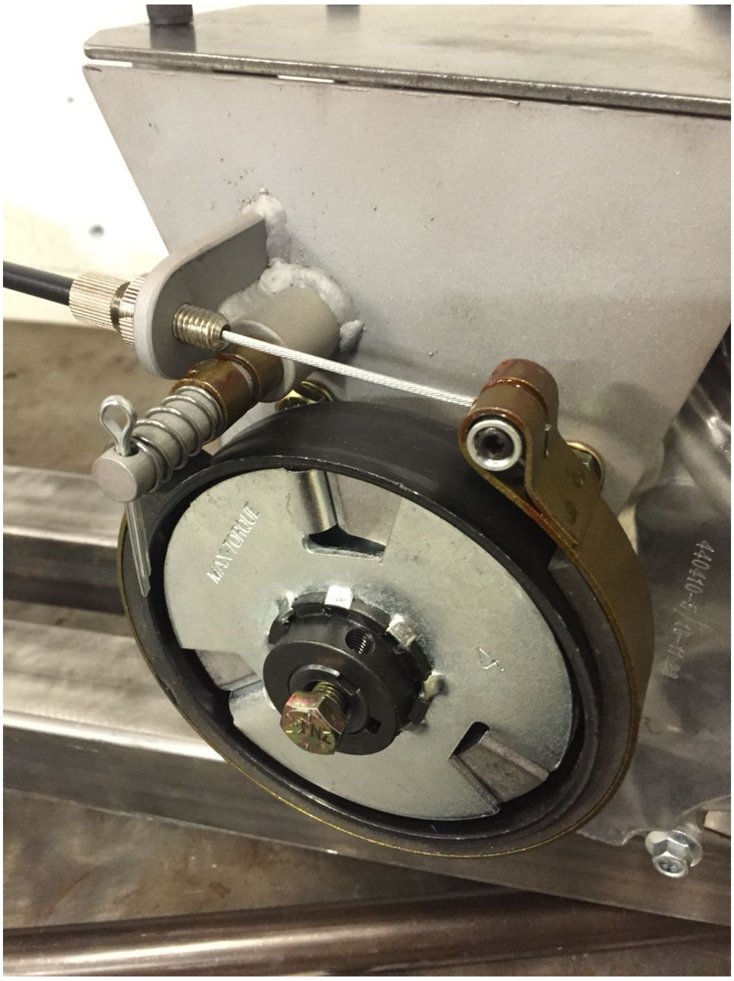
Types of Clutches and Brakes

Drum Brakes

Disc Brakes

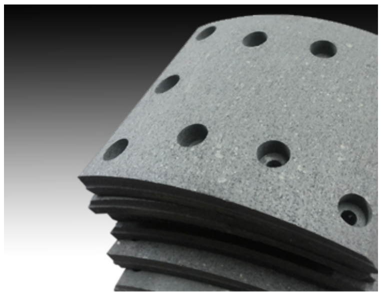


Band Brakes



Brake Linings

Materials



Molded thermosetting polymer or rubber + heat resistant fibers

Woven fibers + brass or zinc woven into fabric + resin

Sintered metal metal powder + inorganic fillers molded and sintered

Dry Linings

Friction Material[a]	Dynamic Friction Coefficient f[b]	Maximum Pressure[c]		Maximum Bulk Temperature	
		psi	kPa	°F	°C
Molded	0.25–0.45	150–300	1030–2070	400–500	204–260
Woven	0.25–0.45	50–100	345–690	400–500	204–260
Sintered metal	0.15–0.45	150–300	1030–2070	450–1250	232–677
Cork	0.30–0.50	8–14	55–95	180	82
Wood	0.20–0.30	50–90	345–620	200	93
Cast iron, hard steel	0.15–0.25	100–250	690–1720	500	260

Wet Linings

Friction Material[a]	Dynamic Friction Coefficient f
Molded	0.06–0.09
Woven	0.08–0.10
Sintered metal	0.05–0.08
Paper	0.10–0.14
Graphitic	0.12 (avg.)
Polymeric	0.11 (avg.)
Cork	0.15–0.25
Wood	0.12–0.16
Cast iron, hard steel	0.03–0.06

Drum Brake

Internal Drum Brake

$$p = \frac{p_{\max}}{(\sin\theta)_{\max}} \sin\theta \tag{61}$$

$$M_n = \int_{\theta_1}^{\theta_2} dN(a\sin\theta) \tag{62}$$

$$dN = p(r\,d\theta)b \tag{63}$$

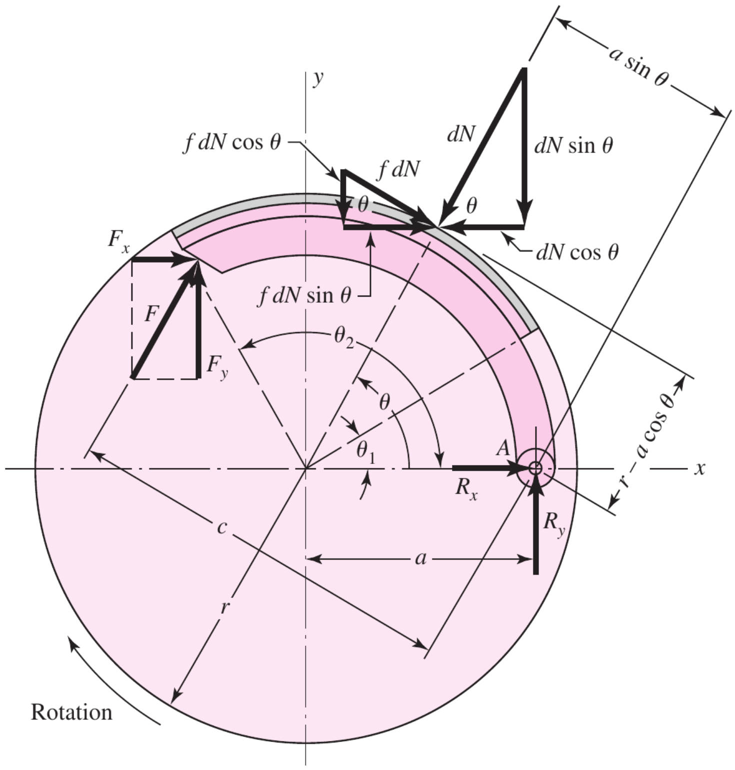


Figure 16: Forces and geometry of an internal drum brake.

Moment Generated on Drum by Normal Force

$$dN = \frac{p_{\max} br \sin\theta d\theta}{(\sin\theta)_{\max}} \tag{64}$$

$$\begin{aligned} M_n &= \int_{\theta_1}^{\theta_2} \frac{p_{\max} bra \sin^2\theta}{(\sin\theta)_{\max}} d\theta \\ &= \frac{p_{\max} bra}{(\sin\theta)_{\max}} \int_{\theta_1}^{\theta_2} \sin^2\theta d\theta \\ &= \frac{p_{\max} bra}{4(\sin\theta)_{\max}} [2(\theta_2 - \theta_1) - \sin 2\theta_2 + \sin 2\theta_1] \end{aligned} \tag{65}$$

Moment Generated on Drum by Friction

$$\begin{aligned} M_f &= \int_{\theta_1}^{\theta_2} \mu dN(r - a\cos\theta) \\ &= \int_{\theta_1}^{\theta_2} \frac{\mu p_{\max} \sin\theta r d\theta b(r - a\cos\theta)}{(\sin\theta)_{\max}} \\ &= \frac{\mu p_{\max} br}{(\sin\theta)_{\max}} \left[r(\cos\theta_1 - \cos\theta_2) + \frac{a}{4}(\cos 2\theta_2 - \cos 2\theta_1) \right] \end{aligned} \tag{66}$$

Self-energizing Brake

- if $M_f \geqslant M_n$, the brake is **self-energizing**
- The shoe sticks to the drum without actuating force F

Torque Generated on the Drum

$$
\begin{aligned}
T &= \int_{\theta_1}^{\theta_2} \mu r dN \\
&= \frac{\mu r^2 b p_{\max}}{(\sin\theta)_{\max}} \int_{\theta_1}^{\theta_2} \sin\theta d\theta \\
&= \frac{\mu r^2 b p_{\max}}{(\sin\theta)_{\max}} (-\cos\theta)\Big|_{\theta_1}^{\theta_2} \\
&= \frac{\mu r^2 b p_{\max}}{(\sin\theta)_{\max}} (\cos\theta_1 - \cos\theta_2) && (67) \\
& && (68)
\end{aligned}
$$

External Drum Brake

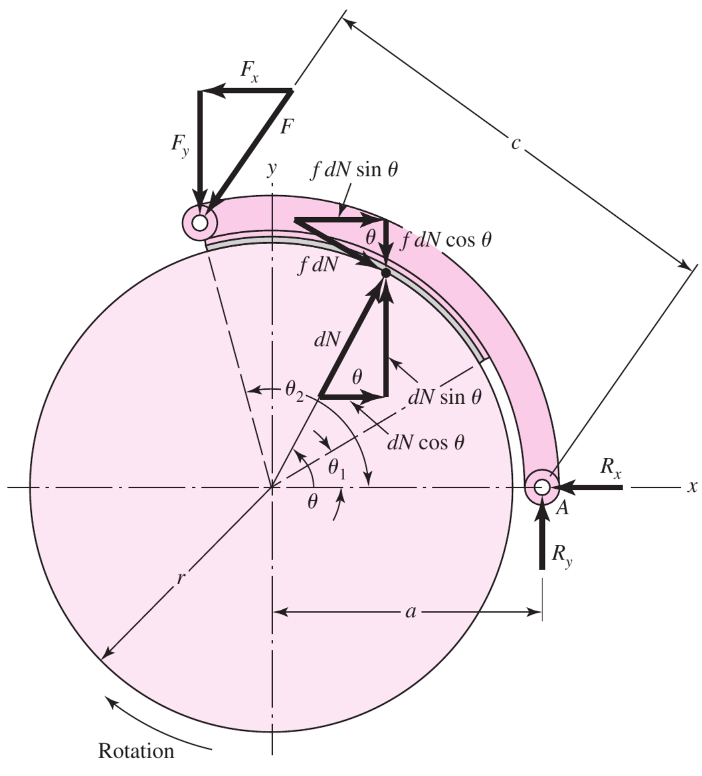
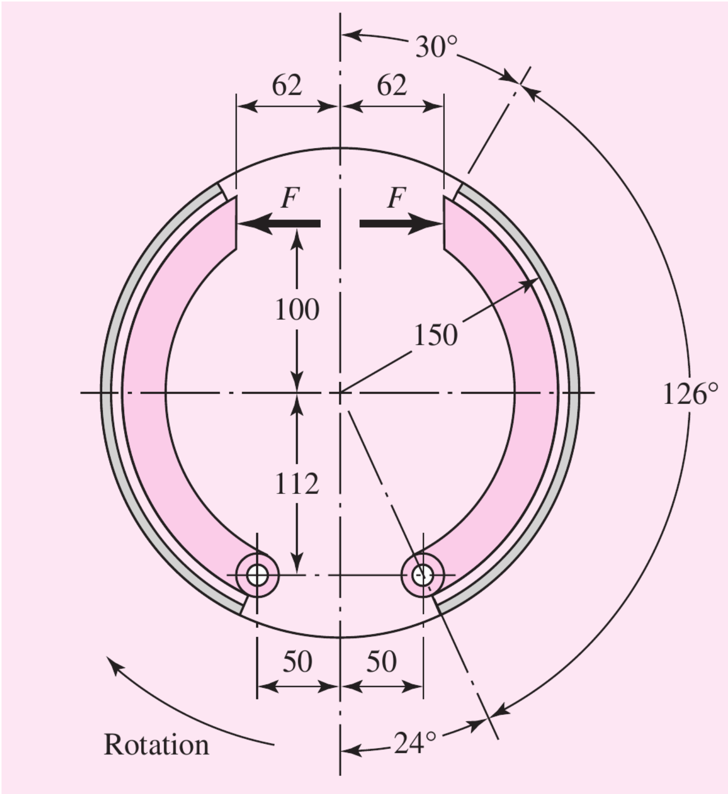


Figure 17: Forces and dimensions of an external drum brake.

Torque Generated on the Drum

- identical equations to internal drum brake, only need to be careful about the direction of actuating force

Example: Braking torque of a drum brake



- F = 2000 N
- μ = 0.3
- b = 3 cm

Determine the braking torque.

Solution

First, we must determine $p_{\max}$ on the right shoe. In this case, M_n and M_f go in opposite directions.

$$Fc = M_n - M_f$$
$$M_n = \frac{p_{\max} bra}{4(\sin\theta)_{\max}}[2(\theta_2 - \theta_1) - \sin 2\theta_2 + \sin 2\theta_1]$$

Let us first find M_n as a function of $p_{\max}$

$$\begin{aligned}
a &= \sqrt{0.112^2 + 0.05^2} = 0.123 \text{ m} \\
M_n &= \frac{p_{\max} bra}{4(\sin\theta)_{\max}}[2(\theta_2 - \theta_1) - \sin 2\theta_2 + \sin 2\theta_1] \\
&= \frac{p_{\max}(0.03)(0.15)(0.123)}{4(\sin 90^\circ)}\left[2(126^\circ(\frac{\pi}{180^\circ})) - \sin(2(126^\circ))\right] \\
&= 7.38 \times 10^{-4} p_{\max}
\end{aligned}$$

Now find M_f as a function of $p_{\max}$

$$\begin{aligned}
M_f &= \frac{\mu p_{\max} br}{(\sin\theta)_{\max}}\left[r(\cos\theta_1 - \cos\theta_2) + \frac{a}{4}(\cos 2\theta_2 - \cos 2\theta_1)\right] \\
&= \frac{0.3 p_{\max}(0.03)(0.15)}{\sin 90^\circ}\left[(0.15)(\cos 0 - \cos 126^\circ) + \frac{0.123}{4}(\cos 2(126^\circ) - \cos 2(0))\right] \\
&= 2.67 \times 10^{-4} p_{\max}
\end{aligned}$$

$$\begin{aligned}
Fc &= M_n - M_f \\
2000(0.212) &= p_{\max}(7.38 - 2.67) \times 10^{-4} \\
p_{\max} &= 9.00 \times 10^5 \text{ Pa}
\end{aligned}$$

Braking torque of the right shoe is

$$\begin{aligned}
T_R &= \frac{\mu r^2 b p_{\max}}{(\sin\theta)_{\max}}(\cos\theta_1 - \cos\theta_2) \\
&= \frac{(0.3)(0.15^2)(0.03)(9.00 \times 10^5)}{1}(\cos 0^\circ - \cos 126^\circ) \\
&= 289 \text{ N-m}
\end{aligned}$$

To calculate braking torque in left shoe, we also must calculate $p_{\max}$. M_n and M_f are now both clockwise.

$$\begin{aligned}
Fc &= M_n + M_f \\
2000(0.212) &= (7.38 + 2.67) \times 10^{-4} p_{\max} \\
p_{\max} &= 4.22 \times 10^5 \text{ Pa}
\end{aligned}$$

Braking torque of the left shoe is

$$
\begin{aligned}
T_L &= \frac{\mu r^2 b p_{\max}}{(\sin\theta)_{\max}}(\cos\theta_1 - \cos\theta_2) \\
&= \frac{(0.3)(0.15^2)(0.03)(4.22\times 10^5)}{1}(\cos 0^\circ - \cos 126^\circ) \\
&= 136 \text{ N-m}
\end{aligned}
$$

Total braking torque is

$$
\begin{aligned}
T &= T_L + T_R \\
&= 289 + 136 = 425 \text{ N-m}
\end{aligned}
$$

Band Brakes

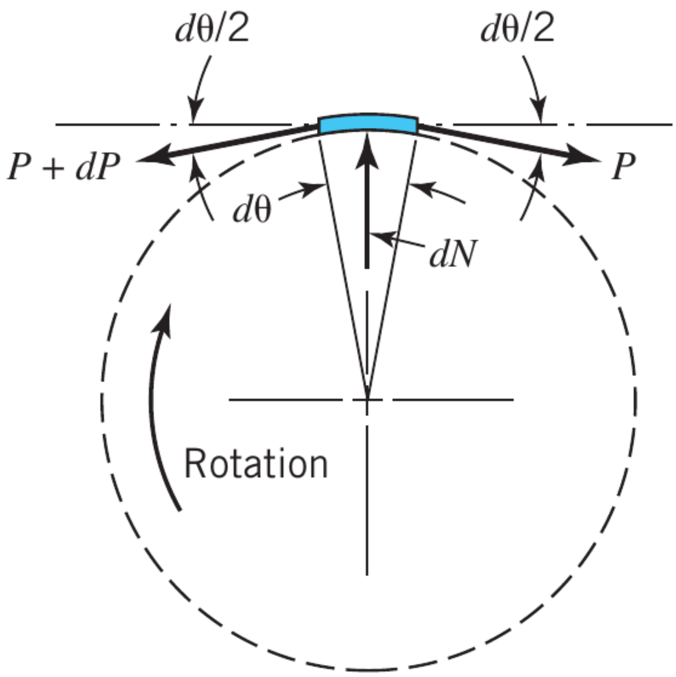
Principles of Band Brakes

- Rely on friction between band and drum

- Similar to pulley-belt system

$$
T = (F_1 - F_2)r \tag{69}
$$

Belt Tension



$$
\begin{aligned}
dF &= \mu dN \\
dN &= 2(Fd\theta/2) = Fd\theta \\
\frac{dF}{F} &= \mu d\theta \\
\ln\frac{F_1}{F_2} &= \mu\theta \\
\frac{F_1}{F_2} &= e^{\mu\theta} \qquad (70)
\end{aligned}
$$

Example: An Exercise Bike

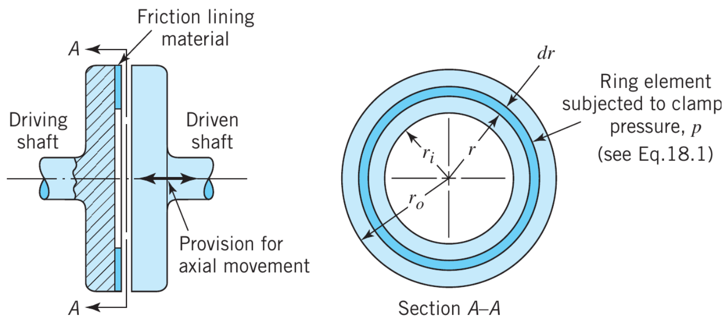
An exercise bike has an adjustable band brake on the wheel to provide different levels of resistance. What should the slack side belt tension be so that the biker can exercise with T = 50 N-m. Take $\theta = 150°$ and $\mu = 0.2$, the bike wheel $r = 50$ cm.

Solution

$$
\begin{aligned}
\frac{F_1}{F_2} &= e^{\mu\theta} \\
T &= (F_1 - F_2)r \\
T &= (e^{\mu\theta} - 1)F_2 r \\
F_2 &= \frac{50}{(e^{0.3(150(\pi/180))}) - 1)(0.5)} \\
&= 83.8 \text{ N}
\end{aligned}
$$

Disc Clutches and Brakes

Working Principles



Disc Wear

- new disc is rigid
- uniform pressure at first, but the outer area wears faster because of higher velocity
- after a while, pressure is no longer uniform, but wear becomes uniform

Torque Calculation

1. Uniform pressure: new disc
2. Uniform wear: old disc

$$
\begin{aligned}
dF &= pdA \\
dT &= \mu rdF = \mu rpdA \\
T &= \int_{r_o}^{r_i} \int_0^{2\pi} \mu rp(rdrd\theta) && (71) \\
&= \frac{2}{3}\mu\pi p \left(r_o^3 - r_i^3\right) && (72)
\end{aligned}
$$

- Taking the actuating force $F = p\pi(r_o^2 - r_i^2)$

$$T = \frac{2\mu F\left(r_o^3 - r_i^3\right)}{3\left(r_o^2 - r_i^2\right)}$$

- For N parallel discs

$$T = \frac{2\mu FN\left(r_o^3 - r_i^3\right)}{3\left(r_o^2 - r_i^2\right)}$$

Uniform Rate of Wear

- Rate of Wear $\propto$ Friction Work Rate

$$pr = C$$

- Max pressure occurs at inside radius, hence the constant is

$$pr = C = p_{\max} r_i$$

Braking Torque

$$
\begin{aligned}
dF &= pdA && (73) \\
dT &= \mu rdF = \mu rpdA = \mu p_{\max} r_i dA && (74) \\
T &= p_{\max} r_i \int_{r_o}^{r_i} \int_0^{2\pi} rdrd\theta && (75) \\
&= \mu\pi p_{\max} r_i \left(r_o^2 - r_i^2\right) && (76)
\end{aligned}
$$

- Taking into account actuating force F

$$T = \mu F\left(\frac{r_o + r_i}{2}\right)$$

- For N parallel discs

$$T = \mu F N \left(\frac{r_o + r_i}{2} \right)$$

Usual Guideline for Disc Brakes/Clutches

1. $0.45 r_o < r_i < 0.8 r_o$
2. Use uniform wear rate, unless for short-term application

Example: Automotive Clutch

- Design a wet clutch to transfer the torque of 100 N-m using the material with μ = 0.08 and $p_{\max}$ = 1500 kPa. Space requirements only allow $r_o \leqslant$ 60 mm. Determine the inner diameter and number of discs.

Solution

- Use r_i = 30 mm,

$$\begin{aligned} N &= \frac{T}{\mu \pi p_{\max} r_i \left(r_o^2 - r_i^2\right)} \\ &= \frac{100}{(0.08)\pi(1500 \times 10^3)(0.03)\left(0.06^2 - 0.03^2\right)} \end{aligned}$$

N = 4 and $d_i = 2r_i$ = 60 mm

Drum Brakes vs Disc Brakes

Drum	Disc
self-energizing possible very sensitive to μ requires larger force once μ goes down	no self-energizing not sensitive to μ well-designed caliper compensate for wear and exert constant pressure

Flexible Mechanical Elements

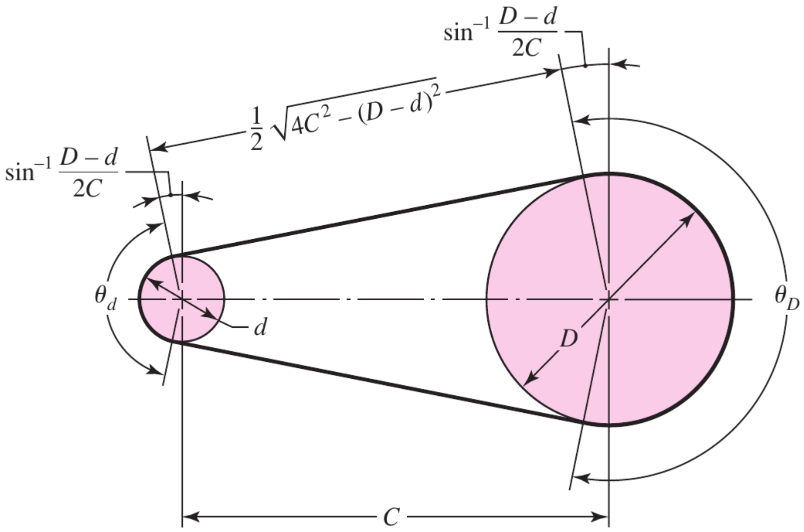
- belts
- ropes
- chains
- used in transmission of power over long distances

Why Flexible?

- Torque capacity: Gears > Belts, Ropes, Chains
- Flexible elements are better against vibration and shock loads
- Important to check for wear, age, and loss of elasticity

Belt – General

Belt - Pulley Geometry

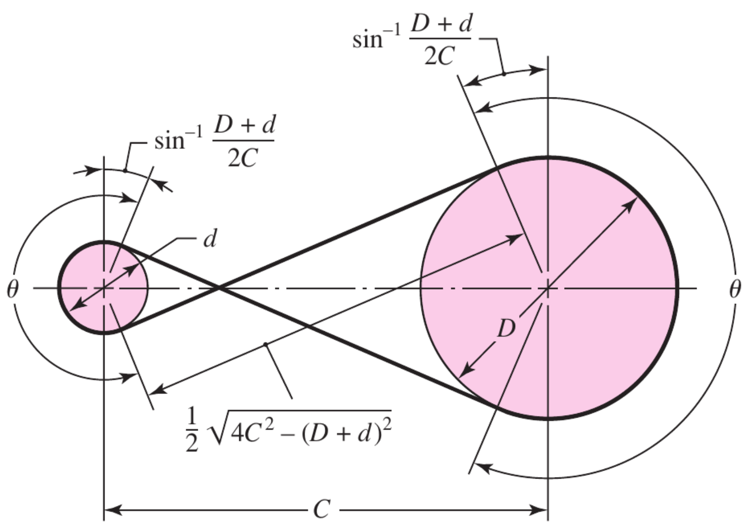


$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C}$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C}$$

$$L = \sqrt{4C^2-(D-d)^2} + \frac{1}{2}(D\theta_D + d\theta_d)$$

Cross Belt - Pulley Geometry



$$\theta_d = \pi + 2\sin^{-1}\frac{D+d}{2C}$$
$$L = \sqrt{4C^2 - (D+d)^2} + \frac{1}{2}(D+d)\theta$$

Flat Belts

Belt Tension

- For low speed belt drive

$$T = (F_1 - F_2)\, r \tag{77}$$

Belt Tension II

- For high speed belt drive

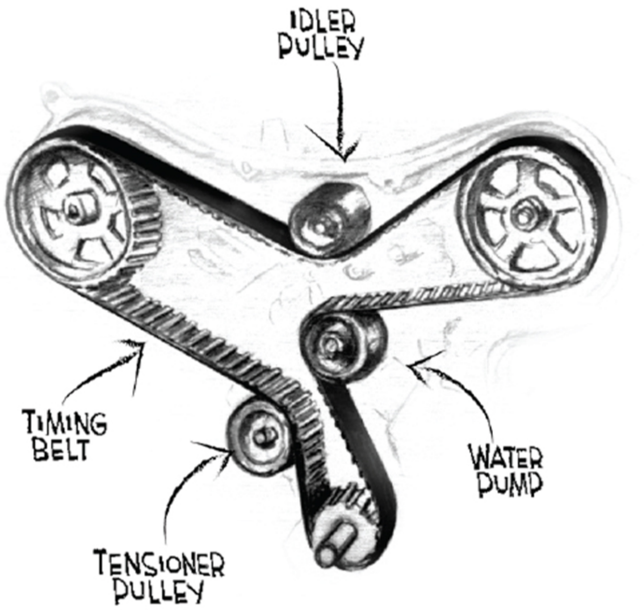
$$\frac{F_1 - F_c}{F_2 - F_c} = e^{\mu\theta} \tag{78}$$

where $F_c = m\omega^2 r^2$ is the centrifugal force on the belt and m is the mass per length of belt

Power and Torque Transmitted

$$\text{Power} = (F_1 - F_2)v \tag{79}$$
$$T = (F_1 - F_2)r \tag{80}$$

Maintaining Belt Tension



Belt Design

$$(F_1)_a = bF_aC_pC_v \tag{81}$$

$(F_1)_a$ = allowable largest tension
b = belt width
F_a = manufacturer's allowed tension (N/m)
C_p = pulley correction factor
C_v = velocity correction factor = 1 except leather belts

Manufacturer's Allowed Tension: F_a

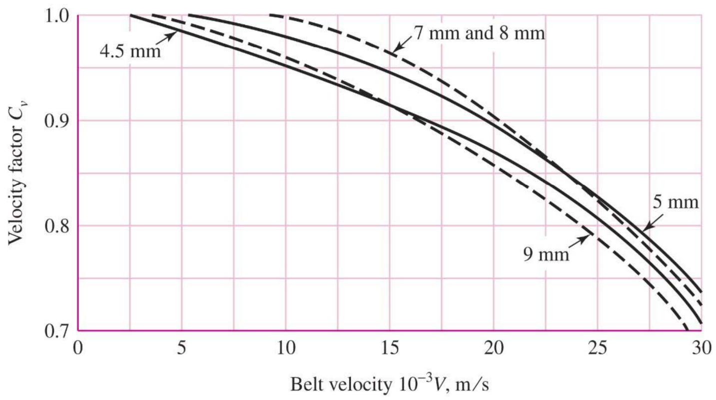
Material	Specification	Size, mm	Minimum Pulley Diameter, mm	Allowable Tension per Unit Width at 3 m/s, (10^3) N/m	Specific Weight, kN/m³	Coefficient of Friction
Leather	1 ply	$t = 4.5$	75	5	9.5–12.2	0.4
		$t = 5$	90	6	9.5–12.2	0.4
	2 ply	$t = 7$	115	7	9.5–12.2	0.4
		$t = 8$	150	9	9.5–12.2	0.4
		$t = 9$	230	10	9.5–12.2	0.4
Polyamide[b]	F–0[c]	$t = 0.8$	15	1.8	9.5	0.5
	F–1[c]	$t = 1.3$	25	6	9.5	0.5
	F–2[c]	$t = 1.8$	60	10	13.8	0.5
	A–2[c]	$t = 2.8$	60	10	10.0	0.8
	A–3[c]	$t = 3.3$	110	18	11.4	0.8
	A–4[c]	$t = 5.0$	240	30	10.6	0.8
	A–5[c]	$t = 6.4$	340	48	10.6	0.8
Urethane[d]	$w = 12.7$	$t = 1.6$	See	1.0[e]	10.3–12.2	0.7
	$w = 19$	$t = 2.0$	Table	1.7[e]	10.3–12.2	0.7
	$w = 32$	$t = 2.3$	17–3	3.3[e]	10.3–12.2	0.7
	Round	$d = 6$	See	1.4[e]	10.3–12.2	0.7
		$d = 10$	Table	3.3[e]	10.3–12.2	0.7
		$d = 12$	17–3	5.8[e]	10.3–12.2	0.7
		$d = 20$		13[e]	10.3–12.2	0.7

Pulley Correction Factor: C_p

Material	Small-Pulley Diameter, mm 40 — 100	115 - 200	220 - 310	355 - 405	460 - 800	Over 800
Leather	0.5	0.6	0.7	0.8	0.9	1.0
Polyamide, F–0	0.95	1.0	1.0	1.0	1.0	1.0
F–1	0.70	0.92	0.95	1.0	1.0	1.0
F–2	0.73	0.86	0.96	1.0	1.0	1.0
A–2	0.73	0.86	0.96	1.0	1.0	1.0
A–3	—	0.70	0.87	0.94	0.96	1.0
A–4	—	—	0.71	0.80	0.85	0.92
A–5	—	—	—	0.72	0.77	0.91

- $C_p = 1$ for urethane belts

Velocity Correction Factor: C_v



Example: Flat belt design

A polyamide A-3 flat belt 15 cm wide is used to transmit 15 hp power. The pulley rotational axes are parallel, the shafts are 2.5 m apart. The driving pulley diameter is 15 cm and rotates at 1750 rpm. The driven pulley diameter is 45 cm.

1. Estimate the centrifugal tension F_c
2. Estimate the allowable F_1 and F_2

Solution: Flat belt design

Find F_c

$$\begin{aligned}
\gamma &= 11.4 \text{ kN/m}^3 \\
\rho &= \frac{\gamma}{g} = 11.4 \times 10^3/9.81 = 1.162 \times 10^3 \text{ kg/m}^3 \\
m &= \rho A = 1.162 \times 10^3(3.3 \times 10^{-3})(0.15) = 0.575 \text{ kg/m} \\
F_c &= m\omega^2 r^2 = 0.575 \times (1750(2\pi/60))^2(0.15/2)^2 = 108.6 \text{ N}
\end{aligned}$$

For $(F_1)_a$ and F_2

$$\begin{aligned}
F_a &= 18 \times 10^3 \text{ N/m} \\
(F_1)_a &= bF_aC_pC_v = 0.15(18 \times 10^3)(0.7)(1) = 1890 \text{ N} \\
T &= \frac{(F_1 - F_2)d}{2} \\
F_1 - F_2 &= \frac{2T}{d} = \frac{2(15(746)/(1750(2\pi/60)))}{0.15} = 814 \text{ N} \\
F_2 &= (F_1)_a - (F_1 - F_2) = 1890 - 814 = 1076 \text{ N}
\end{aligned}$$

V-Belts

Why V Belts?

- Increased tension forces belt further into groove, providing more friction
- Increased torque capacity at a slightly lower efficiency

Belt Tension

$$T = (F_1 - F_2)\, r \tag{82}$$

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{\frac{\mu\theta}{\sin\beta}} \tag{83}$$

V Belt Design Equation

$$N_s = \frac{P_a N}{P_{nom} K_s} \tag{84}$$

N_s safety factor

P_a allowable power per belt

N number of belts (integer)

P_{nom} nominal power = $(F_1 - F_2)v$

K_s service factor

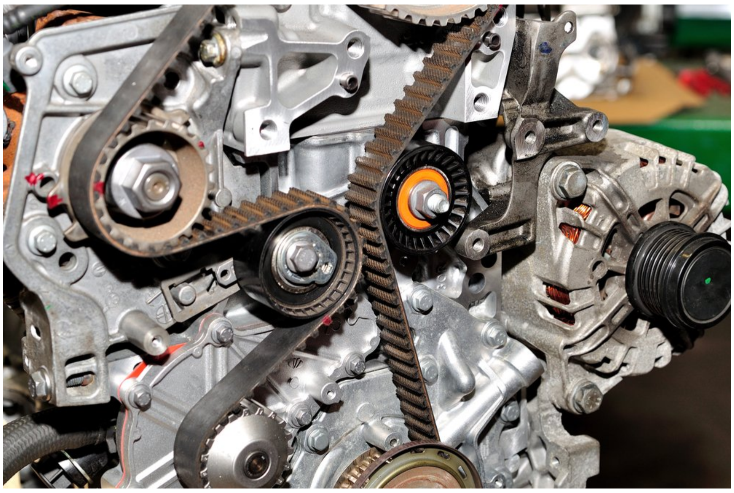
Allowable Power

Belt Section	Sheave Pitch Diameter, mm	Belt Speed, m/s 5	10	15	20	25
A	65	0.35	0.46	0.40	0.11	
	75	0.49	0.75	0.84	0.69	0.28
	85	0.60	0.98	1.17	1.64	0.84
	95	0.69	1.16	1.43	1.49	1.28
	105	0.77	1.30	1.64	1.78	1.63
	115	0.83	1.41	1.82	2.01	1.93
	125 and up	0.87	1.51	1.97	2.21	2.16
B	105	0.80	1.18	1.25	0.94	0.16
	115	0.95	1.48	1.71	1.55	0.92
	125	1.07	1.74	2.09	2.06	1.57
	135	1.19	1.95	2.42	2.49	2.10
	145	1.28	2.14	2.69	2.87	2.57
	155	1.36	2.31	2.94	3.19	2.98
	165	1.43	2.45	3.16	3.48	3.34
	175 and up	1.50	2.58	3.35	3.74	3.66
C	150	1.37	1.98	2.03	1.40	
	175	1.85	2.94	3.46	3.31	2.33
	200	2.21	3.66	4.54	4.74	4.12
	225	2.49	4.21	5.38	5.86	5.51
	250	2.72	4.66	6.05	7.16	6.63
	275	2.89	5.03	6.59	7.46	7.53
	300 and up	3.05	5.33	7.06	8.13	8.28
D	250	3.09	4.57	4.89	3.80	1.01
	275	3.73	5.84	6.80	6.34	4.19
	300	4.26	6.91	8.36	8.50	6.85
	325	4.71	7.83	9.70	10.30	9.10
	350	5.09	8.58	10.89	11.79	11.04
	375	5.42	9.25	11.86	13.13	12.68
	400	5.71	9.85	12.76	14.32	14.17
	425 and up	5.98	10.37	13.50	15.37	15.44
E	400	6.48	10.44	13.06	13.50	11.41
	450	7.40	12.46	15.82	17.16	16.04
	500	8.13	13.95	18.05	20.07	19.69
	550	8.73	15.14	19.84	22.53	22.75
	600	9.25	16.11	21.34	24.54	25.22
	650	9.70	17.01	22.60	26.19	27.38
	700 and up	10.00	17.68	23.72	27.68	29.17

Service Factor

Driven Machinery	Source of Power: Normal Torque Characteristic	High or Nonuniform Torque
Uniform	1.0 to 1.2	1.1 to 1.3
Light shock	1.1 to 1.3	1.2 to 1.4
Medium shock	1.2 to 1.4	1.4 to 1.6
Heavy shock	1.3 to 1.5	1.5 to 1.8

Timing Belts



- No significant stretch or slip $\rightarrow$ power where speed ratio is important
- efficiency of 97 - 99%
- No need for lubrication
- Quieter than chain drives
- Same design equations as V-belts

Example: Design of Belts

Design an F-1 polyamide flat belt to connect two shafts. The driving pulley (d = 4 cm) is connected to a 1-kW motor. The driven pulley has D = 8 cm. The shafts are 20 cm apart. The motor rotates at 500 rpm.

Solution

The required belt tension difference is

$$P = (F_1 - F_2)\, v$$
$$F_1 - F_2 = \frac{1000}{500\pi(0.04)(1/60)} = 955 \text{ N}$$

Actual allowable tension is

$$(F_1)_a = b(6000)(0.7)(1) = 4200b \text{ N}$$

Centrifugal tension F_c is

$$\begin{aligned} F_c &= m\omega^2 r^2 \\ &= \frac{9500b(0.0013)}{10}\left(\frac{500(2\pi)}{60}\right)^2 (0.02)^2 \\ &= 1.35b \text{ N} \end{aligned}$$

Angle on small pulley θ_d is

$$\begin{aligned} \theta_d &= \pi - 2\sin^{-1}\frac{D-d}{2C} \\ &= \pi - 2\sin^{-1}\frac{8-4}{2(20)} \\ &= 2.94 \text{ rad} \end{aligned}$$

$$\begin{aligned} \frac{(F_1)_a - F_c}{F_2 - F_c} &= e^{\mu\theta} \\ \frac{4200b - 1.35b}{4200b - 955 - 1.35b} &= e^{0.5(2.94)} \\ b &= 0.295 \text{ m} \end{aligned}$$

The belt must be at least 29.5 cm in width.

Roller Chains

Roller Chain components

Why Roller Chain (Chain Drives)?

- No slip $\rightarrow$ constant ratio
- Long life
- Can drive multiple shafts from a single source

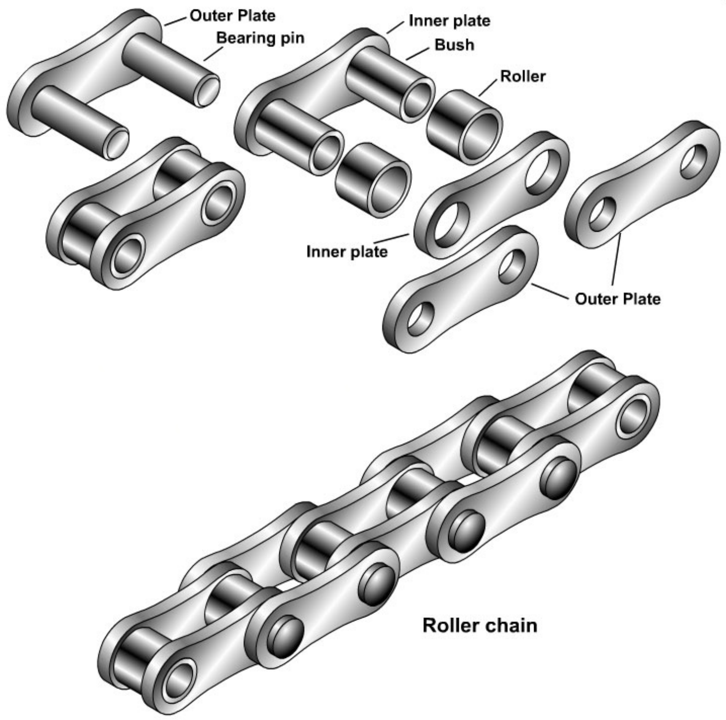


Figure 18: Components of a typical roller chain

Equations on Roller Chains

$$D = \frac{p}{\sin\left(\dfrac{180^\circ}{N}\right)} \tag{85}$$

$$V = Npn \tag{86}$$

- N = number of sprocket teeth
- p = chain pitch
- n = sprocket speed in revolution/min

Design Criteria

- Roller chains rarely fail because of tensile stress
- Need to worry more about fatigue and wear on rollers
- To minimize chordal speed variation. $N \geqslant 17$

Standard Chain Dimensions

ANSI Chain Number	Pitch, in (mm)	Width, in (mm)	Minimum Tensile Strength, lbf (N)	Average Weight, lbf/ft (N/m)	Roller Diameter, in (mm)	Multiple-Strand Spacing, in (mm)
25	0.250 (6.35)	0.125 (3.18)	780 (3 470)	0.09 (1.31)	0.130 (3.30)	0.252 (6.40)
35	0.375 (9.52)	0.188 (4.76)	1 760 (7 830)	0.21 (3.06)	0.200 (5.08)	0.399 (10.13)
41	0.500 (12.70)	0.25 (6.35)	1 500 (6 670)	0.25 (3.65)	0.306 (7.77)	— —
40	0.500 (12.70)	0.312 (7.94)	3 130 (13 920)	0.42 (6.13)	0.312 (7.92)	0.566 (14.38)
50	0.625 (15.88)	0.375 (9.52)	4 880 (21 700)	0.69 (10.1)	0.400 (10.16)	0.713 (18.11)
60	0.750 (19.05)	0.500 (12.7)	7 030 (31 300)	1.00 (14.6)	0.469 (11.91)	0.897 (22.78)
80	1.000 (25.40)	0.625 (15.88)	12 500 (55 600)	1.71 (25.0)	0.625 (15.87)	1.153 (29.29)
100	1.250 (31.75)	0.750 (19.05)	19 500 (86 700)	2.58 (37.7)	0.750 (19.05)	1.409 (35.76)
120	1.500 (38.10)	1.000 (25.40)	28 000 (124 500)	3.87 (56.5)	0.875 (22.22)	1.789 (45.44)
140	1.750 (44.45)	1.000 (25.40)	38 000 (169 000)	4.95 (72.2)	1.000 (25.40)	1.924 (48.87)

Figure 19: Dimensions of standard ANSI roller chain

Power Capacity

- For link-plate limit

$$H_1 = 0.004 N_1^{1.08} n_1^{0.9} p^{3-0.07p} \text{ hp}$$

- For roller limit

$$H_2 = \frac{1000 K_t N_1^{1.5} p^{0.8}}{n_1^{1.5}} \text{ hp}$$

- N_1 = number of teeth in smaller sprocket
- n_1 = sprocker speed [rev/min]
- p = pitch of chain [in]
- K_r = 29 for chain numbers 25,35; 3.4 for 41; and 17 for 40-240

Roller Chain Rated Capacity

Example

Pick a roller chain for a motorcycle whose output is 15 hp. The driving sprocket has 20 teeth and the output has 39 teeth.

Sprocket Speed, rev/min	25	35	40	41	50	60
	ANSI Chain Number					
50	0.05	0.16	0.37	0.20	0.72	1.24
100	0.09	0.29	0.69	0.38	1.34	2.31
150	0.13*	0.41*	0.99*	0.55*	1.92*	3.32
200	0.16*	0.54*	1.29	0.71	2.50	4.30
300	0.23	0.78	1.85	1.02	3.61	6.20
400	0.30*	1.01*	2.40	1.32	4.67	8.03
500	0.37	1.24	2.93	1.61	5.71	9.81
600	0.44*	1.46*	3.45*	1.90*	6.72*	11.6
700	0.50	1.68	3.97	2.18	7.73	13.3
800	0.56*	1.89*	4.48*	2.46*	8.71*	15.0
900	0.62	2.10	4.98	2.74	9.69	16.7
1000	0.68*	2.31*	5.48	3.01	10.7	18.3
1200	0.81	2.73	6.45	3.29	12.6	21.6
1400	0.93*	3.13*	7.41	2.61	14.4	18.1
1600	1.05*	3.53*	8.36	2.14	12.8	14.8
1800	1.16	3.93	8.96	1.79	10.7	12.4
2000	1.27*	4.32*	7.72*	1.52*	9.23*	10.6
2500	1.56	5.28	5.51*	1.10*	6.58*	7.57
3000	1.84	5.64	4.17	0.83	4.98	5.76
Type A			Type B		Type C	

Figure 20: Rate power capacity for standard roller chains

Solution

$$
\begin{aligned}
H_1 &= 0.004(20)^{1.08}(1000)^{0.9}(0.75)^{3-0.07(0.75)} \\
&= 21.8 \text{ hp} \\
H_2 &= \frac{1000(17)(20)^{1.5}(0.75)^{0.8}}{1000^{1.5}} \\
&= 38.2 \text{ hp}
\end{aligned}
$$

This means the chain can deliver the power with the safety factor of 21.8/15 = 1.45

Power Transmission Case Study

Example: Design of a Conveyor Belt System

Let us consider a design for a conveyor belt system. The conveyor belt needs to be moving at 60 cm/s. Both the driving and driven pulleys have the same diameter of 40 cm. The driving pulley is mounted on the same shaft as spur gear B, which is driven by spur gear A connected directly to the driving motor.

Spur gear A has $D_{\text{pitch},A}$ = 5 cm, z = 10 teeth, and pressure angle θ = 20°. The motor is set to operate constantly at 500 rpm and providing the torque of 2 N-m.

Design the following:

1. the driven spur gear B (determine the number of teeth and pitch diameter).
2. width b of the conveyor belt. (Belt material is polyamide F-1, μ = 0.5, thickness t = 1.3 mm, γ = 9.5 kN/m^3, F_a = 6 kN/m, C_v = 1)
3. the radius of the shaft on which spur gear B and the driving pulley are mounted. (E = 210 GPa, S_y = 300 MPa, N_s = 2.5)
4. radial contact bearings for 9 $\times 10^7$ revolutions, 90% reliability (series and bore size).

Solution: Design of a Conveyor Belt System

1. To determine the number of teeth of spur gear B, we need to determine the required pitch line velocity. The belt must be moving at 60 cm/s, which means the pulley must be rotating at

$$\begin{aligned}\omega_{\text{pulley}} &= \frac{0.6}{\pi(0.2)} \\ &= 0.95 \text{ rev/s} = 57.3 \text{ rpm}\end{aligned}$$

since spur gear B is mounted on the same shaft as the pulley, it must have the same ω. The pitch line velocities on two meshing gears must be the same. Spur gear A rotates at 500 rpm, therefore, the pitch radius of spur gear B is

$$\begin{aligned} r_B &= \frac{\omega_A r_A}{\omega_B} \\ &= 0.218 \text{ m} \end{aligned}$$

Its diameter, of course, is 43.6 cm. Given that meshing gears must also have the same module, the number of teeth on B is

$$\begin{aligned} m &= \frac{D_{\text{pitch},A}}{z_A} = \frac{0.05}{10} = 0.005 \\ &= \frac{43.6}{z_B} \\ z_B &= 87.3 \text{ teeth} \end{aligned}$$

2. Assuming no power loss, the power the belt-pulley system needs to transfer is $H = T\omega = 2(500)(2\pi/60)$ = 104.7 W.

   Required torque transfer of the belt is $T = P/\omega = 104.7/(57.3(2\pi/60))$ = 17.4 N-m. The tension difference on the two side needs to be

$$\begin{aligned} T &= (F_1 - F_2) r_{\text{pulley}} \\ F_1 - F_2 &= \frac{T}{r_{\text{pulley}}} = 349 \text{ N} \end{aligned}$$

   We can now write an equation to determine the thickness b of the belt

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{\mu\theta}$$

   Set

$$\begin{aligned} F_1 &= (F_1)_a = bF_aC_vC_p = b(6000)(1)(0.7) = 4200b \\ F_2 &= F_1 - 87 = 4200b - 87 \\ \mu &= 0.5 \\ \theta &= \pi \\ F_c &= (\gamma/g)bt\omega^2 r^2 = (9500/10)b(0.0013)(0.95(2\pi))^2(0.2)^2 \\ &= 1.76b \\ \frac{4200b - 1.76b}{4200b - 87 - 1.76b} &= e^{0.5\pi} \\ b &= 0.0261 = 2.61 \text{ cm} \end{aligned}$$

3. To design the shaft, we must first determine the loads on the shaft, which are exerted by spur gear B and the pulley. Let us determine the load from each component.

   The load from the gear can simply be determined by

$$\begin{aligned} F &= \frac{T}{r\cos\theta} \\ &= \frac{17.4}{(0.2)(\cos 20^\circ)} \\ &= 92.6 \text{ N} \end{aligned}$$

   So the force exerted on the shaft by gear B is 92.6 N in the direction of 20° downward from the tangential line between the two gears.

   The load from the pulley is even simpler

$$\begin{aligned} F_p &= F_1 + F_2 = 4200(0.0261) + 4200(0.0261) - 87 \\ &= 132.24 \text{ N} \end{aligned}$$

   The force exerted on the shaft by the pulley is 132 N to the right.

   Now all we need is to determine the maximum bending moment on the shaft.

   First, bending moment M_g generated by force in the gear is

$$\begin{aligned} M_g &= \frac{F_g x(L-x)}{L} = \frac{92.6(0.1)(0.4-0.1)}{0.4} \\ &= 6.95 \text{ N-m} \end{aligned}$$

   Next, bending moment M_p generaed by force on the pulley is

$$\begin{aligned} M_p &= \frac{F_p x(L-x)}{L} = \frac{132(0.3)(0.4-0.3)}{0.4} \\ &= 9.9 \text{ N-m} \end{aligned}$$

   The maximum bending moment occurs in the middle, where

$$\begin{aligned} M_{\max} &= (6.95 + 9.9)\frac{2}{3} \\ &= 11.2 \text{ N-m} \end{aligned}$$

   The maximum torque is just the required torque transfer, which is T = 17.4 N-m. We can now determine the required shaft size.

$$N_s = \frac{S_y}{\sigma_e} \tag{87}$$

$$2 = \frac{300 \times 10^6}{\sqrt{\left(\frac{4(11.2)}{\pi r^3}\right)^2 + 3\left(\frac{2(17.4)}{\pi r^3}\right)^2}} \tag{88}$$

$$r = 0.0054 = 5.4 \text{ mm} \tag{89}$$

4. Since the problem is not symmetrical, we need to determine the bearing under the higher load. This happens to the bearing closer to the higher force, which is the pulley. Taking the other bearing as a fulcrum, the radial load in the bearing on the pulley side is

$$\begin{aligned} F_r &= \frac{132(3)}{4} + \frac{92.6(1)}{4} \\ &= 122.2 \text{ N} \end{aligned}$$

With $L = 9 \times 10^7$ and $K_r = 1$, $F_e = F_r = 122.2 = C$. This is smaller than the smallest bore which is 10mm for all series. We therefore should pick the smallest bore (10 mm) as our choice.

Bibliography

[1] Joseph Edward Shigley. *Shigley's mechanical engineering design*. Tata McGraw-Hill Education, 2015.