Flexible Mechanical Elements

ME 313: Mechanical Design
Week 8

What are Flexible Mechanical Elements?

- Elastic elements used in conveying and transmission of power
  - Belts, ropes, chains, etc..
- Used to transmit power over long distance
- Replace gears, shafts, and/or bearings

Advantages of Using Flexible Elements

- Absorb shock loads and isolate vibrations
- Simplify design
- Save cost

Belts

- Usually coupled with pulleys
- Cheapest method of power transmission
- Shafts do not have to align
- Very little noise
- No lubrication and small maintenance required

Types of Belts

- Flat belts
  - Simple, flat rectangular cross section

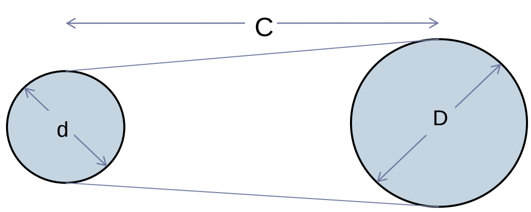
- Round belts
  - Circular cross section

- V belts
  - Trapezoidal cross section

- Timing Belts
  - Have teeth to match toothed pulley

Flat and Round Belt Drives

- Belts usually consists of elastic core surrounded by an elastomer



Contact angles

$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C}$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C}$$

Length of Belt

$$L=[4C^2-(D-d)^2]^{1/2}+\frac{1}{2}(D\theta_D+d\theta_d)$$

- When the belts are crossed, contact angles are the same

$$\theta=\pi+2\sin^{-1}\frac{D+d}{2C}$$

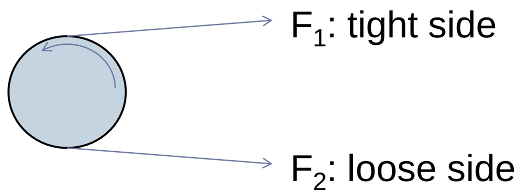
  - And length is

$$L=[4C^2-(D-d)^2]^{1/2}+\frac{\theta}{2}(D+d)$$

Dr. Sappinandana Akamphon

Belt Tension

- Assume belt mass is very small, pulley and belt has coefficient of friction μ and angle of contact θ



$$F_2=F_1e^{-\mu\theta}$$

Dr. Sappinandana Akamphon

Torque Transmission

- Torque transmission is related to the difference in belt tensions

$$F_1 - F_2 = \frac{T}{D/2} = \frac{2T}{D}$$

$$F_1(1 - e^{-\mu\theta}) = \frac{2T}{D}$$

$$F_1 = \frac{2T}{D(1 - e^{-\mu\theta})}$$

Dr. Sappinandana Akamphon

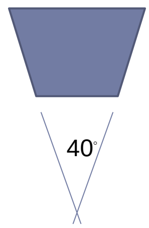
Power Transmission

- If belt is moving at speed *v*, then

$$P = (F_1 - F_2)v$$

Dr. Sappinandana Akamphon

V Belts



- Standard V belt cross section is usually 40 degree trapezoid
- The pulleys also have grooves for belts to fit into
- This allows the belt to fit further into the groove as belt tension increases

Torque Transmission

- Belt friction depends on the angle on the grooves

Groove Angles	Coefficient of Friction
30	0.5
34	0.45
38	0.4

$$F_1 - F_2 = \frac{T}{D/2} = \frac{2T}{D}$$

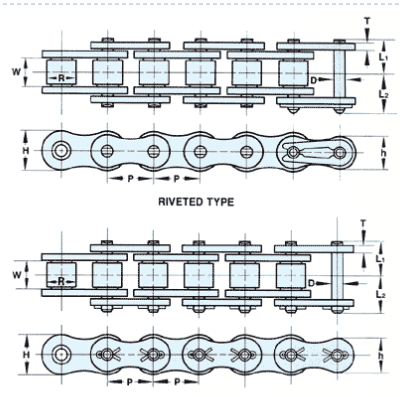
$$F_1 = \frac{2T}{D(1-e^{-\mu\theta})}$$

Dr. Sappinandana Akamphon

Timing Belts

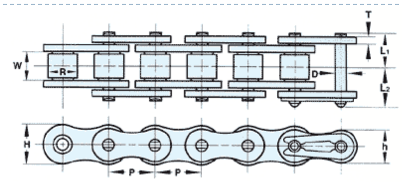
- Belt teeth to fit into grooves of pulley
  - Prevent slipping and eliminate need for initial belt tension
  - Transmits power at constant velocity
- Require no lubrication, quiet operation
- Attractive solution for precision-drive

Roller Chains



- Chains of constant pitch, coupled with a sprocket, for power transmission

Roller Chain Geometry



RIVETED TYPE

- Pitch, *p*
  - Distance between adjacent chains
- Pitch angle, γ
  - Angle between adjacent teeth

$$\sin\frac{\gamma}{2} = \frac{p/2}{D/2}$$

$$D = \frac{p}{\sin(\gamma/2)}$$

Sprocket Dimension

- If there are *N* teeth on the sprocket, and roller chain has pitch *p*

$$D = \frac{p}{\sin(180/N)}$$

Chain Velocity

$$v = Np\omega$$

- v = chain velocity
- N = number of sprocket teeth
- p = chain pitch
- ω = angular velocity

- However, chain velocity is not constant, due to sprocket uneven radius

$$v_{\max} = \frac{\omega p}{2\sin(\gamma/2)}$$

$$v_{\min} = \frac{\omega p \cos(\gamma/2)}{2\sin(\gamma/2)}$$

Dr. Sappinandana Akamphon

Chordal Speed Variation

- The variation in speed is

$$\frac{\Delta v}{v} = \frac{v_{\max} - v_{\min}}{v} = \frac{\pi}{N}\left[\frac{1}{\sin(180/N)} - \frac{1}{\tan(180/N)}\right]$$

  - This is called the *chordal speed variation*

Dr. Sappinandana Akamphon

Chain Failure

- Chains seldom fail because they lack tensile stress
- There mode of failure is usually from long hours of service due to wear on the rollers or pins and fatigue of rollers

Dr. Sappinandana Akamphon

Wire Rope

- Made up of windings of wire into strands, and of strands into rope
- Many uses especially in very long and heavy load carriers
  - Elevators
  - Mineshafts

Dr. Sappinandana Akamphon

Stress in Wire Rope

- From beam bending

$$M = \kappa EI = \frac{EI}{R} \text{ and } M = \frac{\sigma I}{c}$$

  - M = bending moment
  - κ = curvature
  - E = Young's modulus
  - I = moment of inertia
  - R = radius of curvature
  - c = furthest distance from neutral axis

$$\sigma = E_r \frac{d_w}{D}$$

  - E_r = Young's modulus of rope
  - d_w = wire diameter
  - D = sheave diameter (radius of curvature of wire rope)

Importance of Pulley Diameter vs Rope Diameter

- The smaller the pulley diameter, the higher the bending stress
- When analyze stress in the rope, must consider both tensile and bending stress
- General rule:

$$\frac{D}{d_w} \geq 400$$

  - e.g. mine shafts and elevators, the ratio goes from 800-1000

Elevator Problem

- Assume rope is massless, rope tension due to load acceleration-deceleration is

$$F_t = \frac{W}{n}\left(1+\frac{a}{g}\right)$$

  - W is weight at the end of the rope
  - n is the number of wire ropes supporting the load
- Therefore the total stress in the rope is

$$\sigma_{total} = \sigma_{bending} + \sigma_t$$

$$= E_r \frac{d_w}{D} + \frac{W}{nA}\left(1+\frac{a}{g}\right)$$

Questions?