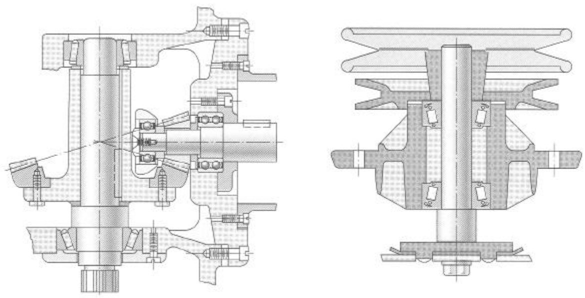

Shafts, Axles, and Couplings

ME 313: Mechanical Design
Week 11

Topics to Cover

- Shafts and Axles
  - Overview
  - Strength considerations
  - Speed considerations
- Couplings
  - Overview
  - Pins and keys
  - Tapered Couplings

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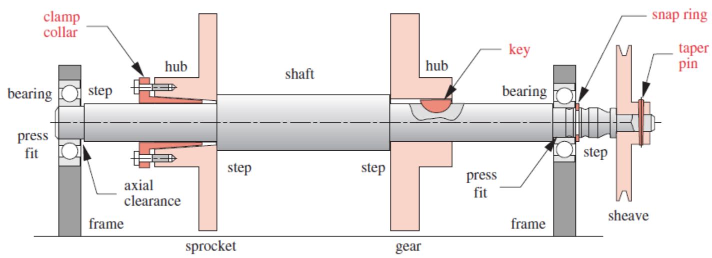
Shafts and Axles

- Shafts
  - transfer of torque or rotation from source to intended component(s)
- Axle
  - nonrotational component used to support rotational parts



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Typical Shaft Designs

- To transmit power from driving device (motor or engine) to machine, gears, pulleys, flywheels, clutches, or sprockets are mounted on shaft
- Torque is transmitted to shaft by press fit, keys, or pins
- Shaft rotates usually on rolling contact bearing or journal bearing

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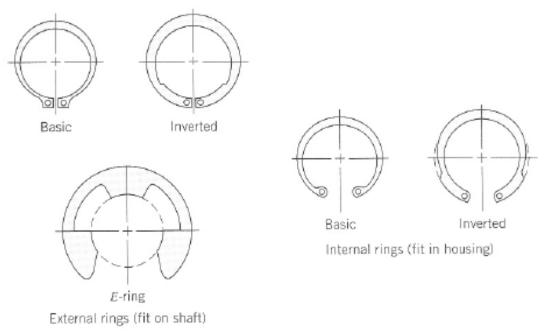
Typical Shaft Designs (Cont)

- Retaining rings
  - take axial load and retain shaft positions



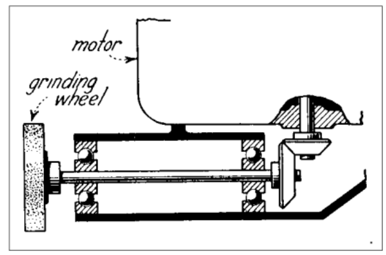
Strength Considerations

- Shaft and axle are subjected to these loads
  - bending
  - torsion
  - shear
- We have to make sure shafts can take these <u>combined</u> loads to operate

Bending Loads in Shafts

- Transverse load away from fulcrum point causes bending
  - radial load from gear
  - belt tension from pulley
- Resulting in bending stress

$$\sigma_{bending} = \frac{Mc}{I}$$
$$= \frac{4M}{\pi r^3} \quad \text{for circular shaft}$$

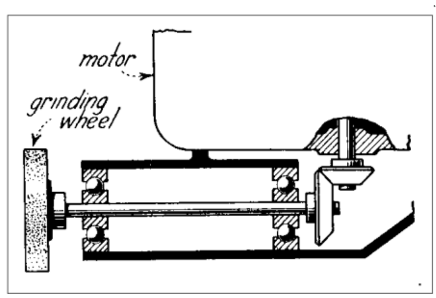


Sectional view showing how grinder shaft is geared to the motor.

Axial Loads on Shaft

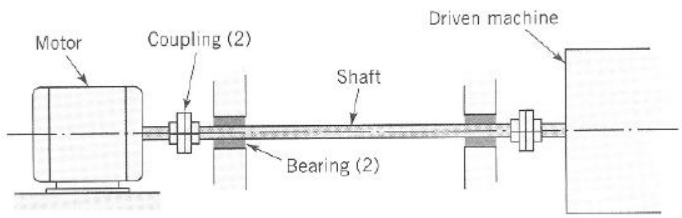
- Compressive/tensile load from axial load
  - axial load from bevel or helical gear

$$\sigma_{axial} = \frac{F_{axial}}{A}$$
$$= \frac{F_{axial}}{\pi r^2}$$



Sectional view showing how grinder shaft is geared to the motor.

Torsional Loads in Shaft



- Shear stress from torsional load

$$\tau_{torsion} = \frac{Tr}{I_p}$$
$$= \frac{2T}{\pi r^3} \text{ for circular shaft}$$

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Combined Loading on Shaft

- In machine applications, shafts are usually subjected to combined loads

$$\sigma = \sigma_{axial} + \sigma_{bending}$$
$$\tau = \tau_{torsion}$$

- Use stress transformation to analyze stress
  - principal stresses

$$\sigma_{1,2} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

  - maximum shear stress

$$\tau_{\max} = \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2}$$

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Speed Consideration

- When shaft is spinning, eccentricity causes centrifugal force deflection
  - for simply supported shaft

$$\omega = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{EI}{m}}$$

  - for clamped shaft

$$\omega = 4\left(\frac{\pi}{l}\right)^2 \sqrt{\frac{EI}{m}}$$

    - m is mass per length
    - l is length of shaft

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Example

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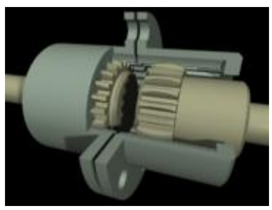
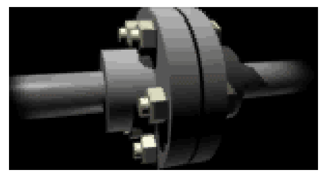
Couplings

- Device used to transfer power from one shaft end to another
- Can allow misalignment and end movement

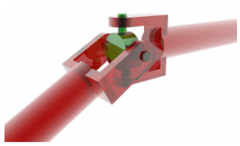
Rigid Couplings

- Can transfer large power
- no misalignment allowed
- no backlash
- example: muff couplings, clamp couplings, flange



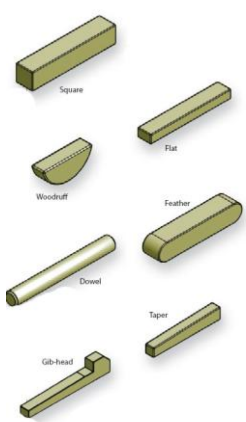
Flexible Couplings

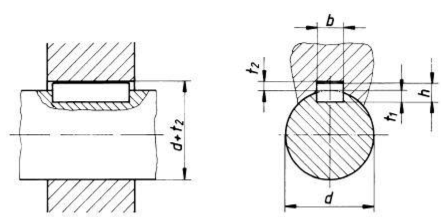
- Misalignment (angle and radial) allowed
- End movement allowed
- Examples: universal coupling, bellow coupling, CV coupling, etc



Keys and Keyways

- small material inserted in slot to transfer torque between driving mechanism and shaft



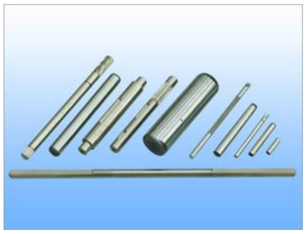


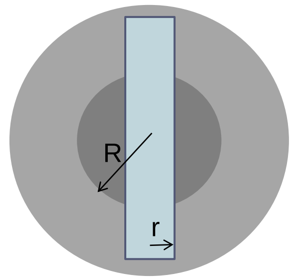
- shear stress in key is

$$\tau = \frac{F_t}{A} = \frac{2T}{dbl}$$

Pins

- Used to locate shaft and transfer torque

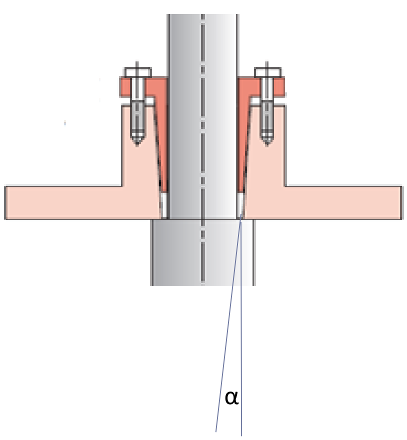




  - Shear stress in pin is due to torque *T* is

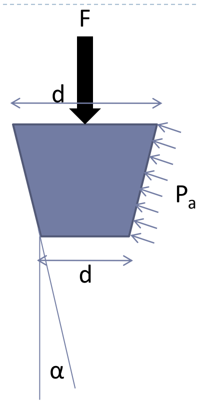
$$\tau = \frac{T}{4\pi r^2 R}$$

Tapered Coupling



- Used to fit a gear or rotating arm to a shaft
  - by using taper, axial force is translated to radial force, generating friction

Tapered Coupling Torque Capacity



- Assuming uniform pressure

$$F = p_a dA \sin\alpha = \int_{d/2}^{D/2} p_a \left(\frac{2\pi r dr}{\sin\alpha}\right)(\sin\alpha)$$

$$= \frac{\pi p_a}{4}(D^2 - d^2)$$

- torque capacity depends on friction force

$$T = \mu r p_a dA = \int_{d/2}^{D/2} \mu r p_a \left(\frac{2\pi r dr}{\sin\alpha}\right)$$

$$T = \frac{\mu F}{3\sin\alpha}\frac{D^3 - d^3}{D^2 - d^2} \qquad = \frac{\mu\pi p_a}{12\sin\alpha}(D^3 - d^3)$$

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Shaft Assembly General Rules

- Shaft should be as short as possible
  - small overhand
  - minimize bending stress
  - increase critical speed
- Bearing support should be as farthest apart as possible
  - provide highest stability

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