Flexible Mechanical Elements

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Outline

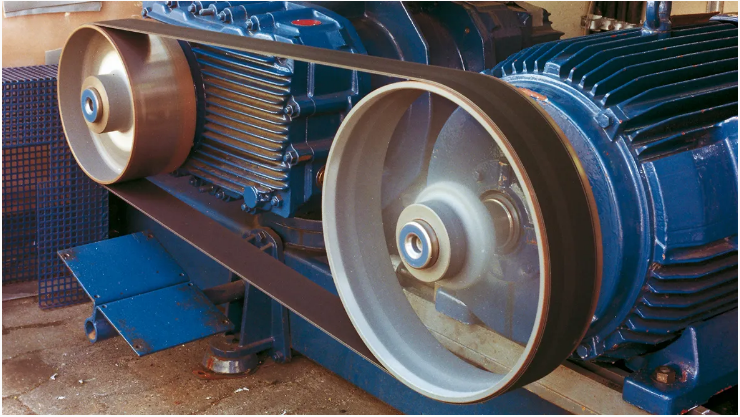
What are Flexible Mechanical Elements

- belts
- ropes
- chains
- used in transmission of power over long distances

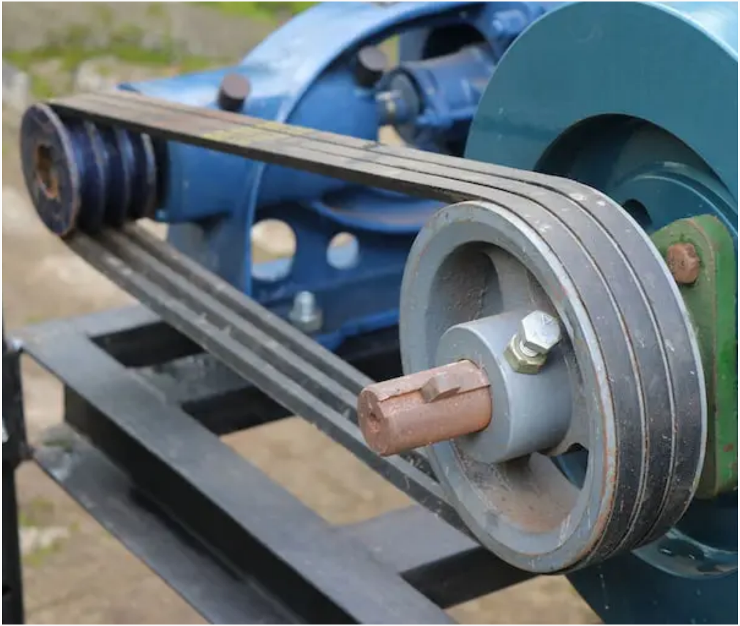
Why Flexible?

- Torque capacity: Gears > Belts, Ropes, Chains
- Flexible elements are better against vibration and shock loads
- Important to check for wear, age, and loss of elasticity

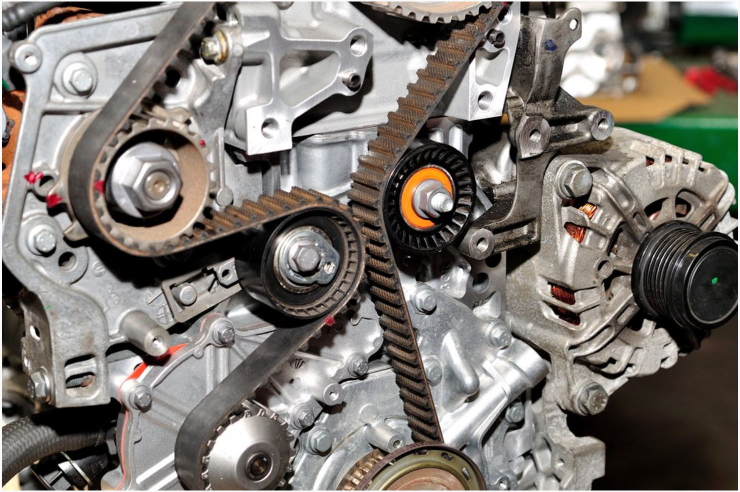
Types of Belts – Flat Belts



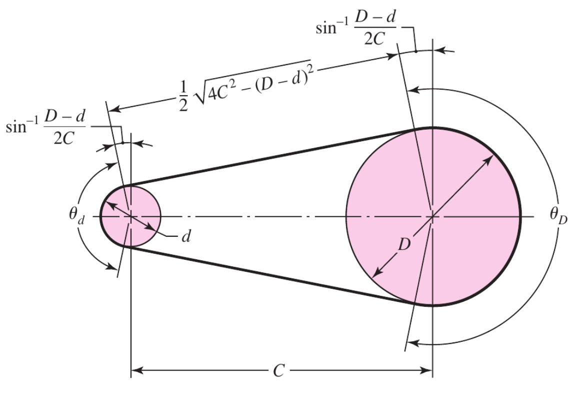
Types of Belts – V Belts



Types of Belts – Timing Belts



Belt - Pulley Geometry

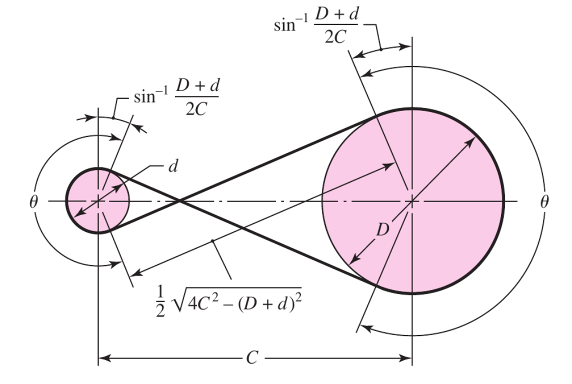


$$\theta_d = \pi - 2\sin^{-1}\frac{D-d}{2C}$$

$$\theta_D = \pi + 2\sin^{-1}\frac{D-d}{2C}$$

$$L = \sqrt{4C^2-(D-d)^2} + \frac{1}{2}(D\theta_D + d\theta_d)$$

Cross Belt - Pulley Geometry



$$\theta_d = \pi + 2\sin^{-1}\frac{D+d}{2C}$$

$$L = \sqrt{4C^2-(D+d)^2} + \frac{1}{2}(D+d)\theta$$

Outline

Flat Belts

Uses: light-duty + long-distance power transmission

Strength:

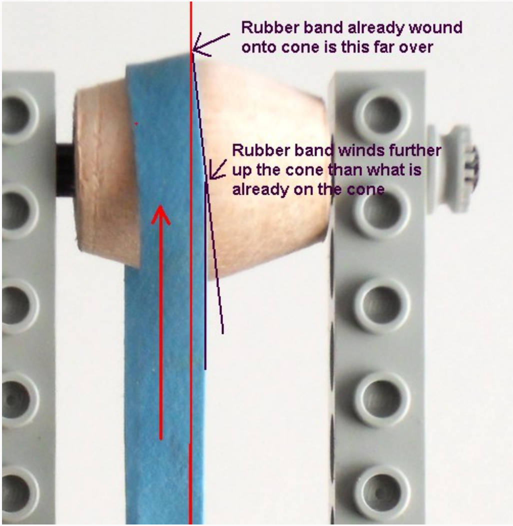
- High performance (less friction involved)
- Easy to install
- Flat surface means less wear and energy loss from friction

Weakness:

- Need high belt tension
- Shortened bearing life
- Tendency to climb

Flat Belt Assembly

- Typically paired with crowned pulleys
- Keep belt centered



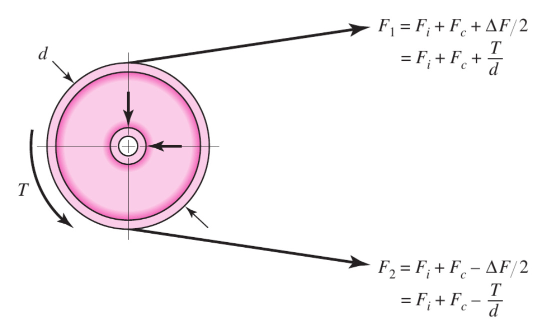
Belt Tension

- For high speed belt drive

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{\mu\theta}$$

where $F_c = m\omega^2 r^2$ is the centrifugal force on the belt and m is the mass per length of belt

Force Balance on Pulley



$$F_1 - F_2 = \frac{2T}{d}$$

$$F_1 + F_2 = 2F_i + 2F_c$$

$$F_i = \frac{F_1 + F_2}{2} + F_c$$

Initial Belt Tension F_i

$$
\begin{aligned}
\frac{F_i}{T/d} &= \frac{(F_1 + F_2)/2 - F_c}{(F_1 - F_2)/2} = \frac{F_1 + F_2 - 2F_c}{F_1 - F_2} \\
&= \frac{(F_1 - F_c) + (F_2 - F_c)}{(F_1 - F_c) - (F_2 - F_c)} \\
&= \frac{(F_1 - F_c)/(F_2 - F_c) + 1}{(F_1 - F_c)/(F_2 - F_c) - 1} \\
&= \frac{e^{\mu\theta} + 1}{e^{\mu\theta} - 1} \\
F_i &= \frac{T}{d}\frac{e^{\mu\theta} + 1}{e^{\mu\theta} - 1}
\end{aligned}
$$

- Note: $F_i = 0 \rightarrow T = 0$

Tight and Slack Side Tensions

$$
\begin{aligned}
F_1 &= F_c + F_i + \frac{T}{d} = F_c + F_i + F_i \frac{e^{\mu\theta} - 1}{e^{\mu\theta} + 1} \\
&= F_c + F_i \frac{\left(e^{\mu\theta} + 1 + e^{\mu\theta} - 1\right)}{e^{\mu\theta} + 1} \\
&= F_c + F_i \frac{2e^{\mu\theta}}{e^{\mu\theta} + 1} \\
F_2 &= F_c + F_i \frac{2}{e^{\mu\theta} + 1}
\end{aligned}
$$

- Power transmitted

$$
H = (F_1 - F_2)v
$$

Belt Design

$$(F_1)_a = bF_aC_pC_v$$

$$\begin{aligned}
(F_1)_a &= \text{allowable largest tension} \\
b &= \text{belt width} \\
F_a &= \text{manufacturer's allowed tension per width (N/m)} \\
C_p &= \text{pulley correction factor} \\
C_v &= \text{velocity correction factor = 1 except leather belts}
\end{aligned}$$

Manufacturer's Allowed Tension: F_a

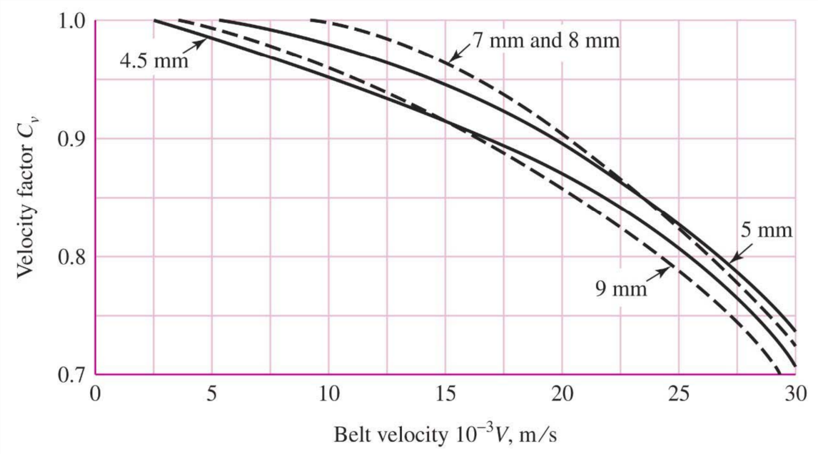
Material	Specification	Size, mm	Minimum Pulley Diameter, mm	Allowable Tension per Unit Width at 3 m/s, (10^3) N/m	Specific Weight, kN/m³	Coefficient of Friction
Leather	1 ply	$t = 4.5$	75	5	9.5–12.2	0.4
		$t = 5$	90	6	9.5–12.2	0.4
	2 ply	$t = 7$	115	7	9.5–12.2	0.4
		$t = 8$	150	9	9.5–12.2	0.4
		$t = 9$	230	10	9.5–12.2	0.4
Polyamide[b]	F–0[c]	$t = 0.8$	15	1.8	9.5	0.5
	F–1[c]	$t = 1.3$	25	6	9.5	0.5
	F–2[c]	$t = 1.8$	60	10	13.8	0.5
	A–2[c]	$t = 2.8$	60	10	10.0	0.8
	A–3[c]	$t = 3.3$	110	18	11.4	0.8
	A–4[c]	$t = 5.0$	240	30	10.6	0.8
	A–5[c]	$t = 6.4$	340	48	10.6	0.8
Urethane[d]	$w = 12.7$	$t = 1.6$	See	1.0[e]	10.3–12.2	0.7
	$w = 19$	$t = 2.0$	Table	1.7[e]	10.3–12.2	0.7
	$w = 32$	$t = 2.3$	17–3	3.3[e]	10.3–12.2	0.7
	Round	$d = 6$	See	1.4[e]	10.3–12.2	0.7
		$d = 10$	Table	3.3[e]	10.3–12.2	0.7
		$d = 12$	17–3	5.8[e]	10.3–12.2	0.7
		$d = 20$		13[e]	10.3–12.2	0.7

Pulley Correction Factor: C_p

Material	Small-Pulley Diameter, mm					
	40 — 100	115 – 200	220 – 310	355 – 405	460 – 800	Over 800
Leather	0.5	0.6	0.7	0.8	0.9	1.0
Polyamide, F–0	0.95	1.0	1.0	1.0	1.0	1.0
F–1	0.70	0.92	0.95	1.0	1.0	1.0
F–2	0.73	0.86	0.96	1.0	1.0	1.0
A–2	0.73	0.86	0.96	1.0	1.0	1.0
A–3	—	0.70	0.87	0.94	0.96	1.0
A–4	—	—	0.71	0.80	0.85	0.92
A–5	—	—	—	0.72	0.77	0.91

- C_p = 1 for urethane belts

Velocity Correction Factor: C_v



- C_v = 1 for polyamide and urethane belts

Example: Flat belt design

A polyamide A-3 flat belt 15 cm wide is used to transmit 15 hp power. The pulley rotational axes are parallel, the shafts are 2.5 m apart. The driving pulley diameter is 15 cm and rotates at 1750 rpm. The driven pulley diameter is 45 cm.

1. Estimate the centrifugal tension F_c
2. Determine the required initial tension F_i
3. Estimate the allowable F_1 and F_2

Solution: Flat belt design (1)

Find F_c

$$\begin{aligned}
\gamma &= 11.4 \text{ kN/m}^3 \\
\rho &= \frac{\gamma}{g} = 11.4 \times 10^3/9.81 = 1.162 \times 10^3 \text{ kg/m}^3 \\
m &= \rho A = 1.162 \times 10^3 (3.3 \times 10^{-3})(0.15) = 0.575 \text{ kg/m} \\
F_c &= m\omega^2 r^2 = 0.575 \times (1750(2\pi/60))^2 (0.15/2)^2 = 108.6 \text{ N}
\end{aligned}$$

Solution: Flat belt design (2)

First, find θ on the smaller pulley (smaller pulley = smaller angle)

$$\begin{aligned}\theta_d &= \pi - 2\sin^{-1}\frac{D-d}{2C} \\ &= \pi - 2\sin^{-1}\frac{45-15}{2(250)} \\ &= 3.02 \text{ rad}\end{aligned}$$

Next, find input torque T

$$\begin{aligned}T = \frac{H}{\omega} &= \frac{15(746)}{1750\frac{2\pi}{60}} \\ &= 61.1 \text{ N-m}\end{aligned}$$

Solution: Flat belt design (2)

Finally,

$$F_i = \frac{T}{d}\frac{e^{\mu\theta}+1}{e^{\mu\theta}-1} = \frac{61.1}{0.15}\frac{e^{0.8(3.02)}+1}{e^{0.8(3.02)}-1} = 487 \text{ N}$$

Solution: Flat belt design (3)

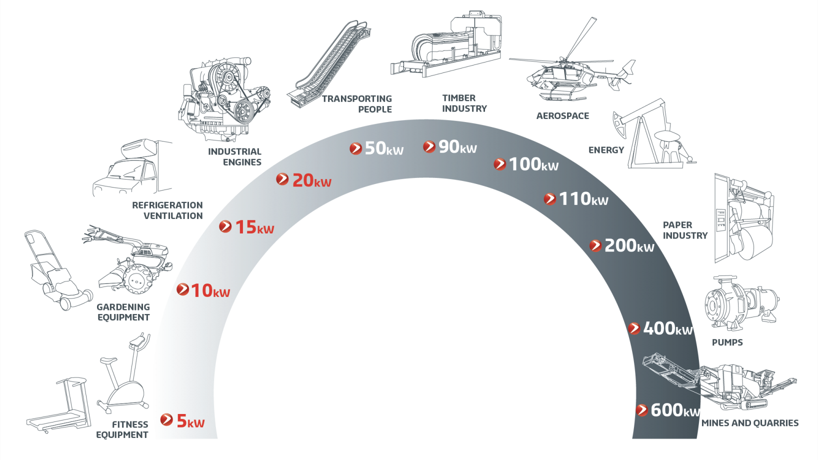
For F_1 and F_2

$$
\begin{aligned}
F_1 &= F_c + F_i \frac{2e^{\mu\theta}}{e^{\mu\theta} + 1} \\
&= 108.6 + 487 \frac{2e^{0.8(3.02)}}{e^{0.8(3.02)} + 1} \\
&= 1003 \text{ N} \\
F_2 &= F_c + F_i \frac{2}{e^{\mu\theta} + 1} \\
&= 108.6 + 487 \frac{2}{e^{0.8(3.02)} + 1} \\
&= 188.4 \text{ N}
\end{aligned}
$$

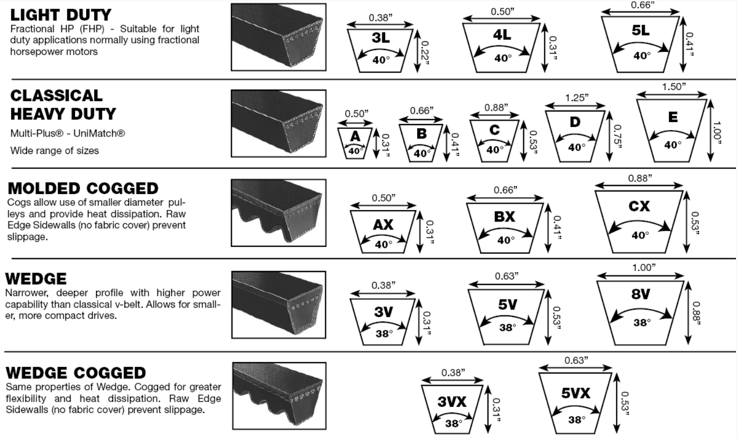
Why V Belts?

- Increased tension forces belt further into groove, providing more friction
- More friction $\rightarrow$ increased torque capacity, but at lower efficiency and increased rate of wear

V Belt Application



V Belt Sections



V Belt Tension

- Similar equations to flat belts
- Minor adjustment for groove angle $\beta \approx 40°$

$$T = (F_1 - F_2)\, r$$

$$H = (F_1 - F_2)\, v$$

$$\frac{F_1 - F_c}{F_2 - F_c} = e^{\frac{\mu\theta}{\sin(\beta/2)}}$$

V Belt Design Equation

- Determine based on belt speed and power requirement

$$N_s = \frac{H_a n}{H K_s}$$

N_s safety factor

H_a allowable power per belt

n number of belts

H actual power = $(F_1 - F_2)v$

K_s service factor

Allowable Power (H_a) in KW per belt

Belt Section	Sheave Pitch Diameter, mm	Belt Speed, m/s 5	10	15	20	25
A	65	0.35	0.46	0.40	0.11	
	75	0.49	0.75	0.84	0.69	0.28
	85	0.60	0.98	1.17	1.64	0.84
	95	0.69	1.16	1.43	1.49	1.28
	105	0.77	1.30	1.64	1.78	1.63
	115	0.83	1.41	1.82	2.01	1.93
	125 and up	0.87	1.51	1.97	2.21	2.16
B	105	0.80	1.18	1.25	0.94	0.16
	115	0.95	1.48	1.71	1.55	0.92
	125	1.07	1.74	2.09	2.06	1.57
	135	1.19	1.95	2.42	2.49	2.10
	145	1.28	2.14	2.69	2.87	2.57
	155	1.36	2.31	2.94	3.19	2.98
	165	1.43	2.45	3.16	3.48	3.34
	175 and up	1.50	2.58	3.35	3.74	3.66
C	150	1.37	1.98	2.03	1.40	
	175	1.85	2.94	3.46	3.31	2.33
	200	2.21	3.66	4.54	4.74	4.12
	225	2.49	4.21	5.38	5.86	5.51
	250	2.72	4.66	6.05	7.16	6.63
	275	2.89	5.03	6.59	7.46	7.53
	300 and up	3.05	5.33	7.06	8.13	8.28
D	250	3.09	4.57	4.89	3.80	1.01
	275	3.73	5.84	6.80	6.34	4.19
	300	4.26	6.91	8.36	8.50	6.85
	325	4.71	7.83	9.70	10.30	9.10
	350	5.09	8.58	10.89	11.79	11.04
	375	5.42	9.25	11.86	13.13	12.68
	400	5.71	9.85	12.76	14.32	14.17
	425 and up	5.98	10.37	13.50	15.37	15.44
E	400	6.48	10.44	13.06	13.50	11.41
	450	7.40	12.46	15.82	17.16	16.04
	500	8.13	13.95	18.05	20.07	19.69
	550	8.73	15.14	19.84	22.53	22.75
	600	9.25	16.11	21.34	24.54	25.22
	650	9.70	17.01	22.60	26.19	27.38
	700 and up	10.00	17.68	23.72	27.68	29.17

Service Factor

	Source of Power	
Driven Machinery	**Normal Torque Characteristic**	**High or Nonuniform Torque**
Uniform	1.0 to 1.2	1.1 to 1.3
Light shock	1.1 to 1.3	1.2 to 1.4
Medium shock	1.2 to 1.4	1.4 to 1.6
Heavy shock	1.3 to 1.5	1.5 to 1.8

Example: V Belt Design

A 5-hp gasoline engine is connected to a water pump by v-belt and sheaves at 2000 rpm. The driving sheave has d = 200 mm. Choose a proper belt section and the number of belts for the job with N_s = 1.5.

Solution: V Belt Design

- Belt speed v is

$$
\begin{aligned}
v &= 2000\left(\frac{2\pi}{60}\right)(0.2/2) \\
&= 20.94 \text{ m/s}
\end{aligned}
$$

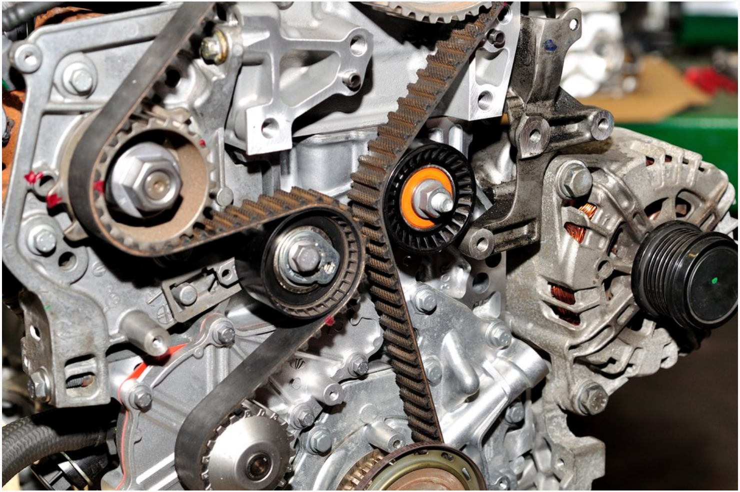
- Gasoline engines are usually classified as 'medium' shock

$$
\begin{aligned}
H_a &= N_s\frac{HK_s}{n} \\
&= 1.5\frac{5(746)(1.4)}{1} \\
&= 7833 \text{ W} = 7.83 \text{ kW}
\end{aligned}
$$

Solution: V Belt Design

- As d = 200 mm, we are limited to sections A, B, and C
- v = 20.94 m/s (round down to 20 m/s) $\rightarrow$ 4th column
- for section C, d = 200, H_a = 4.74 kW $\rightarrow$ 2 belts
- for section B, d = 200, H_a = 3.74 kW $\rightarrow$ 3 belts
- for section A, d = 200, H_a = 2.21 kW $\rightarrow$ 4 belts

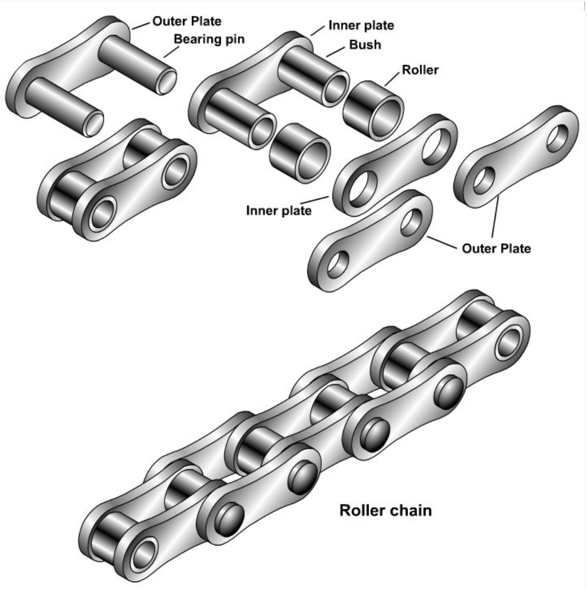
Timing Belts



- No significant stretch or slip → power where speed ratio is important
- efficiency of 97 - 99%

Outline

Roller Chain Components



Why Roller Chain (Chain Drives)?

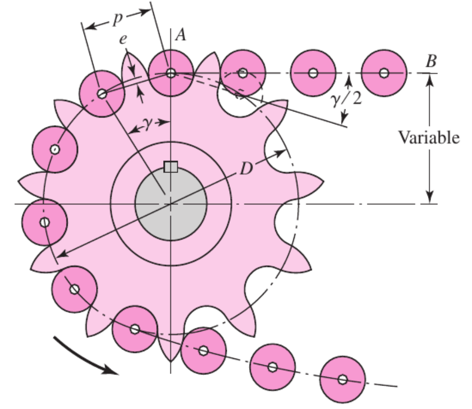
- No slip $\rightarrow$ constant ratio
- Long life
- Can drive multiple shafts from a single source

Forces and Powers

- Chain tension only on tight side

$$T = \frac{F_1 D}{2}$$
$$H = Fv$$

Equations on Roller Chains

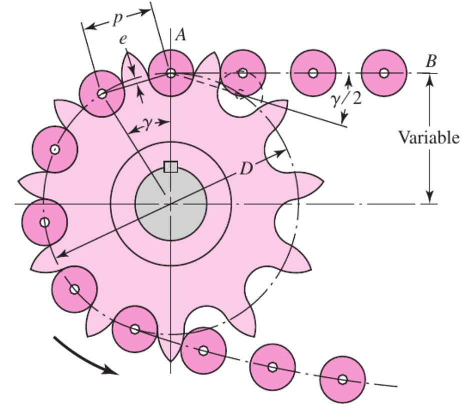


$$D = \frac{p}{\sin\left(\frac{180^\circ}{N}\right)} = \frac{p}{\sin\left(\frac{\gamma}{2}\right)}$$

$$v = Npn$$

- v velocity
- N number of sprocket teeth
- p chain pitch
- n sprocket speed [rev/min]

Chordal Speed Variation



- max chain exit velocity

$$v_{\text{max}} = \pi D n = \frac{\pi n p}{\sin(\gamma/2)}$$

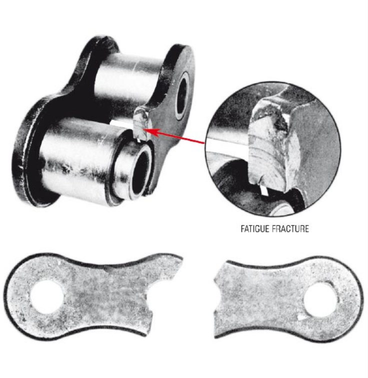
- min chain exit velocity

$$v_{\text{min}} = \pi d n = \pi n p \frac{\cos(\gamma/2)}{\sin(\gamma/2)}$$

- decrease γ to reduce variation

Failure Mode

- Because of speed variation and roller components, chains can fail from fatigue and wear



Link-plate failure from fatigue

Roller wear/fatigue

Design Criteria

- Roller chains rarely fail because of tensile stress
- Need to worry more about fatigue and wear on rollers
- To minimize chordal speed variation. $N \geqslant 17$

Standard Chain Dimensions

ANSI Chain Number	Pitch, in (mm)	Width, in (mm)	Minimum Tensile Strength, lbf (N)	Average Weight, lbf/ft (N/m)	Roller Diameter, in (mm)	Multiple-Strand Spacing, in (mm)
25	0.250 (6.35)	0.125 (3.18)	780 (3 470)	0.09 (1.31)	0.130 (3.30)	0.252 (6.40)
35	0.375 (9.52)	0.188 (4.76)	1 760 (7 830)	0.21 (3.06)	0.200 (5.08)	0.399 (10.13)
41	0.500 (12.70)	0.25 (6.35)	1 500 (6 670)	0.25 (3.65)	0.306 (7.77)	— —
40	0.500 (12.70)	0.312 (7.94)	3 130 (13 920)	0.42 (6.13)	0.312 (7.92)	0.566 (14.38)
50	0.625 (15.88)	0.375 (9.52)	4 880 (21 700)	0.69 (10.1)	0.400 (10.16)	0.713 (18.11)
60	0.750 (19.05)	0.500 (12.7)	7 030 (31 300)	1.00 (14.6)	0.469 (11.91)	0.897 (22.78)
80	1.000 (25.40)	0.625 (15.88)	12 500 (55 600)	1.71 (25.0)	0.625 (15.87)	1.153 (29.29)
100	1.250 (31.75)	0.750 (19.05)	19 500 (86 700)	2.58 (37.7)	0.750 (19.05)	1.409 (35.76)
120	1.500 (38.10)	1.000 (25.40)	28 000 (124 500)	3.87 (56.5)	0.875 (22.22)	1.789 (45.44)
140	1.750 (44.45)	1.000 (25.40)	38 000 (169 000)	4.95 (72.2)	1.000 (25.40)	1.924 (48.87)

Power Capacity H

- For link-plate limit

$$H_1 = 0.004 N_1^{1.08} n_1^{0.9} p^{3-0.07p} \text{ hp}$$

- For roller limit

$$H_2 = \frac{1000 K_t N_1^{1.5} p^{0.8}}{n_1^{1.5}} \text{ hp}$$

- N_1 = number of teeth in smaller sprocket
- n_1 = sprocket speed [rev/min]
- p = pitch of chain [m]
- K_t = 29 for chain numbers 25,35; 3.4 for 41; and 17 for 40-240

Roller Chain Rated Capacity in hp (1)

Sprocket Speed, rev/min	ANSI Chain Number 25	35	40	41	50	60
50	0.05	0.16	0.37	0.20	0.72	1.24
100	0.09	0.29	0.69	0.38	1.34	2.31
150	0.13*	0.41*	0.99*	0.55*	1.92*	3.32
200	0.16*	0.54*	1.29	0.71	2.50	4.30
300	0.23	0.78	1.85	1.02	3.61	6.20
400	0.30*	1.01*	2.40	1.32	4.67	8.03
500	0.37	1.24	2.93	1.61	5.71	9.81
600	0.44*	1.46*	3.45*	1.90*	6.72*	11.6
700	0.50	1.68	3.97	2.18	7.73	13.3
800	0.56*	1.89*	4.48*	2.46*	8.71*	15.0
900	0.62	2.10	4.98	2.74	9.69	16.7
1000	0.68*	2.31*	5.48	3.01	10.7	18.3
1200	0.81	2.73	6.45	3.29	12.6	21.6
1400	0.93*	3.13*	7.41	2.61	14.4	18.1
1600	1.05*	3.53*	8.36	2.14	12.8	14.8
1800	1.16	3.93	8.96	1.79	10.7	12.4
2000	1.27*	4.32*	7.72*	1.52*	9.23*	10.6
2500	1.56	5.28	5.51*	1.10*	6.58*	7.57
3000	1.84	5.64	4.17	0.83	4.98	5.76
Type A		Type B			Type C	

Roller Chain Rated Capacity in hp (2)

Sprocket Speed, rev/min		ANSI Chain Number 80	100	120	140	160	180	200	240
50	Type A	2.88	5.52	9.33	14.4	20.9	28.9	38.4	61.8
100		5.38	10.3	17.4	26.9	39.1	54.0	71.6	115
150		7.75	14.8	25.1	38.8	56.3	77.7	103	166
200		10.0	19.2	32.5	50.3	72.9	101	134	215
300		14.5	27.7	46.8	72.4	105	145	193	310
400		18.7	35.9	60.6	93.8	136	188	249	359
500	Type B	22.9	43.9	74.1	115	166	204	222	0
600		27.0	51.7	87.3	127	141	155	169	
700		31.0	59.4	89.0	101	112	123	0	
800		35.0	63.0	72.8	82.4	91.7	101		
900		39.9	52.8	61.0	69.1	76.8	84.4		
1000		37.7	45.0	52.1	59.0	65.6	72.1		
1200		28.7	34.3	39.6	44.9	49.9	0		
1400		22.7	27.2	31.5	35.6	0			
1600		18.6	22.3	25.8	0				
1800		15.6	18.7	21.6					
2000		13.3	15.9	0					
2500		9.56	0.40						
3000		7.25	0						
Type C				Type C′					

Example

Pick a roller chain for a motorcycle whose output engine is 15 hp at 1000 rpm. The driving sprocket has 20 teeth and the output has 39 teeth.

Solution

For 15 hp at 1000 rpm, try ANSI 60. p = 0.75 in

$$
\begin{aligned}
H_1 &= 0.003(20)^{1.08}(1000)^{0.9}(0.75)^{3-0.07(0.75)} \\
&= 21.8\text{ kW} \\
H_2 &= \frac{1000(17)(20)^{1.5}(0.75)^{0.8}}{1000^{1.5}} \\
&= 38.2\text{ kW}
\end{aligned}
$$

This means the chain can deliver the power with the safety factor of 21.8/15 = 1.45