Plate Design

ME 313: Mechanical Design
Week 5

Definition of Plate

- A flat component that takes the load on its surface
- The main mechanism of load resistance is bending

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Plate Governing Equation

- Much like beam bending, plate bending also has a governing equation

$$\frac{\partial^4 v}{\partial x^4} + \frac{\partial^4 v}{\partial x^2 \partial y^2} + \frac{\partial^4 v}{\partial y^4} = \frac{P}{D}$$

  - where *v (English letter v)* is the deflection, *P* is the load, and *D* is the bending rigidity of plate

$$D = \frac{Et^3}{12(1-\nu^2)}$$

  - where *E* is the modulus of elasticity, *t* is the thickness of the plate, and ν *(Greek letter nu)* is the Poisson's ratio

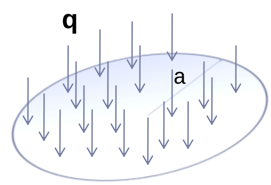
Plate Deflection Under Loads

- Deflection depends on various things
  - Plate shape: circular or rectangular
  - Boundary conditions: fixed (clamped) or simple support
- We will go through the equations for plate deflections for each case

Circular Plate with Uniform Load

- Clamped edges
  - Maximum deflection

$$v_{\max} = \frac{qa^4}{64D}$$



  - Maximum stress

$$\sigma_{\max} = \frac{3}{4}\frac{qa^2}{t^2}$$

  - where q is the magnitude of load per area (pressure)

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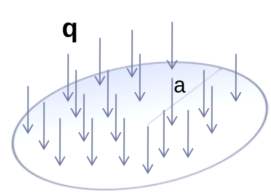
Circular Plate with Uniform Load

- Simply supported edges
  - Maximum deflection

$$v_{\max} = \frac{(5+\nu)qa^4}{64(1+\nu)D}$$



  - Maximum stress

$$\sigma_{\max} = \frac{3(3+\nu)qa^2}{8t^2}$$

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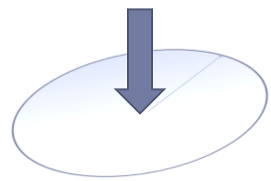
Circular Plate Loaded at the Center

- Clamped edges
  - Maximum deflection

$$v_{\max} = \frac{Pa^2}{16\pi D}$$

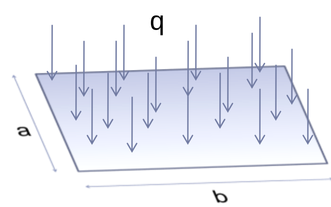


- Simply supported edges

$$v_{\max} = \frac{(3+\nu)Pa^2}{16\pi(1+\nu)D}$$

Rectangular Plate under Uniform Loading

- Clamped edges
  - Maximum deflection

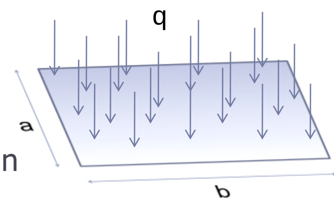


$$v_{\max} = \frac{\alpha q a^4}{D}$$

b/a	α
1.0	0.00126
1.5	0.00220
2.0	0.00254

Rectangular Plate under Uniform Loading

- Clamped edges
  - Maximum stress can be calculated from bending moment
  - There are bending moments both in x and y directions



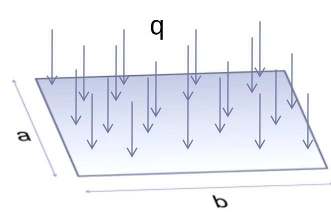
$$\sigma_{\max} = \frac{6M_{bending}}{t^2}$$

$$M_{bending} = \beta q a^2$$

b/a	β_x	β_y
1.0	-0.0513	-0.0513
1.5	-0.0757	-0.0570
2.0	-0.0829	-0.0571

Rectangular Plate under Uniform Loading

- Simply supported edges
  - Maximum deflection

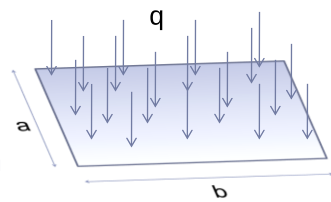


$$v_{\max} = \frac{\alpha q a^4}{D}$$

b/a	α
1.0	0.00406
1.5	0.00772
2.0	0.01013

Rectangular Plate under Uniform Loading

- Simply supported edges
  - Maximum stress can be calculated from bending moment
  - There are bending moment both in x and y directions



$$\sigma_{\max} = \frac{6M_{bending}}{t^2}$$

$$M_{bending} = \beta q a^2$$

b/a	β_x	β_y
1.0	0.0479	0.0479
1.5	0.0812	0.0498
2.0	0.1017	0.0464

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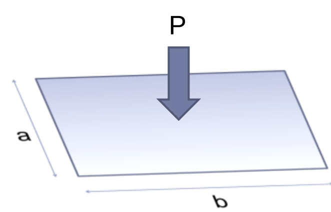
Rectangular Plate under Central Load

- Clamped edges
  - Maximum deflection



$$v_{\max} = \frac{\alpha P a^2}{D}$$

b/a	α
1.0	0.00560
1.5	0.00702
2.0	0.00722

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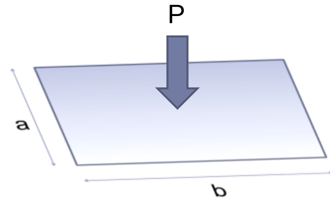
Rectangular Plate under Central Load

- Simply supported edges
  - Maximum deflection

$$v_{\max} = \frac{\alpha P a^2}{D}$$



b/a	α
1.0	0.01160
1.5	0.01527
2.0	0.01651

Plate Design Conclusion

1. Know the requirements: stress and/or deflection
2. Choose the shapes
3. Determine shape parameters: radius, or length x width
4. Compute maximum deflection/stress
5. Is the design satisfactory?
   - If not, make appropriate adjustments

Shell Design

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Shell Structures

- Curved thin component
  - Thickness must be much smaller than radius of curvature

$$t << r$$

  - Used in roofs, walls, containers, …

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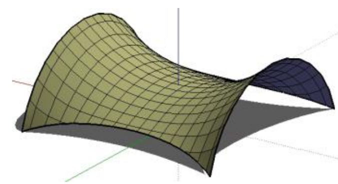
Shell Curvature

- Defined by Gaussian curvature

$$K = \frac{1}{R_x} \times \frac{1}{R_y}$$



- *K > 0*
- *K = 0*
- *K < 0*

Governing Equation

$$\frac{N_x}{R_x} + \frac{N_y}{R_y} = P$$

- Force per length depends on the radius of curvature
- This is called *membrane equation*

Applications of Shell Equation

- Spherical pressure vessels
- Cylindrical pressure vessels

Example: Dented Coke Can