Clutches and Brakes

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Outline

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Clutches and Brakes

- Rely on friction to transfer torque
- Easy to engage/disengage

Clutches vs Brakes

when engaged

Clutches $\omega_{in} = \omega_{out} \neq 0$

Brakes $\omega_{in} = \omega_{out} = 0$

Considerations for Clutch and Brake

Actuating force force to engage clutch/brake

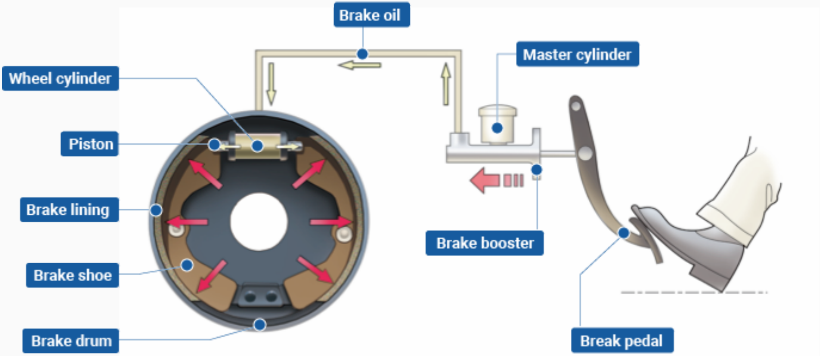
Transmitted torque torque through mechanism

Energy loss energy dissipated before mechanism is fully engaged

Temperature rise temperature increase from energy loss

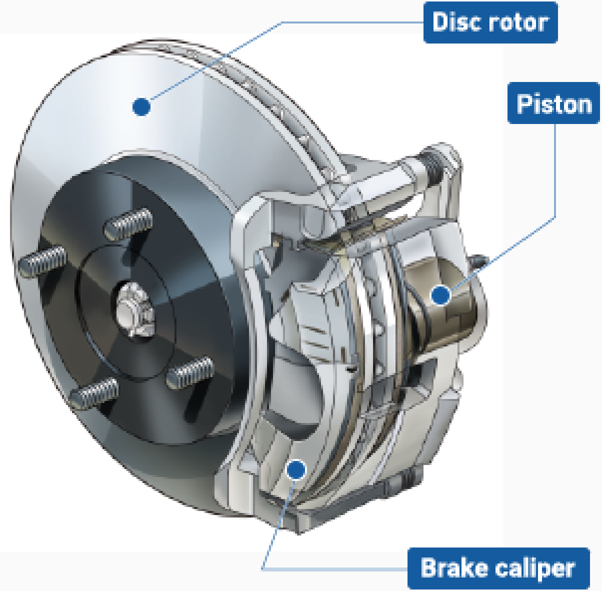
Outline

Drum Brakes

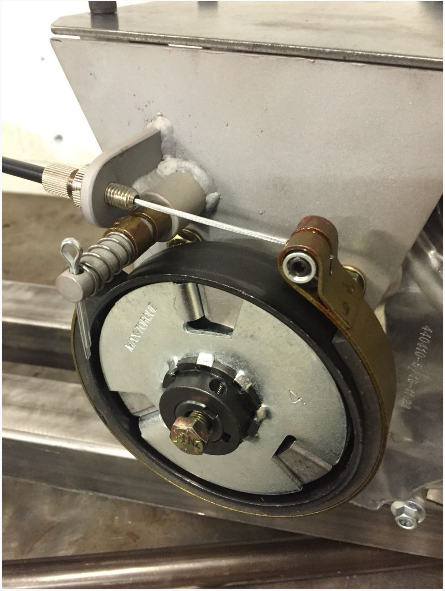


Brake oil
Master cylinder
Wheel cylinder
Piston
Brake lining
Brake booster
Brake shoe
Brake drum
Break pedal

Disc Brakes



Band Brakes



Outline

Materials

Molded thermosetting polymer or rubber + heat resistant fibers

Woven fibers + brass or zinc woven into fabric + resin

Sintered metal metal powder + inorganic fillers molded and sintered

Dry Linings

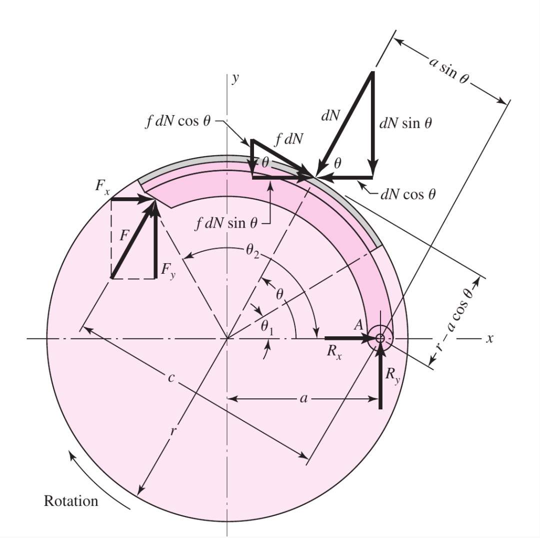
Friction Material[a]	Dynamic Friction Coefficient f[b]	Maximum Pressure[c]		Maximum Bulk Temperature	
		psi	kPa	°F	°C
Molded	0.25–0.45	150–300	1030–2070	400–500	204–260
Woven	0.25–0.45	50–100	345–690	400–500	204–260
Sintered metal	0.15–0.45	150–300	1030–2070	450–1250	232–677
Cork	0.30–0.50	8–14	55–95	180	82
Wood	0.20–0.30	50–90	345–620	200	93
Cast iron, hard steel	0.15–0.25	100–250	690–1720	500	260

Wet Linings

Friction Material[a]	Dynamic Friction Coefficient f
Molded	0.06–0.09
Woven	0.08–0.10
Sintered metal	0.05–0.08
Paper	0.10–0.14
Graphitic	0.12 (avg.)
Polymeric	0.11 (avg.)
Cork	0.15–0.25
Wood	0.12–0.16
Cast iron, hard steel	0.03–0.06

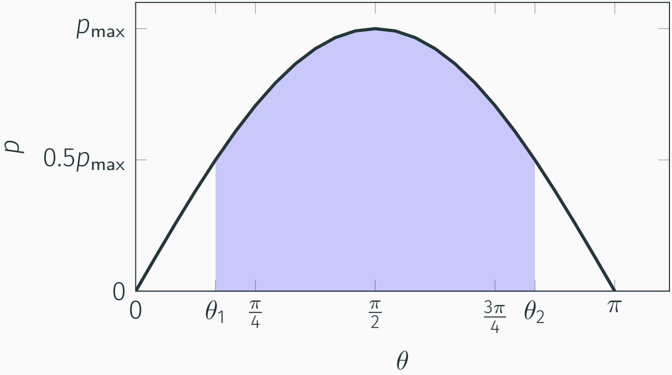
Outline

Internal Drum Brake



- Considering moments about $A \rightarrow 3$ moments
  - moment from normal force, M_n
  - moment from friction, M_f
  - moment from actuating force, Fc

Pressure Distribution on Drum



$$p = \frac{p_{\max}}{(\sin\theta)_{\max}} \sin\theta$$

- $(\sin\theta)_{\max}$ = maximum value of $\sin\theta$ for $\theta_1 \leqslant \theta \leqslant \theta_2$ (not always 1)

Moment Generated on Drum by Normal Force

$$M_n = \int_{\theta_1}^{\theta_2} dN(a \sin\theta)$$

$$dN = p(rd\theta)b$$

$$dN = \frac{p_{\text{max}} br \sin\theta d\theta}{(\sin\theta)_{\text{max}}}$$

$$M_n = \int_{\theta_1}^{\theta_2} \frac{p_{\text{max}} bra \sin^2\theta}{(\sin\theta)_{\text{max}}} d\theta$$

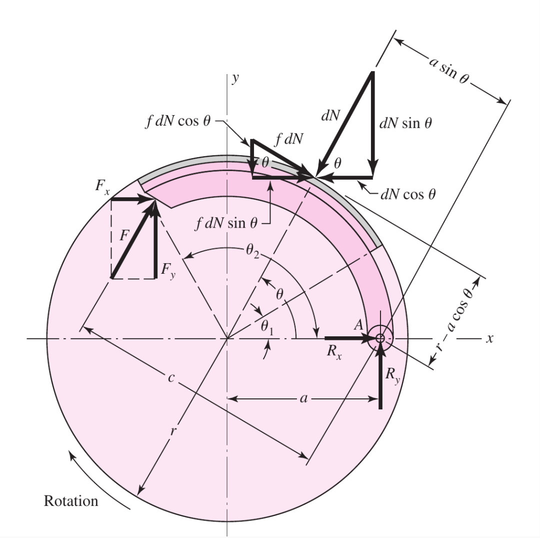
$$= \frac{p_{\text{max}} bra}{(\sin\theta)_{\text{max}}} \int_{\theta_1}^{\theta_2} \sin^2\theta d\theta$$

$$= \frac{p_{\text{max}} bra}{4(\sin\theta)_{\text{max}}} [2(\theta_2 - \theta_1) - \sin 2\theta_2 + \sin 2\theta_1]$$

Moment Generated on Drum by Friction

$$
\begin{aligned}
M_f &= \int_{\theta_1}^{\theta_2} \mu dN(r - a\cos\theta) \\
&= \int_{\theta_1}^{\theta_2} \frac{\mu p_{\max} \sin\theta r d\theta b(r - a\cos\theta)}{(\sin\theta)_{\max}} \\
&= \frac{\mu p_{\max} b r}{(\sin\theta)_{\max}} \left[r(\cos\theta_1 - \cos\theta_2) + \frac{a}{4}(\cos 2\theta_2 - \cos 2\theta_1) \right]
\end{aligned}
$$

Required Actuating Force F



$$F = \frac{M_n - M_f}{c}$$

- If the rotation is reversed, how are the moments changed?

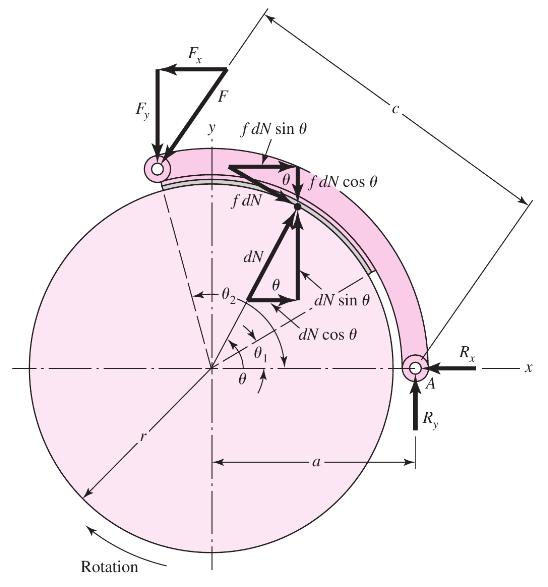
Self-energizing Brake

- if $M_f \geqslant M_n$, the brake is self-energizing
- Moment from friction further presses the shoe against the drum $\rightarrow$ more braking torque
- The shoe sticks to the drum without actuating force F

Torque Generated on the Drum

$$
\begin{aligned}
T &= \int_{\theta_1}^{\theta_2} \mu r dN \\
&= \frac{\mu r^2 b p_{\mathrm{max}}}{(\sin\theta)_{\mathrm{max}}} \int_{\theta_1}^{\theta_2} \sin\theta d\theta \\
&= \frac{\mu r^2 b p_{\mathrm{max}}}{(\sin\theta)_{\mathrm{max}}} (-\cos\theta)\Big|_{\theta_1}^{\theta_2} \\
&= \frac{\mu r^2 b p_{\mathrm{max}}}{(\sin\theta)_{\mathrm{max}}} (\cos\theta_1 - \cos\theta_2)
\end{aligned}
$$

External Drum Brake



Required Actuating Force F

- Normal force flip direction

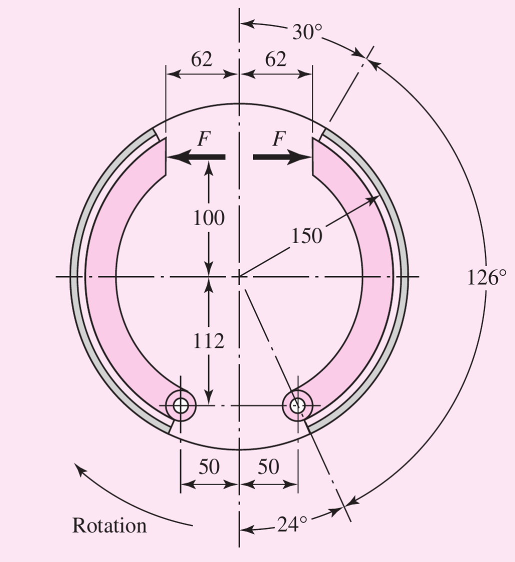
$$F = \frac{M_n + M_f}{c}$$

- Self-energizing NOT possible

Torque Generated on the Drum

- identical equations to internal drum brake, only need to be careful about the direction of actuating force

Example: Braking torque of a drum brake



- F = 2000 N
- μ = 0.3
- b = 3 cm

Determine the braking torque.

Solution

First, we must determine p_{max} on the right shoe. In this case, M_n and M_f go in opposite directions.

$$Fc = M_n - M_f$$
$$M_n = \frac{p_{\mathrm{max}} bra}{4(\sin\theta)_{\mathrm{max}}}[2(\theta_2 - \theta_1) - \sin 2\theta_2 + \sin 2\theta_1]$$

Solution

Let us first find M_n as a function of p_{max}

$$\begin{aligned}
a &= \sqrt{0.112^2 + 0.05^2} = 0.123 \text{ m} \\
M_n &= \frac{p_{\text{max}} bra}{4(\sin\theta)_{\text{max}}}[2(\theta_2 - \theta_1) - \sin 2\theta_2 + \sin 2\theta_1] \\
&= \frac{p_{\text{max}}(0.03)(0.15)(0.123)}{4(\sin 90^\circ)}\left[2(126^\circ(\frac{\pi}{180^\circ})) - \sin(2(126^\circ))\right] \\
&= 7.38 \times 10^{-4} p_{\text{max}}
\end{aligned}$$

Solution

Now find M_f as a function of p_{max}

$$
\begin{aligned}
M_f &= \frac{\mu p_{\text{max}} b r}{(\sin\theta)_{\text{max}}}\left[r(\cos\theta_1 - \cos\theta_2) + \frac{a}{4}(\cos 2\theta_2 - \cos 2\theta_1)\right] \\
&= \frac{0.3 p_{\text{max}}(0.03)(0.15)}{\sin 90^\circ}\,[(0.15)(\cos 0 - \cos 126^\circ) + \\
&\frac{0.123}{4}(\cos 2(126^\circ) - \cos 2(0))\Big] \\
&= 2.67 \times 10^{-4} p_{\text{max}}
\end{aligned}
$$

Solution

$$
\begin{aligned}
Fc &= M_n - M_f \\
2000(0.212) &= p_{\text{max}}(7.38 - 2.67) \times 10^{-4} \\
p_{\text{max}} &= 9.00 \times 10^5 \text{ Pa}
\end{aligned}
$$

Solution

Braking torque of the right shoe is

$$
\begin{aligned}
T_R &= \frac{\mu r^2 b p_{\text{max}}}{(\sin\theta)_{\text{max}}}(\cos\theta_1 - \cos\theta_2) \\
&= \frac{(0.3)(0.15^2)(0.03)(9.00\times 10^5)}{1}(\cos 0^\circ - \cos 126^\circ) \\
&= 289 \text{ N-m}
\end{aligned}
$$

Solution

To calculate braking torque in left shoe, we also must calculate $p_{\max}$.
M_n and M_f are now both clockwise.

$$
\begin{aligned}
Fc &= M_n + M_f \\
2000(0.212) &= (7.38 + 2.67) \times 10^{-4} p_{\max} \\
p_{\max} &= 4.22 \times 10^5 \text{ Pa}
\end{aligned}
$$

Solution

Braking torque of the left shoe is

$$\begin{aligned} T_L &= \frac{\mu r^2 b p_{\text{max}}}{(\sin\theta)_{\text{max}}}(\cos\theta_1 - \cos\theta_2) \\ &= \frac{(0.3)(0.15^2)(0.03)(4.22\times 10^5)}{1}(\cos 0^\circ - \cos 126^\circ) \\ &= 136 \text{ N-m} \end{aligned}$$

Solution

Total braking torque is

$$
\begin{aligned}
T &= T_L + T_R \\
&= 289 + 136 = 425 \text{ N-m}
\end{aligned}
$$

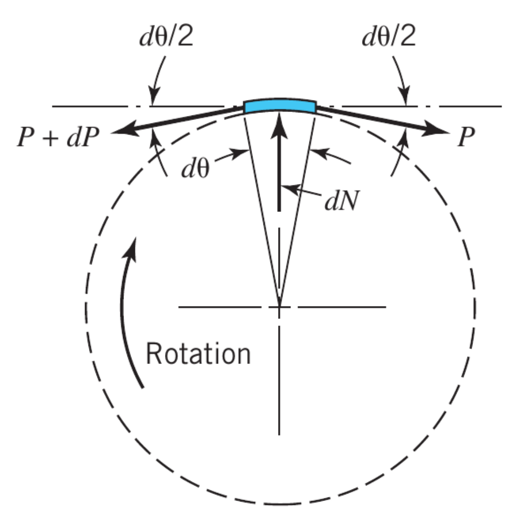
Outline

Principles of Band Brakes

- Rely on friction between band and drum
- Similar to pulley-belt system

$$T = (F_1 - F_2)r$$

Belt Tension



$$dF = \mu dN$$

$$dN = 2(Fd\theta/2) = Fd\theta$$

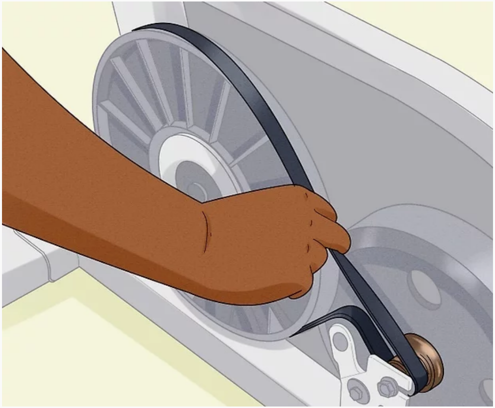
$$\frac{dF}{F} = \mu d\theta$$

$$\ln \frac{F_1}{F_2} = \mu\theta$$

$$\frac{F_1}{F_2} = e^{\mu\theta}$$

Example: An Exercise Bike

An exercise bike has an adjustable band brake on the wheel to provide different levels of resistance. What should the slack side belt tension be so that the biker can exercise with T = 50 N-m. Take $\theta = 150°$ and $\mu = 0.2$, the bike wheel $r = 50$ cm.



Solution

$$\begin{aligned}
\frac{F_1}{F_2} &= e^{\mu\theta} \\
T &= (F_1 - F_2)r \\
T &= (e^{\mu\theta} - 1)F_2 r \\
F_2 &= \frac{50}{(e^{0.2(150(\pi/180))}) - 1)(0.5)} \\
&= 145.3 \text{ N}
\end{aligned}$$

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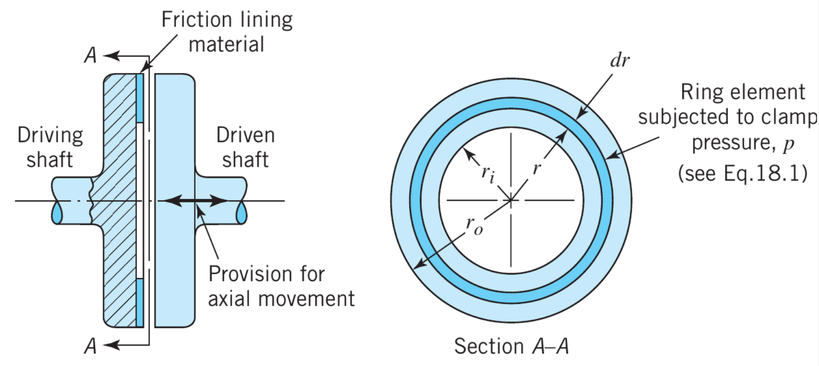
Working Principles



Pressure Distribution

- New disc is flat, resulting in uniform pressure
- Outer area wears faster because of higher velocity
- After a while, pressure is no longer uniform, but wear becomes uniform

Torque Calculation

1. Uniform pressure: new disc
2. Uniform wear: old disc

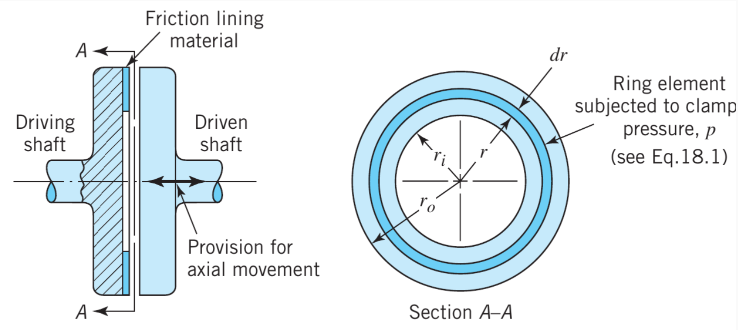
Torque Calculation: Uniform Pressure

$$
\begin{aligned}
dF &= pdA \\
dT &= \mu r dF = \mu r p dA \\
T &= \int_{r_i}^{r_o} \int_0^{2\pi} \mu r p (r dr d\theta) \\
&= \frac{2}{3} \mu \pi p \left(r_o^3 - r_i^3 \right)
\end{aligned}
$$

- For n identical discs

$$
T = \frac{2}{3} n \mu \pi p \left(r_o^3 - r_i^3 \right)
$$

Required Actuating Force for Uniform Pressure



$$
\begin{aligned}
F &= pA \\
&= pn\pi\left(r_o^2 - r_i^2\right)
\end{aligned}
$$

Uniform Pressure (cont.)

- Substitute into $p = F/n\pi(r_o^2 - r_i^2)$ into T equation

$$
\begin{aligned}
T &= \frac{2}{3} n\mu\pi p \left(r_o^3 - r_i^3\right) \\
&= \frac{2}{3} n\mu\pi \left(r_o^3 - r_i^3\right) \frac{F}{n\pi(r_o^2 - r_i^2)} \\
&= \frac{2\mu F \left(r_o^3 - r_i^3\right)}{3\left(r_o^2 - r_i^2\right)}
\end{aligned}
$$

Uniform Rate of Wear

- Rate of Wear $\propto$ Friction Work Rate

$$pr = C$$

- Max pressure occurs at inside radius, hence the constant is

$$pr = C = p_{\mathsf{max}} r_i$$

Required Actuating Force: Uniform Wear

$$
\begin{aligned}
dF &= p dA = p r dr d\theta \\
F &= p_{\text{max}} r_i \int_{r_i}^{r_o} \int_0^{2\pi} dr d\theta \\
&= 2 p_{\text{max}} r_i \pi (r_o - r_i)
\end{aligned}
$$

- For n parallel discs

$$
F = 2 n p_{\text{max}} r_i \pi (r_o - r_i)
$$

Torque Calculation: Uniform Wear

$$
\begin{aligned}
dF &= pdA \\
dT &= \mu r dF = \mu r p dA = \mu p_{\text{max}} r_i dA \\
T &= p_{\text{max}} r_i \int_{r_i}^{r_o} \int_0^{2\pi} r dr d\theta \\
&= \mu \pi p_{\text{max}} r_i \left(r_o^2 - r_i^2 \right)
\end{aligned}
$$

- For n parallel discs

$$
T = \mu \pi n p_{\text{max}} r_i \left(r_o^2 - r_i^2 \right)
$$

Torque Calculation: Uniform Wear (cont)

- Taking into account actuating force by substituting $p_{\max} r_i = F/2n\pi(r_o - r_i)$ in T

$$
\begin{aligned}
T &= \mu\pi n p_{\max} r_i \left(r_o^2 - r_i^2\right) \\
&= \mu\pi n \left(r_o^2 - r_i^2\right) \frac{F}{2n\pi(r_o - r_i)} \\
&= \mu F \left(\frac{r_o + r_i}{2}\right)
\end{aligned}
$$

Usual Guideline for Disc Brakes/Clutches

1. $0.45r_o < r_i < 0.8r_o$
2. Use uniform wear rate, unless for short-term application

Example: Automotive Clutch

Design a wet clutch to transfer the torque of 100 N-m using the material with μ = 0.08 and p_{max} = 1500 kPa. Space requirements only allow $r_o \leqslant 60$ mm. Determine the inner diameter and number of parallel discs.

Solution

- Take $r_i = 0.5r_o$ = 30 mm

$$n = \frac{T}{\left[\mu \pi p_{\text{max}} r_i \left(r_o^2 - r_i^2\right)\right]}$$
$$= \frac{100}{[(0.08)\pi(1500 \times 10^3)(0.03)\left(0.06^2 - 0.03^2\right)]}$$

N = 4 and $d_i = 2r_i$ = 60 mm

Drum Brakes vs Disc Brakes

Drum	Disc
self-energizing possible	no self-energizing
very sensitive to μ	less sensitive to μ
requires larger force once μ goes down	well-designed caliper compensate for wear and exert constant pressure