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Safety Factors in Trees

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The safety factor of trees is determined by cutting windows of a well-defined size and shape into trees. The related stress magnification factors were calculated in comparison to the unnotched solid trunk. As a result, trees can have at least 4·5 times higher stresses in the stem without breakage. This result fits well into the range of safety factors (about 3–4) in mammal bones.

1. What Does Safety Factor Mean?

A mechanical component taken from engineering or biology which is loaded under normal service conditions just below the strength of its material has practically no strength reserves. The smallest accident may lead to breakage. The safety factor of such a design is close to 1, if the safety factor S is defined by the ratio

$$S = \frac{\text{material strength}}{\text{service load}}.$$

For example, a safety factor of $S = 5$ means that the service load of a component can be increased by a factor of 5 before failure will occur. The biological design is always known to be light in weight and of admirable load-bearing capacity. As these two properties act against each other, the question to be asked is: Where is the compromise between light weight and high failure safety located? The answer to this question is given by the safety factor of the respective designs.

2. Safety Factors in Mammal Bones

To the authors' knowledge, little work has been done on safety factors in trees compared to the safety of bones. In the latter area there are some outstanding papers (Alexander, 1981, 1990; Rubin & Lanyon, 1982). More could be mentioned, but the interested reader will find a good overview in Currey (1986). In all the references mentioned above, the safety factors of the mammal bones under consideration varied between $S = 3$ and $S = 4$, whereas the buffalo and the elephant revealed safety factors in the upper range. It is a fact that animals move around and have to burden their own bone mass. They have to accelerate and decelerate and none of these activities

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have to be carried out by trees. Trees do not locomote and their only movements are swaying or very slow corrections of stem and branch orientation, for example by geotropic or phototropic growth.

From these introductory remarks one may already expect that trees have a somewhat higher safety factor to resist storms because they do not move around with those extra materials. However, trees also compete for light reception, especially in dense stands. There they have to grow slender and high in order to avoid overshading by their competing neighbors. This competition will again limit the safety factor in trees (Mattheck, 1991, 1992; Mattheck & Breloer, 1993).

3. Safety Factors in Trees

Nobody who really believes in the success of evolution would ever doubt that the crown sail, the stem girth and the root anchorage in the surrounding soil is well adapted to the wind load, for which the tree is trained early on. This also means that the individual parts of the chain transferring the wind load from the crown to the soil consist of equally strong links (Fig. 1). If not, there would be one weak link and other links which are oversized. From this, one has to conclude that if one is aware of the safety factor of a single chain link, one can expect the same value for the other load-bearing members of the tree. The stem of the tree has, of course, the simplest geometry for the determination of the safety factor.

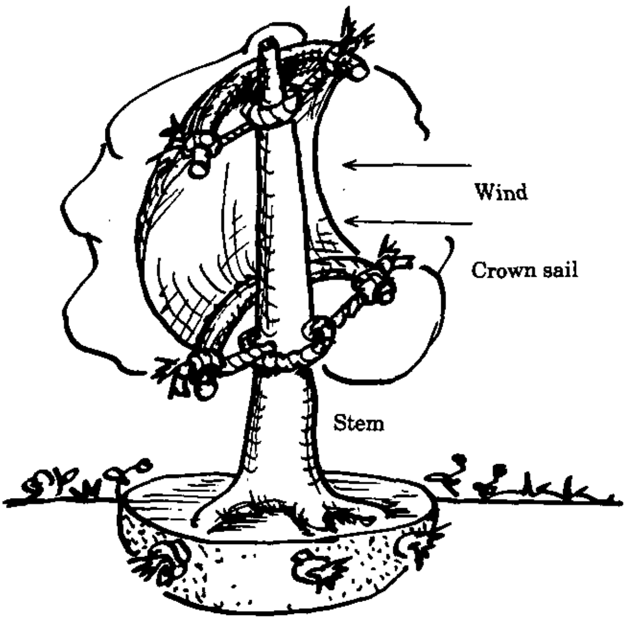

FIG. 1. The tree as a chain of equally loaded links.

In field studies, well-defined windows have been cut into the stems of spruce and beech trees. Some of the trees broke only a few days after treatment [Fig. 2(a)]. Smaller notches did not lead to breakage [Fig. 2(b)] even 18 months after cutting. Now the stress magnification at the corners of these windows, i.e. the notch stresses, have been computed by the use of the standard Finite Element Method (FEM).

For this purpose, a well-defined radius has been cut into the corners of this window in order to have clear boundary conditions. Using this model with smoothed corners, calculated notch stresses have been 4·5 times higher than the maximum bending stress far away from the window.

As these trees did not break, the anticipated safety factor for the tree stems considered are at least $S \geq 4{\cdot}5$ if an isotropic material behavior is assumed as a first approach. Wood is a highly orthotropic material and therefore the effect of an orthotropic material behavior on the stress magnification was also investigated. In the case of a ratio of Young's modulus and Poisson ratio for spruce trees:

$$E_l = 20E_t, \qquad \nu_l = 10 \cdot \nu_t,$$

the stress magnification was even higher, leading to a safety factor of $S \geq 5{\cdot}4$. This may be explained by the laterally reduced stiffness of wood, leading to more lateral contraction of the window when the stem is bent. These "secondary" bending effects of the window frame increase the notch stresses. However, because of the different orthotropic behavior of wood in different species, the authors like to stay with the more general, valid, lower bound safety factor calculated for isotropic material, i.e. $S \geq 4{\cdot}5$.

However, there are other reasons for believing in higher safety factors in trees. Some of the windows cut into the trees have not been smoothed in the corners of the wound. In these trees even higher notch stresses are to be expected. Unfortunately, one cannot cut a much finer corner profile into a green tree under field conditions. Therefore, the authors aim to try a different method in the future. The size of the window will be increased successively until an upper limit of the safety factor will be gained after some years. The radius of the corners will be kept constant in these future studies. The major result of this present study is the lower bound for the safety factor.

A similar value for the safety factor could be gained by looking at trees damaged by beavers near the Mississippi river. The tree shown in Fig. 3(a) was still green and, although slanted, did not break, either by wind or by its own weight. Assuming an approximately semicircular cross-section of the damaged tree and ignoring notch stresses because of the very large notch radius, the safety factor was calculated to be $S \geq 4{\cdot}0$. Broken, smaller reduced cross-sections were found, but there was no evidence of whether the trees broke in a green state with leaves or long after debarking, being already dead, without any leaves and therefore having had a reduced wind load. As for this uncertainty, those trees have been ignored. In the case of Fig. 3(b), a tree was broken at a stress magnification estimated to over 5·0 using the diagram in Fig. 4.

It is plausible to assume similar safety factors for each part of the tree, i.e. also for the branches and even the thinnest branch tips with regard to their respective

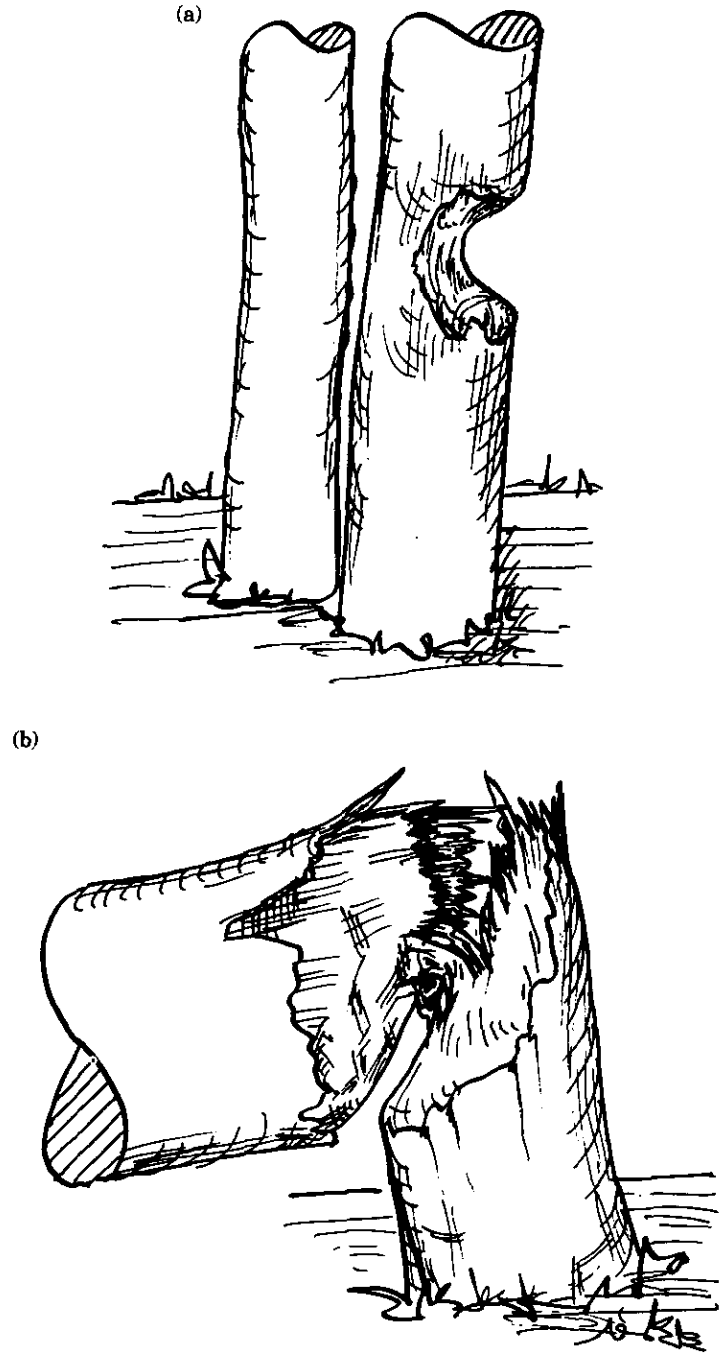

FIG. 3. Field experiments performed by Mississippi beavers (figure after photograph). (a) No breakage at a calculated stress magnification of 4. (b) Breakage after a calculated stress magnification greater than 5.

FIG. 2. Field experiments to determine the safety factor in stems. This page: breakage if window size was 2/3 of stem diameter. Reverse page: no breakage if window size was 1/3 of stem diameter. One-half of the structure has to be generated in the FEM model for reasons of symmetry (isotropic model: $S \geq 4{\cdot}5$).

Isotropic material

$$\frac{\sigma_{max}}{\sigma_0} = 4{\cdot}5$$

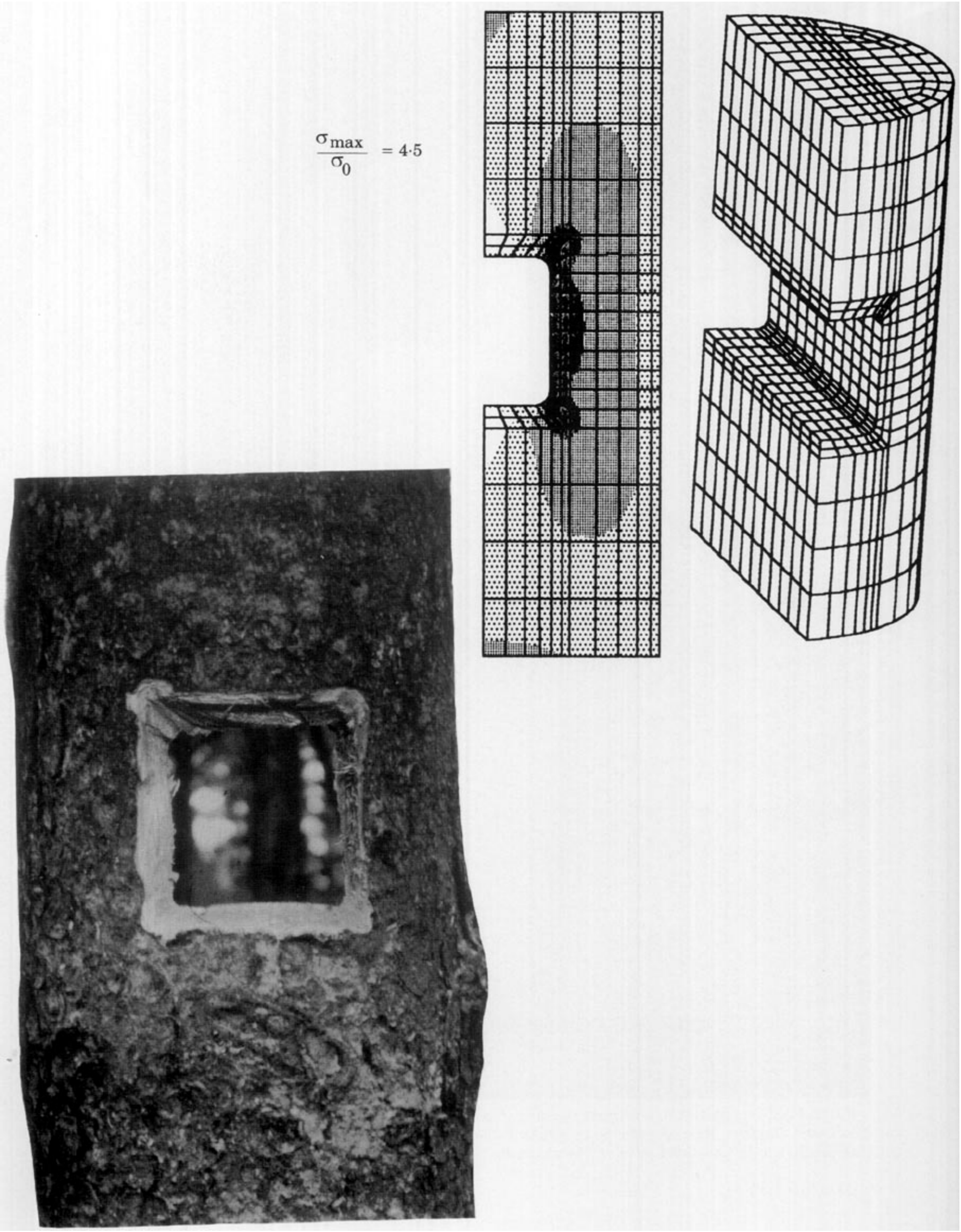

FIG. 2.—*continued.*

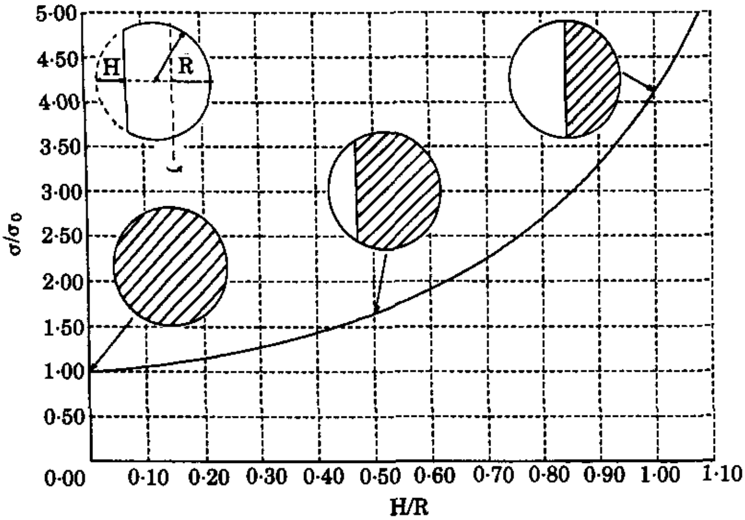

FIG. 4. Figure showing the relative stress σ scaled on the value σ_0 of the full cross-section. (Computing: Dagmar Erb). R: stem radius; H: removed material.

loading. The same should be true for the root system and this is the subject of further research.

4. Conclusions

The major results of this consideration are:

Trees have a safety factor larger than $S = 4{\cdot}5$ in the stem.

Trees therefore have higher safety factors in their structure as they are common in mammal bones.

These considerations were stimulated by Professor McNeill Alexander in Perth, Australia, in 1991. Professor Hans Kübler directed us to the location where beaver-damaged trees could be found.

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