Power Transmission System Design

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ME TSE

What We have Covered So Far

- Shafts
- Gears
- Bearings
- Flexible Elements
- Power screws

Final Goal

- Design a power transmission system …
- with multiple components …
- that interacts with one another …

Where Do We Start?

- Given all the requirements: power, reduction ratio, speed ..
- Which component should be designed first?

System design is typically iterative

- Possibly multiple correct solutions
- Start from input or output
- Proceed to the rest of the system
- When contradiction arises, correct it
- And redesign other parts to fix that mistake
- Lather, rinse, repeat …

Order of Component Design

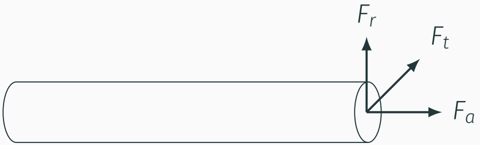
1. Determine the required T, v, ω, H, ...
2. Input - Output $\rightarrow$ gear / chain + sprocket / pulley + belt / power screw
3. Determine forces on shaft
4. Shafts $\rightarrow$ what the input and output are mounted on
5. Determine forces on bearings
6. Bearings $\rightarrow$ what the shaft made contact with
7. Determine forces on joints
8. Joints: springs, bolts, welds, ...

2, 4, 6, 8 $\rightarrow$ already knows
1, 3, 5, 7 $\rightarrow$ need a little dust off

Determine Required T, V, ω, H

- Problem usually gives a subset of mass, weight, torque, velocity, acceleration or ...
- Need to determine F and T on shaft depending on mechanism involved

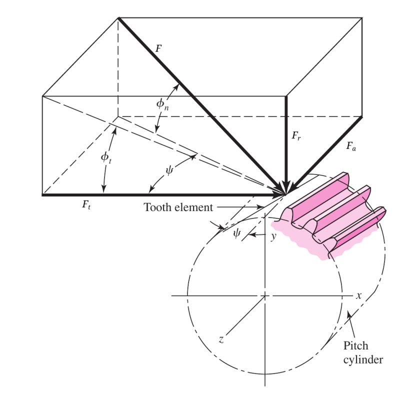
Forces on Shaft



2 directions possible:

- Radial Loads: any forces $\perp$ shaft axis (F_r, F_t)
- Axial Loads: any forces $\parallel$ shaft axis

Determine forces on shaft

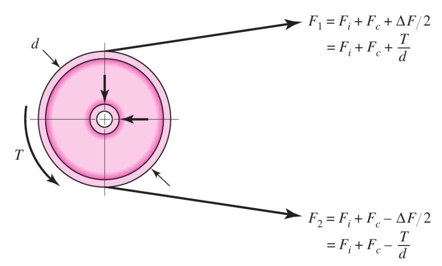


Gears: F_a, F_t, F_r

Forces on shaft:

- $F_a \rightarrow$ Axial load on shaft
- F_r and $F_t \perp$ shaft axis $\rightarrow$ Radial load on shaft = $\sqrt{F_r^2 + F_t^2}$

Determining forces on shaft

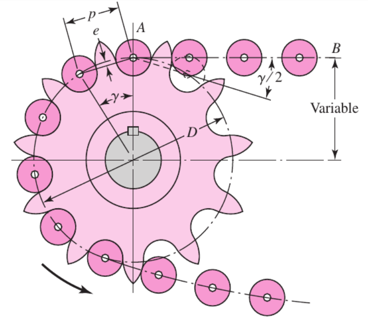


Pulleys + Belts: F_1, F_2

On shaft:

- $F_1, F_2 \perp$ shaft axis $\rightarrow$ Radial load on shaft $\approx F_1 + F_2$

Determining forces on shaft



Chain + Sprocket: F_1

On shaft:

- $F_1 \perp$ shaft axis $\rightarrow$ Radial load on shaft

Shaft Design

- Radial load $\rightarrow F_r \rightarrow$ bending stress
- Axial load $\rightarrow F_a \rightarrow$ axial stress
- Torque $\rightarrow T \rightarrow$ torsion

$$\frac{1}{N_s} = \frac{\sigma_{ae}}{S_e} + \frac{\sigma_{me}}{S_y}$$

$$\sigma_{ae} = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2}$$

$$\sigma_{me} = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2}$$

Bearing Design

- Bearing equation needs radial load F_r and axial load F_a
- Determine forces on bearings using equilibrium equations
- Proceed to design bearings using F_e derived from F_r and F_a

$$L = L_R K_r \left(\frac{C}{F_e} \right)^{10/3}$$

$$C = F_e \left(\frac{L}{K_r L_R} \right)^{0.3}$$

Forces on Joints

- Consider forces of components attached to joints
- Use equilibrium equation to determine the forces

Example

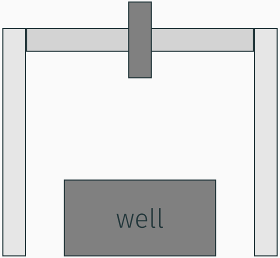
A motor drive is used to operate a well bucket. The maximum water weight the bucket can hold is 100 N. The minimum required speed of the bucket is 50 cm/s. The bucket is held by an unbreakable rope that loops around a 20-cm-radius pulley that is fitted onto the middle of a 1-m shaft.

The shaft is supported by a pair of bearings, one at each end. The input motor provides 100-W at 1200 rpm. The motor can be engaged/disengaged from the shaft with a disc clutch.

Example

Design:

1. Shaft (N_s = 2, S_y = 300 MPa, S_{ut} = 400 MPa, E = 210 GPa)
2. Disc Clutch (μ = 0.1)
3. Bearings



Solution

Required power = (100 N)(0.5 m/s) = 50 W

So the given motor works, assuming 100% efficiency

Assuming constant speed operation, required torque = 100(0.2) = 20 N-m = Torque on shaft

Force on shaft = Rope tension = 100 N

Solution

Shaft:

$$\sigma_a = \frac{100(1)r}{4(\pi/4)r^4} = \frac{31.8}{r^3}$$

$$\sigma_m = 0$$

$$\tau_a = 0$$

$$\tau_m = \frac{20(r)}{(\pi/2)r^4} = \frac{12.7}{r^3}$$

Solution

Calculating equivalent amplitude and mean stresses

$$\sigma_{ae} = \sqrt{\sigma_a^2 + 3\tau_a^2} = \frac{31.8}{r^3}$$
$$\sigma_{me} = \sqrt{\sigma_m^2 + 3\tau_m^2} = \sqrt{3}\frac{12.7}{r^3} = \frac{22}{r^3}$$

Using Soderberg criteria,

$$\frac{1}{N_s} = \frac{\sigma_{ae}}{S_e} + \frac{\sigma_{me}}{S_y}$$
$$\frac{1}{2} = \frac{31.8}{r^3(0.5)(400 \times 10^6)} + \frac{22}{r^3(300 \times 10^6)}$$
$$r^3 = 4.65 \times 10^{-7}$$
$$r = 7.75 \times 10^{-3} \text{ m}$$

Solution

Disc Clutch: $\mu = 0.1$

A clutch – use wet lining.

Friction Material[a]	Dynamic Friction Coefficient f[b]	Maximum Pressure[c]		Maximum Bulk Temperature	
		psi	kPa	°F	°C
Molded	0.25–0.45	150–300	1030–2070	400–500	204–260
Woven	0.25–0.45	50–100	345–690	400–500	204–260
Sintered metal	0.15–0.45	150–300	1030–2070	450–1250	232–677
Cork	0.30–0.50	8–14	55–95	180	82
Wood	0.20–0.30	50–90	345–620	200	93
Cast iron, hard steel	0.15–0.25	100–250	690–1720	500	260

Solution

Pick woven material so that p_{max} = 500 kPa

Take $r_i = 0.6r_o$, number of disc $N = 1$, safety factor N_s = 2.

$$
\begin{aligned}
T_{design} &= N_s T = \mu \pi p_{\text{max}} r_i \left(r_o^2 - r_i^2\right) \\
2(20) &= (0.1)\pi(500 \times 10^3)(0.6r_o)\left(r_o^2 - (0.6r_o)^2\right) \\
r_o &= 0.087 \text{ m}
\end{aligned}
$$

Solution

Bearings:

Radial force = 100 N

Axial force is calculated from actuating force on clutch

$$
\begin{aligned}
F &= p_{\text{max}}\pi\left(r_o^2 - r_i^2\right)\\
&= 7609\ \text{N}
\end{aligned}
$$

Use angular contact since large axial load.

$$
\begin{aligned}
\frac{F_a}{F_r} &= 47.8\\
F_e &= 0.911F_a\\
&= 6932\ \text{N}
\end{aligned}
$$

Solution

Bore (mm)	Radial Ball, $\alpha = 0°$ L00 Xlt (kN)	Radial Ball, $\alpha = 0°$ 200 lt (kN)	Radial Ball, $\alpha = 0°$ 300 med (kN)	Angular Ball, $\alpha = 25°$ L00 Xlt (kN)	Angular Ball, $\alpha = 25°$ 200 lt (kN)	Angular Ball, $\alpha = 25°$ 300 med (kN)	Roller 1000 Xlt (kN)	Roller 1200 lt (kN)	Roller 1300 med (kN)
10	1.02	1.42	1.90	1.02	1.10	1.88			
12	1.12	1.42	2.46	1.10	1.54	2.05			
15	1.22	1.56	3.05	1.28	1.66	2.85			
17	1.32	2.70	3.75	1.36	2.20	3.55	2.12	3.80	4.90
20	2.25	3.35	5.30	2.20	3.05	5.80	3.30	4.40	6.20
25	2.45	3.65	5.90	2.65	3.25	7.20	3.70	5.50	8.50
30	3.35	5.40	8.80	3.60	6.00	8.80	2.40[a]	8.30	10.0
35	4.20	8.50	10.6	4.75	8.20	11.0	3.10[a]	9.30	13.1
40	4.50	9.40	12.6	4.95	9.90	13.2	7.20	11.1	16.5
45	5.80	9.10	14.8	6.30	10.4	16.4	7.40	12.2	20.9
50	6.10	9.70	15.8	6.60	11.0	19.2	5.10[a]	12.5	24.5
55	8.20	12.0	18.0	9.00	13.6	21.5	11.3	14.9	27.1
60	8.70	13.6	20.0	9.70	16.4	24.0	12.0	18.9	32.5

No other specifications, so assume reliability factor = 1, required life = 9×10^7

Application bearings are Xlt(55 mm), lt(35 mm), med(25 mm) = L11, 207, 305

Solution

Recalculate shaft to account for axial load

$$
\begin{aligned}
\sigma_a &= \frac{100(1)r}{4(\pi/4)r^4} = \frac{31.8}{r^3} \\
\sigma_m &= \frac{7609}{\pi r^2} = \frac{2422}{r^2} \\
\tau_a &= 0 \\
\tau_m &= \frac{20(r)}{(\pi/2)r^4} = \frac{12.7}{r^3}
\end{aligned}
$$

Solution

$$\sigma_{ae} = \sqrt{\sigma_a^2 + 3\tau_a^2} = \frac{31.8}{r^3}$$

$$\sigma_{me} = \sqrt{\sigma_m^2 + 3\tau_m^2} = \sqrt{\left(\frac{2422}{r^2}\right)^2 + 3\left(\frac{12.7}{r^3}\right)^2}$$

Using Soderberg criteria,

$$\frac{1}{N_s} = \frac{\sigma_{ae}}{S_e} + \frac{\sigma_{me}}{S_y}$$

$$\frac{1}{2} = \frac{31.8}{r^3(0.5)(400 \times 10^6)} + \frac{\sqrt{\left(\frac{2422}{r^2}\right)^2 + 3\left(\frac{12.7}{r^3}\right)^2}}{(300 \times 10^6)}$$

This must be solved analytically: r = 0.008 m = 8.0 mm

Solution

Check buckling,

$$N_s = \frac{P_{cr}}{F_{comp}}$$

$$P_{cr} = 2(4782) = \frac{\pi^2 EI}{L_e^2}$$

$$I = \frac{\pi r^4}{4} = \frac{2(4782)(1)^2}{\pi^2(210 \times 10^9)}$$

$$r = 8.76 \times 10^{-3} \text{ m}$$

Summary

- Many more variations of possible answers
- Different choices of steel, linings, factors will lead to different answers