Outline

Mechanical Spring Usage

- Springs are used for
- Energy storage
- Impact absorption
- Actuator

Mechanical Spring Types

Helical Spring

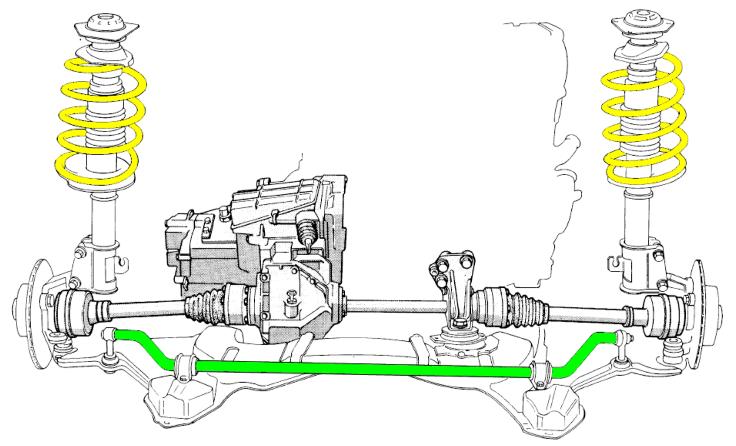
Torsion Spring

Mechanical Spring Types

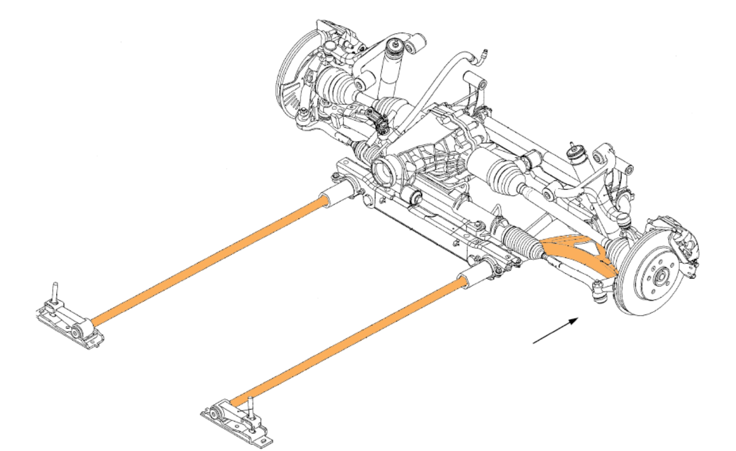
Leaf Spring

Air Spring

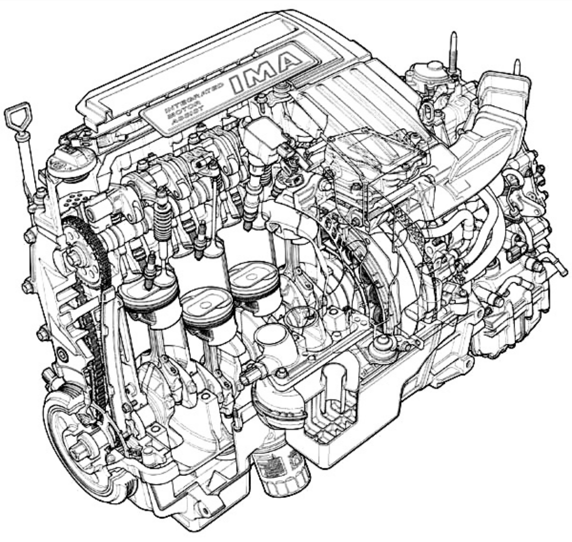
Automotive Applications



Automotive Applications



Automotive Applications



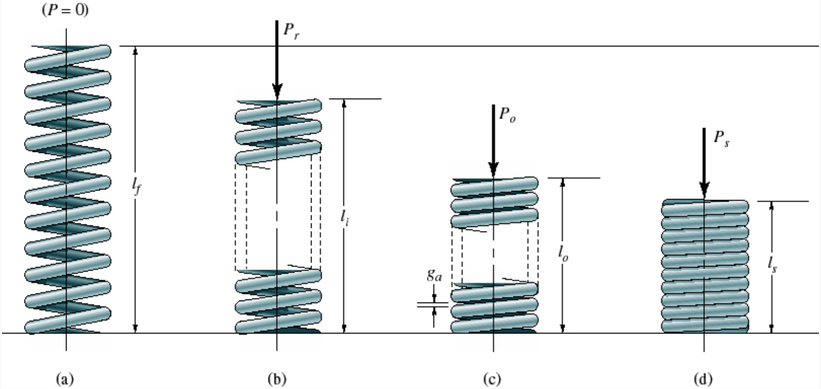
Main Focus: Helical Springs

Compression Springs

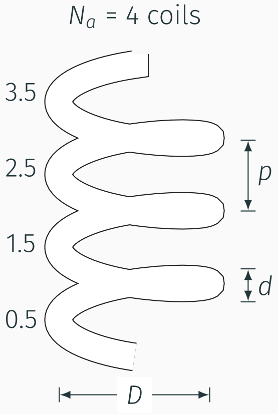
Tension Springs

Outline

Spring Lengths

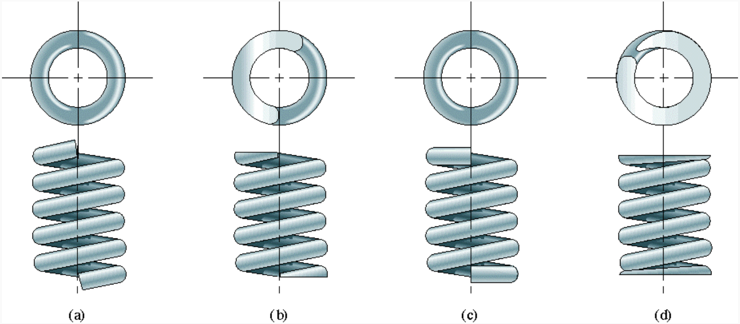


Helical Spring Terminology



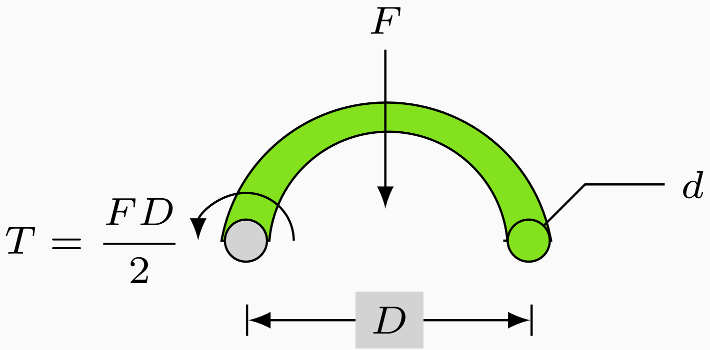
- D = Coil Diameter
- d = Wire Diameter
- p = Pitch
- N_a = Number of Active Coils

Compression Spring End Types



Term	Plain	Plain and ground	Squared or closed	Squared and ground
End coils, N_e	0	1	2	2
Total coils, N_t	N_a	$N_a + 1$	$N_a + 2$	$N_a + 2$
Free length, l_f	$pN_a + d$	$p(N_a + 1)$	$pN_a + 3d$	$pN_a + 2d$
Solid length, l_s	$d(N_t + 1)$	dN_t	$d(N_t + 1)$	dN_t
pitch, p	$(l_f - d)/N_a$	$l_f/(N_a + 1)$	$(l_f - 3d)/N_a$	$(l_f - 2d)/N_a$

Stress Analysis in Compression Springs



$$\tau = \frac{Tr}{J} + \frac{F}{A} = \frac{(FD/2)(d/2)}{\pi d^4/32} + \frac{F}{\pi d^2/4}$$
$$= \frac{8FD}{\pi d^3} + \frac{4F}{\pi d^2}$$

Stress in Helical Compression Springs

- Combine answer to become

$$\tau = K\frac{8FD}{\pi d^3}$$

- where

$$K = \frac{4C+2}{4C-3}$$

$$C = \frac{D}{d}$$

Helical Spring Constant, k

- Using energy method

$$k = \frac{Gd^4}{8D^3N_a} = \frac{Gd}{8C^3N_a}$$

Material Strength

- Material strength varies with wire diameter

$$S_{ut}[MPa] = \frac{A[MPa \cdot mm^m]}{d[mm]^m}$$

- Converting to SI units

$$S_{ut} = \frac{A \cdot 10^6 \cdot 10^{-3m}}{d^m}$$

- Allowable shear stress in spring material

$$\tau_{allow} \approx 0.5 S_{ut}$$

Spring Material Properties

Material	Diameter (mm)	G (GPa)	A (MPa-mm)	m	Relative Cost
Music	0.1 – 6.5	81.7	2211	0.145	2.6
OQ&T	0.5 – 12.7	77.2	1855	0.187	1.3
Hard-drawn	0.7 – 12.7	79.3	1783	0.190	1.0
Chrome-vanadium	0.8 – 11.1	77.2	2005	0.168	3.1
Chrome-silicon	1.6 – 9.5	77.2	1974	0.108	4.0
302 stainless steel	0.3 – 2.5 2.5 – 5 5 – 10	69	1867 2065 2911	0.148 0.263 0.478	7.6 – 11
Phosphor-bronze	0.1 – 0.6 0.6 – 2 2 – 7.5	41	1000 913 932	0 0.028 0.064	8.0

Spring Material Specifications

Materials	Descriptions
Music	Excellent for small springs, repeated loadings
OQ&T*	Good for gen purpose. Not for shock or impact.
Hard-drawn	Cheap. Really cheap.
Chrome-vanadium	Excellent for high stress, fatigue, impact, and shock.
Chrome-silicon	Excellent for high stress and shock. Good longevity.

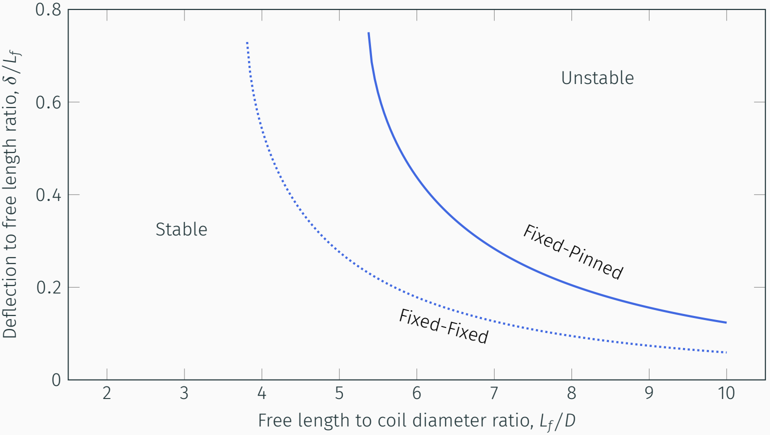
Vibration Issue

- To avoid resonance

$$\omega_{\text{spring}} \geq (15-20)\omega_{\text{sys}}$$
$$\omega_{\text{spring}} = \begin{cases} \frac{1}{2}\sqrt{\frac{k}{m}} & \text{fixed-fixed} \\ \frac{1}{4}\sqrt{\frac{k}{m}} & \text{fixed-free} \end{cases}$$

  - where m is the spring mass

Spring Buckling



Simple Buckling Rule of Thumb

- General rule of thumb is max deflection should be less than 80\

$$\frac{\delta_{\mathsf{max}}}{N_a(p-d)} = 0.8$$

Maximum Compressive Load

- Maximum compressive load must not cause solid length

$$F_{\mathrm{max}} < F_s$$
$$F_s = F_{\mathrm{max}}(1 + \xi)$$
$$\xi \geqslant 0.15$$

Compressive Spring Design for Static Loading

- Allowable compressive stress > stress at solid length

$$\frac{\tau_{allow}}{N_s} = \frac{8KF_sD}{\pi d^3}$$

$$K = \frac{4C+2}{4C-3}$$

$$F_s = F_{\mathrm{max}}(1+\xi)$$

$$\frac{\tau_{allow}}{N_s} = \frac{4C+2}{4C-3}\left[\frac{8F_{\mathrm{max}}(1+\xi)C}{\pi d^2}\right]$$

Solving for Spring Index C

$$\alpha = \frac{\tau_{allow}}{N_s}$$

$$\beta = \frac{8F_{\max}(1+\xi)}{\pi d^2}$$

$$C = \frac{2\alpha - \beta}{4\beta} + \sqrt{\left(\frac{2\alpha - \beta}{4\beta}\right)^2 - \frac{3\alpha}{4\beta}}$$

General Guidelines for Compressive Helical Springs

$$4 \leqslant C \leqslant 12$$

$$3 \leqslant N_a \leqslant 15$$

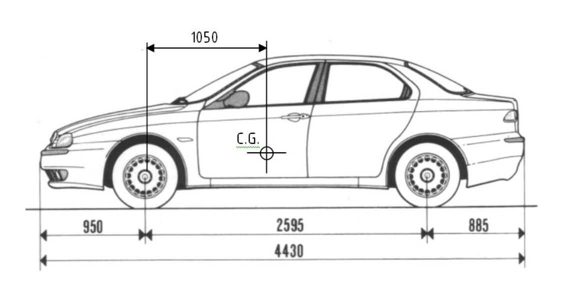
$$N_s \geqslant 1.2$$

$$\xi \geqslant 0.15$$

- Minimize spring mass

$$m = \frac{\rho \pi^2 d^2 N_t D}{4}$$

1. Empty car (1200 kg) should stand 15 cm from the ground.
2. Fully loaded car (add 150 kg to the front seats and 200 kg to the rear seat) should stand 13 cm above the ground (front) and 12 cm above the ground (rear).
   Design the front and rear suspension springs.



Solution

First, we need to find the maximum load in the front and rear suspension. Use static equilibrium. For the empty car

$$
\begin{aligned}
2F_{r1}(2595) &= 12000(1050) \\
F_{r1} &= 2428 \text{ N} \\
2F_{f1} &= 12000 - 2(4856) = 7144 \\
F_{f1} &= 3572 \text{ N}
\end{aligned}
$$

Solution

For the fully loaded car, the added front load goes at the c.g., while the read load goes in the middle of c.g. and rear wheel. The distance from the rear load to the rear wheel is

$$d_r = \frac{2595 - 1050}{2} = 772.5 \text{ mm}$$

The front and rear loads in the fully loaded car are

$$\begin{aligned} 2F_{f2}(2595) &= (12000 + 1500)(2595 - 1050) + 2000(772.5) \\ F_{f2} &= 4316 \text{ N} \\ 2F_{r2} &= 12000 + 1500 + 2000 - 2(4316) \\ F_{r2} &= 3434 \text{ N} \end{aligned}$$

Solution

Setting N_s = 1.25 and ξ = 0.15, if we choose d = 15 mm for the front suspension, the spring index C can be calculated.

$$\alpha = \frac{\tau_{allow}}{N_s} = \frac{0.5}{1.25}\frac{2005}{15^{0.168}} \times 10^6 = 509 \text{ MPa}$$

$$\beta = \frac{8(1+\xi)F_{\max}}{\pi d^2} = \frac{8(1+0.15)4316}{\pi(15 \times 10^{-3})^2} = 56.2 \text{ MPa}$$

$$\begin{aligned} C &= \frac{2\alpha - \beta}{4\beta} + \sqrt{\left(\frac{2\alpha - \beta}{4\beta}\right)^2 - \frac{3\alpha}{4\beta}} \\ &= \frac{2(509) - 56.2}{4(56.2)} + \sqrt{\left(\frac{2(509) - 56.2}{4(56.2)}\right)^2 - \frac{3(509)}{4(56.2)}} \\ &= 7.7 \end{aligned}$$

Solution

We perform the same calculation for the rear springs. Pick d = 13 mm since the maximum force is slightly smaller than the front springs.

$$\alpha = \frac{\tau_{allow}}{N_s} = \frac{0.5}{1.25}\frac{2005}{13^{0.168}} \times 10^6 = 521 \text{ MPa}$$

$$\beta = \frac{8(1+\xi)F_{\max}}{\pi d^2} = \frac{8(1+0.15)3434}{\pi(13 \times 10^{-3})^2} = 59.5 \text{ MPa}$$

$$\begin{aligned} C &= \frac{2\alpha - \beta}{4\beta} + \sqrt{\left(\frac{2\alpha - \beta}{4\beta}\right)^2 - \frac{3\alpha}{4\beta}} \\ &= \frac{2(521) - 59.5}{4(59.5)} + \sqrt{\left(\frac{2(521) - 59.5}{4(59.5)}\right)^2 - \frac{3(521)}{4(59.5)}} \\ &= 7.4 \end{aligned}$$

Solution

D of the front and rear springs can be easily calculated

$$D_f = Cd = 7.7(15) = 116 \text{ mm}$$
$$D_r = 7.4(13) = 96.2 \text{ mm}$$

Spring constants is needed to determine the required number of active coils N_a.

$$\begin{aligned} k_f &= \frac{\Delta F}{\Delta x} = \frac{4316 - 3572}{0.02} \\ &= 37200 \text{ N/m} \\ k_r &= \frac{3434 - 2428}{0.03} \\ &= 33533 \text{ N/m} \end{aligned}$$

Solution

Determine the N_a

$$N_{af} = \frac{Gd}{8C^3k} = \frac{77.2 \times 10^9 (15 \times 10^{-3})}{8(7.7^3)(37200)} = 8.6$$

$$N_{ar} = \frac{Gd}{8C^3k} = \frac{77.2 \times 10^9 (13 \times 10^{-3})}{8(7.4^3)(33533)} = 9.3$$

Solution

Determine pitch p, as a rule of thumb, max deflection should not exceed 80% of the spring compression range, which is $N_a(p-d)$. The max deflection of the front and rear springs are

$$
\begin{aligned}
\delta_f &= \frac{F_{f2}}{k_f} = \frac{4316}{37200} \\
&= 1.16 \times 10^{-1}\ \mathrm{m} \\
\delta_r &= \frac{F_{r2}}{k_r} = \frac{3434}{33533} \\
&= 1.02 \times 10^{-1}\ \mathrm{m}
\end{aligned}
$$

Solution

Finally, the pitches of the front and rear springs are

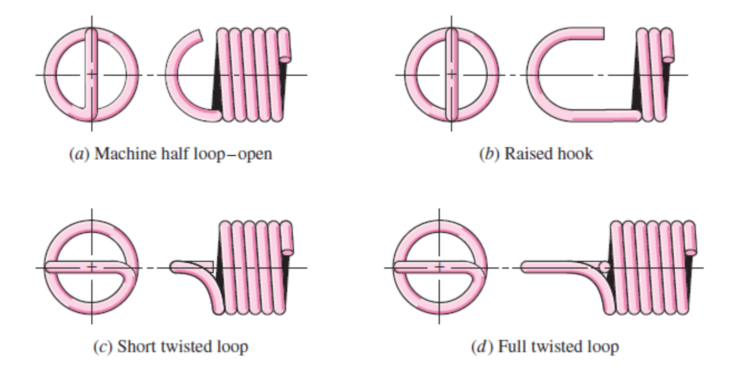
$$\frac{\delta}{N_a(p-d)} = 0.8$$

$$\begin{aligned}
p_f &= \frac{\delta_f}{0.8N_{af}} + d_f \\
&= \frac{1.16 \times 10^{-1}}{0.8(8.6)} + 15 \times 10^{-3} \\
&= 3.19 \times 10^{-2}\ \text{m} \\
p_r &= \frac{\delta_r}{0.8N_{ar}} + d_r \\
&= \frac{1.02 \times 10^{-1}}{0.8(9.3)} + 13 \times 10^{-3} \\
&= 2.67 \times 10^{-2}\ \text{m}
\end{aligned}$$

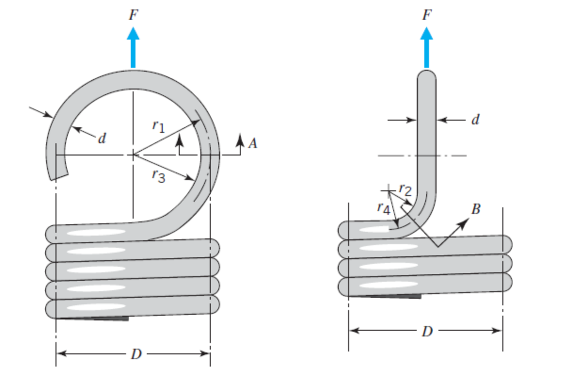
Outline

Helical Extension Springs

- Ends are typically made into hooks or loops



Maximum Stress in Extension Springs



$$\sigma_{\text{max}} = \frac{16FD}{\pi d^3} \frac{r_1}{r_3} \qquad \tau_{\text{max}} = \frac{8FD}{\pi d^3} \frac{r_4}{r_2}$$

Considerations for Extension Springs

- No buckling or solid length
- Only stress

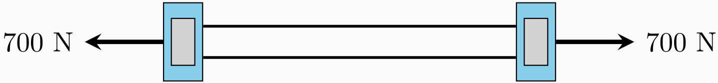
$$\sigma_{\text{max}} \leqslant \frac{S_{ut}}{N_s}$$

$$\tau_{\text{max}} \leqslant \frac{\tau_{allow}}{N_s}$$

$$4 \leqslant C \leqslant 12$$

$$m = \frac{\rho \pi^2 d^2 N_t D}{4}$$

Chest Exercise Springs



The maximum pulling force allowed is 700 N and the unstretched spring is 1 m long. Using a safety factor of 2,

- Determine the proper spring to be used.

Solution

The problem did not mention the maximum stretched length of the spring. However, this is a chest exercise machine and its reasonable maximum length should approximately be of a length of an average human reach, say 1.5 m. We can calculate the required spring stiffness.

$$k = \frac{\Delta F}{\Delta x} = \frac{350 - 0}{1.5 - 1} = 700 \text{ N/m}$$

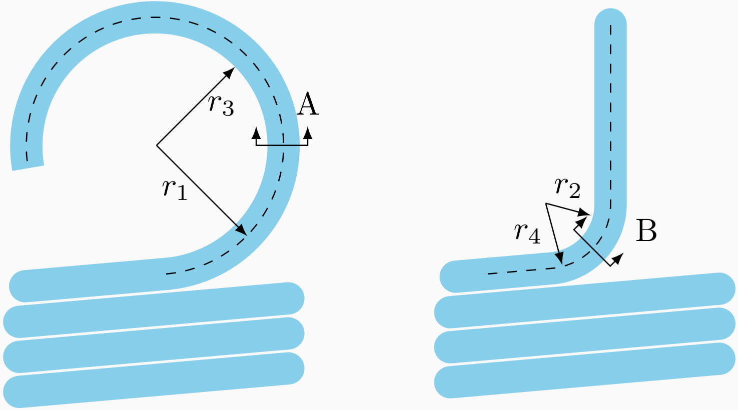
Solution

- Pick extension spring.
- No requirements for shock load or corrosion resistance.
- Pick the inexpensive *hard-drawn* wire

Equations for required diameter is

$$\sigma_{\max} = \frac{16FD}{\pi d^3}\left(\frac{r_1}{r_3}\right) = \frac{16FC}{\pi d^2}\left(\frac{r_1}{r_3}\right)$$
$$\tau_{\max} = \frac{8FD}{\pi d^3}\left(\frac{r_4}{r_2}\right) = \frac{8FC}{\pi d^2}\left(\frac{r_4}{r_2}\right)$$

Another Example?



According to the picture, $r_1 = D/2$, $r_3 = (D - d)/2$. Let us also assume that $r_4 = D/3$, which gives $r_2 = D/3 - d/2$. For a relatively low stiffness spring, we will assume that the spring index $C = 8$, which allows us to solve for d.

Solution

$$\sigma_{\mathsf{max}} = \frac{S_{ut}}{N_s} = \frac{16FC}{\pi d^2}\left(\frac{r_1}{r_3}\right) = \frac{16FC}{\pi d^2}\left(\frac{D}{D-d}\right)$$

$$0.5\frac{A}{d^m} = \frac{16FC}{\pi d^2}\left(\frac{C}{C-1}\right)$$

$$0.5\frac{1783 \times 10^6 \times 10^{-3(0.190)}}{d^{0.190}} = \frac{16(350)(8)(8)}{\pi d^2(8-1)}$$

$$d^{2-0.190} = 6.79 \times 10^{-5}$$

$$d = 4.98 \times 10^{-3}\ \mathrm{m}\ = 4.98\ \mathrm{mm}$$

Solution

$$\tau_{\text{max}} = \frac{\tau_{allow}}{N_s} = \frac{8FC}{\pi d^2}\left(\frac{r_4}{r_2}\right) = \frac{8FC}{\pi d^2}\left(\frac{D/3}{D/3 - d/2}\right)$$

$$\frac{0.5}{2}\frac{A}{d^m} = \frac{8FC}{\pi d^2}\left(\frac{C/3}{C/3 - 1/2}\right)$$

$$\frac{0.5}{2}\frac{1783 \times 10^6 \times 1^{-3(0.190)}}{d^{0.190}} = \frac{8(350)(8)(8/3)}{\pi d^2(8/3 - 1/2)}$$

$$d^{2-0.190} = 8.13 \times 10^{-5}$$

$$d = 5.50 \times 10^{-3}\ \text{m} = 5.50\ \text{mm}$$

Solution

- Choose d = 5.50 mm
- Coil diameter $D = Cd$ = 4.4 cm.
  Finally, the number of required active coils is

$$N_a = \frac{Gd}{8C^3k} = \frac{79.3 \times 10^9(5.5 \times 10^{-3})}{8(8^3)(700)} = 152 \text{ coils}$$