Shaft and Shaft Components

ME 310: Mechanical Design

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Outline

Why Starting off with Shafts?

- Most engineering machines is powered by a rotational machines
- Rotational machines need shafts
- The source of powers... and mistakes

Design Criteria

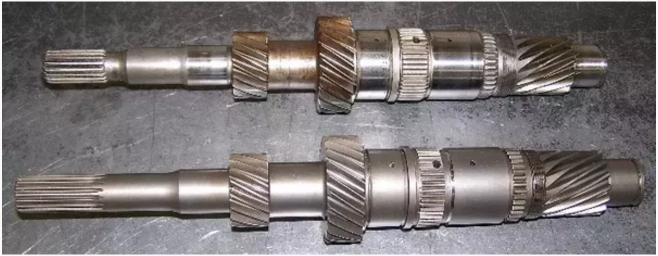
- Material selection
- Layout
- Stress and strength
- Deflection and rigidity
- Vibration

Materials

- Stiffness and deflection: E same for all steels, so material choice does not matter.
- Size:
- For small diameter shafts, use cold drawn steel.
- If heat treatment is required, it should be machined after to provide work hardening.
- Production volume:
    - Low $\rightarrow$ Turning (using lathe or CNC)
    - High $\rightarrow$ hot rolling, cold rolling, casting

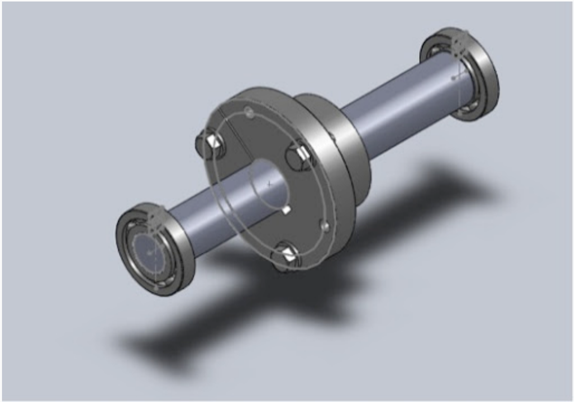
Outline

Layout



- steps to axially locate elements, i.e. gears, pulleys, bearings.
- support axial load using bearings
- provide torque transmission with gears, pulleys, sprockets...

Axial Location of Elements



- 2 bearings per shaft in most cases
- Shortest shaft possible to reduce bending
- Load bearing components should be close to bearings
- Use shoulders or retainer rings to fix axial locations

Axial Load Support

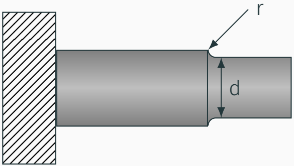
- If significant axial loads are present, support with appropriate bearings.

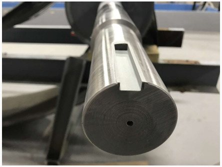
Outline

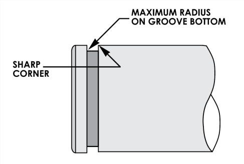
Critical Locations

- Maximum bending moment
- Steps, grooves, and notches

Stress Concentrations in Shafts







	Bending	Torsion	Axial
Shoulder fillet—sharp (r/d = 0.02)	2.7	2.2	3.0
Shoulder fillet—well rounded (r/d = 0.1)	1.7	1.5	1.9
End-mill keyseat (r/d = 0.02)	2.14	3.0	—
Sled runner keyseat	1.7	—	—
Retaining ring groove	5.0	3.0	5.0

Shaft Stresses

- Torsion + Bending
- Axial load usually small and negligible

$$\sigma_a = K_f \frac{M_a c}{I} \qquad \sigma_m = K_f \frac{M_m c}{I}$$
$$\tau_a = K_{fs} \frac{T_a c}{J} \qquad \tau_m = K_{fs} \frac{T_m c}{J}$$

Solid round shafts

$$\sigma_a = K_f \frac{32M_a}{\pi d^3} \qquad \sigma_m = K_f \frac{32M_m}{\pi d^3}$$

$$\tau_a = K_{fs} \frac{16T_a}{\pi d^3} \qquad \tau_m = K_{fs} \frac{16T_m}{\pi d^3}$$

Combine Normal and Shear Stresses

Use MDET

$$\sigma_e = \left(\sigma^2 + 3\tau^2\right)^{1/2}$$

$$\sigma_{ae} = \left(\sigma_a^2 + 3\tau_a^2\right)^{1/2} = \left[\left(\frac{32K_f M_a}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_a}{\pi d^3}\right)^2\right]^{1/2}$$

$$\sigma_{me} = \left(\sigma_m^2 + 3\tau_m^2\right)^{1/2} = \left[\left(\frac{32K_f M_m}{\pi d^3}\right)^2 + 3\left(\frac{16K_{fs} T_m}{\pi d^3}\right)^2\right]^{1/2}$$

Apply Fatigue Limit

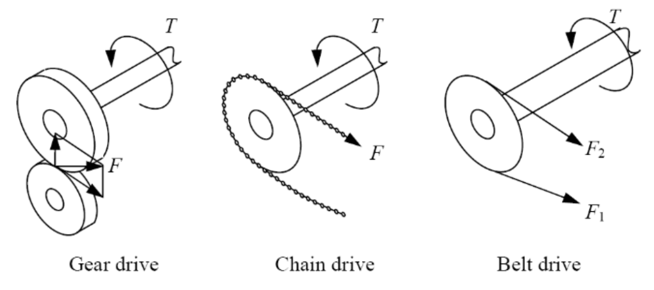
$$\frac{1}{N_s} = \frac{\sigma_{ae}}{S_e} + \frac{\sigma_{me}}{S_y}$$

$$\frac{1}{N_s} = \frac{16}{\pi d^3} \left\{ \frac{1}{S_e} \left[4 \left(K_f M_a \right)^2 + 3 \left(K_{fs} T_a \right)^2 \right]^{1/2} + \frac{1}{S_y} \left[4 \left(K_f M_m \right)^2 + 3 \left(K_{fs} T_m \right)^2 \right]^{1/2} \right\}$$

$$d = \left(\frac{16 N_s}{\pi} \left\{ \frac{1}{S_e} \left[4 \left(K_f M_a \right)^2 + 3 \left(K_{fs} T_a \right)^2 \right]^{1/2} + \frac{1}{S_y} \left[4 \left(K_f M_m \right)^2 + 3 \left(K_{fs} T_m \right)^2 \right]^{1/2} \right\} \right)^{1/3}$$

Shaft Loading Conditions

- Torque
- Bending $\implies$ radial load from torque transmission

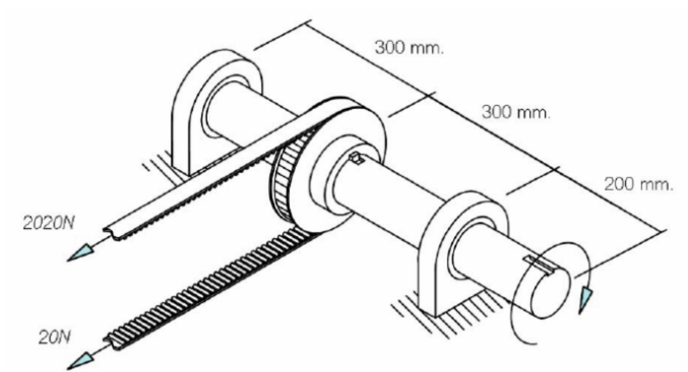


$$F = \frac{T}{r\cos\theta} \qquad F = \frac{T}{r} \qquad F_2 - F_1 = \frac{T}{r}$$

Example: Timing Belt Shaft

Size the shaft (AISI 1040, S_y = 400 MPa, S_{ut} = 600 MPa) using

1. MDET (Static Loading)
2. Soderberg theory (Dynamic Loading)
   Take r_{pulley} = 10 cm and N_s = 3



- The applied torque T is $(2020 - 20)(0.1) = 200$ N-m.
- Midpoint load of 2020 + 20 = 2040 N
- Assuming end-mill keyseat at the sheave: $K_f = 2.14$, $K_{fs} = 3$

Calculating Stresses

$$M = \frac{FL}{4} = \frac{2040(0.6)}{4} = 306$$

$$\sigma_{bending} = K_f \frac{My}{I} = 2.14 \frac{306(d/2)}{(\pi/4)(d/2)^4} = \frac{6670}{d^3}$$

$$\tau_T = K_{fs} \frac{Tr}{J} = 3 \frac{200(d/2)}{(\pi/2)(d/2)^4} = \frac{3056}{d^3}$$

Using MDET, we have that

$$\sigma_e = \sqrt{\left(\frac{6670}{d^3}\right)^2 + 3\left(\frac{3056}{d^3}\right)^2} = \frac{8514}{d^3}$$

$$N_s = 3 = \frac{S_y}{\sigma_e} = \frac{400 \times 10^6}{\sigma_e}$$

$$d^3 = \frac{8514}{400 \times 10^6} = 2.13 \times 10^{-5}$$

$$d = 0.0277$$

Applying Soderberg

- Bending $\rightarrow$ repeated stress (tensile and compressive)
- $\sigma_a = \sigma_{bending}, \sigma_m = 0$
- Torsion $\rightarrow$ constant stress
- $\tau_a = 0, \tau_m = \tau_T$

$$\sigma_{ae} = \sqrt{\sigma_a^2 + 3\tau_a^2} = \sigma_{bending}$$
$$\sigma_{me} = \sqrt{\sigma_m^2 + 3\tau_m^2} = \sqrt{3}\tau_T$$

Applying Soderberg II

$$\frac{1}{N_s} = \frac{1}{3} = \frac{\sigma_{ae}}{S_e} + \frac{\sigma_{me}}{S_y} = \frac{6670}{d^3(0.5)(600 \times 10^6)} + \frac{\sqrt{3}(3056)}{d^3(400 \times 10^6)}$$

$$d^3 = 1.06 \times 10^{-4}$$

$$d = 0.0474$$

General Guidelines

1. shaft should be as short as possible
2. avoid sharp step
3. round shaft if possible
4. to save weight $\rightarrow$ hollow shaft

Outline

Deflection Considerations

- Need geometry for entire shaft
- Should evaluate at gears and bearings – why?
- Maximum deflection < gear teeth size
- In most case, software is needed

Outline

Shaft Whirling or Shaft Whip

- At high speed, the centrifugal force can cause shaft deflection $\sim$ buckling
- For simple shafts:

$$\omega_1 = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{EI}{m}} = \left(\frac{\pi}{l}\right)^2 \sqrt{\frac{EI}{A\rho}}$$

m mass per unit length
ρ density
E Young's modulus
A cross-sectional area

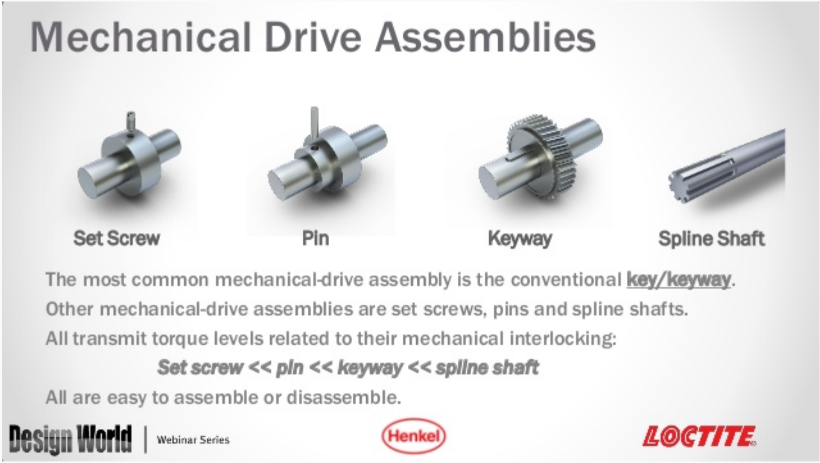
Example: Resize the Shaft

From previous example, use E = 210 GPa and reconsider the proper shaft size if ω_{max} = 10000 rpm

Torque Transmission

- Mechanical drive assembly
- Interference fit assembly
- Welded assembly

Mechanical Drive Assembly



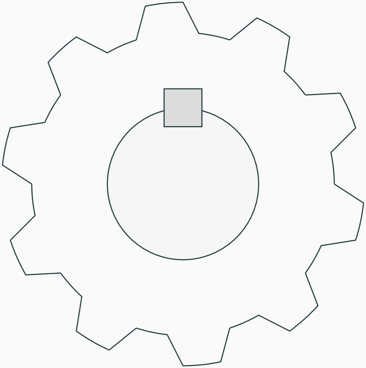
% % - Set screw % - Pin % - Key % - Spline shaft %

** Setscrews}

- Use friction to hold a component on the shaft → *holding power*

Example: Key sizing

A steel shaft whose S_y = 450 MPa has a radius of 5 cm. The shaft rotates at 600 rpm and transmits 40 hp through a gear. Select and appropriate key for the gear. Use safety factor = 3.



Solution: Key sizing

To keep things simple, pick a square key and pick key length = 2 cm.

$$T = \frac{\text{Power}}{\omega} = \frac{40(746)}{600(2\pi/60)}$$
$$= 475 \text{ N-m}$$

For the width (and height) of the key section,

$$N_s T_{\text{max}} = 0.577 S_y b l r_{\text{shaft}}$$
$$b = \frac{3(475)}{0.577(450 \times 10^6)(0.02)(0.05)}$$
$$b = 0.00549 \text{ m}$$

** Retaining Rings}

- Used to axially locate a component on a shaft or a hub.
- Need to cut grooves in shaft to fit $\rightarrow$ stress concentration

Limitation of Mechanical Drive

- Stress concentration
- Backlash
- Machining costs
- Uneven distribution of mass

Interference Fit Assemblies

- Press fit: $d_{\text{shaft}} > d_{\text{hub}}$
- Tapered fit: Taper + Fastener = Fit
- Shrink fit: Hole is heated or shaft is cooled before assembly
- Used to minimize need for shoulders and keyways

Limitations of Interference Fits

- Material, surface, and design restrictions – need sufficient friction
- Close tolerance $\rightarrow$ high machining costs
- Micro-movement causes fretting corrosion
- Surface galling $\rightarrow$ difficult disassembly
- High stress in components

Stress in Interference Fits

- Assumed uniform pressure on shaft and hub

$$p = \frac{d_{\text{shaft}} - d_{\text{hub}}}{\frac{d}{E_o}\left(\frac{d_o^2 + d^2}{d_o^2 - d^2} + \nu_o\right) + \frac{d}{E_i}\left(\frac{d^2 + d_i^2}{d^2 - d_i^2} - \nu_i\right)}$$

- When both are of the same material

$$p = \frac{E(d_{\text{shaft}} - d_{\text{hub}})}{2d^3}\left[\frac{(d_o^2 - d^2)(d^2 - d_i^2)}{d_o^2 - d_i^2}\right]$$

d nominal shaft diameter
d_i inside diameter of shaft
d_o outside diameter of hub

Stress in Interference Fits

- Tangential and radial stresses in shaft and hub are

$$\sigma_{t,\text{shaft}} = -p\frac{d^2 + d_i^2}{d^2 - d_i^2}$$
$$\sigma_{t,\text{hub}} = p\frac{d_o^2 + d^2}{d_o^2 - d^2}$$
$$\sigma_{r,\text{shaft}} = -p$$
$$\sigma_{r,\text{hub}} = -p$$

- Combine σ_t and σ_r using MDET to determine failure

Torque Capacity in Interference Fits

- Depends on friction generated between shaft and hub $\rightarrow$ pressure from interference fits

$$\begin{aligned} f &= \mu N = \mu(pA) \\ &= \pi \mu p l d \\ T &= f d/2 = \pi \mu p l d(d/2) \\ &= \frac{\pi}{2} \mu p l d^2 \end{aligned}$$

Example: Torque Capacity of a Gear on a Shaft

A solid shaft whose diameter is 5 cm is pressed onto a gear whose hub inner diameter is 4.99 cm and outer diameter is 6 cm. If both are made of the same steel whose E = 210 GPa and ν = 0.3, determine the radial and tangential stresses, along with the torque capacity of the fit. Assume steel-on-steel μ = 0.3, and the hub is 7 cm long.

Solution: Torque Capacity of a Gear on a Shaft

$$
\begin{aligned}
p &= \frac{E(d_{\text{shaft}} - d_{\text{hub}})}{2d^3}\left[\frac{(d_o^2 - d^2)(d^2 - d_i^2)}{d_o^2 - d_i^2}\right] \\
&= \frac{210 \times 10^9(0.05 - 0.0499)}{2(0.05)^3}\left[\frac{(0.06^2 - 0.05^2)(0.05^2 - 0)}{0.06^2 - 0}\right] \\
&= 64.2 \text{ MPa} \\
\sigma_{r,\text{shaft}} &= \sigma_{r,\text{hub}} = -64.2 \text{ MPa} \\
\sigma_{t,\text{shaft}} &= -64.2\frac{0.05^2}{0.05^2} = -64.2 \text{ MPa} \\
\sigma_{t,\text{hub}} &= 64.2\frac{0.06^2 + 0.05^2}{0.06^2 - 0.05^2} = 356 \text{ MPa} \\
T &= \frac{\pi}{2}\mu p l d^2 = \frac{\pi}{2}(0.3)64.2 \times 10^6(0.07)(0.05^2) = 5294 \text{ N-m}
\end{aligned}
$$

Welded Assembly

- Connections by welding the part
- Load carried by small welded area

Limitations of welded assembly

- Only compatible materials
- Heating can cause warpage
- Difficult disassembly
- Additional costs
- Need skilled personnel
- Addtional cleaning and grinding afterwards