# Design of Simple Machine Elements

## ME 310: Mechanical Design

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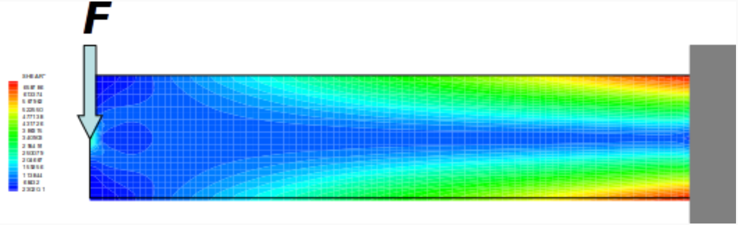
# Outline

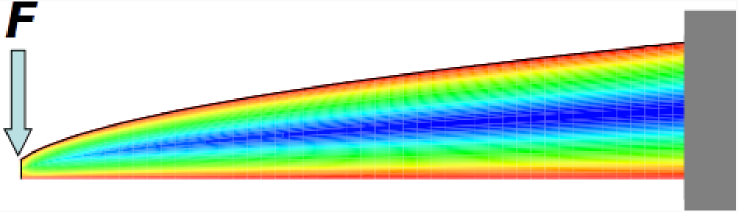
# Not-so-simple Machine Elements

- Most elements have multiple loads at once
- What do we do?
- Relax. That’s what we are here to find out.

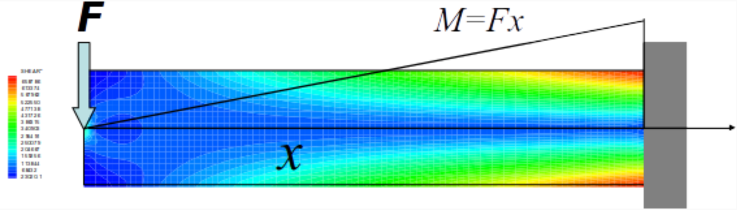
# Outline

# Fully Stressed Beams



# Fully Stressed Cantilever Beam




$$\sigma(x) = \frac{My}{I} = \frac{Fx(h/2)}{bh^3/12} = \frac{6Fx}{bh^2}$$

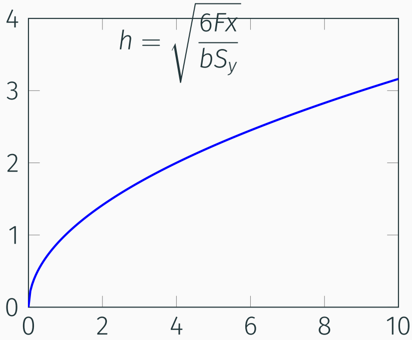
# Shape of Fully Stressed Cantilever Beam

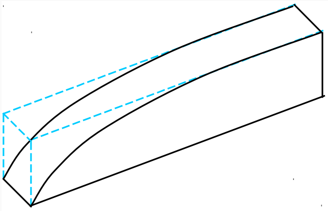
Let $\sigma(x) = S_y$ for all $x$

$$\frac{6Fx}{bh^2} = S_y$$
$$bh^2 = \frac{6Fx}{S_y} = cx$$

# Shape of Fully Stressed Cantilever Beam

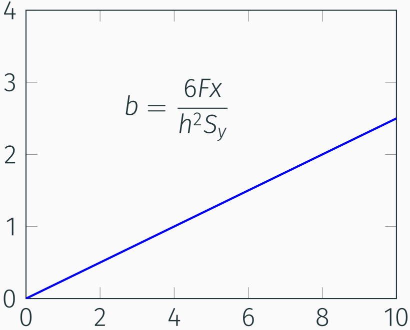
- Set *b* as a constant = constant width

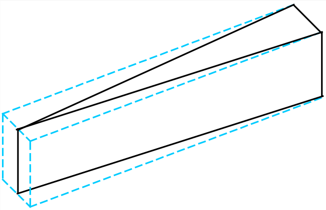


# Shape of Fully Stressed Cantilever Beam

- Set *h* as a constant = constant thickness

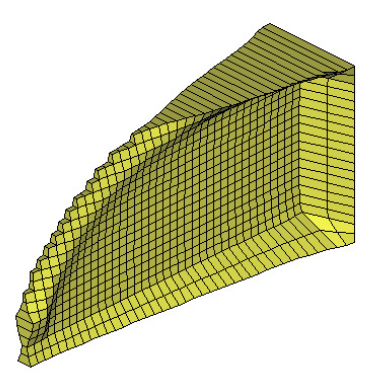


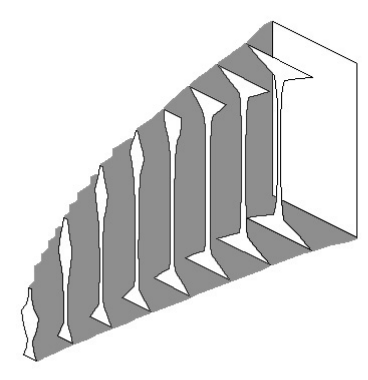



# Exercise: Fully Stressed Simply-supported Beams

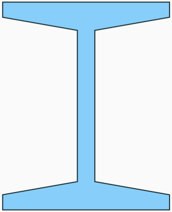
- Find shapes for a simply supported beam with midpoint load
    1. constant $h$
    2. constant $b$

# Fully Stressed Beam in 3D

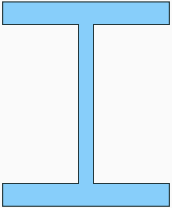




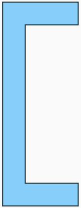
# Structural sections of beams



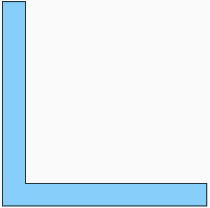
S shape



W shape



C shape



L shape

# Beam design constraints

- Stress
- Deformation

Which of the conditions is mandatory? Why?

# Stress Constraint

From beam and safety factor equations

$$\sigma = \frac{My}{I} = \frac{M}{S}$$

$$N_s = \frac{S_y}{\sigma}$$

We have

$$S = \frac{N_s M}{S_y}$$

# Deformation Constraint

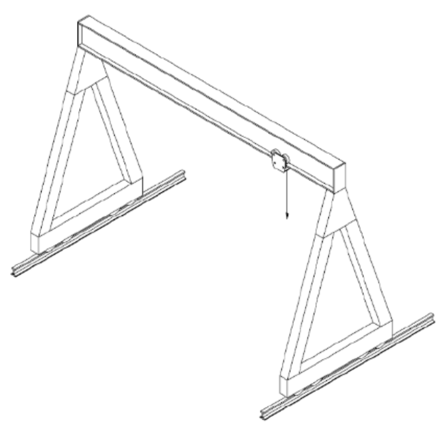
$$\delta = k\frac{FL^3}{EI}$$

$$I = k\frac{FL^3}{E\delta}$$

$k$ depends on loading and support conditions

# Example: Gantry Crane

Select a proper section to build a 3-ton gantry crane of 4-m span. The maximum deflection should be less than 1 cm.



$$S_y = 300 \text{ MPa}$$

$$\delta = \frac{FL^3}{48EI}$$

# Section Properties

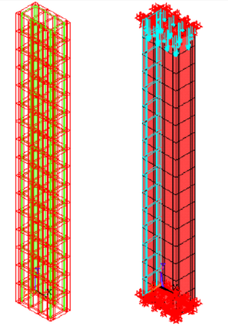
| Designation | Mass per meter | Area | Depth | Web thickness | Flange | | Axis 1-1 | | | Axis 2-2 | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | Width | Average thickness | $I$ | $S$ | $r$ | $I$ | $S$ | $r$ |
| | kg | $mm^2$ | mm | mm | mm | mm | $\times 10^6\ mm^4$ | $\times 10^3\ mm^3$ | mm | $\times 10^6\ mm^4$ | $\times 10^3\ mm^3$ | mm |
| S 610 × 149 | 149 | 18900 | 610 | 18.9 | 184 | 22.1 | 991 | 3260 | 229 | 19.7 | 215 | 32.3 |
| S 610 × 119 | 119 | 15200 | 610 | 12.7 | 178 | 22.1 | 874 | 2870 | 241 | 17.5 | 197 | 34.0 |
| S 510 × 143 | 143 | 18200 | 516 | 20.3 | 183 | 23.4 | 695 | 2700 | 196 | 20.8 | 228 | 33.8 |
| S 510 × 112 | 112 | 14200 | 508 | 16.1 | 162 | 20.2 | 533 | 2100 | 194 | 12.3 | 152 | 29.5 |
| S 460 × 104 | 104 | 13200 | 457 | 18.1 | 159 | 17.6 | 384 | 1690 | 170 | 10.0 | 126 | 27.4 |
| S 460 × 81.4 | 81.4 | 10300 | 457 | 11.7 | 152 | 17.6 | 333 | 1460 | 180 | 8.62 | 113 | 29.0 |
| S 380 × 74 | 74.0 | 9480 | 381 | 14.0 | 143 | 15.8 | 202 | 1060 | 146 | 6.49 | 90.6 | 26.2 |
| S 380 × 64 | 64.0 | 8130 | 381 | 10.4 | 140 | 15.8 | 186 | 973 | 151 | 5.95 | 85.0 | 26.9 |
| S 310 × 74 | 74.0 | 9420 | 305 | 17.4 | 139 | 16.7 | 126 | 829 | 116 | 6.49 | 93.2 | 26.2 |
| S 310 × 52 | 52.0 | 6580 | 305 | 10.9 | 129 | 13.8 | 94.9 | 624 | 120 | 4.10 | 63.6 | 24.9 |
| S 250 × 52 | 52.0 | 6650 | 254 | 15.1 | 125 | 12.5 | 61.2 | 482 | 96.0 | 3.45 | 55.1 | 22.8 |
| S 250 × 37.8 | 37.8 | 4810 | 254 | 7.90 | 118 | 12.5 | 51.2 | 403 | 103 | 2.80 | 47.4 | 24.1 |
| S 200 × 34 | 34.0 | 4360 | 203 | 11.2 | 106 | 10.8 | 26.9 | 265 | 78.5 | 1.78 | 33.6 | 20.2 |
| S 200 × 27.4 | 27.4 | 3480 | 203 | 6.88 | 102 | 10.8 | 23.9 | 236 | 82.8 | 1.54 | 30.2 | 21.0 |
| S 150 × 25.7 | 25.7 | 3260 | 152 | 11.8 | 90.7 | 9.12 | 10.9 | 143 | 57.9 | 0.953 | 21.0 | 17.1 |
| S 150 × 18.6 | 18.6 | 2360 | 152 | 5.89 | 84.6 | 9.12 | 9.16 | 120 | 62.2 | 0.749 | 17.7 | 17.8 |
| S 100 × 14.1 | 14.1 | 1800 | 102 | 8.28 | 71.1 | 7.44 | 2.81 | 55.4 | 39.6 | 0.369 | 10.4 | 14.3 |
| S 100 × 11.5 | 11.5 | 1460 | 102 | 4.90 | 67.6 | 7.44 | 2.52 | 49.7 | 41.7 | 0.311 | 9.21 | 14.6 |

# Outline

# Column constraints

- Theoretically we have $P = \dfrac{\pi^2 EI}{L_e^2}$
- Shouldn't that be enough?
- yielding vs buckling

# Real World Columns



Theory

- perfectly straight
- consistent properties
- centroidal load

Real World

- not true
- never true
- does not happen ever

# Column Dimensions and Design Equations

- Allowable stress depends on beam *slenderness ratio*

$$\lambda = \frac{KL}{r_g}$$

- Need to find a ‘critical slenderness ratio’ $\lambda_c$ where the same load can cause buckling or yielding

# Steel Column Critical Slenderness Ratio

- Critical slenderness ratio $\lambda_c$ where the same load causes buckling and yielding

$$S_y = \frac{\pi^2 E}{\lambda_c^2}$$

$$\lambda_c = \sqrt{\frac{\pi^2 E}{S_y}}$$

- longer beams $\rightarrow$ buckling, shorter beams $\rightarrow$ yielding

# Column with Eccentric Loads (or Bending Loads)

- What if the column is taking a bending load as well?
- For short columns

$$\sigma_{\text{max}} = \frac{P}{A} + \frac{My}{I}$$

# And for longer columns?

- Use interaction formula

$$A_a = \frac{P}{(\sigma_a)_{allow}}$$

$$A_b = \frac{My}{(\sigma_b)_{allow} r_g^2}$$

$$A \geqslant A_a + A_b$$

$$\frac{\sigma_a}{(\sigma_a)_{allow}} + \frac{\sigma_b}{(\sigma_b)_{allow}} \leqslant 1$$

# Example: Column with Bending Load(s)

Reconsider the gantry crane column example, but now the beam is fixed to the columns. The new safety factor should be 3.

# Solution: Column with Bending Loads

- Maximum stress happens when the weight is in the middle.
- The compressive load is that each column takes is $\frac{F}{2} = 50000$ N.
- The bending moment caused by midpoint load is $\frac{FL}{8} = 50000$ N-m

# Calculating the ratio

- We need to determine the ratio $\dfrac{\sigma_a}{(\sigma_a)_{allow}} + \dfrac{\sigma_b}{(\sigma_b)_{allow}}$
- For steel, $(\sigma_a)_{allow} = (\sigma_b)_{allow} = \sigma_{allow}$
- As a sample calculation,

$$\sigma_a = \frac{50000.0}{0.0189} = 2645502.6455026455 \text{ Pa}$$
$$\sigma_b = \frac{50000}{0.00326} = 15337423.312883437 \text{ Pa}$$
$$\frac{\sigma_a + \sigma_b}{\sigma_{allow}} = 0.18$$

# Solution

Use data table to generate results

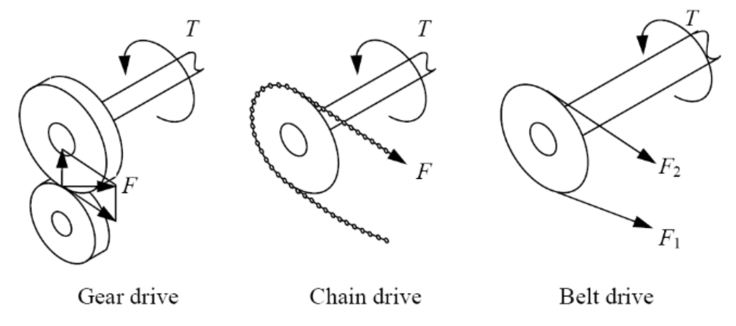
| Designation | Axial Stress | Bending Stress | $\frac{\sigma_a + \sigma_b}{\sigma_{allow}}$ |
|---|---|---|---|
| S610 x 149 | 2.65e+06 | 1.53e+07 | 0.18 |
| S510 x 143 | 2.75e+06 | 1.85e+07 | 0.21 |
| S460 x 104 | 3.79e+06 | 2.96e+07 | 0.33 |
| S380 x 74 | 5.27e+06 | 4.72e+07 | 0.52 |
| S310 x 74 | 5.31e+06 | 6.03e+07 | 0.66 |
| S250 x 52 | 7.52e+06 | 1.04e+08 | 1.11 |

In this case, S310 x 74 is our pick.

# Outline

# Shaft Loading Conditions

- Torque
- Bending $\implies$ radial load from torque transmission

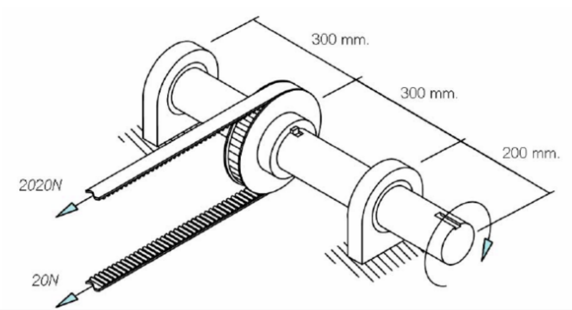



$$F = \frac{T}{r\cos\theta} \qquad F = \frac{T}{r} \qquad F_1 - F_2 = \frac{T}{r}$$

# Example: Timing Belt Shaft

Size the shaft (AISI 1040, $S_y$ = 400 MPa, $S_{ut}$ = 600 MPa) using

1. MDET
2. Soderberg theory



# General Guidelines

1. shaft should be as short as possible
2. avoid sharp step
3. round shaft if possible
4. to save weight $\rightarrow$ hollow shaft

# Typical Shaft Design Equation

Using stress constraints and MDET

$$d = \left( \frac{4N_s}{\pi S_y} \sqrt{(8M + Fd)^2 + 48T^2} \right)^{1/3}$$