Spring Design

ME 313: Mechanical Design
Week 6

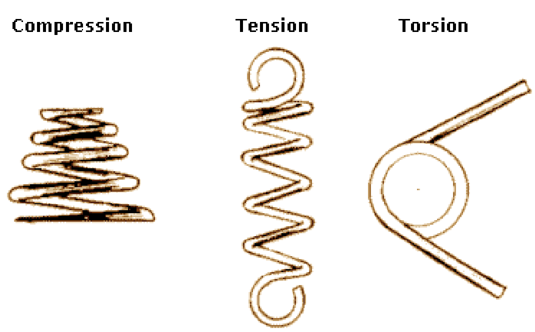
Springs

- Any elastic which exerts resisting force when the shape is changed
- Here, assume that all springs are linear, e.g.

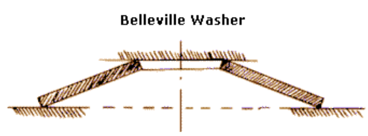
$$F = kx$$

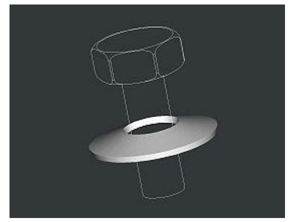
Springs in Daily Life

Types of Springs



Other Types of Springs



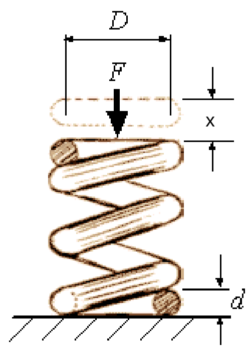


- Belleville washer
  - Used when space is limited

Other Types of Springs (cont)

- Leaf spring
  - Deforms by bending
- Laminated leaf spring
  - Where have you seen this used? Why?

Spring Dimensions



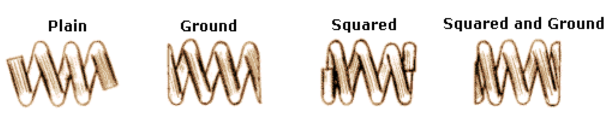
- D = nominal coil diameter
- d = wire diameter

Active Number of Coils, n_a

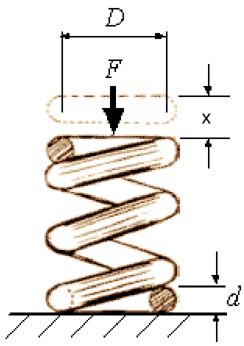
- Number of coils taking the load

$$n_a = n_t - n^*$$

- Depends on end characteristics of spring



Spring Stiffness



$$k = \frac{Gd^4}{8D^3 n_a}$$

Spring Index

- Ratio of nominal coil diameter to wire diameter

$$C = \frac{D}{d}$$

$$k = \frac{Gd}{8C^3 n_a}$$

Length of Active Coil

- Length of wire in active coils

$$L_a = \pi D n_a$$

$$k = \frac{\pi G d^4}{8 D^2 L_a}$$

Spring Failure

- There are two failure modes to consider when designing springs
  - Wire breakage
  - Spring buckling

Wire Breakage

- Since spring is under torsion, spring fails due to … stress
- Maximum shear stress in spring is

$$\tau_{\max} = \frac{8CFW}{\pi d^2} = \frac{8FDW}{\pi d^3}$$

  - where W is Wahl's correction factor

$$W = \frac{4C-1}{4C-4} + \frac{0.615}{C}$$

Spring Buckling

- Occurs in compression springs when spring is too long or too thin
- Basic rule of thumb: length should be no longer than 4 times nominal coil diameter

$$l < 4D$$

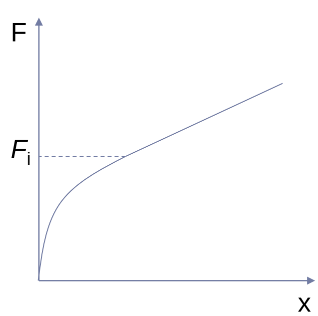
Extension Springs

- Normally manufactured so coils are pressed together

- Initial force F_i must be applied to separate the spring before it starts behaving linearly

Extension Spring Stiffness

- Once the coils separate, extension springs behave much like compression spring



$$x = \frac{8 n_t D^3 (F - F_i)}{G d^4}$$

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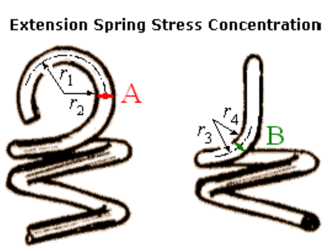
Maximum Shear Stress in Extension Spring

- Maximum shear stress is the sum of stresses from initial force and additional load

$$\tau_{\max} = \frac{8WD(F + F_i)}{\pi d^3}$$

End Stress Concentration



- Hooks create stress concentration at spring ends

$$\sigma_A = \frac{16FD}{\pi d^3}\left(\frac{r_1}{r_2}\right)$$

$$\tau_B = \frac{8FD}{\pi d^3}\left(\frac{r_3}{r_4}\right)$$

Spring Natural Frequency

$$f_{spring} = \frac{1}{2}\sqrt{\frac{k}{m}} = \frac{d}{9D^2 n_t}\sqrt{\frac{G}{\rho}}$$

- Spring natural frequency should be much higher (15-20 times) than the system frequency to prevent resonance

$$f_{system} < \frac{f_{spring}}{15}$$

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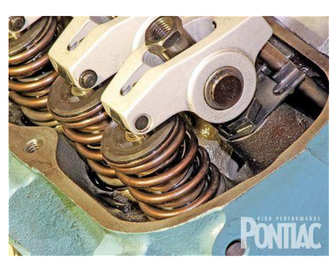
Example: Valve Spring

- If the engine maximum operating speed is 8000 rpm, and we must design the spring so that $d = 0.5$ cm, $D = 3$ cm, $G = 200$ GPa, and material density = 7800 kg/m^3, what is the required length of wire needed to make spring?



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Spring Design: Summary

- Consider these things
  - Spring geometry (*D, d,* n_t, n_a)
  - Spring material (*G, yield)*
  - Operating conditions (natural frequency)