Introduction to Engineering Design

ME 310: Mechanical Design

Sappinandana Akamphon

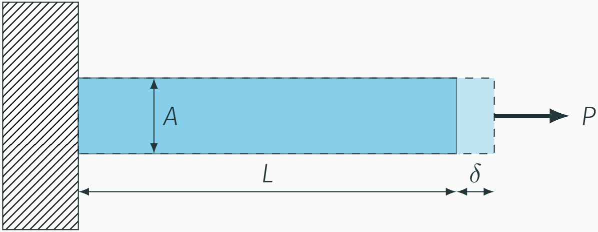
Department of Mechanical Engineering, TSE

Outline

What to Consider in Load Types

- Stresses
- Deformation

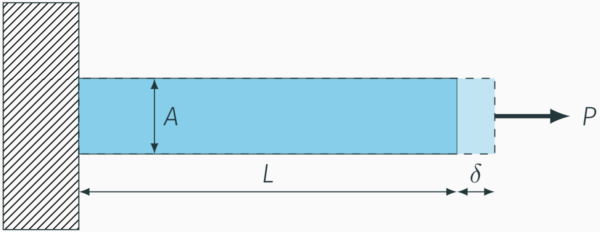
Axial Load



$$\sigma = \frac{P}{A}$$

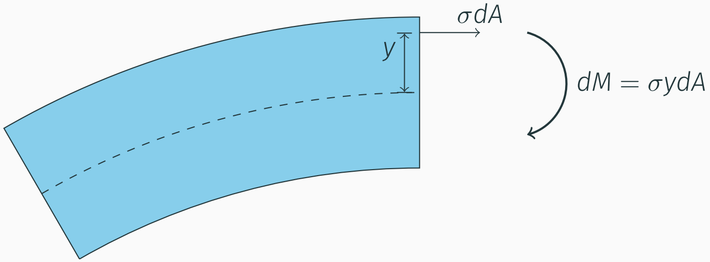
$$\delta = \frac{PL}{AE}$$

Critical Location



Where is the critical point for axially loaded members?

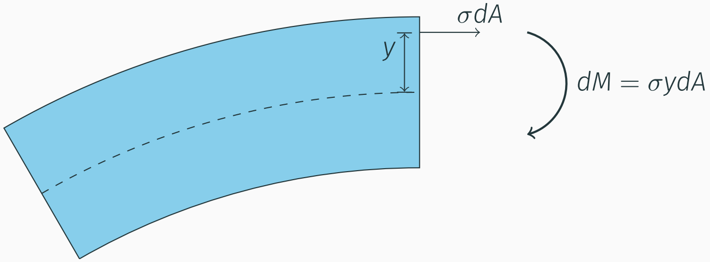
Bending



$$\sigma = \frac{My}{I}$$

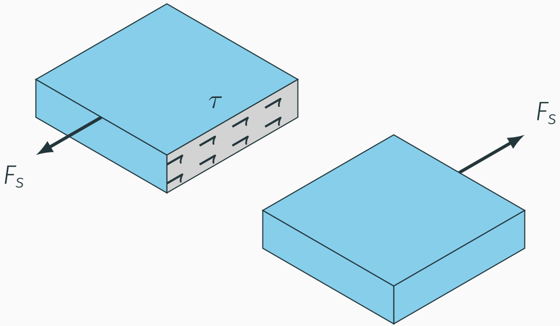
Critical Location

From equation $\sigma = \dfrac{My}{I}$, where is the critical point?



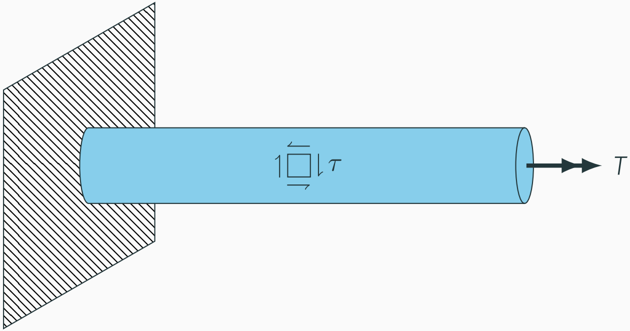
$$\delta = k\frac{FL^3}{EI}$$

Transverse Shear



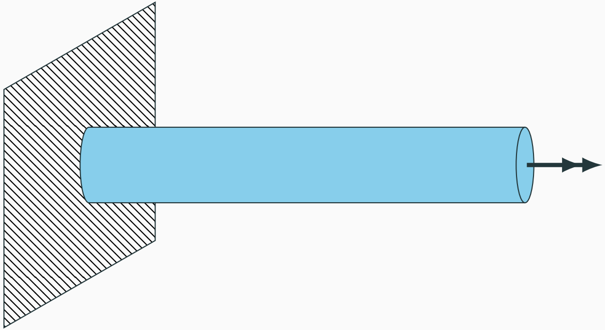
$$\tau = \frac{F}{A}$$

Torsion



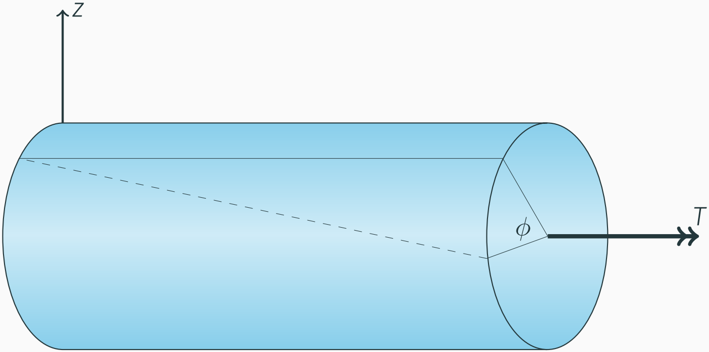
$$\tau = \frac{Tr}{J}$$

Critical Location



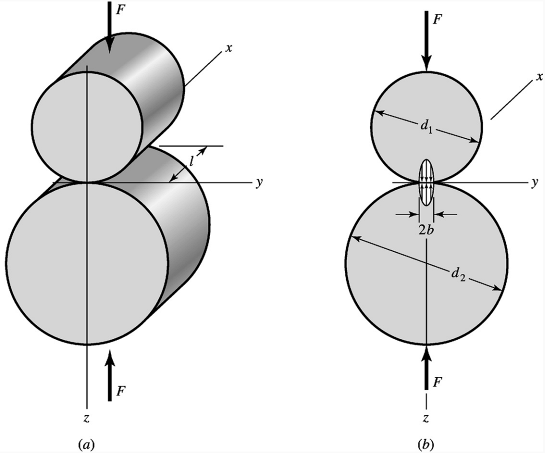
Where is the critical location?

Angle of Twist



$$\phi = \frac{TL}{GJ} \tag{1}$$

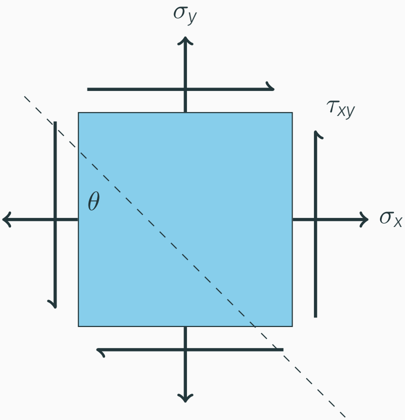
Contact



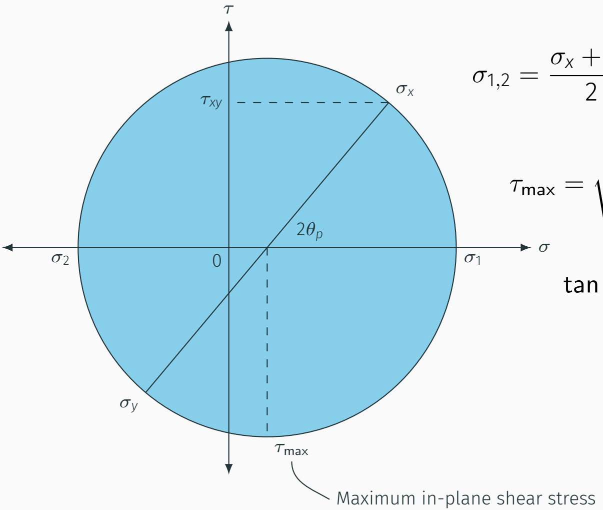
$$\sigma_{avg} = \frac{F}{A} = \frac{F}{2bl}$$

Outline

Stress Transformation



Principal and Maximum Shear Stresses



$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\frac{\sigma_x - \sigma_y}{2}^2 + \tau_{xy}^2}$$

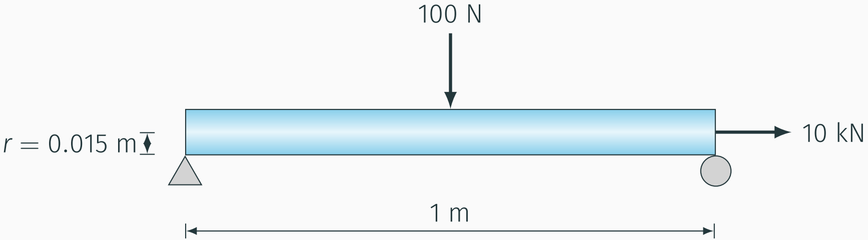
$$\tau_{\text{max}} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Determining Critical Location

- Determine critical points resulting from individual loads
- Find coincident critical points
- Check if there are any reinforcing or canceling stresses

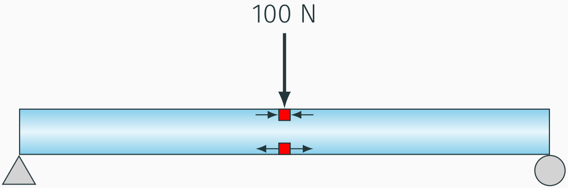
Example: Shaft under multiple loads



Determine the critical point location and its state of stress.

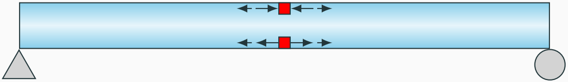
Solution

- Determine critical points from individual loads



Determining Critical Location

The common spots are top and bottom middle. Once taking directions of stresses into account, the bottom middle element is clearly the critical point.



Solution: Determining Critical Location

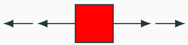
To determine the state of stress, we must first determine resultant stresses from axial load and bending. For axial load, we have

$$\begin{aligned}\sigma_a &= \frac{F}{A} \\ &= \frac{10000}{\pi(0.015)^2} \\ &= 14.15 \text{ MPa}\end{aligned}$$

Solution: Determining Critical Solution

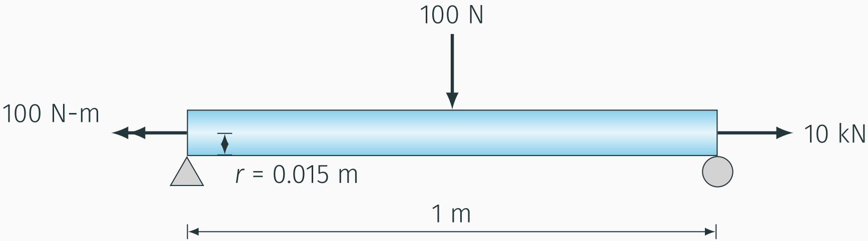
For bending, we have

$$\begin{aligned}\sigma_b &= \frac{My}{I}\\ &= \frac{100(1)(0.015)}{4\frac{\pi}{4}(0.015)^4}\\ &= 9.43 \text{ MPa}\end{aligned}$$



$\sigma_{total} = 14.15 + 9.43$ = 23.58 MPa.

Exercise: Combined Loadings



- Determine the critical point
- Determine the state of stress at the critical point
- Find the principal stresses and principal direction at the critical point

Outline

Basic Types of Loads and Stresses

Multiaxial Analysis

Failure of Materials

Stress Concentration

Failure

1. Yield & Fracture
2. Fatigue
3. Buckling

Fracture & Yield

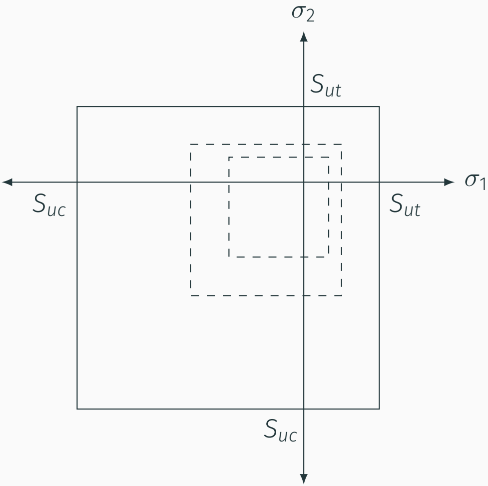
- Sufficiently high stress to overcome intermolecular bonds
- Under uniaxial stress, a material fails when
  - Brittle material - broken molecular bond leads to material separation – Fracture
  - Ductile material - surface slip – Yield

Design Equation for MNST

if $\sigma_1 > \sigma_2$

$$\sigma_1 = \frac{S_{ut}}{N_s}$$

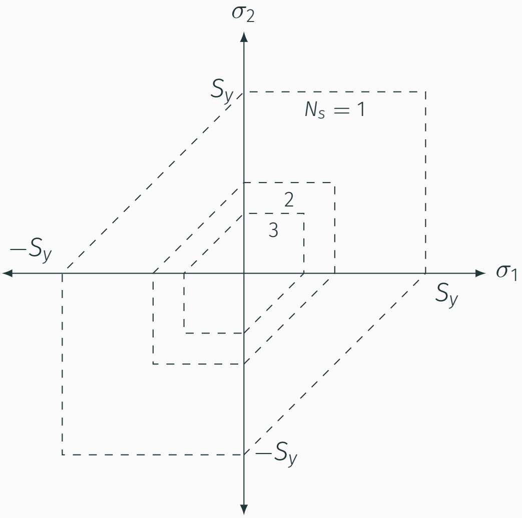
$$\sigma_2 = \frac{S_{uc}}{N_s}$$



Design Equation for MSST

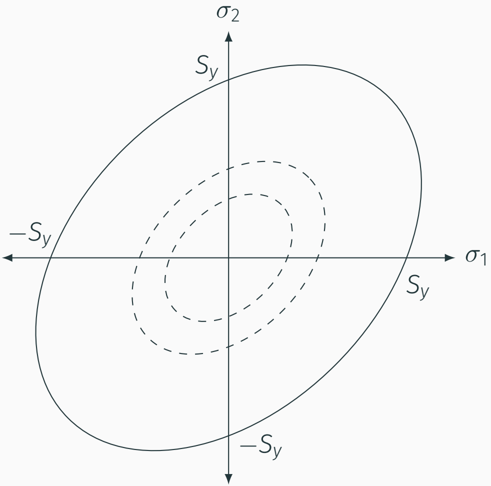
$$\tau_{max} = \frac{S_y}{2N_s}$$

$$\sigma_{1,2} = \frac{S_y}{N_s}$$



Design Equation for MDET

$$\sigma_e = \frac{S_y}{N_s}$$



Fatigue Equations

- Soderberg relation

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_y} = \frac{1}{N_s}$$

- Gerber relation

$$\frac{\sigma_a}{S_e} + \left(\frac{\sigma_m}{S_{ut}}\right)^2 = \frac{1}{N_s}$$

- Goodman relation

$$\frac{\sigma_a}{S_e} + \frac{\sigma_m}{S_{ut}} = \frac{1}{N_s}$$

These are called *constant life lines*.

Buckling

- Instability of column under compressive load

$$P = \frac{n^2 \pi^2 EI}{L^2} \qquad n = 1, 2, \ldots$$

Critical Load and Corresponding Mode Shape

For $n = 1$

$$P_{crit} = \frac{\pi^2 EI}{L^2}$$

This is called *Euler load* or *Critical load*.

What does the corresponding buckled column look like?

$$v(x) = B \sin \sqrt{\frac{P}{EI}}\, x = B \sin \frac{n\pi x}{L}$$

Generalized Critical Load

$$P_{cr} = \frac{\pi^2 EI}{L_e^2}$$

$$L_e = KL$$

L_e = effective length

K = constant depending on supports

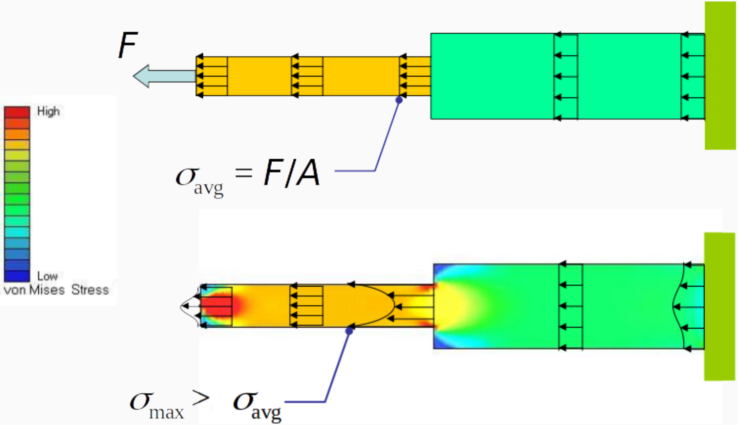
Buckling Design Equation

$$P_{allow} = \frac{P_{cr}}{N_s} = \frac{\pi^2 EI}{N_s L_e^2}$$

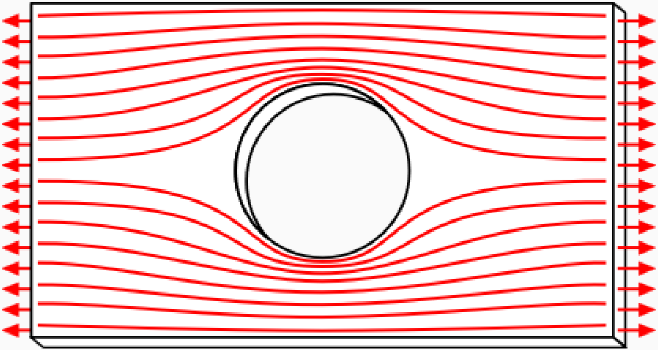
$$\sigma_{allow} = \frac{\sigma_{cr}}{N_s} = \frac{\pi^2 E}{N_s \lambda^2}$$

Outline

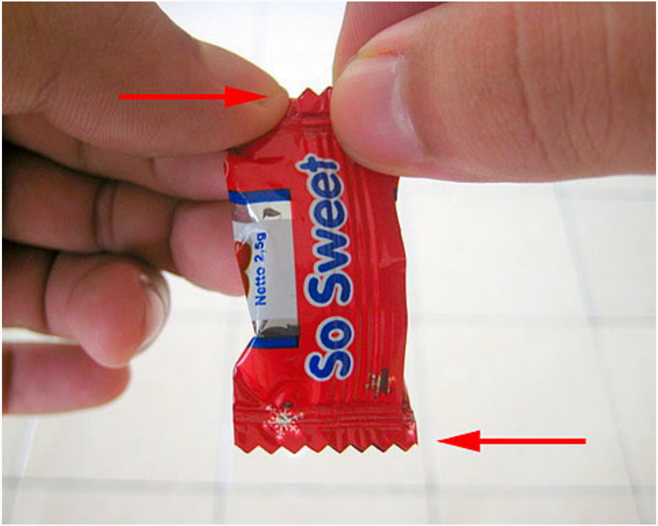
Theory vs Experiment



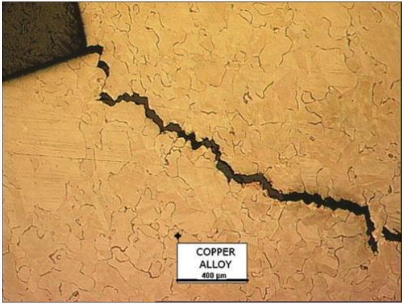
Stress Flow



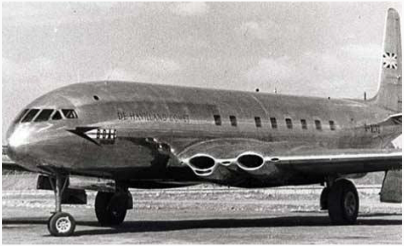
Good Stress Concentration

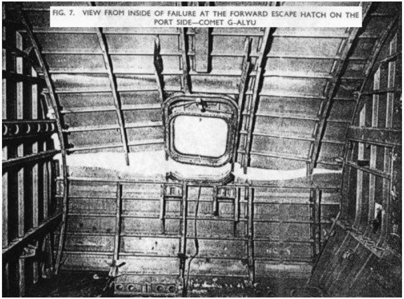


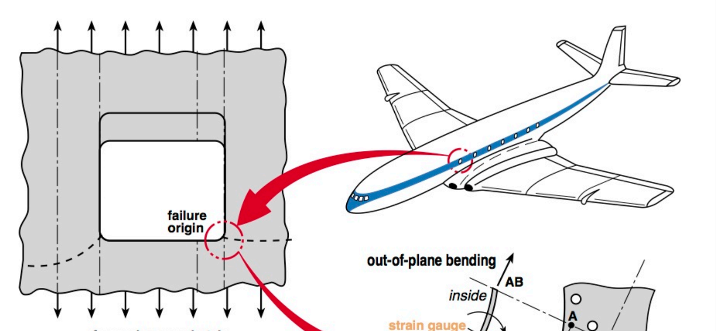
Bad Stress Concentration



Da Havilland Comet Stress







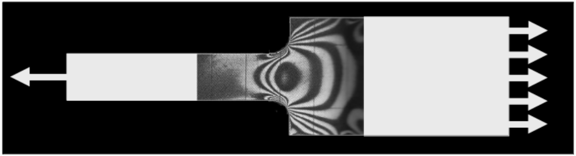
Stress Concentration Factors

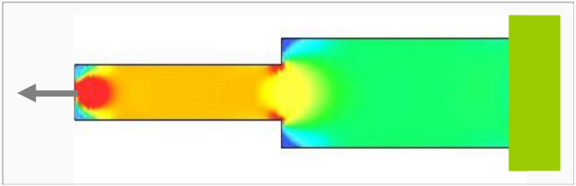
- Stress concentration factor K

$$K = \frac{\sigma_{\mathsf{max}}}{\sigma_{\mathsf{avg}}}$$

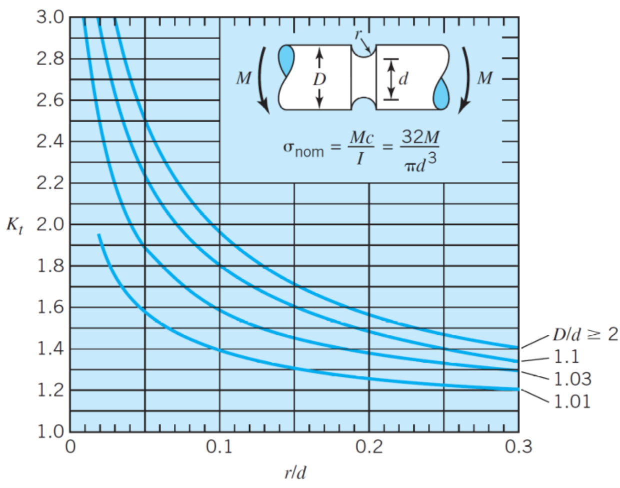
- depends on geometry

Photoelastic vs Simulation



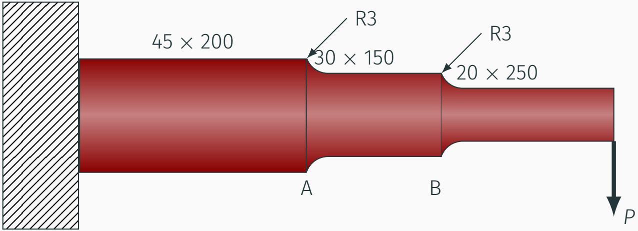


Stress Concentration Factor Chart

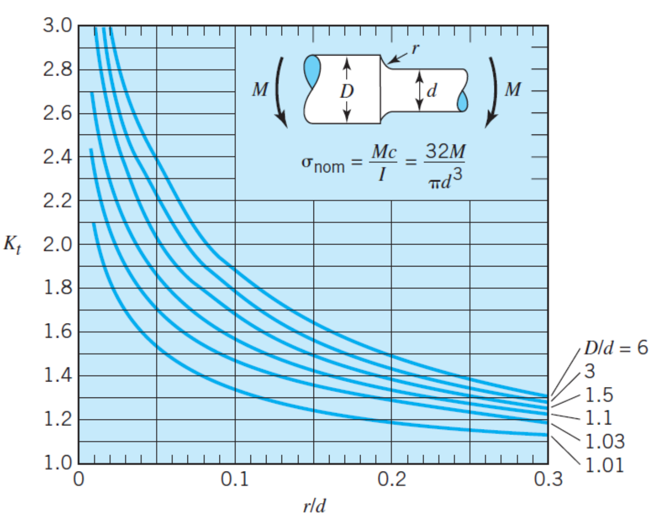


Example

Determine the maximum load this multi-segment cantilever beam can take given that the beam material has σ_{allow} = 150 MPa.



Example: Use this chart



Avoid Stress Concentration Factor