

1. True or False: A recursive definition of the set of integers  $\mathbb{Z}$  is

Basis Step:  $20 \in \mathbb{Z}$   
 Recursive Step: If  $x \in \mathbb{Z}$ , then  $x - 1 \in \mathbb{Z}$

*Key idea: To define a set, we need to specify which elements are in and which elements are out. A description that gives some, but not all, integers is not a definition of the set of integers.*

2. When  $B = \{\text{A}, \text{C}, \text{U}, \text{G}\}$  is the set of RNA bases, consider the recursively defined set  $X$  given by:

Basis Step:  $\text{AA} \in X, \text{CC} \in X, \text{UU} \in X, \text{GG} \in X$   
 Recursive Step: If  $x \in X$  and  $b \in B$  then  $bx b \in X$

where  $bx b$  is the result of string concatenations.

Give three example elements of  $X$  and an English description of the set.

*Key idea: To build new example elements we can start with an element from the basis step and then apply a rule from the recursive step finitely many times.*

3. Write in roster method the set given by the Cartesian product

$$\{1, 2\} \times \{a, b, c\}$$

*Key idea: Cartesian product does not require the sets to have the same size as one another or to have the same types of elements as one another.*

4. Consider the function  $d_2$  : The set of ordered pairs of ratings 4-tuples  $\rightarrow \mathbb{R}$  given by

$$d_2((x_1, x_2, x_3, x_4), (y_1, y_2, y_3, y_4)) = \sqrt{\sum_{i=1}^4 (x_i - y_i)^2}$$

Let  $z = (1, 1, 1, 1)$  be a ratings 4-tuple and consider the set

$$\{x \mid d_2(x, z) = 1\}$$

Rewrite this set using the roster method.

*Key idea: The input to this function is an ordered pair each of whose components is a ratings 4-tuple.*

5. Rewrite the set

$$\{r_{\text{nalen}}(x) \mid x \in S \text{ and } r_{\text{nalen}}(x) < 2\}$$

using the roster method.

*Key idea: The set builder definition for this set can be read as “The collection of all outputs of the function  $r_{\text{nalen}}$  when the input is taken from the set of RNA strands for which  $r_{\text{nalen}}$  gives value less than 2.” Informally, we consider just RNA strands whose  $r_{\text{nalen}}$  value is less than 2 and collect their  $r_{\text{nalen}}$  values into our set.*