

Ratings encoding

In the table below, each row represents a user's ratings of movies: ✓ (check) indicates the person liked the movie, ✗ (x) that they didn't, and • (dot) that they didn't rate it one way or another (neutral rating or didn't watch). Can encode these ratings numerically with 1 for ✓ (check), −1 for ✗ (x), and 0 for • (dot).

| Person | Fyre | Frozen II | Picard | Ratings written as a 3-tuple |
|--------|------|-----------|--------|------------------------------|
| P_1 | ✗ | • | ✓ | |
| P_2 | ✓ | ✓ | ✗ | |
| P_3 | ✓ | ✓ | ✓ | |
| P_4 | • | ✗ | ✓ | |

Definitions

| Term | Notation | Example(s) | We say in English ... |
|---------------------------|---|------------|---|
| sequence | x_1, \dots, x_n | | A sequence x_1 to x_n |
| summation | $\sum_{i=1}^n x_i$ or $\sum_{i=1}^n x_i$ | | The sum of the terms of the sequence x_1 to x_n |
| all reals | \mathbb{R} | | The (set of all) real numbers (numbers on the number line) |
| all integers | \mathbb{Z} | | The (set of all) integers (whole numbers including negatives, zero, and positives) |
| all positive integers | \mathbb{Z}^+ | | The (set of all) strictly positive integers |
| all natural numbers | \mathbb{N} | | The (set of all) natural numbers. Note: we use the convention that 0 is a natural number. |
| piecewise rule definition | $f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$ | | Define f of x to be x when x is nonnegative and to be $-x$ when x is negative |
| function application | $f(7)$ $f(z)$ $f(g(z))$ | | f of 7 or f applied to 7 or the image of 7 under f f of z or f applied to z or the image of z under f f of g of z or f applied to the result of g applied to z |
| absolute value | $ -3 $ | | The absolute value of -3 |
| square root | $\sqrt{9}$ | | The non-negative square root of 9 |

Data types

| Term | Examples: (add additional examples from class) |
|--|---|
| set unordered collection of elements <i>repetition doesn't matter</i> <i>Equal sets agree on membership of all elements</i> | $7 \in \{43, 7, 9\}$ $2 \notin \{43, 7, 9\}$ |
| n-tuple ordered sequence of elements with n “slots” ($n > 0$) <i>repetition matters, fixed length</i> <i>Equal n-tuples have corresponding components equal</i> | |
| string ordered finite sequence of elements each from specified set <i>repetition matters, arbitrary finite length</i> <i>Equal strings have same length and corresponding characters equal</i> | |

Special cases:

When $n = 2$, the 2-tuple is called an **ordered pair**.

A string of length 0 is called the **empty string** and is denoted λ .

A set with no elements is called the **empty set** and is denoted $\{\}$ or \emptyset .

Defining sets

To define sets:

To define a set using **roster method**, explicitly list its elements. That is, start with $\{$ then list elements of the set separated by commas and close with $\}$.

To define a set using **set builder definition**, either form “The set of all x from the universe U such that x is ...” by writing

$$\{x \in U \mid \dots x \dots\}$$

or form “the collection of all outputs of some operation when the input ranges over the universe U ” by writing

$$\{\dots x \dots \mid x \in U\}$$

We use the symbol \in as “is an element of” to indicate membership in a set.

Example sets: For each of the following, identify whether it’s defined using the roster method or set builder notation and give an example element.

$$\{-1, 1\}$$

$$\{0, 0\}$$

$$\{-1, 0, 1\}$$

$$\{(x, x, x) \mid x \in \{-1, 0, 1\}\}$$

$$\{\}$$

$$\{x \in \mathbb{Z} \mid x \geq 0\}$$

$$\{x \in \mathbb{Z} \mid x > 0\}$$

$$\{\text{A, C, U, G}\}$$

$$\{\text{AUG, UAG, UGA, UAA}\}$$

Defining functions ratings

Recall our representation of Netflix users' ratings of movies as n -tuples, where n is the number of movies in the database. Each component of the n -tuple is -1 (didn't like the movie), 0 (neutral rating or didn't watch the movie), or 1 (liked the movie).

Consider the ratings $P_1 = (-1, 0, 1)$, $P_2 = (1, 1, -1)$, $P_3 = (1, 1, 1)$, $P_4 = (0, -1, 1)$

Which of P_1 , P_2 , P_3 has movie preferences most similar to P_4 ?

One approach to answer this question: use **functions** to define distance between user preferences.

For example, consider the function d_0 :

given by

→

$$d_0((x_1, x_2, x_3), (y_1, y_2, y_3)) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2}$$

Extra example: A new movie is released, and P_1 and P_2 watch it before P_3 , and give it ratings; P_1 gives ✓ and P_2 gives ✗. Should this movie be recommended to P_3 ? Why or why not?

Extra example: Define a new function that could be used to compare the 4-tuples of ratings encoding movie preferences now that there are four movies in the database.