Recall the definition of linked lists from class. Consider this (incomplete) definition: **Definition** The function *increment*: ______ that adds 1 to the data in each node of a linked list is defined by:

$$\begin{array}{ccc} increment: \underline{\hspace{1cm}} & \rightarrow \underline{\hspace{1cm}} \\ \text{Basis Step:} & increment([]) & = [] \\ \text{Recursive Step:} & \text{If } l \in L, n \in \mathbb{N} & increment((n,l)) & = (1+n,increment(l)) \end{array}$$

Consider this (incomplete) definition: **Definition** The function $sum: L \to \mathbb{N}$ that adds together all the data in nodes of the list is defined by:

You will compute a sample function application and then fill in the blanks for the domain and codomain of each of these functions.

- 1. Based on the definition, what is the result of increment((4,(2,(7,[]))))? Write your answer directly with no spaces.
- 2. Which of the following describes the domain and codomain of *increment*?
 - (a) The domain is L and the codomain is \mathbb{N}
- (d) The domain is $L \times \mathbb{N}$ and the codomain is \mathbb{N}
- (b) The domain is L and the codomain is $\mathbb{N} \times L$
- (e) The domain is L and the codomain is L
- (c) The domain is $L \times \mathbb{N}$ and the codomain is L
- (f) None of the above
- 3. Assuming we would like sum((5,(6,[]))) to evaluate to 11 and sum((3,(1,(8,[])))) to evaluate to 12, which of the following could be used to fill in the definition of the recursive case of sum?

(a)
$$\begin{cases} 1 + sum(l) & \text{when } n \neq 0 \\ sum(l) & \text{when } n = 0 \end{cases}$$

(c)
$$n + increment(l)$$

(b)
$$1 + sum(l)$$

(d)
$$n + sum(l)$$

- 4. Choose only and all of the following statements that are **well-defined**; that is, they correctly reflect the domains and codomains of the functions and quantifiers, and respect the notational conventions we use in this class. Note that a well-defined statement may be true or false.
 - (a) $\forall l \in L (sum(l))$

(e) $\forall l \in L \, \forall n \in \mathbb{N} \, ((n \times l) \subseteq L)$

(b) $\exists l \in L (sum(l) \land length(l))$

- (f) $\forall l_1 \in L \,\exists l_2 \in L \,(increment(sum(l_1)) = l_2)$
- (c) $\forall l \in L (sum(increment(l)) = 10)$ (d) $\exists l \in L (sum(increment(l)) = 10)$
- (g) $\forall l \in L (length(increment(l)) = length(l))$
- 5. Choose only and all of the statements in the previous part that are both well-defined and true.