

Recall the definition of linked lists from class. Consider this (incomplete) definition: **Definition** The function *increment* : \_\_\_\_\_ that adds 1 to the data in each node of a linked list is defined by:

$$\begin{array}{ll} \text{Basis Step:} & \text{increment} : \text{_____} \rightarrow \text{_____} \\ & \text{increment}(\boxed{\phantom{0}}) = \boxed{\phantom{0}} \\ \text{Recursive Step:} & \text{If } l \in L, n \in \mathbb{N} \quad \text{increment}((n, l)) = (1 + n, \text{increment}(l)) \end{array}$$

Consider this (incomplete) definition: **Definition** The function *sum* :  $L \rightarrow \mathbb{N}$  that adds together all the data in nodes of the list is defined by:

$$\begin{array}{ll} \text{Basis Step:} & \text{sum} : L \rightarrow \mathbb{N} \\ & \text{sum}(\boxed{\phantom{0}}) = 0 \\ \text{Recursive Step:} & \text{If } l \in L, n \in \mathbb{N} \quad \text{sum}((n, l)) = \text{_____} \end{array}$$

You will compute a sample function application and then fill in the blanks for the domain and codomain of each of these functions.

1. Based on the definition, what is the result of  $\text{increment}((4, (2, (7, \boxed{\phantom{0}}))))$ ? Write your answer directly with no spaces.
2. Which of the following describes the domain and codomain of *increment*?
  - (a) The domain is  $L$  and the codomain is  $\mathbb{N}$
  - (b) The domain is  $L$  and the codomain is  $\mathbb{N} \times L$
  - (c) The domain is  $L \times \mathbb{N}$  and the codomain is  $L$
  - (d) The domain is  $L \times \mathbb{N}$  and the codomain is  $\mathbb{N}$
  - (e) The domain is  $L$  and the codomain is  $L$
  - (f) None of the above
3. Assuming we would like  $\text{sum}((5, (6, \boxed{\phantom{0}})))$  to evaluate to 11 and  $\text{sum}((3, (1, (8, \boxed{\phantom{0}}))))$  to evaluate to 12, which of the following could be used to fill in the definition of the recursive case of *sum*?
  - (a)  $\begin{cases} 1 + \text{sum}(l) & \text{when } n \neq 0 \\ \text{sum}(l) & \text{when } n = 0 \end{cases}$
  - (b)  $1 + \text{sum}(l)$
  - (c)  $n + \text{increment}(l)$
  - (d)  $n + \text{sum}(l)$
  - (e) None of the above

4. Choose only and all of the following statements that are **well-defined**; that is, they correctly reflect the domains and codomains of the functions and quantifiers, and respect the notational conventions we use in this class. Note that a well-defined statement may be true or false.

(a)  $\forall l \in L (sum(l))$

(e)  $\forall l \in L \forall n \in \mathbb{N} ((n \times l) \subseteq L)$

(b)  $\exists l \in L (sum(l) \wedge length(l))$

(f)  $\forall l_1 \in L \exists l_2 \in L (increment(sum(l_1)) = l_2)$

(c)  $\forall l \in L (sum(increment(l)) = 10)$

(g)  $\forall l \in L (length(increment(l)) = length(l))$

(d)  $\exists l \in L (sum(increment(l)) = 10)$

5. Choose only and all of the statements in the previous part that are both well-defined and true.