

## Definitions

| Term                      | Notation  | Example(s) | We say in English ...   |
|---------------------------|---|------------|---|
| sequence                  | $x_1, \dots, x_n$   |            | A sequence $x_1$ to $x_n$   |
| summation                 | $\sum_{i=1}^n x_i$ or $\sum_{i=1}^n x_i$  |            | The sum of the terms of the sequence $x_1$ to $x_n$   |
| all reals                 | $\mathbb{R}$  |            | The (set of all) real numbers (numbers on the number line)  |
| all integers              | $\mathbb{Z}$  |            | The (set of all) integers (whole numbers including negatives, zero, and positives)  |
| all positive integers     | $\mathbb{Z}^+$  |            | The (set of all) strictly positive integers   |
| all natural numbers       | $\mathbb{N}$  |            | The (set of all) natural numbers. <b>Note:</b> we use the convention that 0 is a natural number.  |
| piecewise rule definition | $f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$ |            | Define $f$ of $x$ to be $x$ when $x$ is nonnegative and to be $-x$ when $x$ is negative   |
| function application      | $f(7)$<br>$f(z)$<br>$f(g(z))$   |            | $f$ of 7 <b>or</b> $f$ applied to 7 <b>or</b> the image of 7 under $f$<br>$f$ of $z$ <b>or</b> $f$ applied to $z$ <b>or</b> the image of $z$ under $f$<br>$f$ of $g$ of $z$ <b>or</b> $f$ applied to the result of $g$ applied to $z$ |
| absolute value            | $ -3 $  |            | The absolute value of $-3$  |
| square root               | $\sqrt{9}$  |            | The non-negative square root of 9   |

## Defining functions ratings

Recall our representation of Netflix users' ratings of movies as  $n$ -tuples, where  $n$  is the number of movies in the database. Each component of the  $n$ -tuple is  $-1$  (didn't like the movie),  $0$  (neutral rating or didn't watch the movie), or  $1$  (liked the movie).

Consider the ratings  $P_1 = (-1, 0, 1)$ ,  $P_2 = (1, 1, -1)$ ,  $P_3 = (1, 1, 1)$ ,  $P_4 = (0, -1, 1)$

Which of  $P_1$ ,  $P_2$ ,  $P_3$  has movie preferences most similar to  $P_4$ ?

One approach to answer this question: use **functions** to define distance between user preferences.

For example, consider the function  $d_0$  :  
given by

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$$d_0( (x_1, x_2, x_3), (y_1, y_2, y_3) ) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2}$$

*Extra example:* A new movie is released, and  $P_1$  and  $P_2$  watch it before  $P_3$ , and give it ratings;  $P_1$  gives ✓ and  $P_2$  gives ✗. Should this movie be recommended to  $P_3$ ? Why or why not?

*Extra example:* Define a new function that could be used to compare the 4-tuples of ratings encoding movie preferences now that there are four movies in the database.