

# Definitions

Term	Notation	Example(s)	We say in English ...
sequence	$x_1, \dots, x_n$		A sequence $x_1$ to $x_n$
summation	$\sum_{i=1}^n x_i$ or $\sum_{i=1}^n x_i$		The sum of the terms of the sequence $x_1$ to $x_n$
all reals	$\mathbb{R}$		The (set of all) real numbers (numbers on the number line)
all integers	$\mathbb{Z}$		The (set of all) integers (whole numbers including negatives, zero, and positives)
all positive integers	$\mathbb{Z}^+$		The (set of all) strictly positive integers
all natural numbers	$\mathbb{N}$		The (set of all) natural numbers. <b>Note:</b> we use the convention that 0 is a natural number.
piecewise rule definition	$f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$		Define $f$ of $x$ to be $x$ when $x$ is nonnegative and to be $-x$ when $x$ is negative
function application	$f(7)$ $f(z)$ $f(g(z))$		$f$ of 7 <b>or</b> $f$ applied to 7 <b>or</b> the image of 7 under $f$ $f$ of $z$ <b>or</b> $f$ applied to $z$ <b>or</b> the image of $z$ under $f$ $f$ of $g$ of $z$ <b>or</b> $f$ applied to the result of $g$ applied to $z$
absolute value	$ -3 $		The absolute value of $-3$
square root	$\sqrt{9}$		The non-negative square root of 9

# Data types

Term	Examples: (add additional examples from class)
<b>set</b> unordered collection of elements <i>repetition doesn't matter</i> <i>Equal sets agree on membership of all elements</i>	$7 \in \{43, 7, 9\}$ $2 \notin \{43, 7, 9\}$
<b><math>n</math>-tuple</b> ordered sequence of elements with $n$ “slots” ( $n > 0$ ) <i>repetition matters, fixed length</i> <i>Equal <math>n</math>-tuples have corresponding components equal</i>	
<b>string</b> ordered finite sequence of elements each from specified set <i>repetition matters, arbitrary finite length</i> <i>Equal strings have same length and corresponding characters equal</i>	

*Special cases:*

When  $n = 2$ , the 2-tuple is called an **ordered pair**.

A string of length 0 is called the **empty string** and is denoted  $\lambda$ .

A set with no elements is called the **empty set** and is denoted  $\{\}$  or  $\emptyset$ .

# Defining sets

*To define sets:*

To define a set using **roster method**, explicitly list its elements. That is, start with  $\{$  then list elements of the set separated by commas and close with  $\}$ .

To define a set using **set builder definition**, either form “The set of all  $x$  from the universe  $U$  such that  $x$  is ...” by writing

$$\{x \in U \mid \dots x \dots\}$$

or form “the collection of all outputs of some operation when the input ranges over the universe  $U$ ” by writing

$$\{\dots x \dots \mid x \in U\}$$

We use the symbol  $\in$  as “is an element of” to indicate membership in a set.

**Example sets:** For each of the following, identify whether it’s defined using the roster method or set builder notation and give an example element.

$$\{-1, 1\}$$

$$\{0, 0\}$$

$$\{-1, 0, 1\}$$

$$\{(x, x, x) \mid x \in \{-1, 0, 1\}\}$$

$$\{\}$$

$$\{x \in \mathbb{Z} \mid x \geq 0\}$$

$$\{x \in \mathbb{Z} \mid x > 0\}$$

$$\{\text{A, C, U, G}\}$$

$$\{\text{AUG, UAG, UGA, UAA}\}$$

# Set operations

To define a set we can use the roster method, set builder notation, a recursive definition, and also we can apply a set operation to other sets.

## New! Cartesian product of sets and set-wise concatenation of sets of strings

**Definition:** Let  $X$  and  $Y$  be sets. The **Cartesian product** of  $X$  and  $Y$ , denoted  $X \times Y$ , is the set of all ordered pairs  $(x, y)$  where  $x \in X$  and  $y \in Y$

$$X \times Y = \{(x, y) \mid x \in X \text{ and } y \in Y\}$$

**Definition:** Let  $X$  and  $Y$  be sets of strings over the same alphabet. The **set-wise concatenation** of  $X$  and  $Y$ , denoted  $X \circ Y$ , is the set of all results of string concatenation  $xy$  where  $x \in X$  and  $y \in Y$

$$X \circ Y = \{xy \mid x \in X \text{ and } y \in Y\}$$

**Pro-tip:** the meaning of writing one element next to another like  $xy$  depends on the data-types of  $x$  and  $y$ . When  $x$  and  $y$  are strings, the convention is that  $xy$  is the result of string concatenation. When  $x$  and  $y$  are numbers, the convention is that  $xy$  is the result of multiplication. This is (one of the many reasons) why is it very important to declare the data-type of variables before we use them.

*Fill in the missing entries in the table:*

Set	Example elements in this set:			
$B$	A	C	G	U
	(A, C)	(U, U)		
$B \times \{-1, 0, 1\}$				
$\{-1, 0, 1\} \times B$				
			(0, 0, 0)	
$\{A, C, G, U\} \circ \{A, C, G, U\}$				
			GGGG	

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