

## Fundamental theorem proof

**Theorem:** Every positive integer *greater than 1* is a product of (one or more) primes.

**Before we prove, let's try some examples:**

$$20 =$$

$$100 =$$

$$5 =$$

**Proof by strong induction,** with  $b = 2$  and  $j = 0$ .

**Basis step:** WTS property is true about 2.

Since 2 is itself prime, it is already written as a product of (one) prime.

**Recursive step:** Consider an arbitrary integer  $n \geq 2$ . Assume (as the strong induction hypothesis, IH) that the property is true about each of  $2, \dots, n$ . WTS that the property is true about  $n + 1$ : We want to show that  $n + 1$  can be written as a product of primes. Notice that  $n + 1$  is itself prime or it is composite.

*Case 1:* assume  $n + 1$  is prime and then immediately it is written as a product of (one) prime so we are done.

*Case 2:* assume that  $n + 1$  is composite so there are integers  $x$  and  $y$  where  $n + 1 = xy$  and each of them is between 2 and  $n$  (inclusive). Therefore, the induction hypothesis applies to each of  $x$  and  $y$  so each of these factors of  $n + 1$  can be written as a product of primes. Multiplying these products together, we get a product of primes that gives  $n + 1$ , as required.

Since both cases give the necessary conclusion, the proof by cases for the recursive step is complete.

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