

Defining sets

To define sets:

To define a set using **roster method**, explicitly list its elements. That is, start with { then list elements of the set separated by commas and close with }.

To define a set using **set builder definition**, either form “The set of all x from the universe U such that x is ...” by writing

$$\{x \in U \mid ...x...\}$$

or form “the collection of all outputs of some operation when the input ranges over the universe U ” by writing

$$\{...x... \mid x \in U\}$$

We use the symbol \in as “is an element of” to indicate membership in a set.

Example sets: For each of the following, identify whether it’s defined using the roster method or set builder notation and give an example element.

$$\{-1, 1\}$$

$$\{0, 0\}$$

$$\{-1, 0, 1\}$$

$$\{(x, x, x) \mid x \in \{-1, 0, 1\}\}$$

$$\{\}$$

$$\{x \in \mathbb{Z} \mid x \geq 0\}$$

$$\{x \in \mathbb{Z} \mid x > 0\}$$

$$\{\text{A, C, U, G}\}$$

$$\{\text{AUG, UAG, UGA, UAA}\}$$

Rna motivation

RNA is made up of strands of four different bases that encode genomic information in specific ways. The bases are elements of the set $B = \{\mathbf{A}, \mathbf{C}, \mathbf{U}, \mathbf{G}\}$.

Formally, to define the set of all RNA strands, we need more than roster method or set builder descriptions.

Set recursive definition

New! Recursive Definitions of Sets: The set S (pick a name) is defined by:

- Basis Step: Specify finitely many elements of S
- Recursive Step: Give rule(s) for creating a new element of S from known values existing in S , and potentially other values.

The set S then consists of all and only elements that are put in S by finitely many (a nonnegative integer number) of applications of the recursive step after the basis step.

Set recursive examples

Definition The set of nonnegative integers \mathbb{N} is defined (recursively) by:

Basis Step:

Recursive Step:

Examples:

Definition The set of all integers \mathbb{Z} is defined (recursively) by:

Basis Step:

Recursive Step:

Examples:

Definition The set of RNA strands S is defined (recursively) by:

Basis Step: $\mathbf{A} \in S, \mathbf{C} \in S, \mathbf{U} \in S, \mathbf{G} \in S$

Recursive Step: If $s \in S$ and $b \in B$, then $sb \in S$

where sb is string concatenation.

Examples:

Definition The set of bitstrings (strings of 0s and 1s) is defined (recursively) by:

Basis Step:

Recursive Step:

Notation: We call the set of bitstrings $\{0, 1\}^*$.

Examples: