Fundamental theorem proof

Theorem: Every positive integer *greater than 1* is a product of (one or more) primes.

Before we prove, let's try some examples:

$20 =$

$100 =$

$5 =$

Proof by strong induction, with $b = 2$ and $j = 0$.

Basis step: WTS property is true about 2.

Since 2 is itself prime, it is already written as a product of (one) prime.

Recursive step: Consider an arbitrary integer $n \geq 2$. Assume (as the strong induction hypothesis, IH) that the property is true about each of $2, \ldots, n$. WTS that the property is true about $n + 1$: We want to show that $n + 1$ can be written as a product of primes. Notice that $n + 1$ is itself prime or it is composite.

Case 1: assume $n + 1$ is prime and then immediately it is written as a product of (one) prime so we are done.

Case 2: assume that $n + 1$ is composite so there are integers x and y where $n + 1 = xy$ and each of them is between 2 and n (inclusive). Therefore, the induction hypothesis applies to each of x and y so each of these factors of $n + 1$ can be written as a product of primes. Multiplying these products together, we get a product of primes that gives $n + 1$, as required.

Since both cases give the necessary conclusion, the proof by cases for the recursive step is complete.

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