

Definitions

| Term | Notation | Example(s) | We say in English ... |
|---------------------------|---|------------|---|
| sequence | x_1, \dots, x_n | | A sequence x_1 to x_n |
| summation | $\sum_{i=1}^n x_i$ or $\sum_{i=1}^n x_i$ | | The sum of the terms of the sequence x_1 to x_n |
| all reals | \mathbb{R} | | The (set of all) real numbers (numbers on the number line) |
| all integers | \mathbb{Z} | | The (set of all) integers (whole numbers including negatives, zero, and positives) |
| all positive integers | \mathbb{Z}^+ | | The (set of all) strictly positive integers |
| all natural numbers | \mathbb{N} | | The (set of all) natural numbers. Note: we use the convention that 0 is a natural number. |
| piecewise rule definition | $f(x) = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$ | | Define f of x to be x when x is nonnegative and to be $-x$ when x is negative |
| function application | $f(7)$ $f(z)$ $f(g(z))$ | | f of 7 or f applied to 7 or the image of 7 under f f of z or f applied to z or the image of z under f f of g of z or f applied to the result of g applied to z |
| absolute value | $ -3 $ | | The absolute value of -3 |
| square root | $\sqrt{9}$ | | The non-negative square root of 9 |

Data types

| Term | Examples: (add additional examples from class) |
|--|---|
| set unordered collection of elements <i>repetition doesn't matter</i> <i>Equal sets agree on membership of all elements</i> | $7 \in \{43, 7, 9\}$ $2 \notin \{43, 7, 9\}$ |
| n-tuple ordered sequence of elements with n “slots” ($n > 0$) <i>repetition matters, fixed length</i> <i>Equal n-tuples have corresponding components equal</i> | |
| string ordered finite sequence of elements each from specified set <i>repetition matters, arbitrary finite length</i> <i>Equal strings have same length and corresponding characters equal</i> | |

Special cases:

When $n = 2$, the 2-tuple is called an **ordered pair**.

A string of length 0 is called the **empty string** and is denoted λ .

A set with no elements is called the **empty set** and is denoted $\{\}$ or \emptyset .

Defining sets

To define sets:

To define a set using **roster method**, explicitly list its elements. That is, start with $\{$ then list elements of the set separated by commas and close with $\}$.

To define a set using **set builder definition**, either form “The set of all x from the universe U such that x is ...” by writing

$$\{x \in U \mid \dots x \dots\}$$

or form “the collection of all outputs of some operation when the input ranges over the universe U ” by writing

$$\{\dots x \dots \mid x \in U\}$$

We use the symbol \in as “is an element of” to indicate membership in a set.

Example sets: For each of the following, identify whether it’s defined using the roster method or set builder notation and give an example element.

$$\{-1, 1\}$$

$$\{0, 0\}$$

$$\{-1, 0, 1\}$$

$$\{(x, x, x) \mid x \in \{-1, 0, 1\}\}$$

$$\{\}$$

$$\{x \in \mathbb{Z} \mid x \geq 0\}$$

$$\{x \in \mathbb{Z} \mid x > 0\}$$

$$\{\text{A, C, U, G}\}$$

$$\{\text{AUG, UAG, UGA, UAA}\}$$

Set operations

To define a set we can use the roster method, set builder notation, a recursive definition, and also we can apply a set operation to other sets.

New! Cartesian product of sets and set-wise concatenation of sets of strings

Definition: Let X and Y be sets. The **Cartesian product** of X and Y , denoted $X \times Y$, is the set of all ordered pairs (x, y) where $x \in X$ and $y \in Y$

$$X \times Y = \{(x, y) \mid x \in X \text{ and } y \in Y\}$$

Definition: Let X and Y be sets of strings over the same alphabet. The **set-wise concatenation** of X and Y , denoted $X \circ Y$, is the set of all results of string concatenation xy where $x \in X$ and $y \in Y$

$$X \circ Y = \{xy \mid x \in X \text{ and } y \in Y\}$$

Pro-tip: the meaning of writing one element next to another like xy depends on the data-types of x and y . When x and y are strings, the convention is that xy is the result of string concatenation. When x and y are numbers, the convention is that xy is the result of multiplication. This is (one of the many reasons) why is it very important to declare the data-type of variables before we use them.

Fill in the missing entries in the table:

| Set | Example elements in this set: | | | |
|---------------------------------------|-------------------------------|---|--------|---|
| B | A | C | G | U |
| | (A, C) | | (U, U) | |
| $B \times \{-1, 0, 1\}$ | | | | |
| $\{-1, 0, 1\} \times B$ | | | | |
| | (0, 0, 0) | | | |
| $\{A, C, G, U\} \circ \{A, C, G, U\}$ | | | | |
| | GGGG | | | |