Fundamental theorem proof

Theorem: Every positive integer *greater than 1* is a product of (one or more) primes.

Before we prove, let's try some examples:

```
20 = 100 =
```

5 =

Proof by strong induction, with b = 2 and j = 0.

Basis step: WTS property is true about 2.

Since 2 is itself prime, it is already written as a product of (one) prime.

Recursive step: Consider an arbitrary integer $n \geq 2$. Assume (as the strong induction hypothesis, IH) that the property is true about each of $2, \ldots, n$. WTS that the property is true about n + 1: We want to show that n + 1 can be written as a product of primes. Notice that n + 1 is itself prime or it is composite.

Case 1: assume n + 1 is prime and then immediately it is written as a product of (one) prime so we are done.

Case 2: assume that n + 1 is composite so there are integers x and y where n + 1 = xy and each of them is between 2 and n (inclusive). Therefore, the induction hypothesis applies to each of x and y so each of these factors of n + 1 can be written as a product of primes. Multiplying these products together, we get a product of primes that gives n + 1, as required.

Since both cases give the necessary conclusion, the proof by cases for the recursive step is complete.

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