Monday October 11

1 = True   0 = False

Logical operators aka propositional connectives

Conjunction	AND	$\wedge$	`\land`	2 inputs	Evaluates to T exactly when **both** inputs are T
Exclusive or	XOR	$\oplus$	`\oplus`	2 inputs	Evaluates to T exactly when **exactly one** of inputs is T
Disjunction	OR	$\vee$	`\lor`	2 inputs	Evaluates to T exactly when **at least one** of inputs is T
Negation	NOT	$\neg$	`\lnot`	1 input	Evaluates to T exactly when its input is F

Input-output table aka Truth Table

Input		Output		
		Conjunction	**Exclusive or**	**Disjunction**
p	q	$p \wedge q$	$p \oplus q$	$p \vee q$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	F
		AND	XOR	OR

4 rows

Input	Output
	Negation
p	$\neg p$
T	F
F	T
	NOT

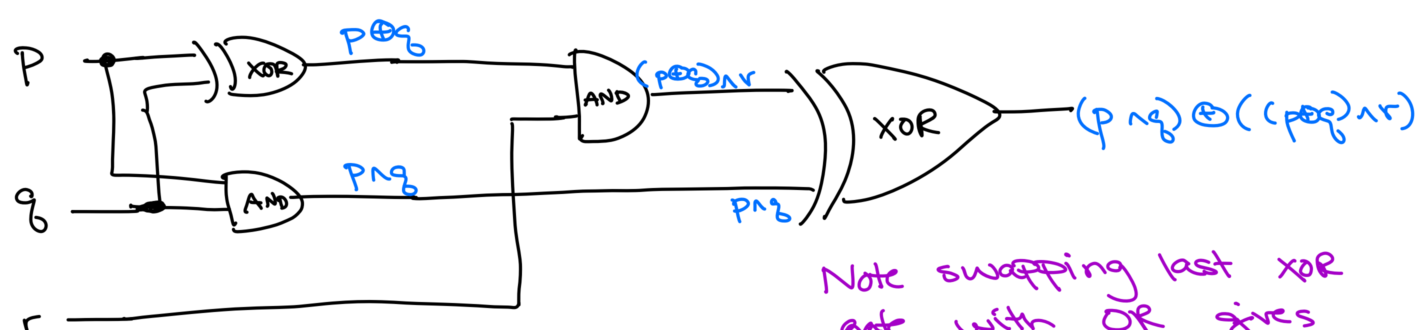
2 rows

expressions

Input						Output	
p	q	r	$p\oplus q$	$p\wedge q$	$(p\oplus q)\wedge r$	$(p \wedge q) \oplus ((p \oplus q) \wedge r)$	$\boxed{(p \wedge q) \vee ((p \oplus q) \wedge r)}$
T	T	T	F	T	$(T\oplus T)\wedge T = F \wedge T = F$	T	T
T	T	F	F	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	T	F	F	F	F
F	T	T	T	F	T	T	T
F	T	F	T	F	F	F	F
F	F	T	F	F	F	F	F
F	F	F	F	F	F	F	F

intermediate columns

Circuits



Note swapping last XOR gate with OR gives circuit for second expression

Given a truth table, how do we find ① an expression using the input variables and logical operators that has the output values specified in this table?

Application: ② design a circuit given a desired input-output relationship.

3 rows are F

Input		Output	
p	q	$mystery_1$	$mystery_2$
T	T	T	F
T	F	T	F
F	T	F	F
F	F	T	T

avoiding p 1st, q 1st $(\neg p \vee \neg q)$

avoiding means $(\neg p \vee q)$

avoiding $(p \vee \neg q)$

$p \vee q$
T
T
T
F

1 row is T for $mystery_2$, i.e. $p=F$, $q=F$

Expressions that have output $mystery_1$ are

① $p \vee \neg q$ evaluates to F exactly when both p and $\neg q$ evaluate to F i.e. p is F and q is T. Note: this is in CNF

② $\neg(q \wedge (q \oplus p))$

p	q	$\neg(q \wedge (q \oplus p))$
T	T	$\neg(T \wedge F) = T$
T	F	$\neg(F \wedge ---) = T$
F	T	$\neg(T \wedge (T \oplus F)) = F$
F	F	$\neg(F \wedge ---) = T$

③ $(p \wedge q) \vee (p \wedge \neg q) \vee (\neg p \wedge \neg q)$ DNF

Expressions that have output $mystery_2$ are

① $\neg p \wedge \neg q$ DNF

② $\neg(p \vee q)$ neither CNF nor DNF

③ $(\neg p \vee \neg q) \wedge (\neg p \vee q) \wedge (p \vee \neg q)$ CNF

IDEA: To get an algorithm for translating truth tables to expressions connect tables to expressions in normal forms

Definition An expression built of variables and logical operators is in **disjunctive normal form** (DNF) means that it is an OR of ANDs of variables and their negations. Pick a row that results in output T.

Definition An expression built of variables and logical operators is in **conjunctive normal form** (CNF) means that it is an AND of ORs of variables and their negations. Avoid all rows that output F.

Extra example: An expression that has output ? is:

Input			Output
p	q	r	?
T	T	T	T
T	T	F	T
T	F	T	F
T	F	F	T
F	T	T	F
F	T	F	F
F	F	T	T
F	F	F	F

DNF: pick a row highlighted green

$(p \wedge q \wedge r) \vee (p \wedge q \wedge \neg r) \vee (p \wedge \neg q \wedge \neg r) \vee (\neg p \wedge \neg q \wedge r)$

CNF: avoid all rows highlighted blue

$(\neg p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg r) \wedge (p \vee \neg q \vee r) \wedge (p \vee q \vee r)$

Wednesday October 13.

Proposition: Declarative sentence that is true or false (not both).

Propositional variable: Variable that represents a proposition.

Compound proposition: New proposition formed from existing propositions (potentially) using logical operators. *Note*: A propositional variable is one example of a compound proposition.

Truth table: Table with one row for each of the possible combinations of truth values of the input and an additional column that shows the truth value of the result of the operation corresponding to a particular row.

Logical equivalence : Two compound propositions are **logically equivalent** means that they have the same truth values for all settings of truth values to their propositional variables.

Tautology: A compound proposition that evaluates to true for all settings of truth values to its propositional variables; it is abbreviated T.

Contradiction: A compound proposition that evaluates to false for all settings of truth values to its propositional variables; it is abbreviated F.

Contingency: A compound proposition that is neither a tautology nor a contradiction.

Label each of the following as a tautology, contradiction, or contingency.

$p \wedge p$

$p \oplus p$

$p \vee p$

$p \vee \neg p$

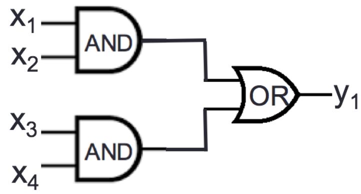
$p \wedge \neg p$

Extra Example: Which of the compound propositions in the table below are logically equivalent?

Input		Output				
p	q	$\neg(p \wedge \neg q)$	$\neg(\neg p \vee \neg q)$	$(\neg p \vee q)$	$(\neg q \vee \neg p)$	$(p \wedge q)$
T	T					
T	F					
F	T					
F	F					

Review: Week 3 Monday

1. (a) Consider the logic circuit



For which of the following settings(s) of input values is the output $y_1 = 0$? (Select all and only those that apply.)

i. $x_1 = 0$, $x_2 = 0$, $x_3 = 0$, and $x_4 = 0$
ii. $x_1 = 1$, $x_2 = 1$, $x_3 = 1$, and $x_4 = 1$
iii. $x_1 = 1$, $x_2 = 0$, $x_3 = 0$, and $x_4 = 1$
iv. $x_1 = 0$, $x_2 = 0$, $x_3 = 1$, and $x_4 = 1$

(b) Consider the logic circuits

For which of the following settings(s) of input values do the outputs of these circuits have the same value, i.e. $y_1 = z_1$? (Select all and only those that apply.)

i. $x_1 = 1$, $x_2 = 1$
ii. $x_1 = 1$, $x_2 = 0$
iii. $x_1 = 0$, $x_2 = 1$
iv. $x_1 = 0$, $x_2 = 0$

2. For each of the following compound propositions, determine if it is in DNF, CNF, both, or neither.

(a) $(x \vee y \vee z) \wedge (x \wedge \neg y \wedge z)$
(b) $\neg(x \wedge y \wedge z) \wedge \neg(\neg x \wedge y \wedge \neg z)$

Wednesday October 13

Proposition: Declarative sentence that is true or false (not both).

Propositional variable: Variable that represents a proposition. ex. p

Compound proposition: New proposition formed from existing propositions (potentially) using logical operators. *Note*: A propositional variable is one example of a compound proposition. ex. $p \vee q$ ex. p

Truth table: Table with one row for each of the possible combinations of truth values of the input and an additional column that shows the truth value of the result of the operation corresponding to a particular row.

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Contradiction: A compound proposition that evaluates to false for all settings of truth values to its propositional variables; it is abbreviated F.

Contingency: A compound proposition that is neither a tautology nor a contradiction.

Label each of the following as a tautology, contradiction, or contingency.

$p \wedge p$ Contingency when $p=T$, $p \wedge p = T \wedge T = T$ and when $p=F$, $p \wedge p = F$

$p \oplus p$ contradiction when $p=T$, $p \oplus p = T \oplus T = F$ and when $p=F$ $p \oplus p = F \oplus F = F$

$p \vee p$ contingency

$p \vee \neg p$ tautology when $p=T$ $p \vee \neg p = T \vee \neg T = T \vee F = T$ and when $p=F$, $p \vee \neg p = F \vee \neg F = F \vee T = T$

$p \wedge \neg p$ contradiction.

Extra Example: Which of the compound propositions in the table below are logically equivalent?

Input		Output				
p	q	$\neg(p \wedge \neg q)$	$\neg(\neg p \vee \neg q)$	$(\neg p \vee q)$	$(\neg q \vee \neg p)$	$(p \wedge q)$
T	T					
T	F					
F	T					
F	F					

Input		Output				
		Conjunction	Exclusive or	Disjunction	Conditional	Biconditional
p	q	$p \wedge q$	$p \oplus q$	$p \vee q$	$p \to q$	$p \leftrightarrow q$
T	T	T	F	T	T	T
T	F	F	T	T	F	F
F	T	F	T	T	T	F
F	F	F	F	F	T	T
		"p and q"	"p xor q"	"p or q"	"if p then q"	"p if and only if q"

The only way to make the conditional statement $p \to q$ false is to have p T and q F

The **hypothesis** of $p \to q$ is p   The **antecedent** of $p \to q$ is p

The **conclusion** of $p \to q$ is q   The **consequent** of $p \to q$ is q

Names for compound propositions related to $p \to q$

The **converse** of $p \to q$ is $q \to p$

The **inverse** of $p \to q$ is $\neg p \to \neg q$

The **contrapositive** of $p \to q$ is $\neg q \to \neg p$

Note: $\to$ in conditionals has a different meaning than the $\to$ in defining function

$(p \wedge q)$ $(p \to q)$ $(p \to (p \to q))$ q $(\neg q)$

p $(\neg p)$ $(\neg(\neg p))$ $(\neg(\neg(\neg p)))$

We can use a recursive definition to describe all compound propositions that use propositional variables from a specified collection. Here's the definition for all compound propositions whose propositional variables are in $\{p, q\}$.

Basis Step: p and q are each a compound proposition

Recursive Step: If x is a compound proposition then so is $(\neg x)$ and if x and y are both compound propositions then so is each of $(x \wedge y), (x \oplus y), (x \vee y), (x \to y), (x \leftrightarrow y)$

Order of operations (Precedence) for logical operators:

Negation, then conjunction / disjunction, then conditional / biconditionals.

Example: $\neg p \vee \neg q$ means $(\neg p) \vee (\neg q)$.

DNF $(\wedge \wedge) \vee (\wedge \wedge) \vee (\wedge \wedge)$

(Some) logical equivalences

Can replace p and q with any compound proposition

$\neg(\neg p) \equiv p$

p	$\neg(\neg p)$	p
T	$\neg(\neg T)=\neg F=T$	T
F	$\neg(\neg F)=\neg T=F$	F

Double negation there are only two truth values

$p \vee q \equiv q \vee p$ $\quad p \wedge q \equiv q \wedge p$ **Commutativity** Ordering of terms

$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $\quad (p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ **Associativity** Grouping of terms

$p \wedge F \equiv F$ $\quad p \vee T \equiv T$ $\quad p \wedge T \equiv p$ $\quad p \vee F \equiv p$ **Domination** aka short circuit evaluation

$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\quad \neg(p \vee q) \equiv \neg p \wedge \neg q$ **DeMorgan's Laws**

$p \to q \equiv \neg p \vee q$ ✓

Conditional

$p \to q \equiv \neg q \to \neg p$ **Contrapositive**

$\neg(p \to q) \equiv p \wedge \neg q$ "The only way to make $p \to q$ false is to have p is T and q is F"

$\neg(p \leftrightarrow q) \equiv p \oplus q$

$p \leftrightarrow q \equiv q \leftrightarrow p$

To formally prove a logical equivalence, draw truth table and compare values in all rows: need to agree for all possible input settings

p	q	$p \to q$	$\neg p \vee q$
T	T	T	$F \vee T = T$
T	F	F	$F \vee F = F$
F	T	T	$T \vee T = T$
F	F	T	$T \vee F = T$

Extra examples: $\neg(p \to q) \equiv \neg(\neg p \vee q) \equiv \neg\neg p \wedge \neg q \equiv p \wedge \neg q$

$p \leftrightarrow q$ is not logically equivalent to $p \wedge q$ because when $p=F, q=F$ $\;p \leftrightarrow q = T$ but $p \wedge q = F$

$p \to q$ is not logically equivalent to $q \to p$ because when $p=T, q=F$ $\;p \to q = T \to F = F$ but $q \to p = F \to T = T$.

converse

p	q	$p \to q$	converse $q \to p$	contrapositive $\neg q \to \neg p$
T	T	T	T	$\neg T \to \neg T = F \to F = T$
T	F	F	T	$\neg F \to \neg T = T \to F$ F
F	T	T	F	$\neg T \to \neg F = F \to T = T$
F	F	T	T	$\neg F \to \neg F = T \to T = T$

Note: converse is logically equivalent to inverse!

Review: Week 3 Wednesday

1. For each of the following propositions, indicate exactly one of:

   - There is no assignment of truth values to its variables that makes it true,
   - There is exactly one assignment of truth values to its variables that makes it true, or
   - There are exactly two assignments of truth values to its variables that make it true, or
   - There are exactly three assignments of truth values to its variables that make it true, or
   - *All* assignments of truth values to its variables make it true.

   (a) $x \wedge y \wedge (x \vee y)$
   (b) $\neg x \wedge y \wedge (x \vee y)$
   (c) $x \wedge \neg y \wedge (x \wedge y)$
   (d) $\neg x \wedge (y \vee \neg y)$
   (e) $x \wedge (y \vee \neg x)$

   For each of the following propositions, indicate exactly one of:

2. - There is no assignment of truth values to its variables that makes it true,
   - There is exactly one assignment of truth values to its variables that makes it true, or
   - There are exactly two assignments of truth values to its variables that make it true, or
   - There are exactly three assignments of truth values to its variables that make it true, or
   - *All* assignments of truth values to its variables make it true.

   (a) $(p \leftrightarrow q) \oplus (p \wedge q)$
   (b) $(p \to q) \vee (q \to p)$
   (c) $(p \to q) \wedge (q \to p)$
   (d) $\neg(p \to q)$

Friday October 15

Common ways to express logical operators in English:

Negation $\neg p$ can be said in English as

- Not p.
- It's not the case that p.
- p is false.

Conjunction $p \wedge q$ can be said in English as

- p and q.
- Both p and q are true.
- p but q.

Exclusive or $p \oplus q$ can be said in English as

- p or q, but not both.
- Exactly one of p and q is true.

Disjunction $p \vee q$ can be said in English as

- p or q, or both.
- p or q (inclusive).
- At least one of p and q is true.

Conditional $p \to q$ can be said in English as

- if p, then q.
- p is sufficient for q.
- q when p.
- q whenever p.
- p implies q.
- q follows from p.
- p is sufficient for q.
- q is necessary for p.
- p only if q.

Biconditional

- p if and only if q.
- p iff q.
- If p then q, and conversely.
- p is necessary and sufficient for q.

Translation: Express each of the following sentences as compound propositions, using the given propositions.

"A sufficient condition for the warranty to be good is that you bought the computer less than a year ago"

w is "the warranty is good"
b is "you bought the computer less than a year ago"

Translation:

$b \to w$

Evaluate:

F because it's possible to have $b = T$ $w = F$ for example if we opened up our device to look inside and voided the warranty.

"Whenever the message was sent from an unknown system, it is scanned for viruses."

s is "The message is scanned for viruses"
u is "The message was sent from an unknown system"

$u \to s$ "u guarantees s"
"if u then s"

$\neg k \to s$

k "The message was sent from a known system"

"I will complete my to-do list only if I put a reminder in my calendar"

d is "I will complete my to-do list"
c is "I put a reminder in my calendar"

d only if c

Option1 $\cancel{c \wedge d}$

Option2 $\cancel{c \Leftrightarrow d}$, i.e. $d \Leftrightarrow c$ i.e. "I will complete my to do list if and only if I put a reminder my calendar"

Option3 $\boxed{d \to c}$

lack of symmetry

Option4 $\cancel{c \to d}$.

Definition: A collection of compound propositions is called **consistent** if there is an assignment of truth values to the propositional variables that makes each of the compound propositions true.

Consistency:

1 Whenever the system software is being upgraded, users cannot access the file system. 2 If users can access the file system, then they can save new files. 3 If users cannot save new files, then the system software is not being upgraded.

1. Translate to symbolic compound propositions

u = system software is being upgraded
a = users can access the system
n = users can save new files

1 $u \rightarrow \neg a$

2 $a \rightarrow n$

3 $\neg n \rightarrow \neg u$

2. Look for some truth assignment to the propositional variables for which all the compound propositions output T

Can use Guess and check, or build truth table with an output column for each compound proposition in the collection. We are looking for a row in the table with T in *each* output column.

u	a	n	$u \rightarrow \neg a$	$a \rightarrow n$	$\neg n \rightarrow \neg u$
T	T	T	F	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	F
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	T
F	F	F	T	T	T

each of these rows gives an example of a truth assignment to the variables that makes all compound propositions true.

Conclude: Consistent!

Review: Week 3 Friday

1. Express each of the following sentences as compound propositions, using the given propositions.

   (a) "If you try to run Zoom while your computer is running many applications, the video is likely to be choppy and laggy." t is "you run Zoom while your computer is running many applications", c is "the video is likely to be choppy", g is "the video is likely to be laggy"

      i. $t \to (c \wedge g)$
      ii. $(c \wedge g) \to t$
      iii. $(c \wedge g) \leftrightarrow t$
      iv. $t \oplus (c \wedge g)$

   (b) "To connect wirelessly on campus without logging in you need to use the UCSD-Guest network." c is "connect wirelessly on campus", g is "logging in", and u is "use UCSD-Guest network".

      i. $c \wedge \neg g \wedge u$
      ii. $(c \wedge \neg g) \vee u$
      iii. $(c \wedge \neg g) \oplus u$
      iv. $(c \wedge \neg g) \to u$
      v. $u \to (c \wedge \neg g)$
      vi. $u \leftrightarrow (c \wedge \neg g)$

For each of the following system specifications, identify the compound propositions that give their translations to logic and then determine if the translated collection of compound propositions is consistent.

2. (a) Specification: If the computer is out of memory, then network connectivity is unreliable. No disk errors can occur when the computer is out of memory. Disk errors only occur when network connectivity is unreliable.

   Translation: M = "the computer is out of memory"; N = "network connectivity is unreliable"; D = "disk errors can occur".

      i.

$$\neg M \to N$$
$$\neg D \to M$$
$$D \to N$$

      ii.

$$M \to \neg N$$
$$\neg D \wedge M$$
$$N \to D$$

      iii.

$$M \to N$$
$$M \to \neg D$$
$$\neg N \to \neg D$$

(b) Specification: Whether you think you can, or you think you can't - you're right. [1]
Translation: $T =$ "you think you can"; $C =$ "you can".

i.

$$T \to C$$
$$\neg T \to \neg C$$

ii.

$$T \wedge C$$
$$\neg T \wedge \neg C$$

iii.

$$T \to \neg T$$
$$C \to \neg C$$

(c) Specification: A secure password must be private and complicated. If a password is complicated then it will be hard to remember. People write down hard-to-remember passwords. If a password is written down, it's not private. The password is secure.
Translation: $S =$ "the password is secure"; $P =$ "the password is private"; $C =$ "the password is complicated"; $H =$ "the password is hard to remember"; $W =$ "the password is written down".

i.

$$\neg(P \wedge C) \to \neg S$$
$$C \to H$$
$$W \wedge H$$
$$W \to \neg P$$
$$S$$

ii.

$$(P \wedge C) \to S$$
$$C \to H$$
$$W \to H$$
$$W \to P$$
$$S$$

iii.

$$S \to (P \wedge C)$$
$$C \to H$$
$$H \to W$$
$$W \to \neg P$$
$$S$$

[1] Henry Ford