Recall the definition of linked lists from class. Consider this (incomplete) definition: **Definition** The function $increment:$ ____________ that adds 1 to the data in each node of a linked list is defined by:

$$\begin{array}{lll} & increment: \_\_\_\_\_\_ & \to \_\_\_\_\_\_ \\ \text{Basis Step:} & increment([]) & = [] \\ \text{Recursive Step:} \quad \text{If } l \in L, n \in \mathbb{N} & increment((n,l)) & = (1+n, increment(l)) \end{array}$$

Consider this (incomplete) definition: **Definition** The function $sum : L \to \mathbb{N}$ that adds together all the data in nodes of the list is defined by:

$$\begin{array}{lll} & sum: L & \to \mathbb{N} \\ \text{Basis Step:} & sum([]) & = 0 \\ \text{Recursive Step:} \quad \text{If } l \in L, n \in \mathbb{N} & sum((n,l)) & = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ \end{array}$$

You will compute a sample function application and then fill in the blanks for the domain and codomain of each of these functions.

1. Based on the definition, what is the result of $increment((4,(2,(7,[]))))$? Write your answer directly with no spaces.

2. Which of the following describes the domain and codomain of $increment$?

   (a) The domain is L and the codomain is $\mathbb{N}$

   (b) The domain is L and the codomain is $\mathbb{N} \times L$

   (c) The domain is $L \times \mathbb{N}$ and the codomain is L

   (d) The domain is $L \times \mathbb{N}$ and the codomain is $\mathbb{N}$

   (e) The domain is L and the codomain is L

   (f) None of the above

3. Assuming we would like $sum((5,(6,[])))$ to evaluate to 11 and $sum((3,(1,(8,[]))))$ to evaluate to 12, which of the following could be used to fill in the definition of the recursive case of sum?

   (a) $\begin{cases} 1 + sum(l) & \text{when } n \neq 0 \\ sum(l) & \text{when } n = 0 \end{cases}$

   (b) $1 + sum(l)$

   (c) $n + increment(l)$

   (d) $n + sum(l)$

   (e) None of the above

4. Choose only and all of the following statements that are **well-defined**; that is, they correctly reflect the domains and codomains of the functions and quantifiers, and respect the notational conventions we use in this class. Note that a well-defined statement may be true or false.

   (a) $\forall l \in L \, (sum(l))$

   (b) $\exists l \in L \, (sum(l) \wedge length(l))$

   (c) $\forall l \in L \, (sum(increment(l)) = 10)$

   (d) $\exists l \in L \, (sum(increment(l)) = 10)$

   (e) $\forall l \in L \, \forall n \in \mathbb{N} \, ((n \times l) \subseteq L)$

   (f) $\forall l_1 \in L \, \exists l_2 \in L \, (increment(sum(l_1)) = l_2)$

   (g) $\forall l \in L \, (length(increment(l)) = length(l))$

5. Choose only and all of the statements in the previous part that are both well-defined and true.