

Least greatest proofs

For a set of numbers X , how do you formalize “there is a greatest X ” or “there is a least X ”?

Prove or disprove: There is a least prime number.

Prove or disprove: There is a greatest integer.

Approach 1, De Morgan’s and universal generalization:

Approach 2, proof by contradiction:

Extra examples: Prove or disprove that \mathbb{N} , \mathbb{Q} each have a least and a greatest element.