Ratings encoding

In the table below, each row represents a user's ratings of movies: \checkmark (check) indicates the person liked the movie, \checkmark (x) that they didn't, and \bullet (dot) that they didn't rate it one way or another (neutral rating or didn't watch). Can encode these ratings numerically with 1 for \checkmark (check), -1 for \checkmark (x), and 0 for \bullet (dot).

| Person | Fyre | Frozen II | Picard | Ratings written as a 3-tuple |
|------------------|------|--------------|--------|------------------------------|
| $\overline{P_1}$ | Х | • | ✓ | |
| P_2 | 1 | \checkmark | X | |
| P_3 | 1 | ✓ | ✓ | |
| P_4 | • | X | ✓ | |

Definitions

| Term | Notation Example(s) | We say in English |
|----------------------------|--|---|
| sequence | x_1, \ldots, x_n | A sequence x_1 to x_n |
| summation | x_1, \dots, x_n $\sum_{i=1}^n x_i \text{ or } \sum_{i=1}^n x_i$ | The sum of the terms of the sequence x_1 to x_n |
| all reals | \mathbb{R} | The (set of all) real numbers (numbers on the number line) |
| all integers | \mathbb{Z} | The (set of all) integers (whole numbers including negatives, zero, and positives) |
| all positive integers | \mathbb{Z}^+ | The (set of all) strictly positive integers |
| all natural numbers | N | The (set of all) natural numbers. Note : we use the convention that 0 is a natural number. |
| piecewise rule definition | $f(x) = \begin{cases} x & \text{if } x \ge 0 \\ -x & \text{if } x < 0 \end{cases}$ | Define f of x to be x when x is nonnegative and to be $-x$ when x is negative |
| function application | f(7) $f(z)$ $f(g(z))$ | f of 7 or f applied to 7 or the image of 7 under f f of z or f applied to z or the image of z under f f of g of z or f applied to the result of g applied to z |
| absolute value square root | $\begin{array}{c} -3 \\ \sqrt{9} \end{array}$ | The absolute value of -3 The non-negative square root of 9 |

Data types

| Term | Examples: | | | | |
|---|----------------------|-------------------------|--|--|--|
| | (add additional | examples from class) | | | |
| set | $7 \in \{43, 7, 9\}$ | $2 \notin \{43, 7, 9\}$ | | | |
| unordered collection of elements | | | | | |
| repetition doesn't matter | | | | | |
| Equal sets agree on membership of all elements | | | | | |
| n-tuple | | | | | |
| ordered sequence of elements with n "slots" $(n > 0)$ | | | | | |
| repetition matters, fixed length | | | | | |
| Equal n-tuples have corresponding components equal | | | | | |

string

ordered finite sequence of elements each from specified set repetition matters, arbitrary finite length $Equal\ strings\ have\ same\ length\ and\ corresponding\ characters\ equal$

$Special\ cases:$

When n = 2, the 2-tuple is called an **ordered pair**.

A string of length 0 is called the **empty string** and is denoted λ .

A set with no elements is called the **empty set** and is denoted $\{\}$ or \emptyset .

Defining sets

To define sets:

To define a set using **roster method**, explicitly list its elements. That is, start with { then list elements of the set separated by commas and close with }.

To define a set using **set builder definition**, either form "The set of all x from the universe U such that x is ..." by writing

$$\{x \in U \mid ...x...\}$$

or form "the collection of all outputs of some operation when the input ranges over the universe U" by writing

$$\{...x... \mid x \in U\}$$

We use the symbol \in as "is an element of" to indicate membership in a set.

Example sets: For each of the following, identify whether it's defined using the roster method or set builder notation and give an example element.

Defining functions ratings

Recall our representation of Netflix users' ratings of movies as n-tuples, where n is the number of movies in the database. Each component of the n-tuple is -1 (didn't like the movie), 0 (neutral rating or didn't watch the movie), or 1 (liked the movie).

Consider the ratings $P_1 = (-1, 0, 1), P_2 = (1, 1, -1), P_3 = (1, 1, 1), P_4 = (0, -1, 1)$

Which of P_1 , P_2 , P_3 has movie preferences most similar to P_4 ?

One approach to answer this question: use **functions** to define distance between user preferences.

For example, consider the function d_0 : given by

 $d_0(((x_1, x_2, x_3), (y_1, y_2, y_3))) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2 + (x_3 - y_3)^2}$

Extra example: A new movie is released, and P_1 and P_2 watch it before P_3 , and give it ratings; P_1 gives \checkmark and P_2 gives \checkmark . Should this movie be recommended to P_3 ? Why or why not?

Extra example: Define a new function that could be used to compare the 4-tuples of ratings encoding movie preferences now that there are four movies in the database.