

## Defining sets

*To define sets:*

To define a set using **roster method**, explicitly list its elements. That is, start with  $\{$  then list elements of the set separated by commas and close with  $\}$ .

To define a set using **set builder definition**, either form “The set of all  $x$  from the universe  $U$  such that  $x$  is ...” by writing

$$\{x \in U \mid \dots x \dots\}$$

or form “the collection of all outputs of some operation when the input ranges over the universe  $U$ ” by writing

$$\{\dots x \dots \mid x \in U\}$$

We use the symbol  $\in$  as “is an element of” to indicate membership in a set.

**Example sets:** For each of the following, identify whether it’s defined using the roster method or set builder notation and give an example element.

$$\{-1, 1\}$$

$$\{0, 0\}$$

$$\{-1, 0, 1\}$$

$$\{(x, x, x) \mid x \in \{-1, 0, 1\}\}$$

$$\{\}$$

$$\{x \in \mathbb{Z} \mid x \geq 0\}$$

$$\{x \in \mathbb{Z} \mid x > 0\}$$

$$\{\text{A, C, U, G}\}$$

$$\{\text{AUG, UAG, UGA, UAA}\}$$

# Rna motivation

RNA is made up of strands of four different bases that encode genomic information in specific ways. The bases are elements of the set  $B = \{\mathbf{A}, \mathbf{C}, \mathbf{U}, \mathbf{G}\}$ .

Formally, to define the set of all RNA strands, we need more than roster method or set builder descriptions.

## Set recursive definition

**New! Recursive Definitions of Sets:** The set  $S$  (pick a name) is defined by:

- Basis Step: Specify finitely many elements of  $S$
- Recursive Step: Give rule(s) for creating a new element of  $S$  from known values existing in  $S$ , and potentially other values.

The set  $S$  then consists of all and only elements that are put in  $S$  by finitely many (a nonnegative integer number) of applications of the recursive step after the basis step.

# Set recursive examples

**Definition** The set of nonnegative integers  $\mathbb{N}$  is defined (recursively) by:

Basis Step:

Recursive Step:

Examples:

**Definition** The set of all integers  $\mathbb{Z}$  is defined (recursively) by:

Basis Step:

Recursive Step:

Examples:

**Definition** The set of RNA strands  $S$  is defined (recursively) by:

Basis Step:  $\mathbf{A} \in S, \mathbf{C} \in S, \mathbf{U} \in S, \mathbf{G} \in S$

Recursive Step: If  $s \in S$  and  $b \in B$ , then  $sb \in S$

where  $sb$  is string concatenation.

Examples:

**Definition** The set of bitstrings (strings of 0s and 1s) is defined (recursively) by:

Basis Step:

Recursive Step:

*Notation:* We call the set of bitstrings  $\{0, 1\}^*$ .

Examples: