Recall the definition of linked lists from class. Consider this (incomplete) definition: **Definition** The function $\mathit{increment} : \_\_\_\_\_\_\_\_\_\_$ that adds 1 to the data in each node of a linked list is defined by:

$$\begin{array}{lll} & \mathit{increment} : \_\_\_\_\_ & \to \_\_\_\_\_ \\ \text{Basis Step:} & \mathit{increment}([]) & = [] \\ \text{Recursive Step: If } l \in L, n \in \mathbb{N} & \mathit{increment}((n, l)) & = (1 + n, \mathit{increment}(l)) \end{array}$$

Consider this (incomplete) definition: **Definition** The function $\mathit{sum} : L \to \mathbb{N}$ that adds together all the data in nodes of the list is defined by:

$$\begin{array}{lll} & \mathit{sum} : L & \to \mathbb{N} \\ \text{Basis Step:} & \mathit{sum}([]) & = 0 \\ \text{Recursive Step: If } l \in L, n \in \mathbb{N} & \mathit{sum}((n, l)) & = \_\_\_\_\_\_\_\_\_\_ \end{array}$$

You will compute a sample function application and then fill in the blanks for the domain and codomain of each of these functions.

1. Based on the definition, what is the result of $\mathit{increment}((4, (2, (7, []))))$? Write your answer directly with no spaces.

2. Which of the following describes the domain and codomain of $\mathit{increment}$?

   (a) The domain is L and the codomain is $\mathbb{N}$

   (b) The domain is L and the codomain is $\mathbb{N} \times L$

   (c) The domain is $L \times \mathbb{N}$ and the codomain is L

   (d) The domain is $L \times \mathbb{N}$ and the codomain is $\mathbb{N}$

   (e) The domain is L and the codomain is L

   (f) None of the above

3. Assuming we would like $\mathit{sum}((5, (6, [])))$ to evaluate to 11 and $\mathit{sum}((3, (1, (8, []))))$ to evaluate to 12, which of the following could be used to fill in the definition of the recursive case of sum?

   (a) $\begin{cases} 1 + \mathit{sum}(l) & \text{when } n \neq 0 \\ \mathit{sum}(l) & \text{when } n = 0 \end{cases}$

   (b) $1 + \mathit{sum}(l)$

   (c) $n + \mathit{increment}(l)$

   (d) $n + \mathit{sum}(l)$

   (e) None of the above

4. Choose only and all of the following statements that are **well-defined**; that is, they correctly reflect the domains and codomains of the functions and quantifiers, and respect the notational conventions we use in this class. Note that a well-defined statement may be true or false.

   (a) $\forall l \in L\,(sum(l))$

   (b) $\exists l \in L\,(sum(l) \wedge length(l))$

   (c) $\forall l \in L\,(sum(increment(l)) = 10)$

   (d) $\exists l \in L\,(sum(increment(l)) = 10)$

   (e) $\forall l \in L\,\forall n \in \mathbb{N}\,((n \times l) \subseteq L)$

   (f) $\forall l_1 \in L\,\exists l_2 \in L\,(increment(sum(l_1)) = l_2)$

   (g) $\forall l \in L\,(length(increment(l)) = length(l))$

5. Choose only and all of the statements in the previous part that are both well-defined and true.