Recall the definition of linked lists from class. Consider this (incomplete) definition: **Definition** The function *increment* : _____ that adds 1 to the data in each node of a linked list is defined by:

		$increment : _____$	$\to _____$
Basis Step:		$increment([])$	$= []$
Recursive Step:	If $l \in L, n \in \mathbb{N}$	$increment((n, l))$	$= (1 + n, increment(l))$

Consider this (incomplete) definition: **Definition** The function $sum : L \to \mathbb{N}$ that adds together all the data in nodes of the list is defined by:

		$sum : L$	$\to \mathbb{N}$
Basis Step:		$sum([])$	$= 0$
Recursive Step:	If $l \in L, n \in \mathbb{N}$	$sum((n, l))$	$= _____$

You will compute a sample function application and then fill in the blanks for the domain and codomain of each of these functions.

1. Based on the definition, what is the result of $increment((4, (2, (7, []))))$? Write your answer directly with no spaces.

2. Which of the following describes the domain and codomain of *increment*?

   (a) The domain is L and the codomain is $\mathbb{N}$
   (b) The domain is L and the codomain is $\mathbb{N} \times L$
   (c) The domain is $L \times \mathbb{N}$ and the codomain is L
   (d) The domain is $L \times \mathbb{N}$ and the codomain is $\mathbb{N}$
   (e) The domain is L and the codomain is L
   (f) None of the above

3. Assuming we would like $sum((5, (6, [])))$ to evaluate to 11 and $sum((3, (1, (8, []))))$ to evaluate to 12, which of the following could be used to fill in the definition of the recursive case of *sum*?

   (a) $\begin{cases} 1 + sum(l) & \text{when } n \neq 0 \\ sum(l) & \text{when } n = 0 \end{cases}$
   (b) $1 + sum(l)$
   (c) $n + increment(l)$
   (d) $n + sum(l)$
   (e) None of the above

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4. Choose only and all of the following statements that are **well-defined**; that is, they correctly reflect the domains and codomains of the functions and quantifiers, and respect the notational conventions we use in this class. Note that a well-defined statement may be true or false.

   (a) $\forall l \in L\,(sum(l))$

   (b) $\exists l \in L\,(sum(l) \wedge length(l))$

   (c) $\forall l \in L\,(sum(increment(l)) = 10)$

   (d) $\exists l \in L\,(sum(increment(l)) = 10)$

   (e) $\forall l \in L\,\forall n \in \mathbb{N}\,((n \times l) \subseteq L)$

   (f) $\forall l_1 \in L\,\exists l_2 \in L\,(increment(sum(l_1)) = l_2)$

   (g) $\forall l \in L\,(length(increment(l)) = length(l))$

5. Choose only and all of the statements in the previous part that are both well-defined and true.

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