CSE 20 Fall 2021
Practice questions for Final Exam

Many additional practice problems can be found in the class notes and in the supplementary textbook for the class. We also recommend that you review your assignments and their solutions from throughout the quarter, along with the grading feedback you received.

1. Algorithms and optimization Our task is to **schedule** jobs on multiple processors. We have access to as many processors as we'd like, and the goal is to **minimize** the number of processors we actually use. The processors are labelled $P_1, P_2, \ldots$. Each job must begin running exactly at its scheduled start time and nothing else can run on the processor running this job until that job's duration has elapsed. A greedy algorithm for this task is described as follows:

> **Input**: Number of jobs n and list of descriptions of jobs $j_1, \ldots, j_n$: for job j_i we have its start time b_i and its duration d_i.
>
> 1. Order the jobs based on start times.
> 2. For each job (starting with the earliest), assign it to the lowest numbered processor that is currently available.

(a) How many processors does this algorithm use when its input is $n = 6$ and

Job	Start time	Duration
j_1	0	3
j_2	0	2
j_3	2	2
j_4	3	4
j_5	3	1
j_6	4	2

(b) Come up with two different sequences of jobs (by specifying their start times and durations) such that at least 5 processors are required. Choose your examples so that they have a different number of total jobs from each other.

2. Number systems and integer operations

(a) Convert $(4217)_8$ to (i) binary, (ii) hexadecimal, and (iii) decimal.

(b) Find a positive integer $b > 1$ which makes the following equation true:

$$(403)_b + (214)_b = (1021)_b$$

Are there other valid solutions for b?

(c) Draw a logic circuit with input signals b_0, b_1, b_2, b_3 which produces output signal s_0 which is 1 if $(b_3b_2b_1b_0)_2$ represents a multiple of 3 (in fixed-width width 4 binary) and is 0 otherwise.

(d) What is the smallest w such that -7 can be written in 2s complement width w?

(e) What is the smallest w such that -7 can be written in sign-magnitude width w?

(f) What is the smallest w such that -8 can be written in 2s complement width w?

(g) What is the smallest w such that -8 can be written in sign-magnitude width w?

3. Propositional logic For each of the following formulas, decide whether it is a **tautology**, **contradiction**, or **contingency**. Then, find a logically equivalent formula in DNF and in CNF.

(a) $\neg p \vee (p \to q)$

(b) $(p \oplus q) \oplus r$

(c) $(p \wedge q) \vee (\neg p \wedge q) \vee \neg q$

4. Predicates and Quantifiers Express each of the following statements symbolically using quantifiers, variables, propositional connectives, and predicates (make sure to define the domain and the meaning of any predicates you introduce). Then, express its negation as a logically equivalent compound proposition without a $\neg$ in front. Decide whether the statement or its negation is true, and prove it.

(a) If m is an integer, $m^2 + m + 1$ is odd.

(b) For all integers n greater than 5, $2^n - 1$ is not prime.

(c) If n and m are even integers, then $n - m$ is even.

5. Proof strategies Prove each of the following claims. Use only basic definitions and general proof strategies in your proofs; do not use any results proved in class / the textbook as lemmas. You may use basic properties of real numbers that we stated (without proof) in class, e.g. that the difference of two integers is an integer.

(a) The difference of any rational number and any irrational number is irrational.

(b) If a, b, c are integers and $a^2 + b^2 = c^2$, then at least one of a and b is even.

(c) For any integer n, $n^2 + 5$ is not divisible by 4.

(d) For all positive integers a, b, c, if $a \nmid bc$ then $a \nmid b$. (The notation $a \nmid b$ means "a does not divide b").

6. Sets Prove or disprove each of the following statements.

(a) For all sets A and B, $\mathcal{P}(A) \cup \mathcal{P}(B) = \mathcal{P}(A \cup B)$.

(b) For all sets A and B, $\mathcal{P}(A) \cap \mathcal{P}(B) = \mathcal{P}(A \cap B)$.

(c) For any sets A, B, C, D, if the Cartesian products $A \times B$ and $C \times D$ are disjoint then either A and C are disjoint or B and D are disjoint (or both).

(d) There are sets A, B such that $A \in B$ and $A \subseteq B$.

(e) For all sets A, B, C: $A \cap B = \emptyset$ and $B \cap C = \emptyset$ if and only if $(A \cap B) \cap C = \emptyset$.

7. Induction and Recursion Prove the following statements:

(a) $\forall s \in S\ (\ \mathit{rnalen}(s) = \mathit{basecount}(s, \mathtt{A}) + \mathit{basecount}(s, \mathtt{C}) + \mathit{basecount}(s, \mathtt{U}) + \mathit{basecount}(s, \mathtt{G})\ )$ where S RNA strands and the functions *rnalen* and *basecount* are define recursively as:

		$\mathit{rnalen} : S$	$\to \mathbb{Z}^+$
Basis Step:	If $b \in B$ then	$\mathit{rnalen}(b)$	$= 1$
Recursive Step:	If $s \in S$ and $b \in B$, then	$\mathit{rnalen}(sb)$	$= 1 + \mathit{rnalen}(s)$

		$\mathit{basecount} : S \times B$	$\to \mathbb{N}$
Basis Step:	If $b_1 \in B, b_2 \in B$	$\mathit{basecount}(b_1, b_2)$	$= \begin{cases} 1 & \text{when } b_1 = b_2 \\ 0 & \text{when } b_1 \neq b_2 \end{cases}$
Recursive Step:	If $s \in S, b_1 \in B, b_2 \in B$	$\mathit{basecount}(sb_1, b_2)$	$= \begin{cases} 1 + \mathit{basecount}(s, b_2) & \text{when } b_1 = b_2 \\ \mathit{basecount}(s, b_2) & \text{when } b_1 \neq b_2 \end{cases}$

(b) $\forall l_1 \in L\, \forall l_2 \in L\, (sum(concat(l_1, l_2)) = sum(l_1) + sum(l_2))$, where the function $concat$ is defined as:

		$concat : L \times L$	$\to L$
Basis Step:	If $l \in L$	$concat([], l)$	$= l$
Recursive Step:	If $l, l' \in L$ and $n \in \mathbb{N}$, then	$concat((n, l), l')$	$= (n, concat(l, l'))$

and the function $sum : L \to \mathbb{N}$ that sums all the elements of a list and is defined by:

		$sum : L$	$\to \mathbb{N}$
Basis Step:		$sum([])$	$= 0$
Recursive Step:	If $l \in L, n \in \mathbb{N}$	$sum((n, l))$	$= n + sum(l)$

8. Induction and Recursion Define a set of bit strings, S, recursively by *BASIS STEP*: $0 \in S$; *RECURSIVE STEP*: if $x \in S$, then $xx \in S$. In this definition, xx is the concatenation of x with itself.

Prove using structural induction that, for every $x \in S$, $|x|$ is a power of 2, where $|x|$ is the number of 0's in x, or its length. You may assume in your proof that $\forall x \in S\, (|x| + |x| = |xx|)$.

9. Induction and Recursion Prove that every positive integer has a base 3 expansion. *Hint: use strong induction.*

10. Functions & Cardinalities of sets Prove each of the following claims.

(a) For the "function" $f : \mathbb{Z} \to \{0, 1, 2, 3\}$ given by $f(x) = x \textbf{ mod } 5$, it is not the case that every element of the domain maps to exactly one element of the codomain (that is, it is not a well-defined function).

(b) The "function" $f : \mathcal{P}(\mathbb{Z}^+) \to \mathbb{Z}$ given by

$$f(A) = \text{the maximum element in A}$$

is not well-defined.

(c) There is a one-to-one function with domain $\{a, b, c\}$ and codomain $\mathbb{R}$.

(d) There is an onto function with domain R and codomain $\{\pi, \frac{1}{17}\}$, where R is the set of user ratings in a database with 5 movies.

(e) The Cartesian product $\mathbb{Z}^+ \times \{a, b, c\}$ is countable.

(f) The interval of real numbers $\{x \in \mathbb{R} \mid 5 \leq x \leq 8\}$ is uncountable. *Hint: you may use the fact that the unit interval $\{x \in \mathbb{R} \mid 0 \leq x \leq 1\}$ is uncountable.*

11. Binary relations (a) Recall the definition of the binary relation $R_{(\textbf{mod } n)}$ on the set $\mathbb{Z}$: for each $a, b \in \mathbb{Z}|$, we define $(a, b) \in R_{(\textbf{mod } n)}$ to mean $a \textbf{ mod } n = b \textbf{ mod } n$. Give a counterexample to the claim

$$\forall a \in \mathbb{Z}\, \forall n \in \mathbb{Z}^+ \left(((a \textbf{ div } 2) \textbf{ mod } n, ((a \textbf{ mod } n) \textbf{ div } 2) \textbf{ mod } n) \in R_{(\textbf{mod } n)} \right)$$

(b) Let $M_{=2}$ be a binary relation on the set of RNA strands S where a pair of strands are in $M_{=2}$ if they are both length two or greater and start with the same TWO bases, OR if both are length one and have the same first base. *Bonus: Prove that $M_{=2}$ is an equivalence relation on S.*

(a) Give an example of an equivalence class for $M_{=2}$ that is finite-sized, or argue why one cannot exist.

(b) Give an example of an equivalence class for $M_{=2}$ that is countably infinite, or argue why one cannot exist.

(c) Give an example of an equivalence class for $M_{=2}$ that is uncountably infinite, or argue why one cannot exist.