Defining sets

To define sets:

To define a set using **roster method**, explicitly list its elements. That is, start with $\{$ then list elements of the set separated by commas and close with $\}$.

To define a set using **set builder definition**, either form "The set of all x from the universe U such that x is ..." by writing

$$\{x \in U \mid ...x...\}$$

or form "the collection of all outputs of some operation when the input ranges over the universe U" by writing

$$\{...x... \mid x \in U\}$$

We use the symbol $\in$ as "is an element of" to indicate membership in a set.

Example sets: For each of the following, identify whether it's defined using the roster method or set builder notation and give an example element.

$\{-1, 1\}$

$\{0, 0\}$

$\{-1, 0, 1\}$

$\{(x, x, x) \mid x \in \{-1, 0, 1\}\}$

$\{\}$

$\{x \in \mathbb{Z} \mid x \geq 0\}$

$\{x \in \mathbb{Z} \mid x > 0\}$

$\{\mathtt{A}, \mathtt{C}, \mathtt{U}, \mathtt{G}\}$

$\{\mathtt{AUG}, \mathtt{UAG}, \mathtt{UGA}, \mathtt{UAA}\}$

Rna motivation

RNA is made up of strands of four different bases that encode genomic information in specific ways. The bases are elements of the set $B = \{\texttt{A}, \texttt{C}, \texttt{U}, \texttt{G}\}$.

Formally, to define the set of all RNA strands, we need more than roster method or set builder descriptions.

Set recursive definition

New! Recursive Definitions of Sets: The set S (pick a name) is defined by:

Basis Step:	Specify finitely many elements of S
Recursive Step:	Give rule(s) for creating a new element of S from known values existing in S, and potentially other values.

The set S then consists of all and only elements that are put in S by finitely many (a nonnegative integer number) of applications of the recursive step after the basis step.

Set recursive examples

Definition The set of nonnegative integers $\mathbb{N}$ is defined (recursively) by:

Basis Step:
Recursive Step:

Examples:

Definition The set of all integers $\mathbb{Z}$ is defined (recursively) by:

Basis Step:
Recursive Step:

Examples:

Definition The set of RNA strands S is defined (recursively) by:

Basis Step:	$\mathtt{A} \in S, \mathtt{C} \in S, \mathtt{U} \in S, \mathtt{G} \in S$
Recursive Step:	If $s \in S$ and $b \in B$, then $sb \in S$

where sb is string concatenation.

Examples:

Definition The set of bitstrings (strings of 0s and 1s) is defined (recursively) by:

Basis Step:
Recursive Step:

Notation: We call the set of bitstrings $\{0,1\}^*$.

Examples: