Name - Akanksha Dubey   Univ Roll no. - 2016600
Section - C
Class Roll No. 42

Tutorial - 6

1. Minimum Spanning Tree
It is a subset of edges of a connected edge-weighted undirected graph that connects all the vertices together without any cycles & with minimum possible edge weighted.

Applications
1) consider n stations are to be linked using a communication network and lying of communication link between any two stations involves a cost. The ideal solution would be to extract a subgraph termed as minimum cost spanning tree.
2) Designing LAN.
3) Suppose you want to construct highways or railways spanning several cities, then we can use concept of MST.

2. (i) Prim's Algorithm

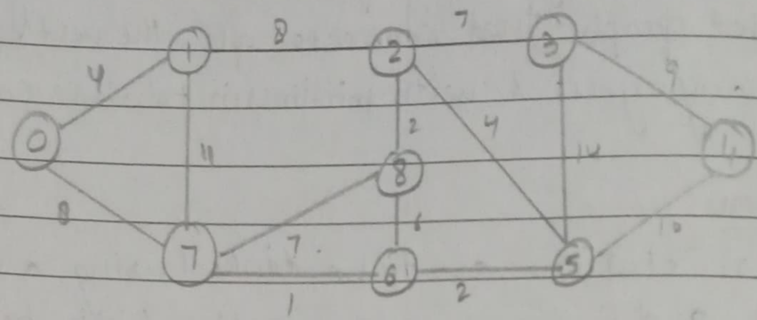
Time complexity :- $O(|E|\log|V|)$
Space complexity :- $O(|V|)$

(ii) Krushkal Algorithm
Time complexity : $O(|E|\log|E|)$
Space complexity : $O(|V|)$

(iii) Dijkstra's Algorithm
Time complexity :- $O(V^2)$
Space complexity :- $O(V^2)$

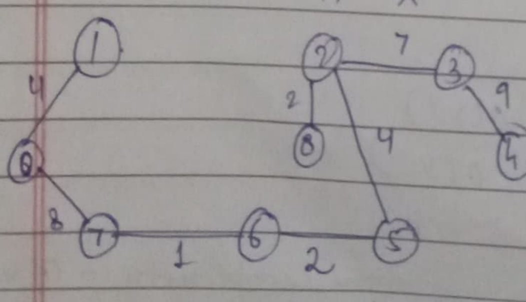
(iv) Bellman Ford's Algorithm   Time complexity :- $O(VE)$
Space complexity :- $O(E)$

3. Apply Krushkal & Prim's Algo on given graph. to compute MST and its weight



Krushkal's Algorithm

0	V	W	
6	7	1	✓
5	6	2	✓
2	8	2	✓
0	1	4	✓
2	5	4	✓
6	8	6	✗
2	3	7	✓
7	8	7	✗
0	7	8	✓
1	2	8	✗
4	3	9	✓
4	5	10	✗
1	7	11	✗
3	5	14	✗



weight = 1+2+2+4+4+7+8+9
MST = 37

Prim's Algo

0	1	2	3	4	5	6	7	8
-1	-1	-1	-1	-1	-1	-1	-1	-1

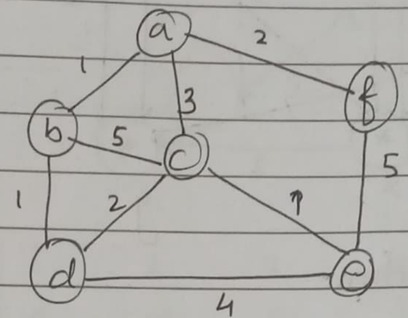
Parent Array

Weight = 4+8+2+4+2+7+9+3
= 37

4. Given a directed grap. You are also given the shortest path from a source vertex 's' to a destination vertex 't'. Does the shortest path remain same in following cases.

(i) If weighted of every edge is increased by 10 units.

(ii) If weight of every edge is multiplied by 10 units.



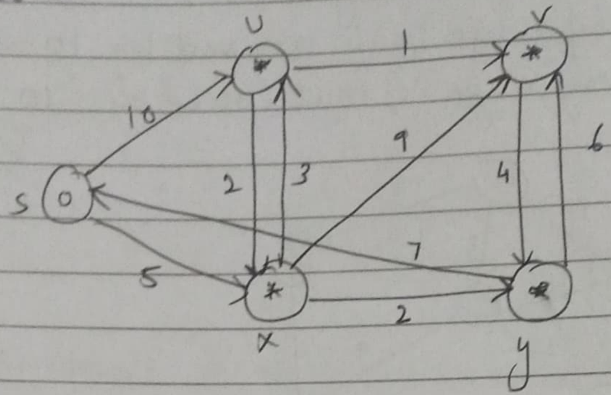
(i) The shortest path may change. The reason is that there may be different no. of edges in different paths from 's' to 't'.

For eg: Let the shortest path of weight is and has edge's. Let there another path with 2 edges and total weigh 2s.

The weight of shortest path is increased by s^10 and becomes 1s + 50. weight of other path is increased by 2^10 & becomes 2s + 20, so, the shortest path changes to other path with weight as 4s.

(ii) If we multiply all edges weight by 10, the shortest path doesn't change. The reason is that weight of all path from 's' to 't' gets multiplied by same unit. the no. of edges or path doesn't matter.

5. Apply Dijkstra & Bellman ford algorithm on graph given ~~right~~ to compute shortest path to all nodes from node s.



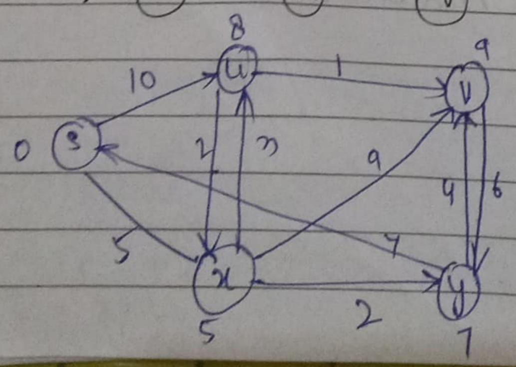
Dijkstra's Algorithm

Node	Shortest Distance from source Node
U	8
X	5
V	9
y	7

Bellman ford Algorithm

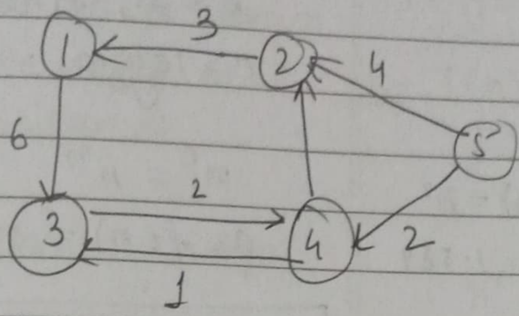
	S	U	V	X	Y
1st →	0	~~∞~~ 10	∞	~~∞~~ 5	∞
2nd →	0	10	~~∞~~ 11	5	∞
3rd →	0	~~10~~ 8	~~11~~ 9	5	~~∞~~ 7
4th →	0	8	9	5	7

The graph doesn't have -ve cycle



→ final graph.

Q. Apply all pair shortest path algorithm (Floyd warshall) on graph. Also analyse space & time complexity of it.



$$D_0 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & \infty \\ 2 & 2 & 0 & \infty & \infty & \infty \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & \infty & 1 & 1 & 0 & \infty \\ 5 & \infty & 4 & \infty & 2 & 0 \end{array}$$

$$D_1 = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & \infty \\ 2 & 2 & 0 & 8 & 5 & \infty \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & \infty & 1 & 1 & 0 & \infty \\ 5 & \infty & 4 & \infty & 2 & 0 \end{array}$$

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & \infty \\ 2 & 2 & 0 & 8 & 5 & \infty \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & 3 & 1 & 1 & 0 & \infty \\ 5 & 6 & 4 & 12 & 2 & 0 \end{array}$$

$$\begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 0 & \infty & 6 & 3 & \infty \\ 2 & 2 & 0 & 8 & 5 & \infty \\ 3 & \infty & \infty & 0 & 2 & \infty \\ 4 & 3 & 1 & 1 & 0 & \infty \\ 5 & 6 & 4 & 12 & 2 & 0 \end{array} \quad \text{ans}$$

Time complexity $\rightarrow O(|V|^3)$

Space complexity $\rightarrow O(|V|^2)$