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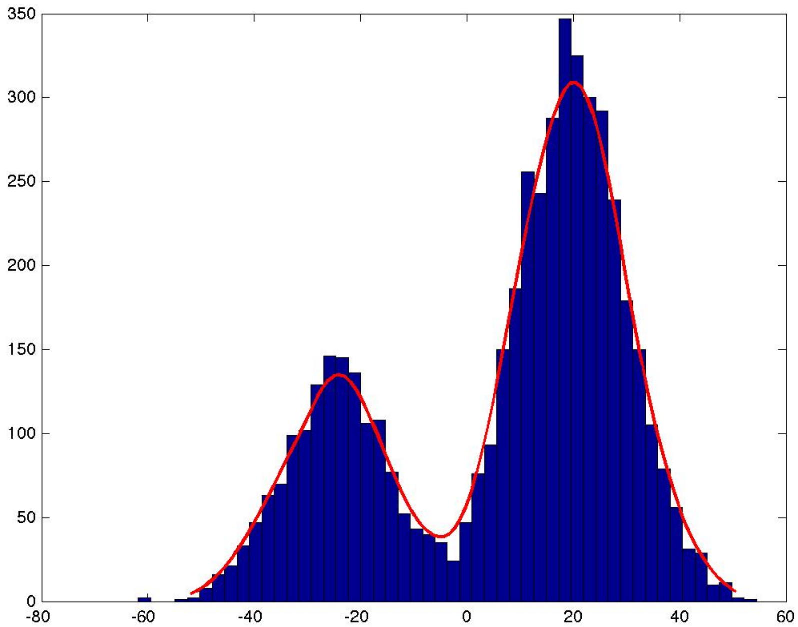
Assignment 3
Feb 21, 2015

Metropolis-Hastings The distribution estimate for estimate of $p(x)$ is shown below.



where : $p(x) = 0.3 * N(\mu_1, \tau) + 0.7 * N(\mu_2, \tau)$

The posterior probability is given by:

$P(\mu_1; \mu_2|x) = \frac{1}{Z} P(\mu_1; \mu_2) * P(\mu_1; \mu_2)$

$= \frac{1}{Z} \prod_{i=1}^{N} [0.3 * N(x_i|\mu_1, 1) + 0.7 * N(x_i|\mu_2, 1)] * N(\mu_1|0, \tau) N(\mu_2|0, \tau)$

The normalization term doesn't really matter for the Metropolis-Hastings algorithm, since we only consider the ratio of probabilities for the acceptance probability of a sample:

$A(x; x') = min(1; \frac{p(x')q(x';x)}{p(x)q(x;x')})$

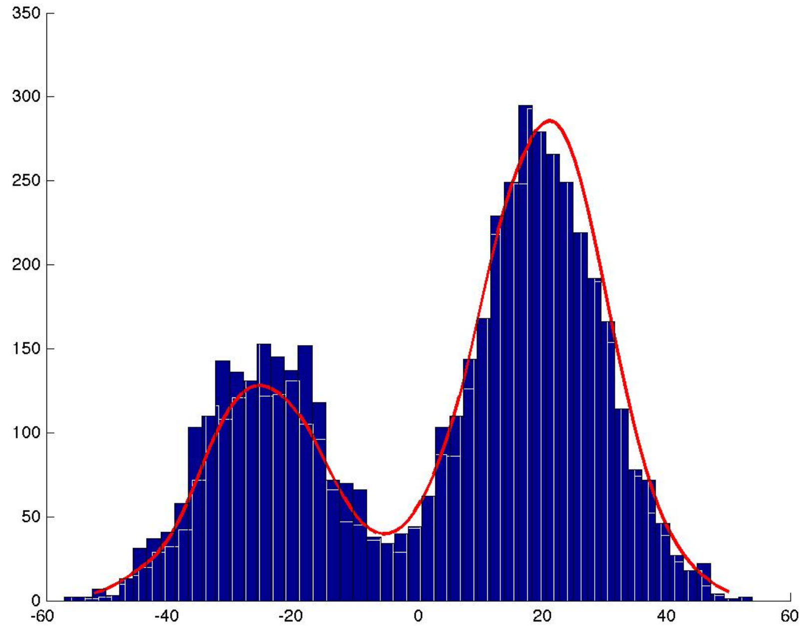
We'll choose as the proposal distribution

$q(\mu^{(t)} \sim N(\mu^{(t-1)}, 10I_2)$

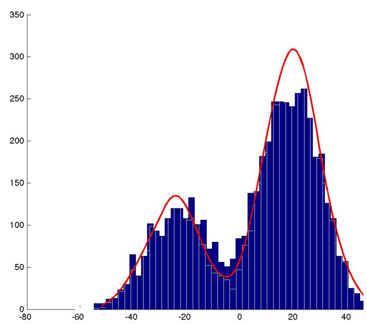
As this distribution is symmetrical and thus the acceptance probability can be reduced to

$A(x; x') = min(1; \frac{p(x')}{p(x)})$

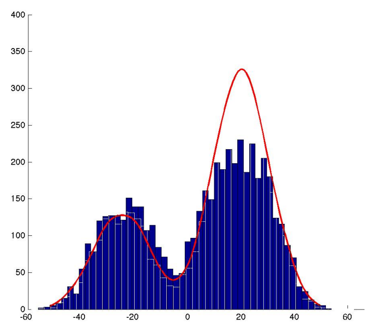
Best Sampling Output : Generated with SD = 10



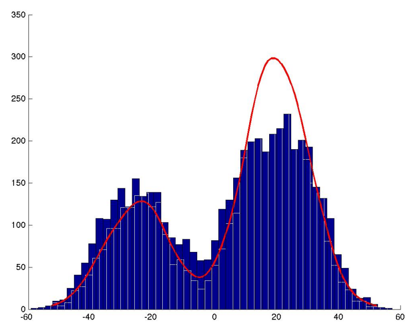
Only with a huge change in the variance when compared with the actual variance we see a drastic change in the sampling accuracy. as can be seen from the images given below. Increasing the SD tends to normalize the data more.



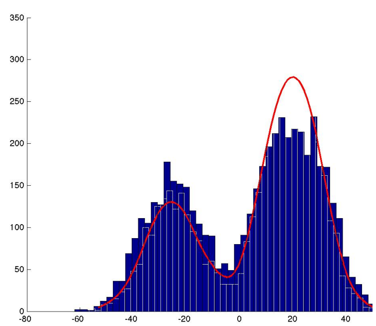
(a) Sd = 20



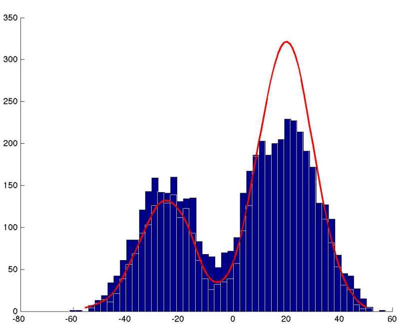
(b) Sd = 50



(c) Sd = 100



(d) Sd = 500



(e) Sd = 1000

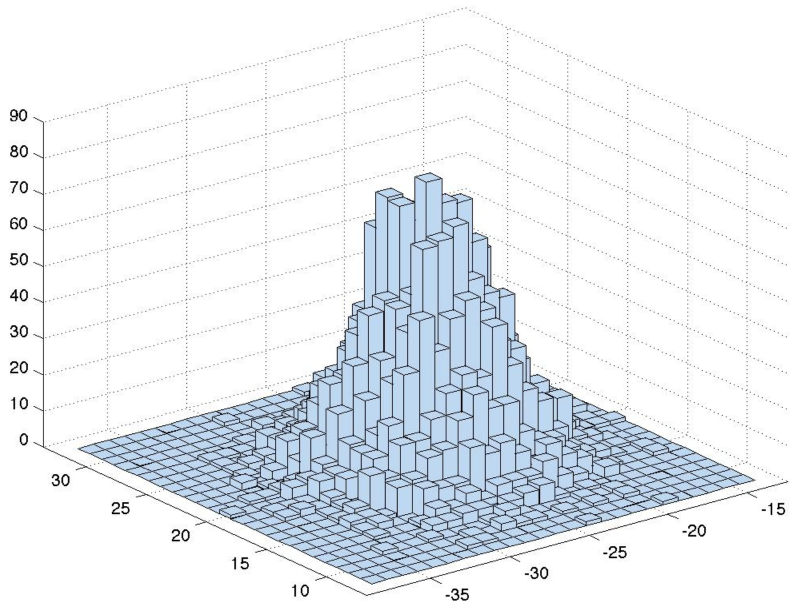
Figure 1: Pictures indicating effect of SD on sampling output

2-Dimension Using the similar tactics as used for the 1-Dimension case, we generate the data for the 2-Dimension Normal data using the matlab function *mvnrnd* and then make use of the function *mvnpdf* to get an estimate of the probability density.

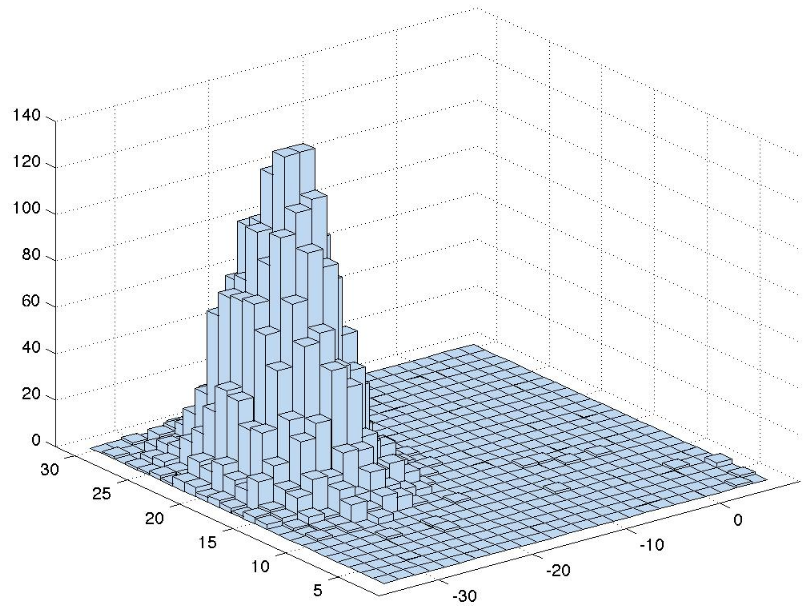
Original graph for the expression :

$p(x) = N_x(\mu_1, \tau) + N_x(\mu_2, \tau)$

$p(y) = N_y(\mu_1, \tau) + N_y(\mu_2, \tau)$



Graph Indicating Sampling Output :



140
120
100
80
60
40
20
0
30
25
20
15
10
5
-30
-20
-10
0