Homework 1

- Akankshi Mody (MSBA Class of 2020)

Problem 1

Problem 1 :

$$(A^{-1})^T = (A^T)^{-1}$$

We know, if B is inverse of A^T,

$$A^T B = B A^T = I.$$

lets consider $(A^{-1})^T$ as B.

$$A^T (A^{-1})^T = (A^{-1} \cdot A)^T = I^T = I \quad [\because A^T B^T = (BA)^T]$$

$$(A^{-1})^T A^T = (A A^{-1})^T = I^T = I.$$

$$\therefore A^T (A^{-1})^T = (A^{-1})^T A^T = I.$$

Thus $(A^{-1})^T$ is inverse of A^T

$$\therefore (A^{-1})^T = (A^T)^{-1}$$

Hence proved.

Problem 2

Problem 2 :

let ~~first~~ first & second mortgage, home improvement, personal overdraft be denoted by f, s, h and p respectively.

Given:

$f + s + h + p = 250{,}000{,}000$ — (1)

$f = 0.55(f + s)$

$0.45f - 0.55s = 0$ — (2)

$s = 0.25(250{,}000{,}000)$

$s = 62{,}500{,}000$ — (3)

$14f + 20s + 20h + 10p = 3{,}750{,}000{,}000$ — (4)

In matrix form:

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0.45 & -0.55 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 14 & 20 & 20 & 10 \end{bmatrix} \begin{bmatrix} f \\ s \\ h \\ p \end{bmatrix} = \begin{bmatrix} 250{,}00{,}000 \\ 0 \\ 62{,}500{,}000 \\ 3{,}750{,}000{,}000 \end{bmatrix}$$

$Ax = b$

$\therefore x = A^{-1}b$.

$$x = \begin{bmatrix} 76{,}388{,}889 \\ 62{,}500{,}000 \\ 31{,}944{,}444 \\ 79{,}166{,}667 \end{bmatrix}$$

$\therefore$ f = \$ 76,388,889 h = \$ 31,944,444

s = \$62,500,000 p = \$ 79,166,667.

Problem 3

Problem 3

Find $v1, v2, v3, v4$

To max $1.5(v1) + 2.5(v2) + 3(v3) + 4.5(v4)$

Subject to :

$$2(v1) + 4(v2) + 3(v3) + 7(v4) \leq 100000$$
$$3(v1) + 2(v2) + 3(v3) + 4(v4) \leq 50000$$
$$2(v1) + 3(v2) + 2(v3) + 5(v4) \leq 60000$$
$$v1, v2, v3, v4 \geq 0$$

Problem 4

Part A

```
#Creating a nxn matrix of zeroes
n<-20
A <- matrix(0, nrow = n, ncol = n)
#A

for (i in 1:n){
  for (j in 1:n){
    if(i<=j) A[i,j] = i/j
    else A[i,j] = j/i
  }
}
#A
```

Part B

```
isSymmetric(A)

## [1] TRUE
```

Part C

```
#Checking if determinant !=0
cat("Is Determinant!=0 ?",det(A)!=0,"\n")
```

```
## Is Determinant!=0 ? TRUE
```

```
#C is inverse of A
C <- solve(A)
#Creating an nxn identity matrix
I <- diag(n)
#Checking if the inverse is correct
cat("Is Inverse correct?", all.equal(C%*%A,I))
```

```
## Is Inverse correct? TRUE
```

Part D

```
d <- c((1:10),(10:1))
d
```

```
##  [1]  1  2  3  4  5  6  7  8  9 10 10  9  8  7  6  5  4  3  2  1
```

Part E

```
#Ax = Cd
solve(A,C%*%d)
```

```
##                [,1]
##  [1,] -3.404684e-15
##  [2,]  9.118632e-15
##  [3,]  1.652012e-14
##  [4,] -4.618528e-14
##  [5,]  2.220446e-14
##  [6,] -2.295600e-14
##  [7,]  7.320412e-14
##  [8,] -5.305386e-14
##  [9,] -2.481203e+01
## [10,]  2.006424e+01
## [11,]  3.581375e+01
## [12,] -3.006263e+01
## [13,] -3.736996e-04
## [14,] -2.772044e-04
## [15,] -2.099688e-04
## [16,] -1.619541e-04
## [17,] -1.269228e-04
## [18,] -1.008779e-04
## [19,]  9.505933e+01
## [20,] -1.000629e+02
```