

Homework 1

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Non-graded Policy

We will grade this assignment as Credit/No-Credit: if you complete the homework to an acceptable level, you will get credit. If not, you will get no credit. Getting credit is required to obtain a grade for the group project that follows.

Problem 1

Use definitions to prove

$$(A^{-1})^T = (A^T)^{-1},$$

where A is an invertible square matrix and A^T means the transpose of matrix A .

Problem 2

A bank makes four kinds of loans to its customers, and these loans yield the following annual interest rates to the bank:

- First mortgage 14%
- Second mortgage 20%
- Home improvement 20%
- Personal overdraft 10%

We are interested in the bank's lending strategy. The information we know is as following:

1. In total \$250 million is lent out.
2. First mortgages are 55% of all mortgages (i.e., first and second mortgage) issued.
3. Second mortgages are 25% of all loans issued.
4. The average interest rate on all loans is 15%.

Calculate the lending strategy using matrix inversion.

Problem 3

A company manufactures four variants of the same product, and in the final part of the manufacturing process, there are assembly, polishing and packing operations. For each variant, the time required for these operations is shown below (in minutes) as is the profit per unit sold.

	Assembly	Polish	Pack	Profit(\$)
Variant 1	2	3	2	1.50
2	4	2	3	2.50
3	3	3	2	3.00
4	7	4	5	4.50

Given the current state of the labor force the company estimate that they have 100000 minutes of assembly time, 50000 minutes of polishing time and 60000 minutes of packing time available.

Add the non-negative constraints which ensure a positive number of units manufactured for each type of variant. Formulate this as a Linear Programming problem with appropriate decision variables, constraints, and an objective. Do not solve the program.

Problem 4

Part a

Use “for loop(s)” and “if statement” within the loop to generate a 20 by 20 Lehmer matrix A where

$$A_{ij} = \begin{cases} \frac{i}{j}, & j \geq i \\ \frac{j}{i}, & j < i. \end{cases}$$

Hint: First generate a 4-by-4 matrix with all the elements being 0. Then use for loop(s) and if statement to definite the Lehmer matrix. Find the 4-by-4 Lehmer matrix and use the [Wikipedia](#) to check. Then you can change the code to run a 20 by 20

Part b

Test whether A is symmetric or not. Namely, is A equal to A^T ?

Part c

Calculate $C = A^{-1}$ in R. Test whether the inverse is correct. That is, calculate $C \times A$ in R and see whether the product is an identity matrix.

Hint: Identity matrix can be generated using function `diag()`. You may use function `all.equal()` to compare $C \times A$ and the identity matrix.

Part d

Assign `[1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 10, 9, 8, 7, 6, 5, 4, 3, 2, 1]` to d in R.

Hint: This is a combination of sequence `1:10` and `10:1`.

Part e

Solve for x in the equation $Ax = Cd$.

Deliverables

Name your homework as `homework1.pdf` and submit it online.

[Optional] Rmarkdown

This document is generated using [Rmarkdown](#). You might consider using it to write the homeworks and projects.