Problem 1 :

Max $Z = -x_1 + 4x_2$

subject to : $-10x_1 + 20x_2 \leq 22$

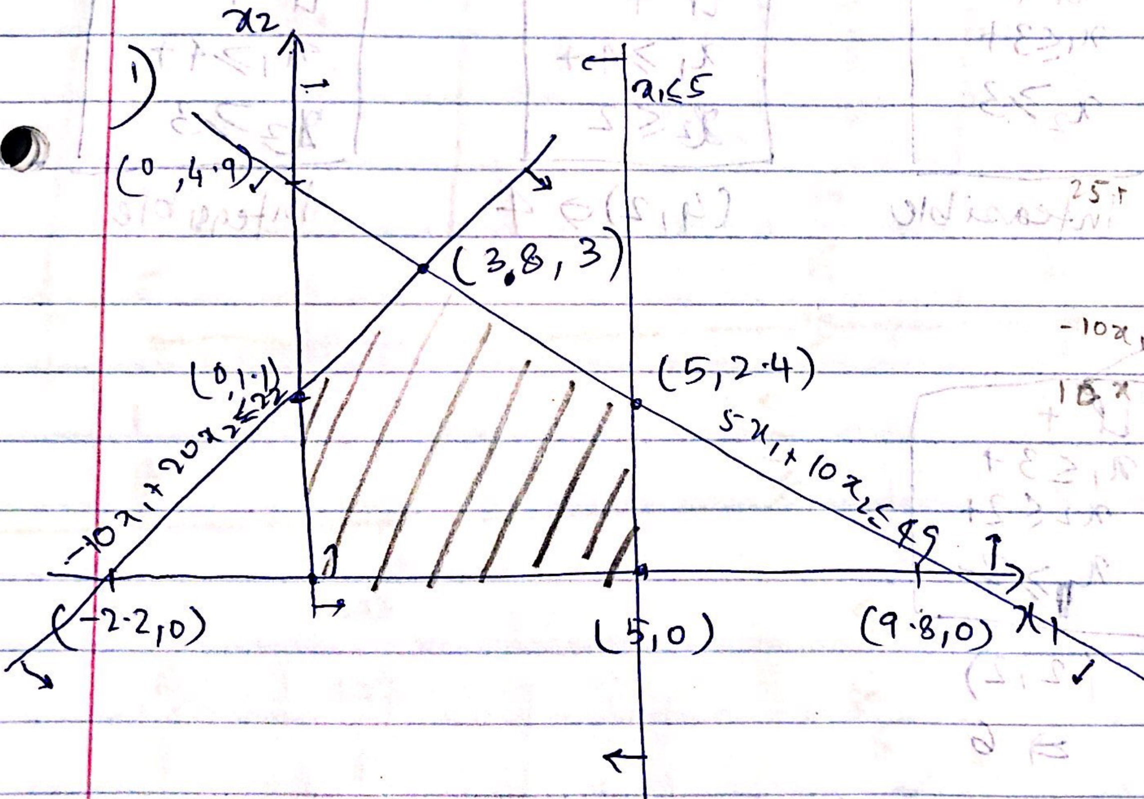
$5x_1 + 10x_2 \leq 49$

$x_1 \leq 5$

$x_i \geq 0$, x_i are integers.

$x_1 = 0 \quad x_2 = 1.1$
$x_1 = -2.2 \quad x_2 = 0$
$x_1 = 0 \quad x_2 = 4.9$
$x_1 = 9.8 \quad x_2 = 0$

1)



$25 + 10x_2 = 49$
$10x_2 = 24$

$-10x_1 + 20x_2 = 22$
$10x_1 + 20x_2 = 98$
$40x_2 = 120$
$x_2 = \cancel{\frac{7}{4}} \cancel{1.75}\ 3$
$-10x_1 = 22 - 60$
$= -38$
3.8

Checking obj values at corners:

$(5, 0) \Rightarrow -5$

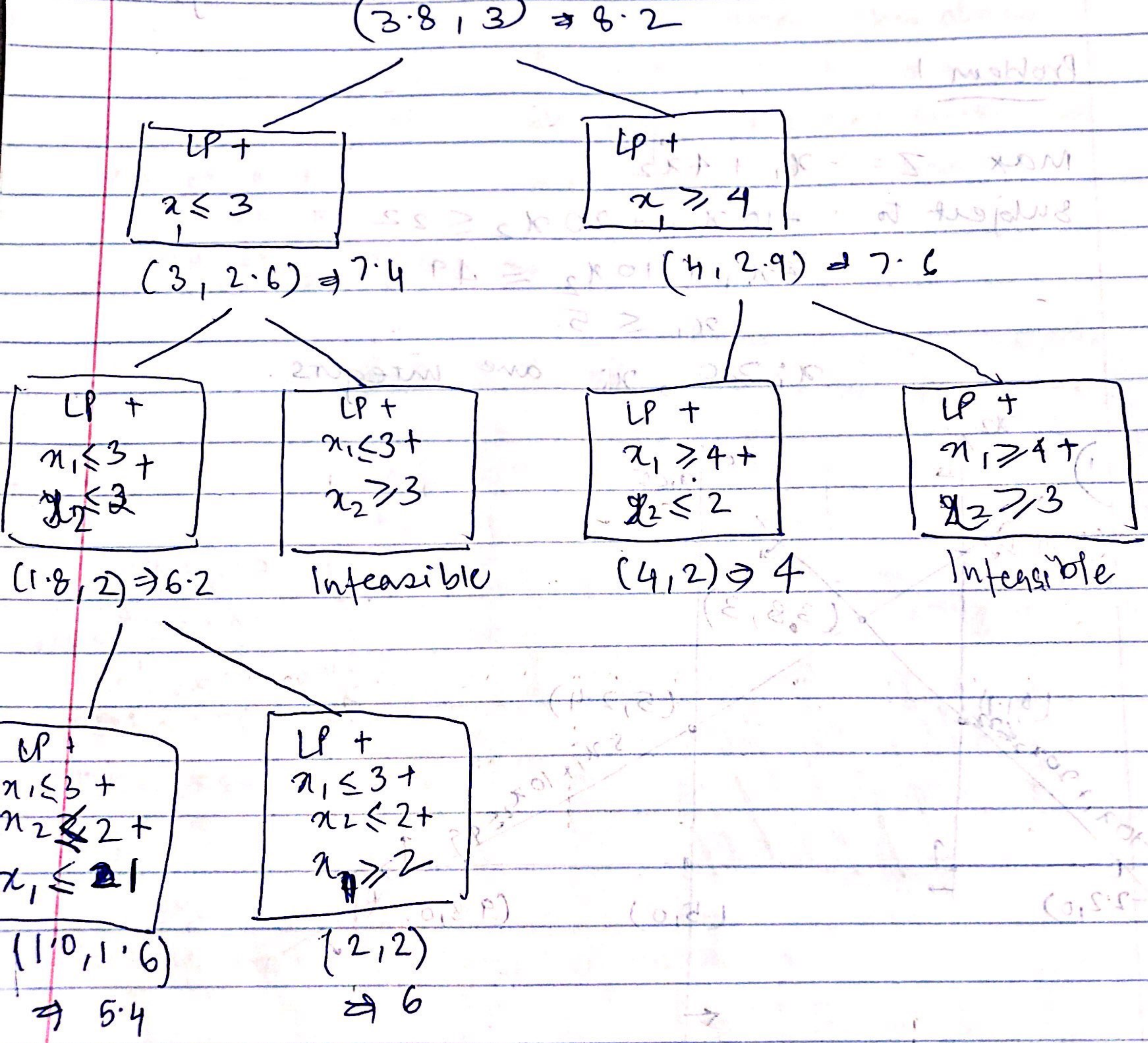
$(5, 2.4) \Rightarrow \cancel{5.4}\ 4.6$

$(3.8, 3) \Rightarrow \cancel{9}\ 8.2$

$(0, 1.1) \Rightarrow 4.4$

Thus solution to LP is $(3.8, 3)$

Applying branch and bound to get the solution to IP:



The solution is (2, 2) with obj value is 6

Problem 2

· Find $\underbrace{x_1, x_2}_{\text{Factory}}$, $\underbrace{y_1, y_2}_{\text{warehouse}}$

objective $9x_1 + 5x_2 + 6y_1 + 4y_2$

subject to: $6x_1 + 3x_2 + 5y_1 + 2y_2 \leq 11$

$$y_1 + y_2 \leq 1$$

$$x_1 + x_2 \geq 1$$

x_i, y_i are binary

$x_i \geq 0, y_i \geq 0$

$$C^T x = \begin{bmatrix} 9 & 5 & 6 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{bmatrix}$$

$$\underset{A}{\begin{bmatrix} 6 & 3 & 5 & 2 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \end{bmatrix}} \quad \underset{x}{\begin{bmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{bmatrix}} \quad \underset{dir}{\begin{bmatrix} \leq \\ \leq \\ \geq \end{bmatrix}} \quad \underset{b}{\begin{bmatrix} 11 \\ 1 \\ 1 \end{bmatrix}}$$

Optimal solution $\Rightarrow$ ~~opt~~ Build x_1, x_2 & y_2 to get obj. val $18M

Problem 3:

Let each city be represented as: $x_1 \to ATL$, $x_2 \to BOS$, $x_3 \to CHI$, $x_4 \to DEN$, $x_5 \to HOU$, $x_6 \to LAX$, $x_7 \to NO$, $x_8 \to NY$, $x_9 \to PIT$, $x_{10} \to SLC$, $x_{11} \to SF$, $x_{12} \to SEA$

Find $x_1, x_2 \ldots x_{12}$

To min $x_1 + x_2 + x_3 \ldots + x_{12}$

subject to

$x_3 + x_5 + x_7 + x_8 + x_9 \geq 1$ (ATL) 1

$x_8 + x_9 \geq 1$ (BOS) 2

$x_1 + x_7 + x_8 + x_9 \geq 1$ (CHI) 3

$x_{10} \geq 1$ (DEN) 4

$x_1 + x_7 \geq 1$ (HOU) 5

$x_{10} + x_{11} \geq 1$ (LAX) 6

$x_1 + x_3 + x_5 \cancel{+ x} \geq 1$ (NO) 7

$x_1 + x_2 + x_3 + x_9 \geq 1$ (NY) 8

$x_1 + x_2 + x_3 + x_8 \geq 1$ (PIT) 9

$x_4 + x_6 + x_{11} + x_{12} \geq 1$ (SLC) 10

$x_6 + x_{10} + x_{12} \geq 1$ (SF) 11

$x_{10} + x_{11} \geq 1$ (SEA) 12

x_i are binary

Scanned with CamScanner

For R:

$$C^T x \Rightarrow [1\ 1\ 1\ 1\ \cdots\ 1] \begin{bmatrix} x_1 \\ \vdots \\ x_{12} \end{bmatrix}$$

$$\overset{A}{\begin{array}{cccccccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \end{array}}$$

$$\begin{bmatrix}
0 & 0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0
\end{bmatrix}
\overset{x}{\begin{bmatrix} x_1 \\ \vdots \\ x_2 \end{bmatrix}}
\overset{\text{dir}}{\geq}
\overset{b}{\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}}$$

Thus we build hubs at:
BOS, DEN, HOUS, SLC

problem 4 :

$I \Rightarrow$ set of order widths.

$J \Rightarrow$ set of patterns.

$\therefore I = \{25, 37, 54\}$

$J = \{(25, 37, 54)^{(4)}, (25, 25, 25, 25)^{(20)}, (37, 37, 37)^{(9)}, (54, 54)^{(12)}, \cancel{(25, 25, 37, 54)}, (25, 25, 25, 37)^{(8)}, (25, 25, 54)^{(16)}, (25, 37, 37)^{(21)}\}$

a_{ij} = number of rolls of width i cut in pattern j

$a_{11} = 1$ $\quad a_{12} = 4$ $\quad a_{13} = 0$ $\quad a_{14} = 0$ $\quad a_{15} = 3$ $\quad a_{16} = 2$ $\quad a_{17} = 1$

$a_{21} = 1$ $\quad a_{22} = 0$ $\quad a_{23} = 3$ $\quad a_{24} = 0$ $\quad a_{25} = 1$ $\quad a_{26} = 0$ $\quad a_{27} = 2$

$a_{31} = 1$ $\quad a_{32} = 0$ $\quad a_{33} = 0$ $\quad a_{34} = 2$ $\quad a_{35} = 0$ $\quad a_{36} = 1$ $\quad a_{37} = 0$

$b_i \Rightarrow$ demand for order width i

$b_1 = 233$ $\quad b_2 = 148$ $\quad b_3 = 106$

Decision variables:

x_j = number of rolls cut using pattern j.
$(j = 1 \dots 7)$

Objective: min no. of rolls (waste)

$$\Rightarrow \min \sum_{j=1}^{7} x_j$$

subject to $\sum_{j=1}^{7} a_{ij} x_j \geq b_i$ for each i

x_j are integers

$$C^T x = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_7 \end{bmatrix}$$

$$\overset{A}{\begin{bmatrix} 1 & 4 & 0 & 0 & 3 & 2 & 1 \\ 1 & 0 & 3 & 0 & 1 & 0 & 2 \\ 1 & 0 & 0 & 2 & 0 & 1 & 0 \end{bmatrix}} \overset{x}{\begin{bmatrix} x_1 \\ \vdots \\ x_7 \end{bmatrix}} \overset{dir}{\begin{bmatrix} \geq \\ \geq \\ \geq \end{bmatrix}} \overset{b}{\begin{bmatrix} 233 \\ 148 \\ 106 \end{bmatrix}}$$

Solution:
105 rolls using (25, 37, 54)
2 rolls using (25, 25, 25, 25)
1 roll using (37, 37, 37)
1 roll using (54, 54)
40 rolls using (25, 25, 25, 37)

To give minimum ~~waste~~ no of rolls of 149

Problem 5

Find $x_1, x_2, x_3, x_4, x_5, x_6, x_7$ where x_i is number of workers on day pattern i

To Min $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7$.

Subject to
$$\begin{aligned}
x_1 + x_4 + x_5 + x_6 + x_7 &\geq 5 \\
x_1 + x_2 + x_5 + x_6 + x_7 &\geq 13 \\
x_1 + x_2 + x_3 + x_6 + x_7 &\geq 12 \\
x_1 + x_2 + x_3 + x_4 + x_7 &\geq 10 \\
x_1 + x_2 + x_3 + x_4 + x_5 &\geq 14 \\
x_2 + x_3 + x_4 + x_5 + x_6 &\geq 8 \\
x_3 + x_4 + x_5 + x_6 + x_7 &\geq 6
\end{aligned}$$

x_i are integers.

$$C^T x = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_7 \end{bmatrix}$$

$$\begin{array}{cccc} A & x & \text{dir} & b \\
\begin{bmatrix}
1 & 0 & 0 & 1 & 1 & 1 & 1 \\
1 & 1 & 0 & 0 & 1 & 1 & 1 \\
1 & 1 & 1 & 0 & 0 & 1 & 1 \\
1 & 1 & 1 & 1 & 0 & 0 & 1 \\
1 & 1 & 1 & 1 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 0 \\
0 & 0 & 1 & 1 & 1 & 1 & 1
\end{bmatrix} &
\begin{bmatrix} x_1 \\ \vdots \\ x_7 \end{bmatrix} &
\begin{bmatrix} \geq \\ \geq \\ \vdots \\ \geq \end{bmatrix} &
\begin{bmatrix} 5 \\ 13 \\ 12 \\ 10 \\ 14 \\ 8 \\ 6 \end{bmatrix}
\end{array}$$

Soln: $\begin{bmatrix} 1 & 8 & 2 & 0 & 3 & 1 & 0 \end{bmatrix}$