Homework 2

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Problem 1 :

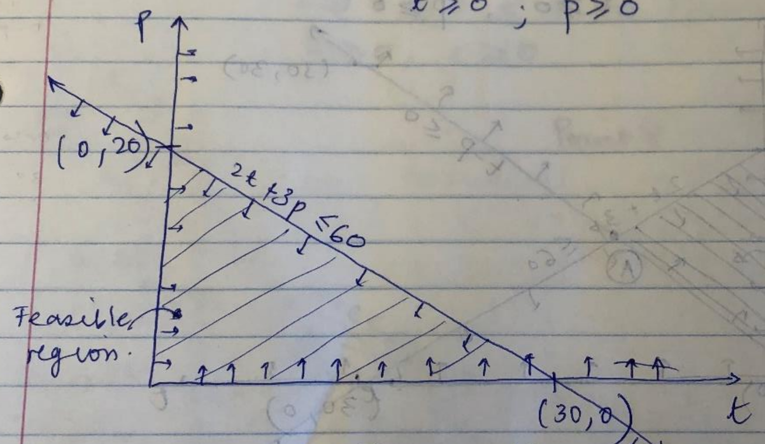
Stochastic Control & Optimization
Homework 2

Problem 1:

Let t & p be the number of tortes and pies Max eats respectively.

Decision variables: t, p

Find t, p
To maximize $4t + 5p$
Subject to $2t + 3p \leq 60$
$t \geq 0$; $p \geq 0$



Testing the objective at the corners:

At (0,20) ; $4t + 5p = 100$
At (30,0) ; $4t + 5p = 120$.

Thus, Max should eat 30 tortes and 0 pies to get most points.

Now, if he would like to stick to his preference of eating at least as many pies as tortes, the linear programming problem will be as follows:

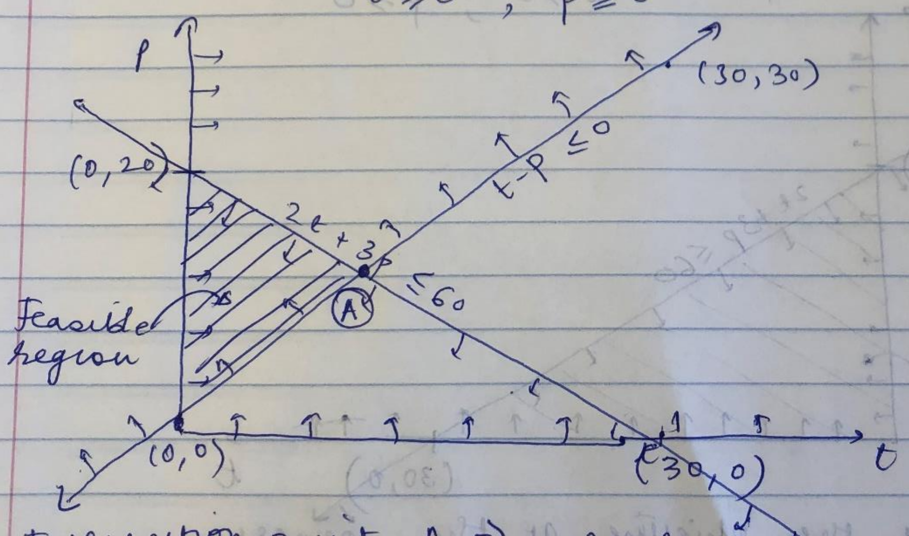
Find t, p

To max $4t + 5p$

Subject to $2t + 3p \leq 60$

$p \geq t \Rightarrow t - p \leq 0$

$t \geq 0, \; p \geq 0$



Intersection point $A \Rightarrow$

$$\begin{array}{r} 2t + 3p = 60 \\ 3t - 3p = 0 \\ \hline 5t = 60 \end{array} \qquad t = 12 \; ; \; p = 12 ;$$

$\therefore \boxed{A \Rightarrow (12, 12)}$

checking objective at ~~[illegible]~~ corners:

① $(0, 20) \Rightarrow 4t + 5p = 100$

② $(12, 12) \Rightarrow 4t + 5p = 108$

Thus, to get max points, Max should eat 12 tortes & 12 pies

Problem 2:

Problem 2:

Let number of acres planted for wheat and corn be w, c.

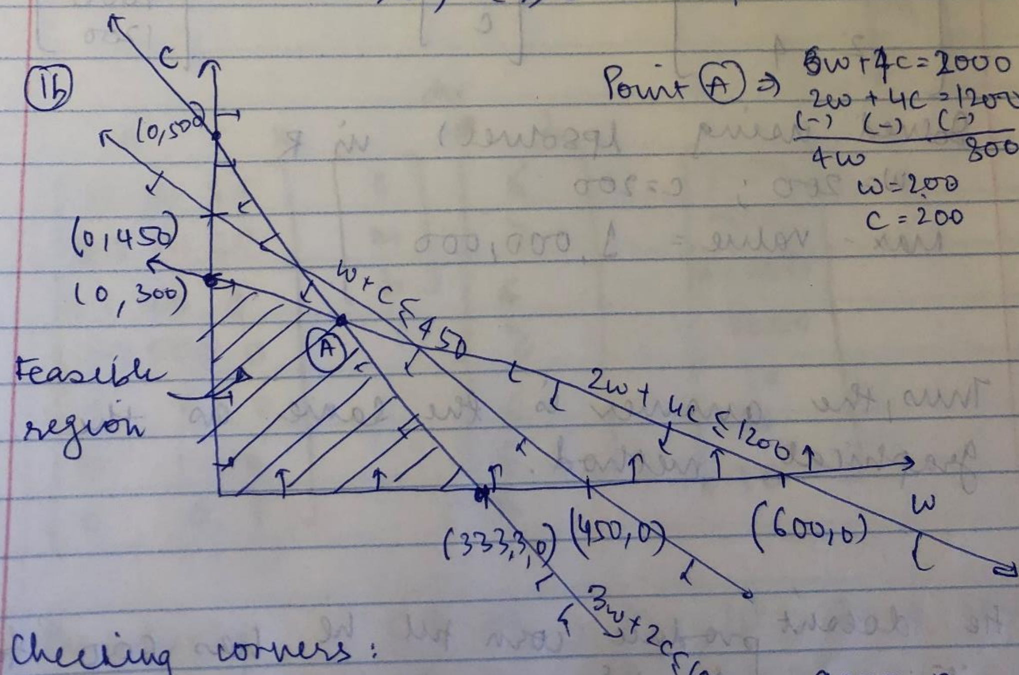
(1a) Find w, c

To max $2000w + 3000c$

Subject to

$w + c \leq 450$	$(0,450)$, $(450,0)$
$3w + 2c \leq 1000$	$(333.3,0)$, $(0,500)$
$2w + 4c \leq 1200$	$(600,0)$, $(0,300)$
$w \geq 0, \ c \geq 0$	

(1b)



Point (A) $\Rightarrow$ $6w + 4c = 2000$

$2w + 4c = 1200$

$(-) \quad (-) \quad (-)$

$4w = 800$

$w = 200$

$c = 200$

Checking corners:

$(0,300) \Rightarrow 2000w + 3000c = 900{,}000$

$(200,200) \Rightarrow 2000w + 3000c = 1{,}000{,}000$

$(333.3,0) \Rightarrow 2000w + 3000c = 666{,}666.$

$\boxed{\therefore w = 200 \ ; \ c = 200}$

(2) Find w, c

Max $C^T x$

Subject to $Ax \leq b$

$C^T x \Rightarrow \begin{bmatrix} 2000 & 3000 \end{bmatrix} \begin{bmatrix} w \\ c \end{bmatrix}$

$$\begin{array}{cccc} A & x & \leq & b \\ \begin{bmatrix} 1 & 1 \\ 3 & 2 \\ 2 & 4 \end{bmatrix} & \begin{bmatrix} w \\ c \end{bmatrix} & \leq & \begin{bmatrix} 450 \\ 1000 \\ 1200 \end{bmatrix} \end{array}$$

Solved using lpsolve() in R

$W = 200$; $C = 200$

Max - value = 1,000,000

Thus, the answer is the same as the graphical method.

(3) He doesn't produce corn till he has 600 tons of fertilizer and discontinues producing wheat when he has 1800 tons of fertilizer.

R CODE:

```
c <- c(2000,3000)
A <- matrix(c(1,3,2,1,2,4),3,2)
dir <- c("<=","<=","<=")
b <- c(450, 1000, 1200)

s<-lp("max",c,A,dir,b)
s$solution

## [1] 200 200

s$objval

## [1] 1e+06

for(i in seq(from=200, to=2200, by=100)){
  c <- c(2000,3000)
  A <- matrix(c(1,3,2,1,2,4),3,2)
  dir <- c("<=","<=","<=")
  b <- c(450, 1000, i)
  s<-lp("max",c,A,dir,b)
  cat("Solution for",i," fertilizer is:",s$solution)
  print(s$objval)

}

## Solution for 200  fertilizer is: 100 0[1] 2e+05
## Solution for 300  fertilizer is: 150 0[1] 3e+05
## Solution for 400  fertilizer is: 200 0[1] 4e+05
## Solution for 500  fertilizer is: 250 0[1] 5e+05
## Solution for 600  fertilizer is: 300 0[1] 6e+05
## Solution for 700  fertilizer is: 325 12.5[1] 687500
## Solution for 800  fertilizer is: 300 50[1] 750000
## Solution for 900  fertilizer is: 275 87.5[1] 812500
## Solution for 1000  fertilizer is: 250 125[1] 875000
## Solution for 1100  fertilizer is: 225 162.5[1] 937500
## Solution for 1200  fertilizer is: 200 200[1] 1e+06
## Solution for 1300  fertilizer is: 175 237.5[1] 1062500
## Solution for 1400  fertilizer is: 150 275[1] 1125000
## Solution for 1500  fertilizer is: 125 312.5[1] 1187500
## Solution for 1600  fertilizer is: 100 350[1] 1250000
## Solution for 1700  fertilizer is: 50 400[1] 1300000
## Solution for 1800  fertilizer is: 0 450[1] 1350000
## Solution for 1900  fertilizer is: 0 450[1] 1350000
## Solution for 2000  fertilizer is: 0 450[1] 1350000
## Solution for 2100  fertilizer is: 0 450[1] 1350000
## Solution for 2200  fertilizer is: 0 450[1] 1350000
```

As we can see from the output above, the farmer does’nt produce corn till he has 600 tons of fertilizer and discontinues producing wheat when he has 1800 tons of fertilizer.

Problem 3:

Problem 3 :

let x_1, x_2, x_3, x_4, x_5 be respective investments

Find x_1, x_2, x_3, x_4, x_5

To max $13x_1 + 16x_2 + 16x_3 + 14x_4 + 39x_5$

Subject to $11x_1 + 53x_2 + 5x_3 + 5x_4 + 29x_5 \leq 40$

$3x_1 + 6x_2 + 5x_3 + x_4 + 34x_5 \leq 20$

$x_1 \geq 0$, $x_2 \geq 0$, $x_3 \geq 0$, $x_4 \geq 0$, $x_5 \geq 0$

Problem 4:

Problem 4:

Find c, m, b

To min $0.18c + 0.23m + 0.05b$

subject to
$$2000 \le 72c + 121m + 65b \le 2250$$
$$5000 \le 107c + 500m \le 50{,}000$$
$$0 \le c \le 10$$
$$0 \le m \le 10$$
$$0 \le b \le 10$$

For lpsolve: $c^T x \Rightarrow \begin{bmatrix} 0.18 & 0.23 & 0.05 \end{bmatrix} \begin{bmatrix} c \\ m \\ b \end{bmatrix}$

$$\begin{array}{cccc} A & x & \text{dir} & b \\ \begin{bmatrix} 72 & 121 & 65 \\ 72 & 121 & 65 \\ 107 & 500 & 0 \\ 107 & 500 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} & \begin{bmatrix} c \\ m \\ b \end{bmatrix} & \begin{bmatrix} \le \\ \ge \\ \le \\ \ge \\ \le \\ \le \\ \le \end{bmatrix} & \begin{bmatrix} 2250 \\ 2000 \\ 50000 \\ 5000 \\ 10 \\ 10 \\ 10 \end{bmatrix} \end{array}$$

On solving, $c = 10$, $m = 10$, $b = 4.923077$

with minimum cost = \$4.35

Problem 5:

1.6
3
4.8

Problem 5:

Find $\underbrace{x_1, x_2, x_3}_{\text{Forest 1}}$ and $\underbrace{y_1, y_2, y_3}_{\text{Forest 2}}$

To max $x_1 + x_2 + x_3 + y_1 + y_2 + y_3$

Subject to

$$0 \le x_1 \le 2 \qquad 0 \le y_1 \le 3$$
$$0 \le x_2 \le 2.6 \qquad 0 \le y_2 \le 3.6$$
$$0 \le x_3 \le 2.8 \qquad 0 \le y_3 \le 4.8$$
$$2 \ge x_1 + y_1 \ge 1.2$$
$$2 \ge x_2 + y_2 \ge 1.5$$
$$3 \ge x_3 + y_3 \ge 2$$

$$C^T x = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\underset{A}{\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}} \underset{x}{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ y_1 \\ y_2 \\ y_3 \end{bmatrix}} \begin{matrix} \le \\ \le \\ \le \\ \le \\ \le \\ \le \\ \le \\ \le \\ \le \\ \ge \\ \ge \\ \ge \end{matrix} \underset{b}{\begin{bmatrix} 2 \\ 2.6 \\ 2.8 \\ 3 \\ 3.6 \\ 4.8 \\ 2 \\ 2 \\ 3 \\ 1.2 \\ 1.5 \\ 2 \end{bmatrix}}$$

Thus, x1 = 2, x2 = 2, x3 = 2.8, y1 = 0, y2 = 0, y3 = 0.2 with the total tons of weight extracted = 7