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Optimized Singular Value Decomposition Methods for Image compression

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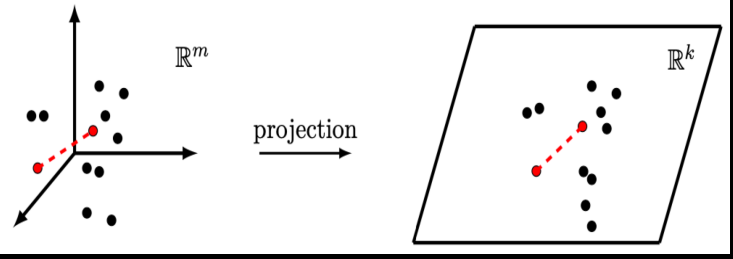


PROTOTYPE FOR RANDOMIZED SVD

Given an $m \times n$ matrix $\boldsymbol{A}$, a target number k of singular vectors, and an exponent q (say $q = 1$ or $q = 2$), this procedure computes an approximate rank-$2k$ factorization $\boldsymbol{U\Sigma V}^*$, where $\boldsymbol{U}$ and $\boldsymbol{V}$ are orthonormal, and $\boldsymbol{\Sigma}$ is nonnegative and diagonal.

Stage A:

1 Generate an $n \times 2k$ Gaussian test matrix $\boldsymbol{\Omega}$.
2 Form $\boldsymbol{Y} = (\boldsymbol{AA}^*)^q \boldsymbol{A\Omega}$ by multiplying alternately with $\boldsymbol{A}$ and $\boldsymbol{A}^*$.
3 Construct a matrix $\boldsymbol{Q}$ whose columns form an orthonormal basis for the range of $\boldsymbol{Y}$.

Stage B:

4 Form $\boldsymbol{B} = \boldsymbol{Q}^* \boldsymbol{A}$.
5 Compute an SVD of the small matrix: $\boldsymbol{B} = \tilde{\boldsymbol{U}} \boldsymbol{\Sigma V}^*$.
6 Set $\boldsymbol{U} = \boldsymbol{Q}\tilde{\boldsymbol{U}}$.

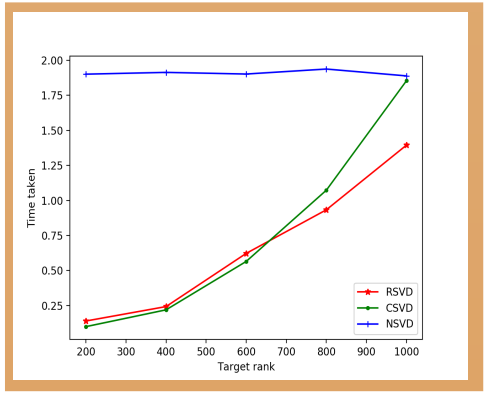
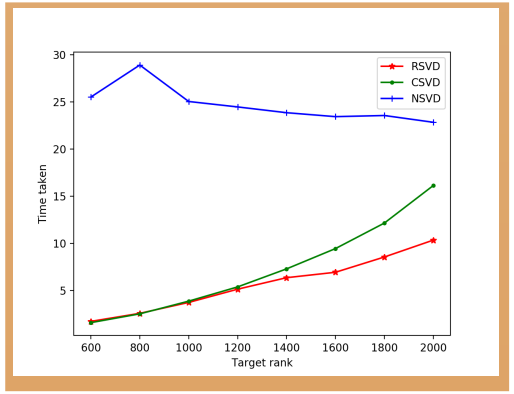
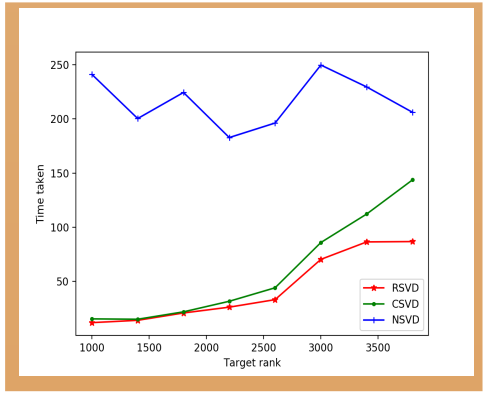
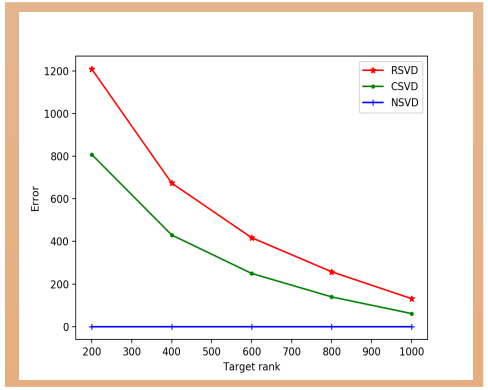
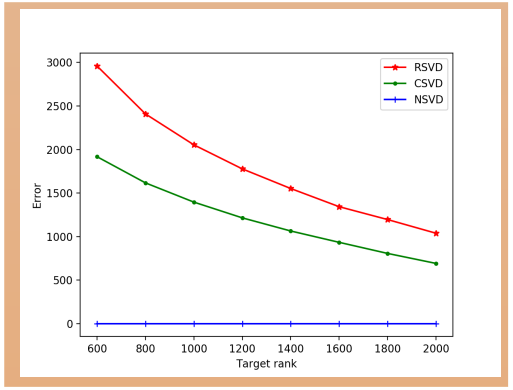
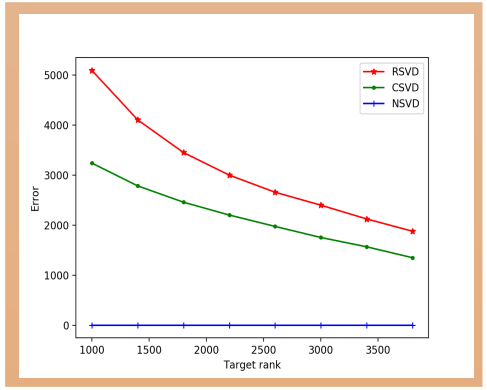
Algorithm 1 Compressed SVD (cSVD)

Input: Input matrix $\mathbf{X}$ of dimension $m \times n$, and target rank k.

Optional: Parameters p to control oversampling.

(1)	$l \leftarrow k + p$	Slight oversampling
(2)	$\mathbf{\Phi} \leftarrow \texttt{rand}(l, m)$	Generate $l \times m$ random test matrix.
(3)	$\mathbf{Y} \leftarrow \mathbf{\Phi} * \mathbf{X}$	Sketch input matrix.
(4)	$\mathbf{B} \leftarrow \mathbf{Y} * \mathbf{Y}^\top$	Form smaller $l \times l$ matrix.
(5)	$\mathbf{B} \leftarrow \frac{1}{2} \cdot (\mathbf{B} + \mathbf{B}^\top)$	Ensure symmetry.
(6)	$\mathbf{T}, \mathbf{D} \leftarrow \texttt{eig}(\mathbf{B}, k)$	Truncated eigendecomposition.
(7)	$\tilde{\mathbf{S}} \leftarrow \sqrt{\mathbf{D}}$	Rescale eigenvalues.
(8)	$\tilde{\mathbf{V}} \leftarrow \mathbf{Y}^\top * \mathbf{T} * \tilde{\mathbf{S}}^{-1}$	Approx. right singular values.
(9)	$\tilde{\mathbf{U}} \leftarrow \mathbf{X} * \tilde{\mathbf{V}}$	Approx. unscaled left singular vals.
(10)	$\mathbf{U}, \mathbf{S}, \mathbf{Q}^\top \leftarrow \texttt{svd}(\tilde{\mathbf{U}})$	Update left singular vectors and vals.
(11)	$\mathbf{V} \leftarrow \tilde{\mathbf{V}}\mathbf{Q}$	Update right singular vectors.

Return: $\mathbf{U} \in \mathbb{R}^{m \times k}$, $\mathbf{S} \in \mathbb{R}^{k \times k}$ and $\mathbf{V} \in \mathbb{R}^{n \times k}$

	1079x1553 (k=600)	2848x4096 (k=1000)	5696x8191 (k=1500)
Image reconstruc-ted using CSVD			
Time taken vs target rank (k)			
Error vs target rank (k)			

References

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3) Erichson, N. B., Voronin, S., Brunton, S. L., & Kutz, J. N. (2016). Randomized matrix decompositions using R. *arXiv preprint arXiv:1608.02148*.