Optimization for Machine Learning

Lecture 11: Proximal Gradient Methods

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Lecture Outline

Composite Optimization Problems

Projected Gradient Descent

Proximal Gradient Descent

Stochastic Proximal Gradient Descent

Composite Optimization Problem

$$\min_{\mathbf{x}\in\mathbb{R}^d} f(\mathbf{x}) + \psi(\mathbf{x})$$

- $f\colon \mathbb{R}^d \to \mathbb{R}$, L-smooth
- $\psi\colon \mathbb{R}^d \to \mathbb{R} \cup \{+\infty\}$ proper, closed and convex regularizer

Example: Constrained Minimization

Let $X \subseteq \mathbf{dom}(f)$ be a convex set.

$$\min_{\mathbf{x} \in X} f(\mathbf{x}) = \min_{\mathbf{x} \in \mathbb{R}^d} f(\mathbf{x}) + \psi(\mathbf{x})$$

where $\psi(\mathbf{x}) := \mathbf{1}_X(\mathbf{x})$

Indicator Function: Given a closed convex set X, the indicator function of the set X is given as the convex function

$$\begin{aligned} \mathbf{1}_X : \mathbb{R}^d &\to \mathbb{R} \cup +\infty \\ \mathbf{x} \mapsto \mathbf{1}_X(\mathbf{x}) &:= \begin{cases} 0 & \text{if } \mathbf{x} \in X, \\ +\infty & \text{otherwise.} \end{cases} \end{aligned}$$

Example: Regularization

Lasso: Sparsity inducing regularization

$$\min_{\mathbf{x}\in\mathbb{R}^d} f(\mathbf{x}) + \lambda \|\mathbf{x}\|_1$$

with $\|\mathbf{x}\|_1 := \sum_{i=1}^d |\mathbf{x}_i|$.

Ridge regression:

$$\min_{\mathbf{x}\in\mathbb{R}^d} f(\mathbf{x}) + \frac{\lambda}{2} \|\mathbf{x}\|_2^2$$

with $\|\mathbf{x}\|_2^2 := \sum_{i=1}^d |\mathbf{x}_i|^2$.

Example: Consensus Formulation

Distributed optimization:

$$\min_{\mathbf{x}\in\mathbb{R}^d}\left[f(\mathbf{x}) := \frac{1}{n}\sum_{i=1}^{n} f_i(\mathbf{x})\right] = \min_{\mathbf{x}_1,\dots,\mathbf{x}_n\in\mathbb{R}^d} \frac{1}{n}\sum_{i=1}^{n} f_i(\mathbf{x}_i) + \psi(\mathbf{x}_1,\dots,\mathbf{x}_n)\,,$$

where $\psi(\mathbf{x}_1,\dots,\mathbf{x}_n) := \begin{cases} 0, & \text{if } \mathbf{x}_1 = \cdots = \mathbf{x}_n \\ +\infty, & \text{otherwise} \end{cases}.$

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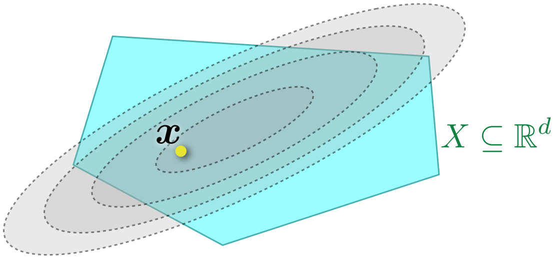
Stochastic Proximal Gradient Descent

Constrained Optimization

$$\begin{array}{ll} \text{minimize} & f(\mathbf{x}) \\ \text{subject to} & \mathbf{x} \in X \end{array}$$



Constrained Minimization

Definition 11.1

Let $f : \mathbf{dom}(f) \to \mathbb{R}$ be convex and let $X \subseteq \mathbf{dom}(f)$ be a convex set. A point $\mathbf{x} \in X$ is a minimizer of f over X if

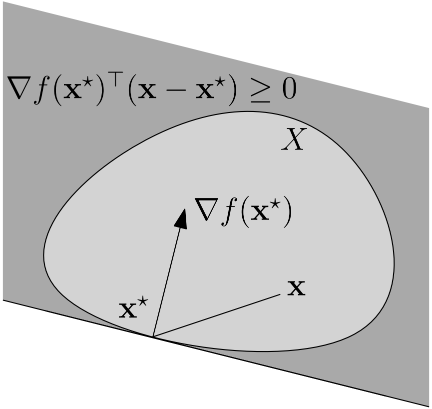
$$f(\mathbf{x}) \leq f(\mathbf{y}) \quad \forall \mathbf{y} \in X.$$

Lemma 11.2

Suppose that $f : \mathbf{dom}(f) \to \mathbb{R}$ is convex and differentiable over an open domain $\mathbf{dom}(f) \subseteq \mathbb{R}^d$, and let $X \subseteq \mathbf{dom}(f)$ be a convex set. Point $\mathbf{x}^\star \in X$ is a minimizer of f over X if and only if

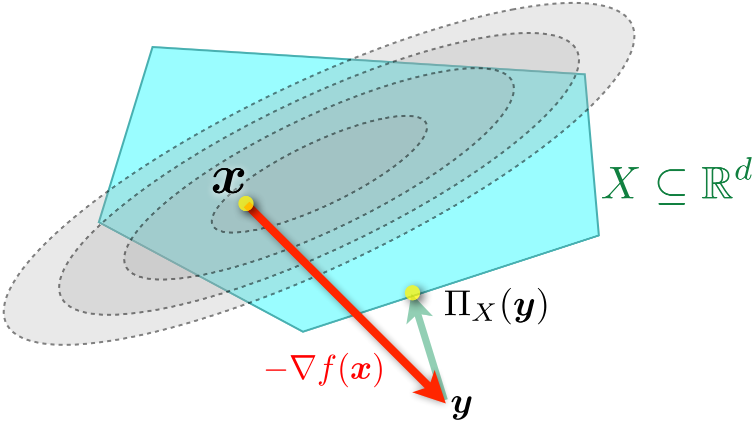
$$\nabla f(\mathbf{x}^\star)^\top (\mathbf{x} - \mathbf{x}^\star) \geq 0 \quad \forall \mathbf{x} \in X.$$

Constrained Minimization



Projected Gradient Descent

Idea: project onto X after every step: $\Pi_X(\mathbf{y}) := \operatorname{argmin}_{\mathbf{x} \in X} \|\mathbf{x} - \mathbf{y}\|$



Projected gradient descent: $\mathbf{x}_{t+1} := \Pi_X\left[\mathbf{x}_t - \gamma \nabla f(\mathbf{x}_t)\right]$

The Algorithm

Projected gradient descent:

$$
\begin{aligned}
\mathbf{y}_{t+1} &:= \mathbf{x}_t - \gamma \nabla f(\mathbf{x}_t), \\
\mathbf{x}_{t+1} &:= \Pi_X(\mathbf{y}_{t+1}) := \operatorname*{argmin}_{\mathbf{x} \in X} \|\mathbf{x} - \mathbf{y}_{t+1}\|^2.
\end{aligned}
$$

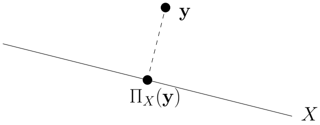
for **timesteps** $t = 0, 1, \ldots,$ and **stepsize** $\gamma \geq 0$.

The Projection Step: $\Pi_X(\mathbf{y}) := \operatorname{argmin}_{\mathbf{x}\in X} \|\mathbf{x}-\mathbf{y}\|$

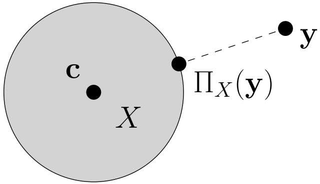
Computing $\Pi_X(\mathbf{y})$ is an optimization problem itself.

It can efficiently be solved in relevant cases:

- Projecting onto an affine subspace (leads to system of linear equations, similar to least squares)

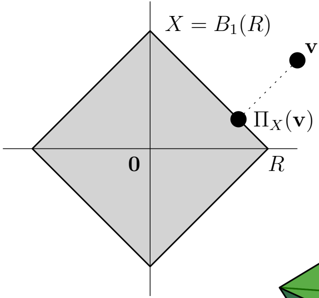


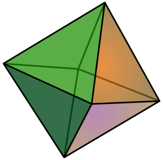
- Projecting onto a Euclidean ball with center $\mathbf{c}$ (simply scale the vector $\mathbf{y}-\mathbf{c}$)



Projecting onto ℓ_1-balls (needed in Lasso)

W.l.o.g. restrict to center at $\mathbf{0}$: $B_1(R) = \{\mathbf{x} \in \mathbb{R}^d : \|\mathbf{x}\|_1 = \sum_{i=1}^{d} |x_i| \leq R\}$.



$B_1(R)$ is the cross polytope ($2d$ vertices, 2^d facets).  (octahedron, $d = 3$)

Section 4.5: projection can be computed in $\mathcal{O}(d \log d)$ time

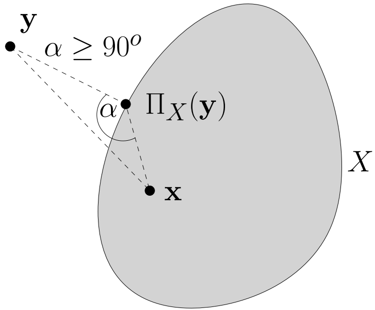
Properties of Projection

Fact 11.3

Let $X \subseteq \mathbb{R}^d$ be closed and convex, $\mathbf{x} \in X, \mathbf{y} \in \mathbb{R}^d$. Then

(i) $(\mathbf{x} - \Pi_X(\mathbf{y}))^\top (\mathbf{y} - \Pi_X(\mathbf{y})) \leq 0$.

(ii) $\|\mathbf{x} - \Pi_X(\mathbf{y})\|^2 + \|\mathbf{y} - \Pi_X(\mathbf{y})\|^2 \leq \|\mathbf{x} - \mathbf{y}\|^2$.



Properties of Projection II

Fact 11.4

Let $X \subseteq \mathbb{R}^d$ *be closed and convex,* $\mathbf{x} \in X, \mathbf{y} \in \mathbb{R}^d$*. Then*

(i) $(\mathbf{x} - \Pi_X(\mathbf{y}))^\top (\mathbf{y} - \Pi_X(\mathbf{y})) \leq 0$.

(ii) $\|\mathbf{x} - \Pi_X(\mathbf{y})\|^2 + \|\mathbf{y} - \Pi_X(\mathbf{y})\|^2 \leq \|\mathbf{x} - \mathbf{y}\|^2$.

Proof.

(i) $\Pi_X(\mathbf{y})$ is minimizer of (differentiable) convex function $d_{\mathbf{y}}(\mathbf{x}) = \|\mathbf{x} - \mathbf{y}\|^2$ over X. By first-order characterization of optimality (**Lemma 2.28**),

$$\begin{array}{rrcl} & 0 & \leq & \nabla d_{\mathbf{y}}(\Pi_X(\mathbf{y}))^\top (\mathbf{x} - \Pi_X(\mathbf{y})) \\ & & = & 2(\Pi_X(\mathbf{y}) - \mathbf{y})^\top (\mathbf{x} - \Pi_X(\mathbf{y})) \\ \Leftrightarrow & 0 & \geq & 2(\mathbf{y} - \Pi_X(\mathbf{y}))^\top (\mathbf{x} - \Pi_X(\mathbf{y})) \\ \Leftrightarrow & 0 & \geq & (\mathbf{x} - \Pi_X(\mathbf{y}))^\top (\mathbf{y} - \Pi_X(\mathbf{y})) \end{array}$$

$\square$

Properties of Projection III

Fact 11.5

Let $X \subseteq \mathbb{R}^d$ be closed and convex, $\mathbf{x} \in X, \mathbf{y} \in \mathbb{R}^d$. Then

(i) $(\mathbf{x} - \Pi_X(\mathbf{y}))^\top (\mathbf{y} - \Pi_X(\mathbf{y})) \leq 0$.

(ii) $\|\mathbf{x} - \Pi_X(\mathbf{y})\|^2 + \|\mathbf{y} - \Pi_X(\mathbf{y})\|^2 \leq \|\mathbf{x} - \mathbf{y}\|^2$.

Proof.

(ii)

$$\mathbf{v} := (\mathbf{x} - \Pi_X(\mathbf{y})), \quad \mathbf{w} := (\mathbf{y} - \Pi_X(\mathbf{y})).$$

By (i),

$$\begin{aligned} 0 \geq 2\mathbf{v}^\top \mathbf{w} &= \|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 - \|\mathbf{v} - \mathbf{w}\|^2 \\ &= \|\mathbf{x} - \Pi_X(\mathbf{y})\|^2 + \|\mathbf{y} - \Pi_X(\mathbf{y})\|^2 - \|\mathbf{x} - \mathbf{y}\|^2. \end{aligned}$$

□

Results for projected gradient descent over closed and convex X

The same number of steps as gradient over $\mathbb{R}^d$!

- Lipschitz convex functions over X: $\mathcal{O}(1/\varepsilon^2)$ steps
- Smooth convex functions over X: $\mathcal{O}(1/\varepsilon)$ steps
- Smooth and strongly convex functions over X: $\mathcal{O}(\log(1/\varepsilon))$ steps

We will adapt (one) of the previous proofs for gradient descent.

BUT:

- Each step involves a projection onto X
- may or may not be efficient (in relevant cases, it is)...

Smooth convex functions over X: $\mathcal{O}(1/\varepsilon)$ steps

Theorem 11.6

Let $f : \mathbb{R}^d \to \mathbb{R}$ be convex and differentiable. Let $X \subseteq \mathbb{R}^d$ be a closed convex set, and assume that there is a minimizer $\mathbf{x}^\star$ of f over X; furthermore, suppose that f is smooth over X with parameter L. Choosing stepsize

$$\gamma := \frac{1}{L},$$

projected gradient descent yields

$$\frac{1}{T}\sum_{t=1}^{T} f(\mathbf{x}_t) - f(\mathbf{x}^\star) \leq \frac{L}{2T}\|\mathbf{x}_0 - \mathbf{x}^\star\|^2, \quad T > 0.$$

(**Exercise 29** in the lecture notes ask you to prove $f(\mathbf{x}_T) - f(\mathbf{x}^\star) \leq \frac{L}{2T}\|\mathbf{x}_0 - \mathbf{x}^\star\|^2$).

Step I: Sufficient decrease for projected gradient descent

Lemma 11.7

Let $f : \mathbb{R}^d \to \mathbb{R}$ be differentiable and smooth with parameter L over X. Choosing stepsize

$$\gamma := \frac{1}{L},$$

projected gradient descent with arbitrary $\mathbf{x}_0 \in X$ satisfies

$$f(\mathbf{x}_{t+1}) \leq f(\mathbf{x}_t) - \frac{1}{2L}\|\nabla f(\mathbf{x}_t)\|^2 + \frac{L}{2}\|\mathbf{y}_{t+1} - \mathbf{x}_{t+1}\|^2, \quad t \geq 0.$$

Sufficient decrease II

$$f(\mathbf{x}_{t+1}) \leq f(\mathbf{x}_t) - \frac{1}{2L}\|\nabla f(\mathbf{x}_t)\|^2 + \frac{L}{2}\|\mathbf{y}_{t+1} - \mathbf{x}_{t+1}\|^2.$$

Proof.

Use smoothness, $\mathbf{y}_{t+1} - \mathbf{x}_t = -\nabla f(\mathbf{x}_t)/L$, $2\mathbf{v}^\top \mathbf{w} = \|\mathbf{v}\|^2 + \|\mathbf{w}\|^2 - \|\mathbf{v} - \mathbf{w}\|^2$:

$$\begin{aligned}
f(\mathbf{x}_{t+1}) &\leq f(\mathbf{x}_t) + \nabla f(\mathbf{x}_t)^\top (\mathbf{x}_{t+1} - \mathbf{x}_t) + \frac{L}{2}\|\mathbf{x}_t - \mathbf{x}_{t+1}\|^2 \\
&= f(\mathbf{x}_t) - L(\mathbf{y}_{t+1} - \mathbf{x}_t)^\top (\mathbf{x}_{t+1} - \mathbf{x}_t) + \frac{L}{2}\|\mathbf{x}_t - \mathbf{x}_{t+1}\|^2 \\
&= f(\mathbf{x}_t) - \frac{L}{2}\left(\|\mathbf{y}_{t+1} - \mathbf{x}_t\|^2 + \underline{\|\mathbf{x}_{t+1} - \mathbf{x}_t\|^2} - \|\mathbf{y}_{t+1} - \mathbf{x}_{t+1}\|^2\right) + \frac{L}{2}\underline{\|\mathbf{x}_t - \mathbf{x}_{t+1}\|^2} \\
&= f(\mathbf{x}_t) - \frac{L}{2}\|\mathbf{y}_{t+1} - \mathbf{x}_t\|^2 + \frac{L}{2}\|\mathbf{y}_{t+1} - \mathbf{x}_{t+1}\|^2 \\
&= f(\mathbf{x}_t) - \frac{1}{2L}\|\nabla f(\mathbf{x}_t)\|^2 + \frac{L}{2}\|\mathbf{y}_{t+1} - \mathbf{x}_{t+1}\|^2.
\end{aligned}$$

$\square$

Proof I

- By convexity:
$$f(\mathbf{x}_t) - f(\mathbf{x}^\star) \leq \nabla f(\mathbf{x}_t)^\top (\mathbf{x}_t - \mathbf{x}^\star)$$
- With $\mathbf{y}_{t+1} = \mathbf{x}_t - \gamma \nabla f(\mathbf{x}_t)$ we have
$$\nabla f(\mathbf{x}_t)^\top (\mathbf{x}_t - \mathbf{x}^\star) = \frac{1}{2\gamma} \left(\gamma^2 \|\nabla f(\mathbf{x}_t)\|^2 + \|\mathbf{x}_t - \mathbf{x}^\star\|^2 - \|\mathbf{y}_{t+1} - \mathbf{x}^\star\|^2 \right).$$
- Use Fact (ii): $\|\mathbf{x} - \Pi_X(\mathbf{y})\|^2 + \|\mathbf{y} - \Pi_X(\mathbf{y})\|^2 \leq \|\mathbf{x} - \mathbf{y}\|^2$.
- With $\mathbf{x} = \mathbf{x}^\star, \mathbf{y} = \mathbf{y}_{t+1}$, we have $\Pi_X(\mathbf{y}) = \mathbf{x}_{t+1}$, and hence
$$\|\mathbf{x}^\star - \mathbf{x}_{t+1}\|^2 + \underline{\|\mathbf{y}_{t+1} - \mathbf{x}_{t+1}\|^2} \leq \|\mathbf{x}^\star - \mathbf{y}_{t+1}\|^2$$
- This saving term is crucial to make telescoping work again!
$$\nabla f(\mathbf{x}_t)^\top (\mathbf{x}_t - \mathbf{x}^\star) \leq \frac{1}{2\gamma} \left(\gamma^2 \|\nabla f(\mathbf{x}_t)\|^2 + \|\mathbf{x}_t - \mathbf{x}^\star\|^2 - \|\mathbf{x}_{t+1} - \mathbf{x}^\star\|^2 - \underline{\|\mathbf{y}_{t+1} - \mathbf{x}_{t+1}\|^2} \right)$$

- Set $\gamma = \frac{1}{L}$ and use the sufficient decrease lemma to bound $\|\nabla f(\mathbf{x}_t)\|^2$:

$$\begin{aligned}\nabla f(\mathbf{x}_t)^\top(\mathbf{x}_t - \mathbf{x}^\star) &\leq \frac{1}{2L}\|\nabla f(\mathbf{x}_t)\|^2 + \frac{L}{2}\|\mathbf{x}_t - \mathbf{x}^\star\|^2 - \frac{L}{2}\|\mathbf{x}_{t+1} - \mathbf{x}^\star\|^2 - \frac{L}{2}\|\mathbf{y}_{t+1} - \mathbf{x}_{t+1}\|^2 \\ &\leq f(\mathbf{x}_t) - f(\mathbf{x}_{t+1}) + \frac{L}{2}\|\mathbf{x}_t - \mathbf{x}^\star\|^2 - \frac{L}{2}\|\mathbf{x}_{t+1} - \mathbf{x}^\star\|^2\end{aligned}$$

- This "trick" makes telescoping work again!

$$\sum_{t=0}^{T} f(\mathbf{x}_t) - f(\mathbf{x}^\star) \leq \sum_{t=0}^{T}\left(f(\mathbf{x}_t) - f(\mathbf{x}_{t+1}) + \frac{L}{2}\|\mathbf{x}_t - \mathbf{x}^\star\|^2 - \frac{L}{2}\|\mathbf{x}_{t+1} - \mathbf{x}^\star\|^2\right)$$

Hence

$$\frac{1}{T}\sum_{t=1}^{T} f(\mathbf{x}_t) - f(\mathbf{x}^\star) \leq \frac{L}{2T}\|\mathbf{x}_0 - \mathbf{x}^\star\|^2$$

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Projected Gradient Descent

Proximal Gradient Descent

Stochastic Proximal Gradient Descent

Composite optimization problems

Consider objective functions composed as

$$F(\mathbf{x}) := f(\mathbf{x}) + \psi(\mathbf{x})$$

where f is a "nice" function, where as ψ is a "simple" additional term, which however doesn't satisfy the assumptions of niceness which we used in the convergence analysis so far.

In particular, an important case is when ψ is not differentiable.

Idea

The classical gradient step for minimizing f:

$$\mathbf{x}_{t+1} = \operatorname*{argmin}_{\mathbf{y}} \; f(\mathbf{x}_t) + \nabla f(\mathbf{x}_t)^\top (\mathbf{y} - \mathbf{x}_t) + \frac{1}{2\gamma} \|\mathbf{y} - \mathbf{x}_t\|^2 \; .$$

For the stepsize $\gamma := \frac{1}{L}$ it exactly minimizes the local quadratic model of g at our current iterate $\mathbf{x}_t$, formed by the smoothness property with parameter L.

Now for $F = f + \psi$, keep the same for f, and add ψ unmodified.

$$\begin{aligned} \mathbf{x}_{t+1} &:= \operatorname*{argmin}_{\mathbf{y}} \; f(\mathbf{x}_t) + \nabla f(\mathbf{x}_t)^\top (\mathbf{y} - \mathbf{x}_t) + \frac{1}{2\gamma} \|\mathbf{y} - \mathbf{x}_t\|^2 + \psi(\mathbf{y}) \\ &= \operatorname*{argmin}_{\mathbf{y}} \; \frac{1}{2\gamma} \|\mathbf{y} - (\mathbf{x}_t - \gamma \nabla f(\mathbf{x}_t))\|^2 + \psi(\mathbf{y}) \; , \end{aligned}$$

the proximal gradient descent update.

The proximal gradient descent algorithm

An iteration of proximal gradient descent is defined as

$$\mathbf{x}_{t+1} := \operatorname{prox}_{\psi,\gamma}(\mathbf{x}_t - \gamma \nabla f(\mathbf{x}_t)) \ .$$

where the proximal mapping for a given function ψ, and parameter $\gamma > 0$ is defined as

$$\operatorname{prox}_{\psi,\gamma}(\mathbf{z}) := \operatorname*{argmin}_{\mathbf{y}} \left\{ \frac{1}{2\gamma} \|\mathbf{y} - \mathbf{z}\|^2 + \psi(\mathbf{y}) \right\} \ .$$

A generalization of gradient descent?

- $\psi \equiv 0$: recover gradient descent
- $\psi \equiv \mathbf{1}_X$: recover projected gradient descent!
  Proximal mapping becomes

$$\operatorname{prox}_{h,\gamma}(\mathbf{z}) := \underset{\mathbf{y}}{\operatorname{argmin}} \left\{ \frac{1}{2\gamma} \|\mathbf{y} - \mathbf{z}\|^2 + \mathbf{1}_X(\mathbf{y}) \right\} = \underset{\mathbf{y} \in X}{\operatorname{argmin}} \ \|\mathbf{y} - \mathbf{z}\|^2$$

  which is the projection onto X.

Convergence in $\mathcal{O}(1/\varepsilon)$ steps

For many classes of function f, it can be shown that proximal gradient descent on $f(\mathbf{x}) + \psi(\mathbf{x})$ converges in the same number of steps, as gradient descent on $f(\mathbf{x})$.

The the additional complexity is “hidden” in the proximal step, as it is assumed that the proximal update can be computed efficiently.

Lecture Outline

Stochastic Proximal Gradient Method

$$\mathbf{x}_{t+1} = \operatorname*{argmin}_{\mathbf{x} \in \mathbb{R}^d} \mathbf{g}_t{}^\top \mathbf{x} + \psi(\mathbf{x}) + \frac{1}{2\gamma} \|\mathbf{x} - \mathbf{x}_t\|^2 \ ,$$

where $\mathbb{E}\mathbf{g}_t = \nabla f(\mathbf{x}_t)$ with bounded variance:

$$\mathbb{E} \|\mathbf{g}_t - \nabla f(\mathbf{x}_t)\|^2 \leq \sigma^2.$$

Be careful with stochastic prox!

- Again, we would expect that the Stochastic Proximal Gradient Method works similarly as the Stochastic Gradient Method.
- However, the proximal step with a stochastic gradients could amplify the stochastic variance.

$$\mathbf{x}_{t+1} = \operatorname*{argmin}_{\mathbf{x}\in\mathbb{R}^d} \mathbf{g}_t{}^\top \mathbf{x} + \psi(\mathbf{x}) + \frac{1}{2\gamma} \left\|\mathbf{x} - \mathbf{x}_t\right\|^2$$

- In practice, this is often addressed with large batches. In theory, the batch size sometimes needs to be taken as large as $\frac{1}{\epsilon}$!

SPG with momentum

Large batches can be avoided with momentum.

SPG with momentum:
For an initialization $\mathbf{m}_{-1} \in \mathbb{R}^d$, and a momentum parameter η:

$$\mathbf{m}_t = (1-\eta)\mathbf{m}_{t-1} + \eta \mathbf{g}_t$$

$$\mathbf{x}_{t+1} = \operatorname*{argmin}_{\mathbf{x} \in \mathbb{R}^d} \mathbf{m}_t{}^\top \mathbf{x} + \psi(\mathbf{x}) + \frac{1}{2\gamma} \|\mathbf{x} - \mathbf{x}_t\|^2 \ ,$$

where again $\mathbb{E}\mathbf{g}_t = \nabla f(\mathbf{x}_t)$ denotes a stochastic gradient.

SPG with momentum [GRS24]

Theorem 11.8

If $\mathbf{m}_0$ is initialized such that $\mathbb{E}\left\|\mathbf{m}_0 - \nabla f(\mathbf{x}_0)\right\|^2 = \mathcal{O}(LF_0)$ with $F_0 = f(\mathbf{x}_0) - f^\star$, $\mathbb{E}\left\|\mathbf{g}_t - \nabla f(\mathbf{x}_t)\right\|^2 \leq \sigma^2$, f is L-smooth, and the momentum parameter $\eta = \frac{3L\gamma}{1-L\gamma}$, and $\gamma = \min\left\{\frac{1}{4L}, \frac{C}{\sqrt{T}}\right\}$ (for a constant C), then

$$\sum_{t=0}^{T} \mathbb{E}\left\|\nabla f(\mathbf{x}_t)\right\|^2 \leq \mathcal{O}\left(\frac{LF_0}{T} + \frac{\sigma\sqrt{LF_0}}{\sqrt{T}}\right).$$

The initialization condition can for instance be reached for $\mathbf{m}_0 = \frac{1}{|B_0|}\sum_{i\in B_0}\mathbf{g}(\mathbf{x}_0)$ with a mini-batch of size $\max\left\{\frac{\sigma^2}{LF_0}, 1\right\}$. This batch size does not depend on ϵ.

Recommended reading: [GRS24]

Discussion

- composite problems $f(\mathbf{x}) + \psi(\mathbf{x})$
- under the assumption that $\psi(\mathbf{x})$ is simple, composite problems can usually be solved with proximal methods in the same number of iterations as it takes to minimize $f(\mathbf{x})$ alone

Bibliography I

Yuan Gao, Anton Rodomanov, and Sebastian U Stich.
Non-convex stochastic composite optimization with polyak momentum.
arXiv preprint arXiv:2403.02967, 2024.