Optimization for Machine Learning

Lecture 9: Finite Sum Optimization

Variance-reduced Stochastic Methods

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Course Outlook

- course format: mini-survey
- course project: on plan? questions?

Group Project Timeline

- Group registration between May 28 – June 4 (register on CMS)
  - groups of 2–3
- before June 18: get in touch with the contact person and schedule a meeting!
  - read the related literature
  - prepare a list of **research goals and tasks**
- before June 25: meet with your contact person
  - zoom meeting, 30-60min, can also be in-person
  - **discuss your research plan**
  - **ask questions** about things you do not understand
- July 16: Poster presentation (& suggested report submission)
  - (note that the **poster printing deadline** is a bit earlier, TBA!)
- July 26: last possible date to submit the report

There will be no exercise sheets in the weeks of June 25/July 2 — you can also discuss the project ~~in the exercise session~~ with your contact person.

Recap

- We formulated (& simplified) federated learning as a finite sum optimization problem:

$$\min_{\mathbf{x}\in\mathbb{R}^d} \quad F(\mathbf{x}) := \frac{1}{n}\sum_{i=1}^{n} f_i(\mathbf{x}) \qquad \text{(FS)}$$

n ... clients
"data on client i"
loss for datapoint i

- We observed optimization difficulties in the heterogeneous setting: $f_i(\mathbf{x}^\star) \neq f_j(\mathbf{x}^\star)$.
- Specialized methods, like SCAFFOLD can mitigate "drift". In this lecture, we will see that drift correction can be seen as a special case of variance reduction.

SGD vs. GD for Finite Sum Problem

$$\nabla F(x) = \frac{1}{n}\sum_{i=1}^{n} \nabla f_i(x)$$

$$\min_{\mathbf{x}\in\mathbb{R}^d} \quad F(\mathbf{x}) := \frac{1}{n}\sum_{i=1}^{n} f_i(\mathbf{x})$$

Table: Complexity for smooth and strongly convex problems: $\kappa = L/\mu$

	iteration complexity	per-iteration cost	total cost
GD	$O(\kappa \cdot \log 1/\epsilon)$	$O(n)$	$O(\kappa \cdot n \cdot \log 1/\epsilon)$
SGD	$O(\frac{\sigma^2}{\mu\epsilon})$	$O(1)$	$O(\frac{\sigma^2}{\mu\epsilon})$

- GD converges faster but with expensive iteration cost
- SGD converges slowly but with cheap iteration cost
- SGD is more appealing for large n and moderate accuracy ϵ.

Can we achieve both worlds?

- GD: deterministic, linear rate, $O(n)$ iteration cost, fixed stepsize.
- SGD: stochastic, sublinear rate, $O(1)$ iteration cost, diminishing stepsize.

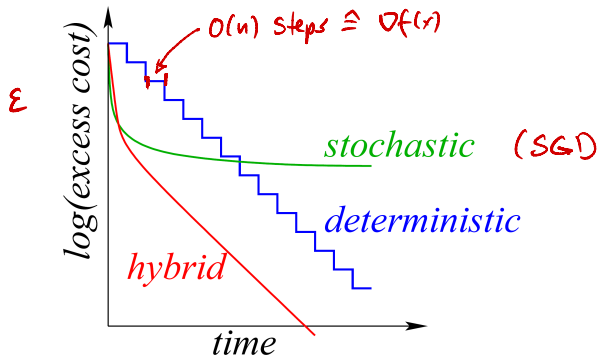


Figure from Bach's NeurIPS 2016 tutorial

Observation: reducing variance is the key

$$\mathbb{E}[\|\nabla f_{i_t}(\mathbf{x}_t) - \nabla F(\mathbf{x}_t)\|_2^2] \leq \sigma^2$$

A high variance slows down the convergence when seeking high accuracy:

$$\mathcal{O}\left(\kappa \log \frac{1}{\epsilon} + \frac{\sigma^2}{\mu\epsilon}\right)$$

Q: Can we design gradient estimators with reduced variance?

Stochastic Varianced-reduced Methods

Stochastic variance-reduced methods are as cheap to update as SGD, but have as fast convergence as full gradient descent.

Popular algorithms:

- SAG (stochastic average gradient) [Le Roux et al., 2012]
- SVRG (stochastic variance-reduced gradient) [Johnson and Zhang, 2013]
- SDCA (stochastic dual coordinate ascent) [Shalev-Shwartz and Zhang, 2013]
- SAGA (stochastic average gradient amélioré) [Defazio et al., 2014]
- Many many others: MISO, Finito, Catalyst-SVRG, S2GD, etc.
- Recent variants for nonconvex setting: SPIDER, SARAH, STORM, PAGE, etc.

Preview of VR Methods

Algorithm	# of Iterations	Per-iteration Cost
GD	$O\left((\kappa \log \frac{1}{\epsilon}\right)$	$O(n)$
SGD	$O\left(\frac{\kappa}{\epsilon}\right)$	$O(1)$
VR	$O\left((n+\kappa) \log \frac{1}{\epsilon}\right)$	$O(1)$

Table: Complexity of strongly convex and smooth finite-sum optimization

Preview of VR Methods

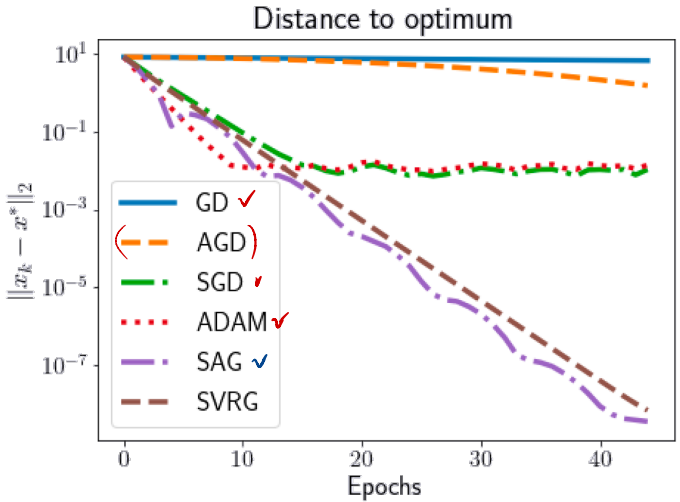


Figure: Logistic regression on `mushrooms` dataset with $n = 8124$ [Gow20]

Lecture Outline

Variance Reduction Techniques

Stochastic Variance-reduced Methods

- SAG/SAGA
- SVRG

Classical Variance Reduction Techniques

$$\min_{\mathbf{x}\in\mathbb{R}^d} \quad F(\mathbf{x}) := \frac{1}{n}\sum_{i=1}^{n} f_i(\mathbf{x})$$

- Mini-batching: Use the average of gradients from a random subset

$$\mathbf{x}_{t+1} = \mathbf{x}_t - \gamma_t \frac{1}{|B_t|}\sum_{i\in B_t} \nabla f_i(\mathbf{x}_t)$$

NB: Variance reduction comes at a computational cost.

- Momentum: add momentum to the gradient step

$$\mathbf{x}_{t+1} = \mathbf{x}_t - \gamma_t \hat{\mathbf{m}}_t, \text{ where } \hat{\mathbf{m}}_t = c \cdot \sum_{\tau=1}^{t} \alpha^{t-\tau} \nabla f_{i_\tau}(\mathbf{x}_\tau)$$

NB: Here $\mathbf{m}_t$ is the weighted average of the past stochastic gradients.

A Modern Variance Reduction Technique

Suppose we want to estimate $\theta = \mathbb{E}[X]$, X is a random variable.
Consider the **point estimator** for θ:

$$\hat{\Theta} := X - Y$$

- $\mathbb{E}[X - Y] = \theta$ if and only if $\mathbb{E}[Y] = 0$
- $\mathbb{V}[X - Y]$ is less than $\mathbb{V}[X]$ if Y is highly positively correlated with X.

A Modern Variance Reduction Technique

Suppose X is positively correlated with Y and we can compute $\mathbb{E}[Y]$.

Point Estimator:

$$\hat{\Theta}_\alpha = \alpha(X - Y) + \mathbb{E}[Y], \quad (0 \leq \alpha \leq 1).$$

$$\mathbb{E}[\hat{\Theta}_\alpha] = \alpha\mathbb{E}[X] + (1-\alpha)\mathbb{E}[Y]$$

($\alpha = 1$ ... unbiased!)

$$\mathbb{V}[\hat{\Theta}_\alpha] = \alpha^2(\mathbb{V}[X] + \mathbb{V}[Y] - 2\mathrm{Cov}[X, Y])$$

α small $\Rightarrow$ low variance

if $\mathrm{Cov}[X, Y]$ is large $\Rightarrow$ low variance

- If covariance is sufficiently large, then $\mathbb{V}[\hat{\Theta}_\alpha] \leq \mathbb{V}[X]$.

Motivation

Q: Can we design cheap gradient estimators with reduced variance?

Key Idea: if $\mathbf{x}_t$ is not too far away from previous iterates, then we can leverage previous gradient information to construct positively correlated control variates.

- SGD: estimate $\nabla F(\mathbf{x}_t)$ by $\nabla f_{i_t}(\mathbf{x}_t)$
- VR: estimate $\nabla F(\mathbf{x}_t)$ by $\mathbf{g}_t := \alpha(\nabla f_{i_t}(\mathbf{x}_t) - Y) + \mathbb{E}[Y]$ such that

$$\mathbb{E}[\|\mathbf{g}_t - \nabla F(\mathbf{x}_t)\|^2] \to 0, \text{ as } t \to \infty. \qquad \text{(VR property)}$$

So how to design Y?

Design Ideas

Goal: Construct Y that is positively correlated to $X = \nabla f_{i_t}(\mathbf{x}_t)$:

Choice I: $Y = \nabla f_{i_t}(\mathbf{x}^\star)$, where $\mathbf{x}^\star$ is the optimal solution

- $\mathbb{E}[Y] = 0$, unrealistic but conceptually useful

Choice II: $Y = \nabla f_{i_t}(\bar{\mathbf{x}}_{i_t})$, where $\bar{\mathbf{x}}_i$ is the last point for which we evaluated $\nabla f_i(\bar{\mathbf{x}}_i)$

- $\mathbb{E}[Y] = \frac{1}{n}\sum_{i=1}^n \nabla f_i(\bar{\mathbf{x}}_i)$, requires storage of $\{\bar{\mathbf{x}}_i\}_{i=1}^n$ or $\{\nabla f_i(\bar{\mathbf{x}}_i)\}_{i=1}^n$

Choice III: $Y = \nabla f_{i_t}(\tilde{\mathbf{x}})$, where $\tilde{\mathbf{x}}$ is some fixed reference point

- $\mathbb{E}[Y] = \frac{1}{n}\sum_{i=1}^n \nabla f_i(\tilde{\mathbf{x}})$, requires computing the full gradient at $\tilde{\mathbf{x}}$

Lecture Outline

Variance Reduction Techniques

Stochastic Variance-reduced Methods

- SAG/SAGA
- SVRG

Variance Reduction Techniques for Finite Sum Problems

Goal: estimate $\theta = \nabla F(\mathbf{x}_t)$, $X = \nabla f_{i_t}(\mathbf{x}_t)$

- SGD: $\mathbf{g}_t = \nabla f_{i_t}(\mathbf{x}_t)$ $\quad [\alpha = 1, Y = 0]$
- SAG: $\mathbf{g}_t = \frac{1}{n}(\nabla f_{i_t}(\mathbf{x}_t) - \mathbf{v}_{i_t}) + \frac{1}{n}\sum_{i=1}^n \mathbf{v}_i$ $\quad [\alpha = \frac{1}{n}, Y = \mathbf{v}_{i_t}]$
- SAGA: $\mathbf{g}_t = (\nabla f_{i_t}(\mathbf{x}_t) - \mathbf{v}_{i_t}) + \frac{1}{n}\sum_{i=1}^n \mathbf{v}_i$ $\quad [\alpha = 1, Y = \mathbf{v}_{i_t}]$

  Here $\{\mathbf{v}_i, i = 1, \ldots, n\}$ are the past stored gradients for each component.
- SVRG: $\mathbf{g}_t = \nabla f_{i_t}(\mathbf{x}_t) - \nabla f_{i_t}(\tilde{\mathbf{x}}) + \nabla F(\tilde{\mathbf{x}})$ $\quad [\alpha = 1, Y = \nabla f_{i_t}(\tilde{\mathbf{x}})]$

reference point

Stochastic Average Gradient (SAG)

Idea: keep track of the average of $\mathbf{v}_i$ as an estimate of the full gradient

$$\mathbf{g}_t = \frac{1}{n}\sum_{i=1}^{n} \mathbf{v}_i^t \qquad \approx \qquad \frac{1}{n}\sum_{i=1}^{n} \nabla f_i(\mathbf{x}_t) = \nabla F(\mathbf{x}_t)$$

- The past gradients are updated as:

$$\mathbf{v}_i^t = \begin{cases} \nabla f_{i_t}(\mathbf{x}_t), & \text{if } i = i_t, \\ \mathbf{v}_i^{t-1}, & \text{if } i \neq i_t. \end{cases}$$

- Equivalently, we have

$$\mathbf{g}_t = \mathbf{g}_{t-1} - \frac{1}{n}\mathbf{v}_{i_t}^{t-1} + \frac{1}{n}\nabla f_{i_t}(\mathbf{x}_t)$$

Stochastic Average Gradient (SAG, continued)

$$\mathbf{x}_{t+1} = \mathbf{x}_t - \frac{\gamma}{n}\sum_{i=1}^{n}\mathbf{v}_i^t, \text{ where } \mathbf{v}_i^t = \begin{cases} \nabla f_{i_t}(\mathbf{x}_t), & \text{if } i = i_t \\ \mathbf{v}_i^{t-1}, & \text{otherwise} \end{cases}$$

Algorithm SAG (Le Roux et al., 2012)

Initialize $\mathbf{v}_i = 0, i = 1, \ldots, n$
for $t = 1, 2, \ldots, T$ **do**
  Randomly pick $i_t \in \{1, 2, \ldots, n\}$
  $\mathbf{g}_t = \mathbf{g}_{t-1} - \frac{1}{n}\mathbf{v}_{i_t}$
  $\mathbf{v}_{i_t} = \nabla f_{i_t}(\mathbf{x}_t)$
  $\mathbf{g}_t = \mathbf{g}_t + \frac{1}{n}\mathbf{v}_{i_t}$
  $\mathbf{x}_{t+1} = \mathbf{x}_t - \gamma\mathbf{g}_t$
end for

- Biased gradient
- Cheap iteration cost
- $O(nd)$ memory cost
- Hard to analyze

Stochastic Average Gradient (SAG, continued)

- **Linear convergence:** The first stochastic methods to enjoy linear rate using a constant stepsize for strongly-convex and smooth objectives.

  If F is μ-strongly convex and each f_i is L_i-smooth and convex, setting $\gamma = 1/(16L_{\max})$, one can show that

  $$\mathbb{E}[F(\mathbf{x}_t) - F(\mathbf{x}^\star)] \leq C \cdot \left(1 - \min\{\frac{1}{8n}, \frac{\mu}{16L_{\max}}\}\right)^t.$$

  Here $L_{\max} := \max\{L_1, \ldots, L_n\}$.

- **Memory cost:** $O(n)$ times higher than SGD/SVRG
- **Per-iteration cost**: one gradient evaluation
- Total complexity: $O\left((n + \kappa_{\max}) \log(\frac{1}{\epsilon})\right)$.

SAGA

$E[g_t] = \nabla f(x_t) - \frac{1}{n}\sum_{i=1}^{n} v_i^{t-1} + \frac{1}{n}\sum_{i=1}^{n} v_i^{t-1} = \nabla f(x_t)$, with $\frac{1}{n}\sum_{i=1}^{n} v_i^{t-1} = 0$

SAGA (Defazio, Bach, Lacoste-Julien, 2016):

$$\mathbf{x}_{t+1} = \mathbf{x}_t - \gamma \left[(\nabla f_{i_t}(\mathbf{x}_t) - \mathbf{v}_{i_t}^{t-1}) + \frac{1}{n}\sum_{i=1}^{n} \mathbf{v}_i^{t-1} \right]$$

g_t

Store past gradients $\forall i$

- Unbiased update, while SAG is biased
- Same $O(nd)$ memory cost as SAG
- Similar linear convergence rate as SAG, but has a much simpler proof

Stochastic Variance Reduced Gradient (SVRG)

Key idea: Build covariates based on fixed reference point; balance the frequency of reference point update and the variance reduction.

Algorithm Stochastic Variance Reduced Gradient (Johnson & Zhang '13)

for $s = 1, 2, \ldots$ **do** — outer loop
  Set $\tilde{\mathbf{x}} = \tilde{\mathbf{x}}^{s-1}$ and compute $\nabla F(\tilde{\mathbf{x}}) = \frac{1}{n}\sum_{i=1}^{n} \nabla f_i(\tilde{\mathbf{x}})$ (update snapshot)
  Initialize $\mathbf{x}_0 = \tilde{\mathbf{x}}$
  for $t = 0, 1, \ldots, m-1$ **do** — inner loop
    Randomly pick $i_t \in \{1, 2, \ldots, n\}$ and update
    $\mathbf{x}_{t+1} = \mathbf{x}_t - \eta\left(\nabla f_{i_t}(\mathbf{x}_t) - \nabla f_{i_t}(\tilde{\mathbf{x}}) + \nabla F(\tilde{\mathbf{x}})\right)$ (cheap cost)
  end for
  Update $\tilde{\mathbf{x}}^s = \frac{1}{m}\sum_{t=0}^{m-1} \mathbf{x}_t$ ← average (in practice can use $\tilde{x}^s = x_m$)
end for

SVRG: Key Features

Intuition: the closer $\tilde{\mathbf{x}}$ is to $\mathbf{x}_t$, the smaller the variance of the gradient estimator

$$\mathbb{E}[\|\mathbf{g}_t - \nabla F(\mathbf{x}_t)\|^2] \leq \mathbb{E}[\|\nabla f_{i_t}(\mathbf{x}_t) - \nabla f_{i_t}(\tilde{\mathbf{x}})\|^2] \leq L_{\max}^2 \|\mathbf{x}_t - \tilde{\mathbf{x}}\|^2$$

Two-loop structure:

- Outer loop: update reference point and compute its full gradient at $O(n)$ cost
- Inner loop: update iterates with variance-reduced gradient for m steps
- Total of $O(n + 2m)$ component gradient evaluations at each epoch

Compare to SAG/SAGA $\Rightarrow m \approx n$

(+) Cheap memory cost, no need to store past gradients or past iterates

(−) More parameter tuning, two gradient computation per iteration

Convergence of SVRG

Theorem 9.1 (Johnson & Zhang, 2013)

Assume each $f_i(\mathbf{x})$ is convex and L_i-smooth, $F(\mathbf{x})$ is μ-strongly convex. Assume m is sufficiently large and $\eta < \frac{1}{2L_{max}}$ such that $\rho = \frac{1}{\mu\eta(1-2\eta L_{max})m} + \frac{2\eta L_{max}}{1-2\eta L_{max}} < 1$, then

$$\mathbb{E}[F(\tilde{\mathbf{x}}^s) - F(\mathbf{x}^*)] \leq \rho^s \, [F(\tilde{\mathbf{x}}^0) - F(\mathbf{x}^*)].$$

s ... # outer loops

- **Linear convergence:** choose $m = O(\frac{L_{\max}}{\mu})$, $\eta = O(\frac{1}{L_{\max}})$ such that $\rho \in (0, \frac{1}{2})$.
- **Total complexity**:

$$O\left((2m+n)\log\frac{1}{\epsilon}\right) = O\left(\left(n + \frac{L_{\max}}{\mu}\right)\log\frac{1}{\epsilon}\right).$$

SVRG vs. SAG/SAGA

Table: Comparisons between SVRG and SAG/SAGA

	SVRG	SAG/SAGA
memory cost	$O(d)$	$O(nd)$
epoch-based	yes	no
# gradients per step	at least 2	1
parameters	stepsize & epoch length	stepsize
unbiasedness	yes	yes/no
total complexity	$O\left((n+\kappa_{\max})\log\frac{1}{\epsilon}\right)$	$O\left((n+\kappa_{\max})\log\frac{1}{\epsilon}\right)$

just the reference point

Loopless-SVRG: [Hofmann et al., 2015]

Numerical Illustration

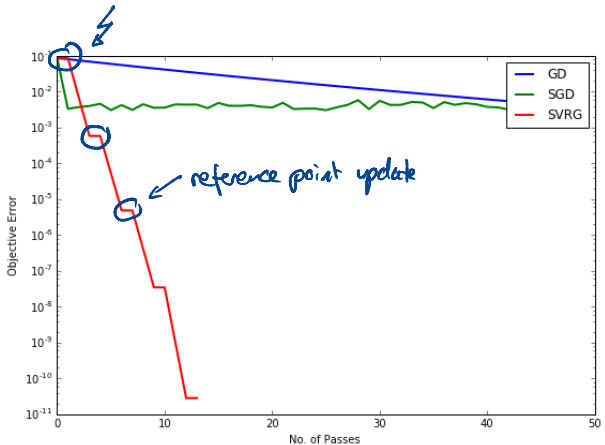


Figure: Numerical illustration among GD, SGD, SVRG on logistic regression.

convex!

Convergence Analysis of SVRG: Key Lemma

Lemma 9.2 (Exercise, property of smoothness)

$$\frac{1}{n}\sum_{i=1}^{n}\|\nabla f_i(\mathbf{x})-\nabla f_i(\mathbf{x}^\star)\|_2^2 \le 2L_{max}(F(\mathbf{x})-F(\mathbf{x}^\star))$$

Lemma 9.3 (Bound of variance)

Denote $\mathbf{g}_t = \nabla f_{i_t}(\mathbf{x}^t) - \nabla f_{i_t}(\tilde{\mathbf{x}}) + \nabla F(\tilde{\mathbf{x}})$. *We have*

$$\mathbb{E}[\|\mathbf{g}_t\|_2^2] \le 4L_{max}[F(\mathbf{x}_t) - F(\mathbf{x}^\star) + F(\tilde{\mathbf{x}}) - F(\mathbf{x}^\star)].$$

Convergence Analysis of SVRG: Proof

For notation simplicity, denote $L = L_{\max}$. From Lemma 9.3, we have

$$\begin{aligned}
&\mathbb{E}\left[\|\mathbf{x}_{t+1} - \mathbf{x}^\star\|_2^2\right] \\
&= \|\mathbf{x}_t - \mathbf{x}^\star\|_2^2 - 2\eta(\mathbf{x}_t - \mathbf{x}^\star)^\top \mathbb{E}\left[\mathbf{g}_t\right] + \eta^2 \mathbb{E}\left[\|\mathbf{g}_t\|_2^2\right] \\
&\leq \|\mathbf{x}_t - \mathbf{x}^\star\|_2^2 - 2\eta(1 - 2L\eta)(F(\mathbf{x}_t) - F(\mathbf{x}^\star)) + 4L\eta^2\left[F(\tilde{\mathbf{x}}) - F(\mathbf{x}^\star)\right]
\end{aligned}$$

We can then establish the contraction after telescoping the sum and invoking the definition for $\tilde{\mathbf{x}}$.

Convergence Analysis of SVRG: Proof (continued)

It follows that

$$
\begin{aligned}
&\mathbb{E}\left[\|\mathbf{x}_m - \mathbf{x}_\star\|^2\right] + 2\eta(1-2L\eta)m\mathbb{E}\left[f(\tilde{\mathbf{x}}^s) - f(\mathbf{x}^\star)\right] \\
&\leq \mathbb{E}\left[\|\mathbf{x}_m - \mathbf{x}_\star\|^2\right] + 2\eta(1-2L\eta)\sum\nolimits_{t=0}^{m-1}\mathbb{E}\left[f(\mathbf{x}_t) - f(\mathbf{x}^\star)\right] && \text{(by convexity)} \\
&\leq \mathbb{E}\left[\|\mathbf{x}_0 - \mathbf{x}^\star\|^2\right] + 4Lm\eta^2\mathbb{E}\left[f(\tilde{\mathbf{x}}^{s-1}) - f(\mathbf{x}^\star)\right] && \text{(by telescoping)} \\
&\leq \mathbb{E}\left[\|\tilde{\mathbf{x}}^{s-1} - \mathbf{x}^\star\|^2\right] + 4Lm\eta^2\mathbb{E}\left[f(\tilde{\mathbf{x}}^{s-1}) - f(\mathbf{x}^\star)\right] && \text{(by definition of } \mathbf{x}_0\text{)} \\
&\leq \frac{2}{\mu}\mathbb{E}\left[f(\tilde{\mathbf{x}}^{s-1}) - f(\mathbf{x}^\star)\right] + 4Lm\eta^2\mathbb{E}\left[f(\tilde{\mathbf{x}}^{s-1}) - f(\mathbf{x}^\star)\right] && \text{(by } \mu \text{ strongly convexity)}
\end{aligned}
$$

This further implies

$$
\mathbb{E}\left[f(\tilde{\mathbf{x}}^s) - f(\mathbf{x}^\star)\right] \leq \left[\frac{1}{\mu\eta(1-2L\eta)m} + \frac{2L\eta}{1-2L\eta}\right]\mathbb{E}\left[f(\tilde{\mathbf{x}}^{s-1}) - f(\mathbf{x}^\star)\right].
$$

□

Summary: Finite Sum Optimization

$$\min_{\mathbf{x}\in\mathbb{R}^d} \quad F(\mathbf{x}) := \frac{1}{n}\sum_{i=1}^{n} f_i(\mathbf{x})$$

(f_i is L_i-smooth and convex, F is L-smooth and μ-strongly convex)

Algorithm	# of Iterations	Per-iteration Cost
GD	$O\left(\kappa \log \frac{1}{\epsilon}\right)$	$O(n)$
SGD	$O\left(\frac{\kappa_{\max}}{\epsilon}\right)$	$O(1)$
SAG/SAGA/SVRG	$O\left((n+\kappa_{\max}) \log \frac{1}{\epsilon}\right)$	$O(1)$

Table: Complexity of finite-sum optimization, $\kappa = \frac{L}{\mu}$, $\kappa_{\max} = \frac{L_{\max}}{\mu}$

Remarks

- Variance reduction technique is crucial for finite sum problems.
- In general, $L \leq L_{\max} \leq nL$. VR methods are always superior in terms of total runtime than GD.
- If $L_i = L, \forall i$, then $\kappa = \kappa_{\max}$, VR methods are much faster than GD especially when $\kappa = O(n)$.
- SGD has much worse dependency on ϵ than VR methods, which explain its poor performance when ϵ is small.

Can we further improve the VR methods?

- Non-uniform sampling: improve to $O\left((n+\kappa_{\text{avg}})\log\frac{1}{\epsilon}\right)$

$$P(i_t = i) = \frac{L_i}{\sum_{i=1}^{n} L_i}$$

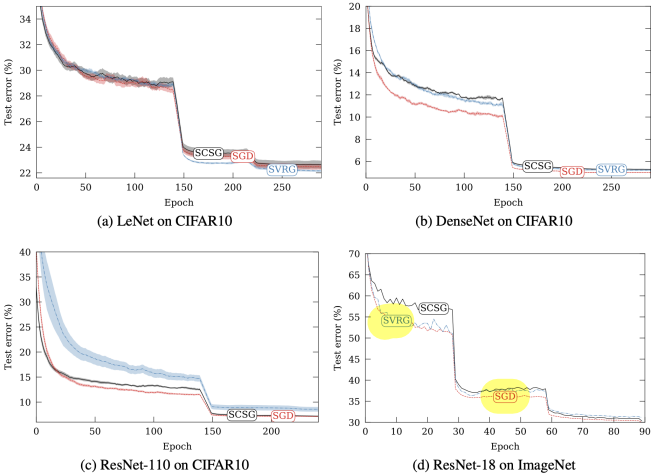
- Incorporating acceleration: can improve to $O\left((n+\sqrt{n\kappa_{\max}})\log\frac{1}{\epsilon}\right)$.

- Lower complexity bound: $O\left((n+\sqrt{n\kappa_{\max}})\log\frac{1}{\epsilon}\right)$ for the strongly-convex and smooth finite-sum problems considered
(Woodworth and Srebro, 2016; Lan and Zhou, 2018)

Limitations?

- Challenges with practical implementations: learning rate and sampling
- Less advantage beyond smooth or strongly convex objectives or finite-sum setting
- VR may be ineffective for training neural networks [Defazio and Bottou, 2019].

$\eta = 0.1$ for epochs 1-30
$\eta = 0.01$ for epoch > 30



(a) LeNet on CIFAR10

(b) DenseNet on CIFAR10

(c) ResNet-110 on CIFAR10

(d) ResNet-18 on ImageNet

Lecture 9 Recap

- Finite-sum problems allow for variance reduction
  - reason: it is possible to query the stochastic oracle twice!
- Discussed several variance reduction techniques, with pros & cons

SVAG

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