

# Image Processing and Computer Vision

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Summer Term 2023

<http://www.mia.uni-saarland.de/Teaching/ipcv23.shtml>

*Welcome to this class!*

*Please do not hesitate to pose questions (in English or German).*

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## Image Processing and Computer Vision 2023 – Introduction

### Lecture 1: Introduction

#### Contents

1. Motivation and Important Areas within Visual Computing
2. Organisational Issues
3. Introductory Examples
4. Forthcoming Topics

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# Motivation

## Images and Applications

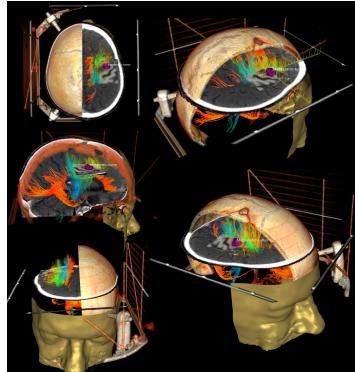
- ◆ in our everyday lives: increasing amount of devices that create images and videos
- ◆ Numerous applications require to analyse, improve, or modify those images.

autonomous driving/robotics



ESA/RAL Space/ESO

medical imaging



A. Jakab, University of Debrecen

image enhancement



Rombach et al. 2021

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## Synthetic Images

- ◆ also: fully synthetic images
- ◆ current trend: AI for image-to-text
- ◆ examples generated with stable diffusion (Rombach et al. 2021, DreamStudio)
- ◆ prompts below the images



a researcher with glasses, short beard, sitting in front of a screen, creating an image with stable diffusion, highly detailed, soft lighting



a student traveling through a perilous landscape made of exam papers, style: fantasy art

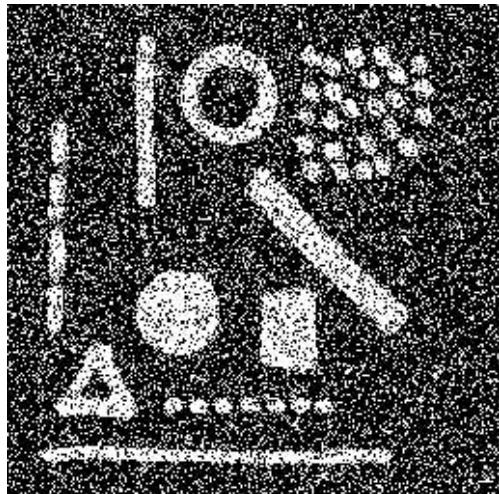


a cat studying computer science, style: pixel art

## Important Areas within Visual Computing

### Image Processing (Bildverarbeitung)

- ◆ transforms a digital image to another digital image that allows a better interpretation by humans or computers



Example of an image processing application. **Left:** Noisy image. **Right:** Filtered. Author: J. Weickert.

### Computer Vision, Image Understanding (Rechnersehen, Bildverstehen)

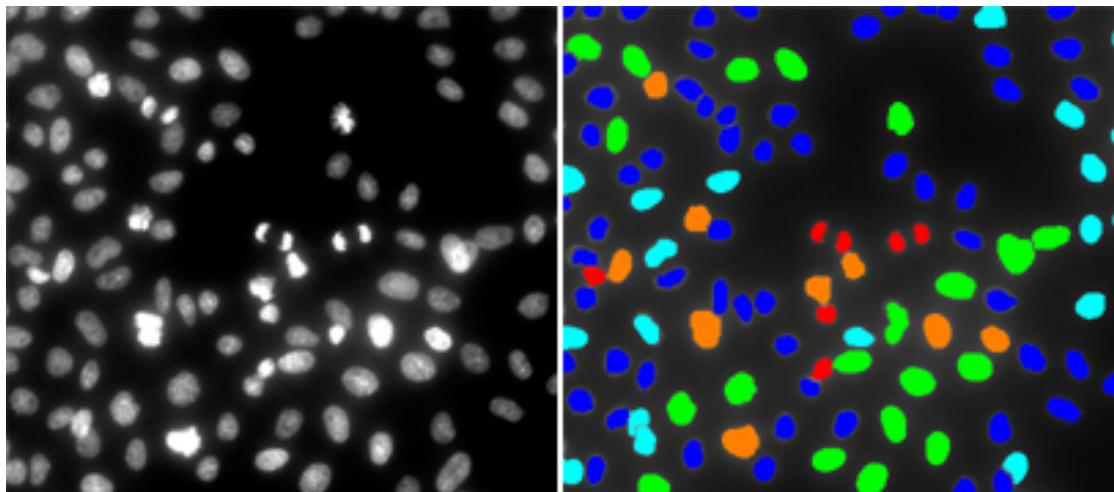
- ◆ extraction of information about the 3-D world from 2-D images



Example of a computer vision application. **Left:** Stereo image pair. **Right:** 3-D reconstruction. Authors: L. Alvarez, R. Deriche, J. Sánchez, J. Weickert.

## Pattern Recognition (Mustererkennung)

- ◆ labelling of image structures to certain classes



Example of a pattern recognition application. **Left:** Cells. **Right:** Classification into different cell cycle phases. Source: MetaMorph Analysis, <http://www.moleculardevices.com/>.

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## Computer Graphics (Computergrafik)

- ◆ synthesis of a digital image that is supposed to look like an image of a 3-D scene



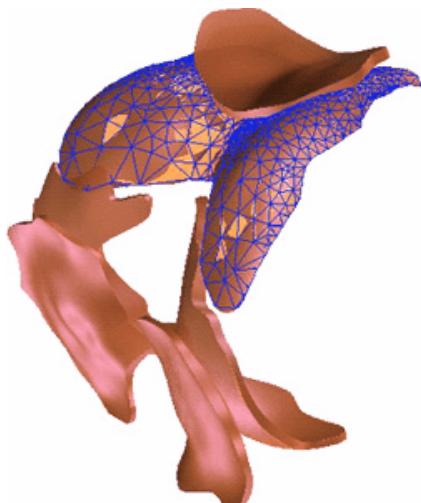
Example of a computer graphics application: Realistically looking simulation of a forest using ray tracing.  
Author: P. Slusallek.

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### Geometric Modelling (Geometrische Modellierung)

- ◆ mathematical representation of curves and surfaces
- ◆ allows to synthesise 2-D and 3-D objects on a computer

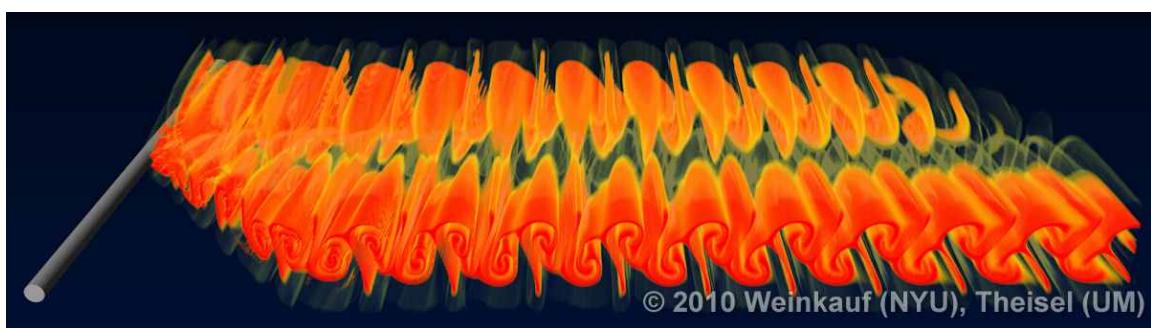


Example of a geometric modelling application: A surface model of a tissue in a human knee using a triangulation. Source: [http://www.scorec.rpi.edu/research\\_biomechanical.html](http://www.scorec.rpi.edu/research_biomechanical.html).

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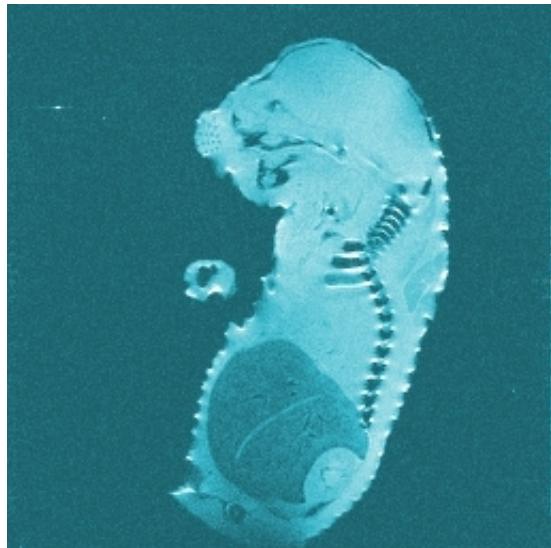
### Scientific Visualisation (Wissenschaftliche Visualisierung)

- ◆ useful visual representation of data from real experiments or computer simulations



Example of a scientific visualisation application: Three-dimensional flow visualisation. Authors: T. Weinkauf, H. Theisel.

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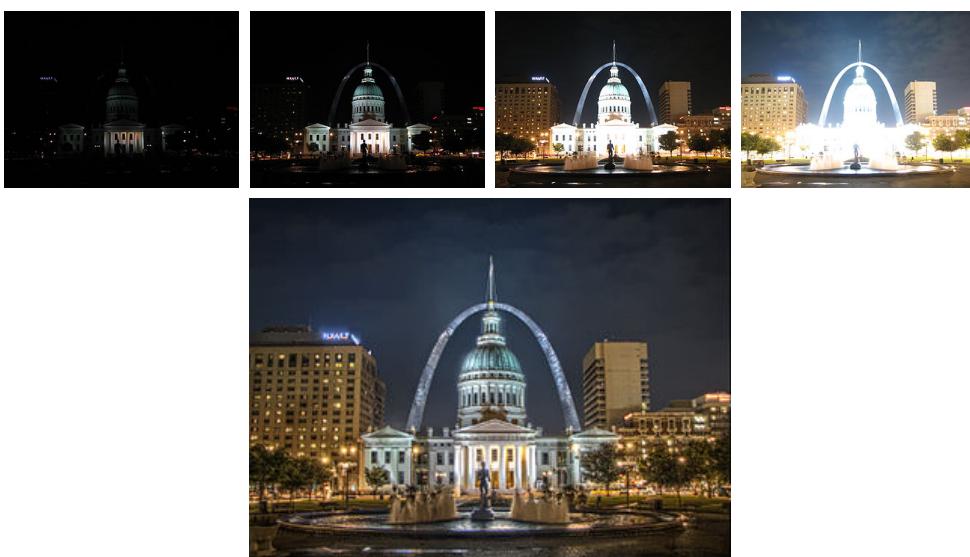


Example of an imaging application: Magnetic resonance image of a mouse embryo. Author: F. Volke.

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### Computational Photography

- ◆ combines image capture, processing and manipulation



Example of a computational photography application: **Top row:** Four differently exposed images. **Bottom row:** Fusion to a single image with local tone mapping. Source: [en.wikipedia.org/wiki/High\\_dynamic\\_range\\_imaging](http://en.wikipedia.org/wiki/High_dynamic_range_imaging).

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## Collective Terms

- ◆ **Image Analysis (Bildanalyse):**

includes image processing, computer vision, pattern recognition

- ◆ **Image Synthesis (Bildsynthese):**

includes computer graphics, geometric modelling, scientific visualisation

- ◆ **Visual Computing, Image Sciences (Bildwissenschaften):**

comprises image acquisition and the analysis and synthesis of digital images

We focus on image analysis, in particular on image processing and computer vision.

There is a growing confluence between the individual areas:

- ◆ Human motion capture and embedding into virtual scenes combines image analysis and image synthesis.
- ◆ Computational photography combines image acquisition, image analysis, and image synthesis.

## Outline

### Lecture 1: Introduction

### Contents

1. Motivation and Important Areas within Visual Computing
2. **Organisational Issues**
3. Introductory Examples
4. Forthcoming Topics

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# Organisatorial Issues

## Why is this Class Important?

- ◆ introduction to mathematically well-founded, model-based areas of image processing and computer vision
- ◆ important in numerous applications, e.g. medical imaging, computer-aided quality control, driver assistance systems, robotics, bioinformatics, computer graphics, multimedia, and artificial intelligence
- ◆ beautiful application area for almost all branches of mathematics
- ◆ qualifies for starting a bachelor thesis in our group;  
for a master thesis, one also has to attend  
*Differential Equations in Image Processing and Computer Vision*

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## How Can I Make Use of These Classes?

- ◆ 4 hours classroom lectures, 2 hours tutorials (9 ECTS points)
- ◆ can be used in numerous bachelor or master programs, e.g.:

Study Program	Usage
Visual Computing (VC)	VC core area / subarea image analysis
Computer Science (CS)	core class (Stammvorlesung) or mathematics class if minor is mathematics
Mathematics	mathematics core class or CS class if minor is CS
Mathematics & CS	core class in mathematics or CS
DSA1	DSA1 core class (bachelor and master)
Embedded Systems	master core class
Media Informatics	master core class
Bioinformatics	master core class
Physics	non-physical free elective master class

### Which Prerequisites are Required?

- ◆ **undergraduate mathematics** (e.g. *Mathematik für Informatiker I–III*)

examples of suitable textbooks:

- E. Kreyszig: *Advanced Engineering Mathematics*. Wiley, Chichester, 2010.
- M. Wolff, P. Hauck und W. Küchlin: *Mathematik für Informatik und Bioinformatik*. Springer, Berlin, 2004.

- ◆ **elementary C knowledge** (for the programming assignments)

can be learned e.g. in

- R. Kirsch, U. Schmitt: *Programmieren in C – Eine mathematikorientierte Einführung*. Springer, Berlin, 2013.
- *C Programming*  
<https://www.learn-c.org/>

- ◆ **reasonable working knowledge of English**

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### Tutorials

- ◆ combined theoretical and programming assignments
- ◆ coordinated by Michael Ertel (Teams and ertel@mia.uni-saarland.de)

Head Tutor



Michael Ertel



Aseer Ahmad  
Ansari



Cameron  
Braunstein

Tutors



Soumava  
Paul



Chengjiangrong  
Peng



Aheli Saha

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### Tutorials

- ◆ **Register for the tutorials in Teams until April 20.**  
**This is mandatory for course participation.**
- ◆ multiple options for in person and online tutorials:
  - Group T1: Tue, 12:15 – 14:00
  - Group T2: Tue, 14:15 – 16:00
  - Group T3: Tue, 16:15 – 18:00
  - Group W1: Wed, 10:15 – 12:00
  - Group W2: Wed, 12:15 – 14:00
  - Group W3: Wed, 14:15 – 16:00
  - Group W4: Wed, 16:15 – 18:00
- ◆ Optional Guided Programming Helpdesk: Tue, 18:15 – 20:00  
contact Michael Ertel ([ertel@mia.uni-saarland.de](mailto:ertel@mia.uni-saarland.de)) for online appointments

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### Homework

- ◆ Each week: theory or programming assignments worth 24 points in total.
- ◆ Submit assignments by posting your solutions in your private channel in teams.
- ◆ Both handwritten (scanned) and digitally typeset solutions in pdf format are fine for theory submissions, as long as they are readable.

### Classroom Work

- ◆ There are additional assignments each week intended for group work.
- ◆ You get 12 points for participation with either of these 3 options:
  - Option 1:** Attend in-person tutorials.
  - Option 2:** Cooperate online in your private channels and report to your tutor.
  - Option 3:** Submit solution attempts (not graded, do not have to be complete).

### Written Exams

- ◆ Self-test problems (Probeklausur) with sample solutions will be available.
- ◆ admission: 312/468 tutorial points (13 tutorials)
- ◆ Two written exams (physical presences at UdS required):
  - Tuesday, July 25, 2023, 2 pm, Building E2.2, Günter Hotz Lecture Theatre.
  - Monday, October 9, 2023, 2 pm, Building E2.2, Günter Hotz Lecture Theatre
- ◆ You need to register for the exams on LSF.
- ◆ Two exams count as two attempts.  
If you participate in both, the better grade counts.
- ◆ Both exams will be closed book exams.  
You are allowed and obliged to bring three things to your desk only:
  - your student ID card (Studierendenausweis)
  - a ball-pen that has no function other than writing
  - one double-sided, hand-written A4 cheat sheet  
(will be collected after the exam and returned at the inspection meeting)
- ◆ You must not take anything with you that contains information about the exam.

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### Questions and Problems

- ◆ Please feel free to ask me during the lectures, in the break, or in the office hour  
(Tue, 12:30–13:30, send message or mail for an appointment).
- ◆ Also your tutor and Michael Ertel will be happy to help.
  - Michael Ertel's office hour is on Tuesday, 15:00–16:00.
  - Tutor office hours will be provided on the IPCV webpage.

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## Planned Contents



Foundations and Transformations

Image Processing

Computer Vision and Image Understanding

The IPCV class resembles a triptychon, consisting of three parts of roughly equal size. Image source: <https://commons.wikimedia.org/wiki/File:Welterbe-Wein-Tryptychon.jpg>.

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### Part I: Foundations and Transformations

#### 1. Foundations

- 1.1 Definitions, Image Types, Discretisation
- 1.2 Degradations in Digital Images
- 1.3 Colour Perception and Colour Spaces

#### 2. Image Transformations

- 2.1 Continuous Fourier Transform
- 2.2 Discrete Fourier Transform
- 2.3 Discrete Cosine Transform
- 2.4 Image Pyramids
- 2.5 Wavelet Transform

#### 3. Image Compression

#### 4. Image Interpolation

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### Part II: Image Processing

#### 5. Point Operations

#### 6. Linear Filters

- 6.1 System Theory
- 6.2 Derivative Approximations
- 6.3 Edge and Corner Detection

#### 7. Nonlinear Filters

- 7.1 Morphology and Median Filters
- 7.2 Wavelet Shrinkage
- 7.3 Bilateral Filters
- 7.4 NL Means
- 7.5 Nonlinear Diffusion Filtering

#### 8. Global Filters

- 8.1 Discrete Variational Methods
- 8.2 Continuous Variational Methods
- 8.3 Deconvolution Methods

#### 9. Texture Analysis

## Planned Contents (4)



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### Part III: Computer Vision and Image Understanding

#### 10. Image Sequence Analysis

- 10.1 Local Methods
- 10.2 Variational Methods

#### 11. 3-D Reconstruction

- 11.1 Camera Geometry
- 11.2 Stereo Reconstruction
- 11.3 Shape-from-Shading

#### 12. Object Recognition

- 12.1 Segmentation
- 12.2 Hough Transform
- 12.3 Invariants
- 12.4 Eigenspace Methods
- 12.5 Neural Networks and Deep Learning

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## References

We do **not** rely on a single textbook:  
 Our didactic concept differs from all of them.  
 However, here are some of the more useful books:

### Textbooks on Image Processing

- ◆ J. Bigun: *Vision with Direction*. Springer, Berlin, 2010.  
*One of most systematic books on image processing, with a clear focus on matrix representations.*
- ◆ R. C. Gonzalez, R. E. Woods: *Digital Image Processing*. Global Edition, Pearson Prentice Hall, Upper Saddle River, 2017.  
*A classical book on image processing. Comprehensive and fairly well readable.*
- ◆ H. Süße, E. Rodner: *Bildverarbeitung und Objekterkennung*. Springer, Wiesbaden, 2014.  
*German book on selected aspects of image processing, computer vision, and machine learning.*

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### Textbooks on Computer Vision

- ◆ R. Klette: *Concise Computer Vision*. Springer, London, 2014.  
*Covers also some more recent methods, but not everything.*
- ◆ R. Szeliski: *Computer Vision: Algorithms and Applications*. Springer, New York, Second Edition, 2022.  
[http://szeliski.org/Book/drafts/SzeliskiBook\\_20100903\\_draft.pdf](http://szeliski.org/Book/drafts/SzeliskiBook_20100903_draft.pdf)  
*Covers many things, but not very detailed.*
- ◆ E. Trucco, A. Verri: *Introductory Techniques for 3-D Computer Vision*. Prentice-Hall, Upper Saddle River, 1998.  
*Good selection of some important aspects, but not up to date in others.*
- ◆ B. K. P. Horn: *Robot Vision*. MIT Press, Cambridge, MA, 1986.  
*Very old, but stood the test of time. Excellent w.r.t. continuous modelling.*

These and other textbooks can be found in the course reading (Semesterapparat) of our Computer Science and Mathematics Library (Building E2.3).

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### Useful Internet Resources

◆ **CV Online**

<https://homepages.inf.ed.ac.uk/rbf/CVonline/>

Online compendium on numerous image processing and computer vision topics.

Often very useful.

◆ **Annotated Computer Vision Bibliography**

[www.visionbib.com/bibliography/contents.html](http://www.visionbib.com/bibliography/contents.html)

helpful when searching for specific references for a certain topic

◆ **Google Scholar**

<https://scholar.google.com>

provides links to highly cited articles of all scientific areas

# Sampling and Quantisation

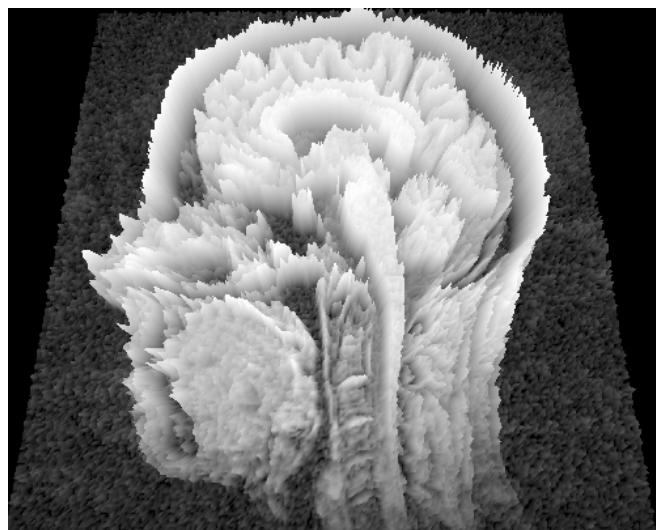
## Continuous Greyscale Image (Kontinuierliches Grauwertbild)

- ◆ mapping  $f$  from a rectangular domain (Definitionsbereich)  $\Omega = (0, a_1) \times (0, a_2)$  to a co-domain (Wertebereich)  $\mathbb{R}$ :

$$f : \mathbb{R}^2 \supset \Omega \rightarrow \mathbb{R}$$

- ◆ domain  $\Omega$  is called *image domain* or *image plane*
- ◆ co-domain  $\mathbb{R}$  specifies *grey value*
- ◆ Low grey values are dark and high grey values bright.

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**Left:** Magnetic resonance (MR) image of a human head. **Right:** Representation as a function  $f(x, y)$  over a rectangular image domain  $\Omega$ . Authors: J. Weickert, C. Schnörr.

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### Sampling (Abtastung)

- ◆ discretisation of the *domain*  $\Omega$
- ◆ Image data are only given on a rectangular point grid within the image domain  $\Omega$ .
- ◆ creates a digital image

$$\{f_{i,j} \mid i = 1, \dots, N; j = 1, \dots, M\}$$

- ◆ The grid point / cell  $(i, j)$  is called *pixel* (picture element).
- ◆ 2-D images often have equal pixel distances in both directions.
- ◆ Lazy image processing people often normalise these grid sizes to 1.  
**This can be dangerous and is not recommended!**
- ◆ If the sampling is too coarse, the image quality degrades severely.

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## Sampling and Quantisation (4)

256 × 256 pixels



128 × 128 pixels



64 × 64 pixels



32 × 32 pixels



Digital test image with different sampling rates. Author: J. Weickert.

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### Quantisation (Quantisierung)

- ◆ discretisation of the *co-domain*
- ◆ saves disk space
- ◆ Often a grey value is encoded with a single byte.  
Then the discrete co-domain is given by  $\{0, 1, \dots, 255\}$ .
- ◆ Binary images have the co-domain  $\{0, 1\}$ .
- ◆ Humans can distinguish only about 40 greyscales.  
They are even very good in recognising the content of binary images.

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256 greyscales



32 greyscales



8 greyscales



2 greyscales



Digital test image ( $256 \times 256$  pixels) with different quantisation rates. Author: J. Weickert.

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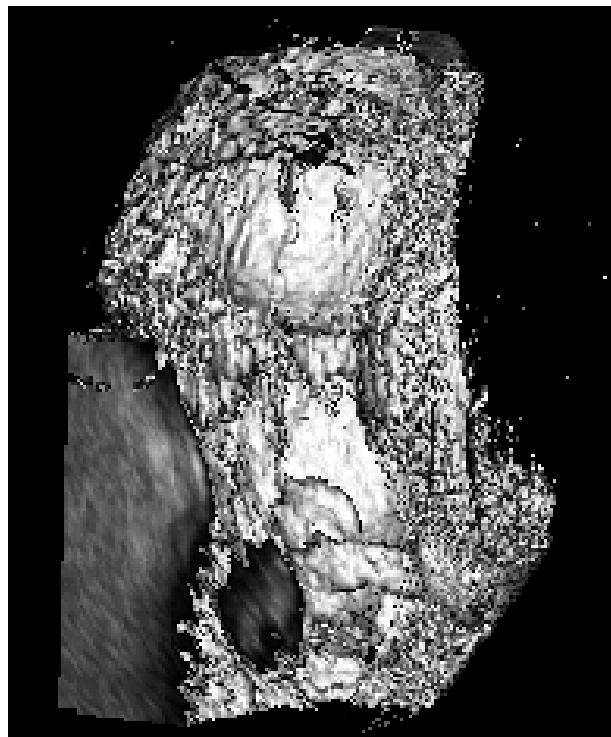
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## Types of Images

### *m*-dimensional Images

- ◆ have their domain in  $\mathbb{R}^m$
- ◆  $m = 1$ : signals
- ◆  $m = 2$ : (two-dimensional) images
- ◆  $m = 3$ : three-dimensional images
  - important in medical imaging, e.g. computerised tomography (CT, Computertomographie), magnetic resonance imaging (MRI, Kernspintomographie).
  - Image points/cells in 3-D are called *voxels* (volume elements).
  - Voxel dimensions usually differ in different directions!

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Rendering of a 3-D ultrasound image of a human fetus in its 10th week. Authors: J. Weickert, K. Zuiderweld, B.M. ter Haar Romeny, W. Niessen.

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## Vector-Valued Images

- ◆ have their co-domain in  $\mathbb{R}^n$ , containing  $n$  channels

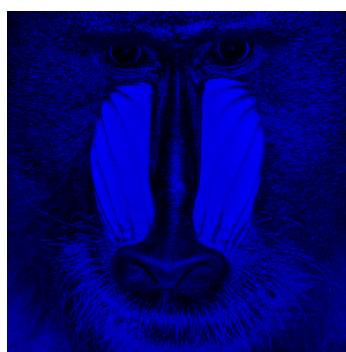
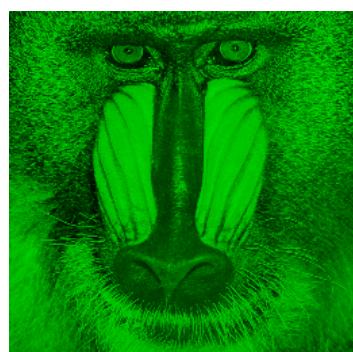
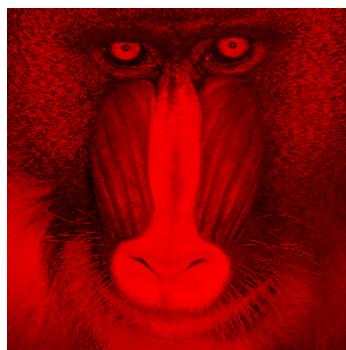
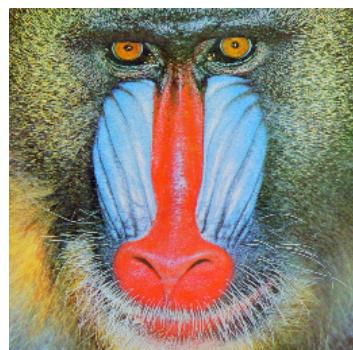
### ◆ Example 1: Colour Images

- three channels: R (red), G (green), B (blue)
- Humans can distinguish two million colours!

### ◆ Example 2: Satellite Images

- Different channels represent different frequency bands.
- Multispectral images use only a few channels.  
Hyperspectral images involve hundreds or thousands of channels.

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Colour image as example for a vector-valued image. **Top left:** Original image. **Top right:** Red channel. **Bottom left:** Green channel. **Bottom right:** Blue channel. Author: J. Weickert.

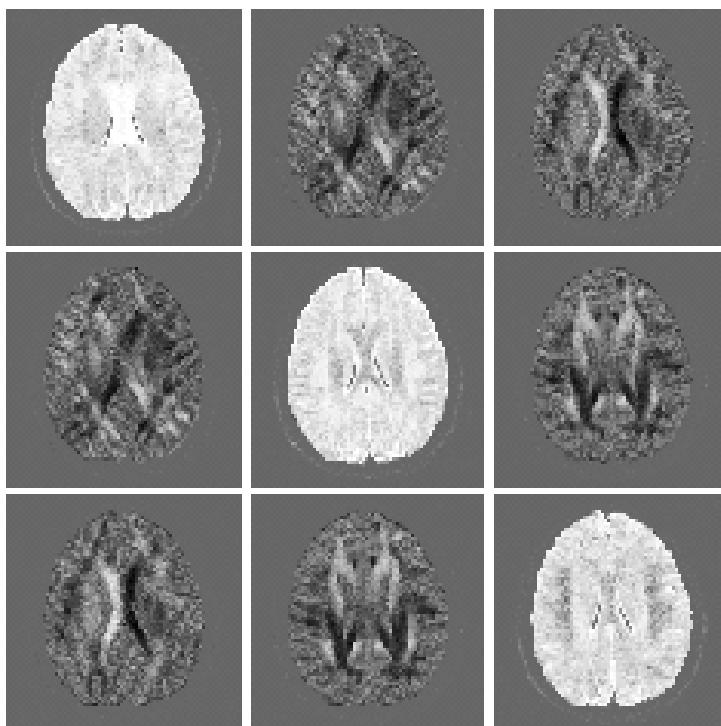
### Matrix-Valued Images

- ◆ have their co-domain in  $\mathbb{R}^{n \times n}$
- ◆ **Example: Diffusion Tensor Magnetic Resonance Imaging (DT-MRI)**
  - measures in each voxel in 3-D the diffusion properties of water molecules
  - described by a symmetric positive definite  $3 \times 3$  matrix
- ◆ may create additional constraints, e.g.:
 

A reasonable filter should not destroy physically relevant properties, such as positive definiteness of the matrix field.

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Images of the nine coefficients of a DT-MRI data set of a human brain. Since the diffusion matrix is a symmetric  $3 \times 3$  matrix, only 6 out of 9 images differ. Authors: D. Weinstein, G. Kindlmann, E. Lundberg.

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### Image Sequences (Bildfolgen)

- ◆ One can consider image sequences for any of the above mentioned types of images.
- ◆ The time variable increases the dimensionality of the domain from  $m$  to  $m+1$ .
- ◆ **Example: 3-D Echo Cardiography (Echokardiographie)**
  - creates a sequence of 3-D scalar-valued ultrasound images (can be regarded as 4-D image)

### Relevant Images in This Class

- ◆ We mainly focus on 2-D scalar-valued (i.e. greyscale) images, and their image sequences.
- ◆ This keeps the description as simple as possible.
- ◆ Many of our methods can be generalised to other types of images. Often this is not very challenging.

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**Form left to right and from top to bottom:** Four subsequent frames of an image sequence (the famous Hamburg taxi scene, 256 × 190 pixels). Can you recognise what has moved in which direction?  
Source: [i21www.ira.uka.de/image\\_sequences/](http://i21www.ira.uka.de/image_sequences/)

## Summary

- ◆ Digital images are discretised in two ways:  
in the domain (sampling) and the co-domain (quantisation).
- ◆ generalisation of the domain:  
 $m$ -dimensional images, image sequences
- ◆ generalisation of the co-domain:  
vector-valued images, matrix-valued images

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