Lecture 10: Point Operations

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Recently on IPCV ...

- Interpolation recovers continuous data from discrete samples.
- Classical interpolation involves
  - the function values,
  - a synthesis function that satisfies the interpolation condition.
  - Examples: nearest neighbour, linear, Keys, sinc interpolation
- Generalised interpolation renounces the interpolation condition.
  - It requires to compute the weight coefficients.
  - This extra effort can be rewarded by better quality.
  - Example: Cubic spline interpolation.
    It offers a favourable cost–performance ratio.

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What are Point Operations?

An image processing operation transforms an image into an improved version.
The result is more useful for a human– or computer–based interpretation.

Point Operations

- simplest transformations for image enhancement: global greyscale modification
- do not take into account grey value configuration in the neighbourhood
- For a greyscale image $f(x,y)$, all point operations have the structure

$$\phi : \quad f(x,y) \quad \longmapsto \quad g(x,y) = \phi(f(x,y))\,.$$

- Note that the location (x,y) does not matter for ϕ, only its grey value $f(x,y)$.
- Often ϕ is *nonlinear*, i.e. it may violate the superposition principle

$$\phi\big(\alpha\, f_1(x,y) + \beta\, f_2(x,y)\big) \;=\; \alpha\, \phi(f_1(x,y)) \;+\; \beta\, \phi(f_2(x,y)) \qquad \forall\, \alpha, \beta \in \mathbb{R}.$$

- Point operations can be useful in many applications and should be tried first.

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Affine Greyscale Transformations

- have a simple structure: $g(x,y) \;=\; a\, f(x,y) + b \quad (a, b \in \mathbb{R})$.
- For $b = 0$, they are also called *linear* greyscale transformations.
- allow numerous possibilities:

(1)	*identity:*	$a = 1,$	$b = 0$
(2)	*contrast enhancement:*	$a > 1$	
(3)	*contrast attenuation:*	$0 \leq a < 1$	
(4)	*brightening:*	$a = 1,$	$b > 0$
(5)	*darkening:*	$a = 1,$	$b < 0$
(6)	*greyscale reversion:*	$a = -1,$	$b = g_{\max}$

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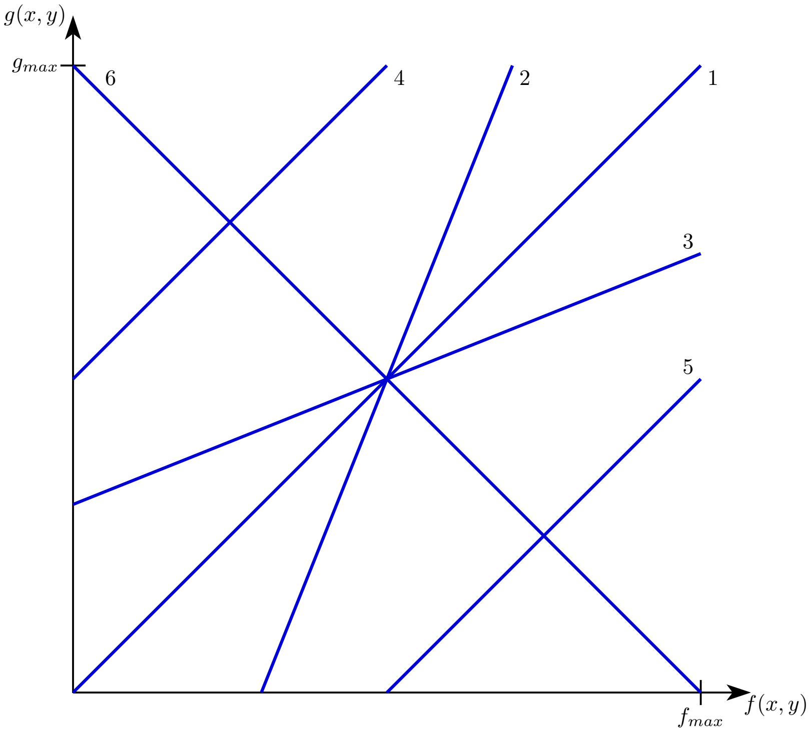

Affine greyscale transformations. **(1)** Identity. **(2)** Contrast enhancement. **(3)** Contrast attenuation. **(4)** Brightening. **(5)** Darkening. **(6)** Greyscale reversion. Author: T. Schneevoigt.

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Important Affine Transformation

- greyscale transformation to the interval $[0, 255]$
  (which is easy to display and to store)
- If $f(x, y)$ has the range $[f_{\min}, f_{\max}]$, this transformation is defined as

$$g(x, y) \;=\; 255 \cdot \frac{f(x, y) - f_{\min}}{f_{\max} - f_{\min}}\,.$$

- disadvantage:
  - single outlier can spoil result
  - does not take into account how often a grey value is present
- remedy:
  - histogram equalisation (later)

Affine Greyscale Transformations (4)

Left: Underexposed original image. **Right:** After affine greyscale mapping to the interval $[0, 255]$. Author: J. Weickert.

Outline

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1. What are Point Operations?
2. Affine Greyscale Operations
3. **Thresholding**
4. Logarithmic Dynamic Compression
5. Gamma Correction
6. Histogram Equalisation
7. Pseudocolour Representation
8. False Colour Representation
9. Adding Images
10. Subtracting Images

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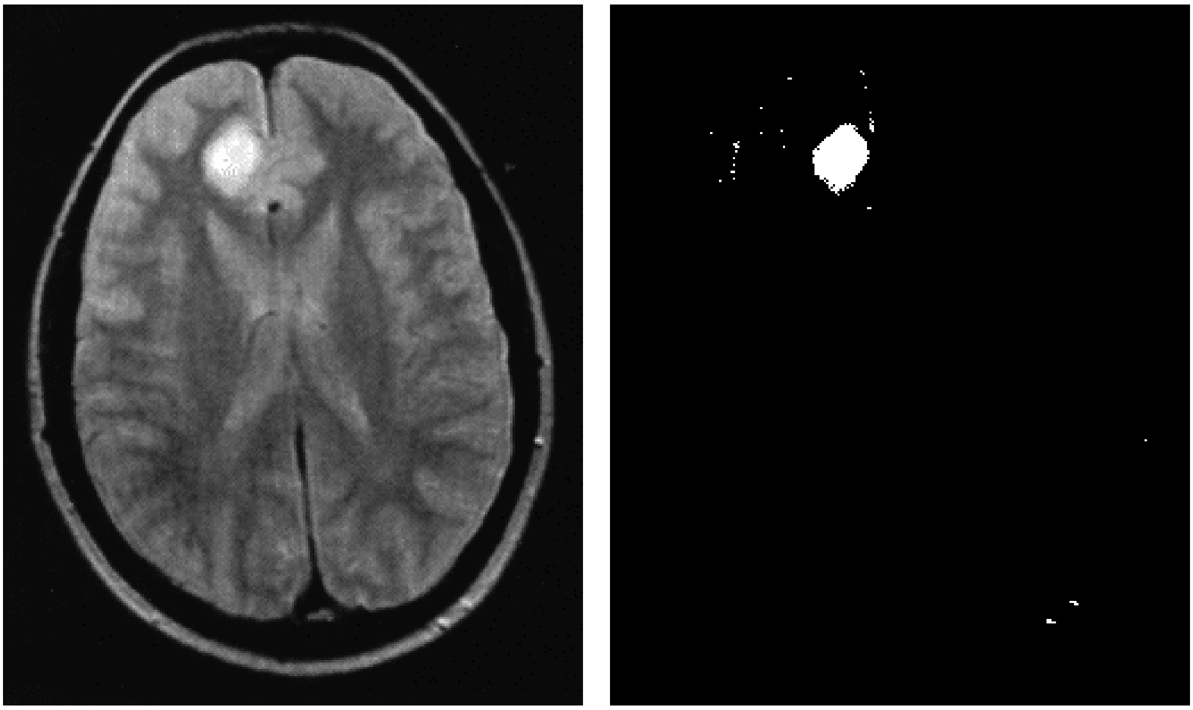
Thresholding (Binarisation, Schwellwertbildung)

- can be described by the nonlinear transformation

$$\phi(f(x,y)) := \begin{cases} g_{\max} & \text{for } f(x,y) \geq T, \\ 0 & \text{else}, \end{cases}$$

  where $g_{\max}$ is usually 1 or 255
- simplest method for segmentation



Thresholding. **Left:** MR image of a human brain with a tumour. Greyscale range: $[0, 255]$. **Right:** Thresholding with $T = 180$ allows to segment the tumour. Author: J. Weickert.

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Logarithmic Dynamic Compression

- useful transformation if
  - the greyscale ranges over many orders of magnitude
  - the *ratio* between two grey values is more important than their *difference*
- example: visualisation of the Fourier spectrum (cf. Lecture 5)
- For an image $f(x, y)$ with range $[0, f_{\max}]$, one computes

$$\phi(f(x,y)) \;:=\; c \ln\left(1 + f(x,y)\right) \qquad (c > 0).$$

  Adding 1 ensures that $\phi(0) = 0$.
- Often the constant c is chosen such that $\phi(f_{\max}) = 255$:

$$c \;:=\; \frac{255}{\ln\left(1 + f_{\max}\right)}\,.$$

  Then all transformed grey values are in $[0, 255]$.

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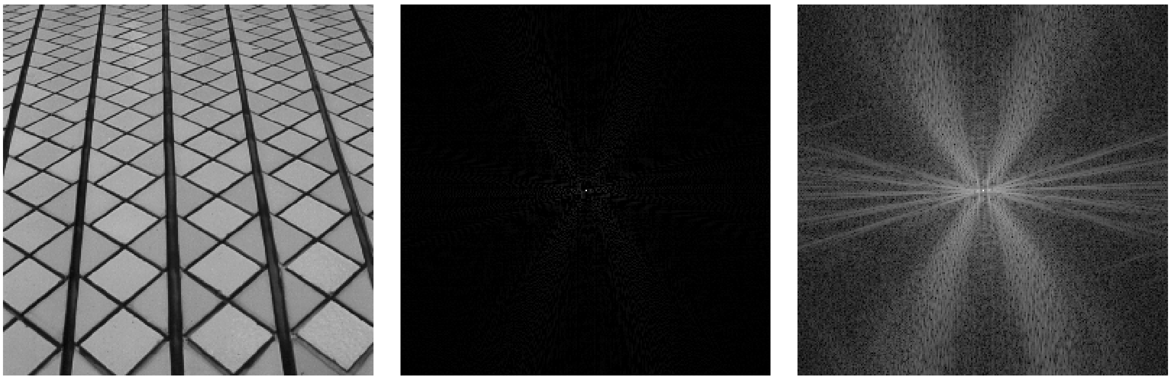
Logarithmic Dynamic Compression (2)



Logarithmic dynamic compression. **Left:** Original image, 256×256 pixels. **Middle:** Fourier spectrum without logarithmic dynamic compression. The white pixel in the centre corresponds to the sum of all grey values. It dominates over all other Fourier coefficients. **Right:** After logarithmic dynamic compression, the entire Fourier spectrum is well visible. The constant c is chosen such that the range of the transformed spectrum coincides with the interval $[0, 255]$. Author: J. Weickert.

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Gamma Correction (Gammakorrektur)

- Many video cameras transform an incoming light intensity I into a grey value f that is proportional to I^γ.
  Often $\gamma \approx 0.4$ (compresses dynamic range, similar to our visual system).
- Similar transformations are also used in computer monitors.
  Here the value for γ may vary from brand to brand.
- Sometimes the γ value is even changed by software.
- As a result, an image may look unpleasant on a specific monitor or printer.
- To compensate these effects, a so-called *gamma correction* can be used.
  For an image $f(x, y)$ with greyscale range $[0, f_{\max}]$ it is defined as

$$\phi(f(x,y)) \;:=\; f_{\max} \cdot \left(\frac{f(x,y)}{f_{\max}}\right)^{1/\gamma} \qquad (\gamma > 0).$$

- Thus, the transformed image has the same range $[0, f_{\max}]$.
  Choosing $\gamma < 1$ gives a darker image, while $\gamma > 1$ creates a brighter image.

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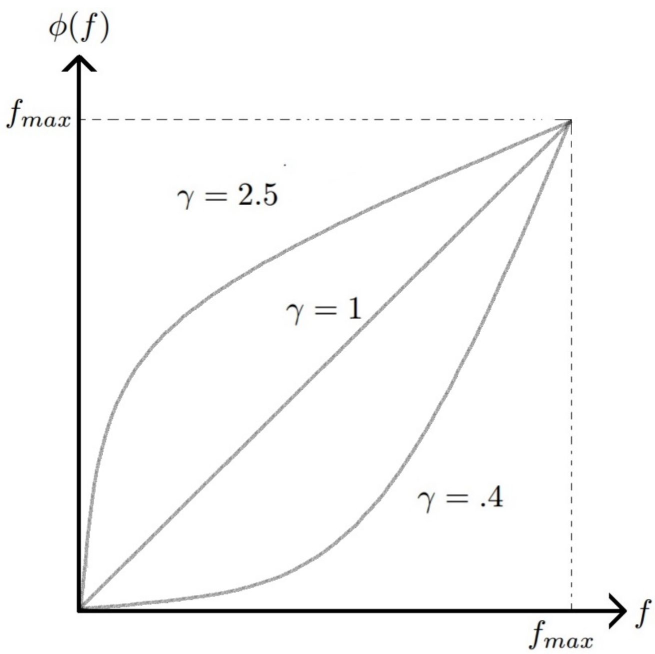
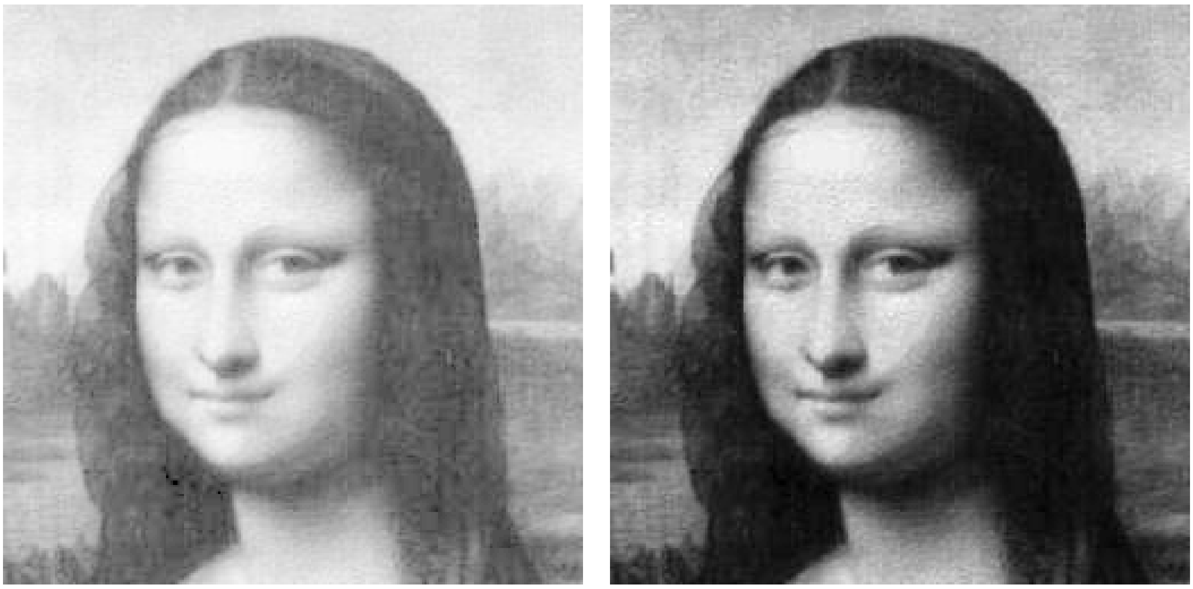


Visualisation of the gamma correction curve. Author: A. Goswami.

Gamma Correction (3)



Gamma correction. **Left:** Although the entire greyscale range $[0, 255]$ is used, the Mona Lisa image appears pale and not very rich in contrast. **Right:** A gamma correction with $\gamma = 0.4$ is a remedy. Authors: L. da Vinci, J. Weickert.

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Histogram Equalisation (Histogrammegalisierung)

Basic Idea

- another important nonlinear point operation
- goal: transformation such that all grey values occur equally often
- may give dramatic improvements in the subjective image quality

Histogram

- specifies how often a certain grey value appears within an image
- spatial context does not matter: any pixel permutation gives same histogram

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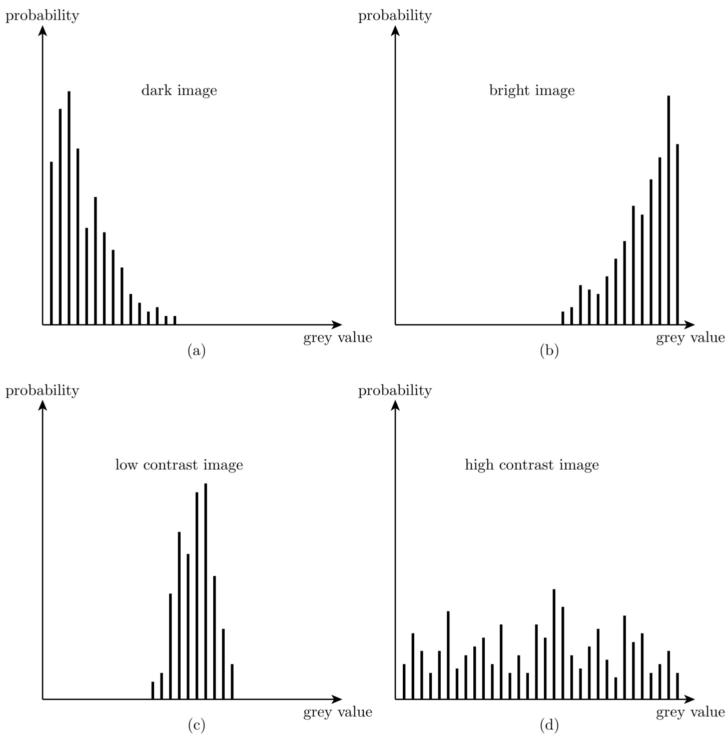


Histograms of different types of images. **(a)** Dark image. **(b)** Bright image. **(c)** Low contrast image. **(d)** High contrast image. Author: T. Schneevoigt.

Histogram Equalisation (3)

Algorithm for Direct Histogram Equalisation of a Discrete Image

- Often one aims at a balanced histogram with equal grey value probabilities.
- Basic idea: area-based mapping from a histogram (p_i) to a histogram (q_j)
- Given: p_i: number of pixels of image f having grey value v_i $(i = 1, ..., m)$
  q_j: desired number of pixels of g with grey value w_j $(j = 1, ..., n)$
  (for N pixels and 256 greyscales: $q_j := \frac{N}{256}$)
- Set $k_0 := 0$.
- For $r = 1, ..., n$:
  /* fill bin number r in target histogram by comparing cumulative histograms */
  Find the largest index $k_r \leq m$ with

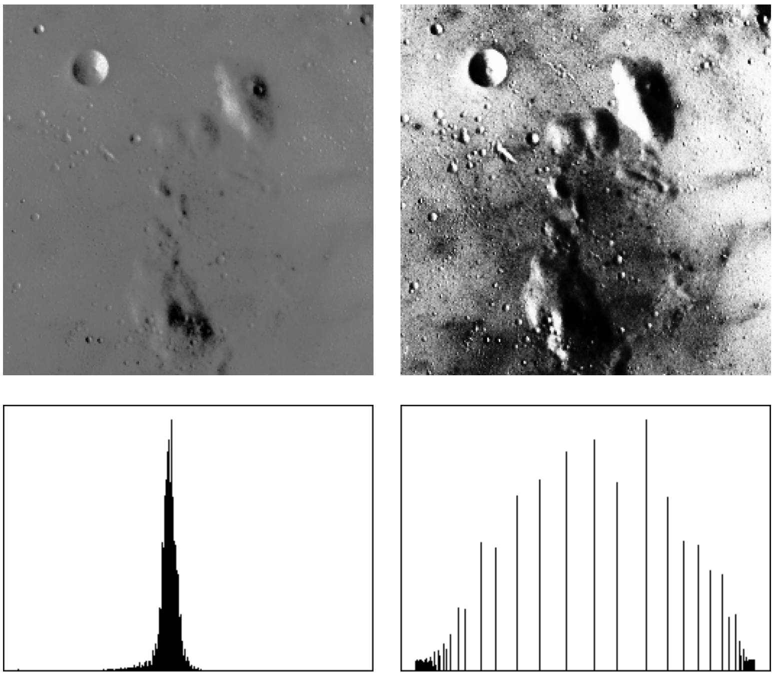
$$\sum_{i=1}^{k_r} p_i \;\leq\; \sum_{j=1}^{r} q_j \,.$$

  Then map the grey values $v_{k_{r-1}+1}, \ldots, v_{k_r}$ to w_r.

Histogram Equalisation (4)

Remarks

- The algorithm does not only perform histogram equalisation:
  - It can also transform a histogram to any other histogram:
    This application is called *histogram specification.*
  - All one has to do is to use other values for $q_1, \ldots, q_n$.
- For a general discrete image, histogram equalisation can only be approximated (see next page).



Histogram equalisation. **Top left:** Original image of the surface of the moon. **Top right:** After discrete histogram equalisation. **Bottom left:** Histogram of the original image. **Bottom right:** Histogram of the equalised image. Since the heights of the histogram bars cannot be decreased, the algorithm tries to equalise the histogram by spreading their distances. Author: J. Weickert.

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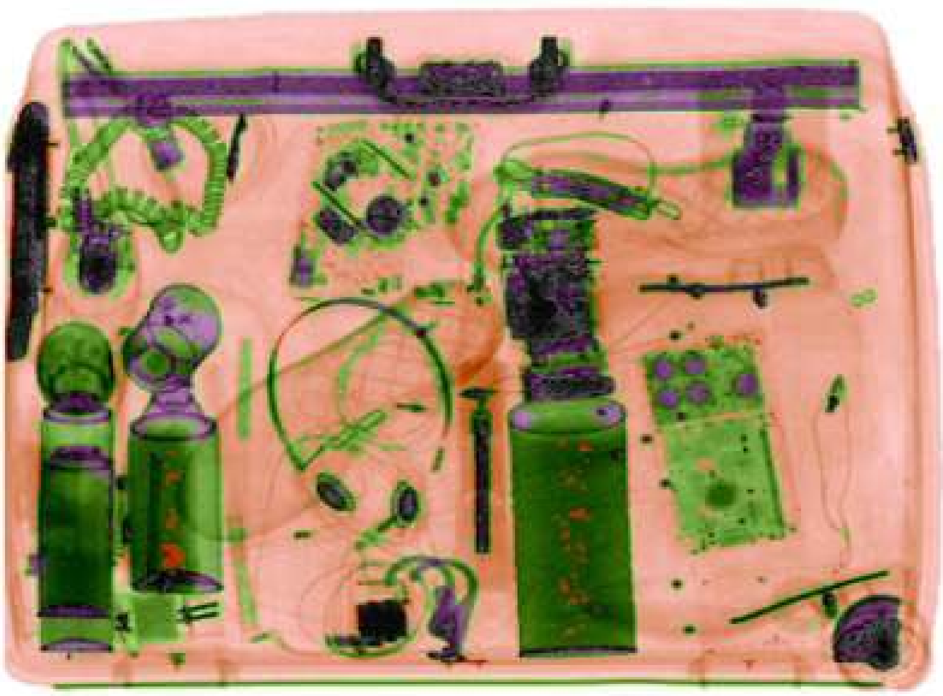
Pseudocolour Representation of Greyscale Images

- Humans can distinguish only ca. 40 greyscales, but 2 million colours.
- Thus, colouring grey values allows better visual discrimination.
- There are numerous possibilities to design mappings of type

$$f(x,y) \quad \longmapsto \quad \begin{pmatrix} \phi_r(f(x,y)) \\ \phi_g(f(x,y)) \\ \phi_b(f(x,y)) \end{pmatrix}.$$

- used e.g. in X-ray scanners at airports and for thermographic images



Colouring X-ray images at airport security checks allows a human observer to distinguish objects in a better way. Source: `http://static.howstuffworks.com/gif/airport-security-xray2.jpg`.

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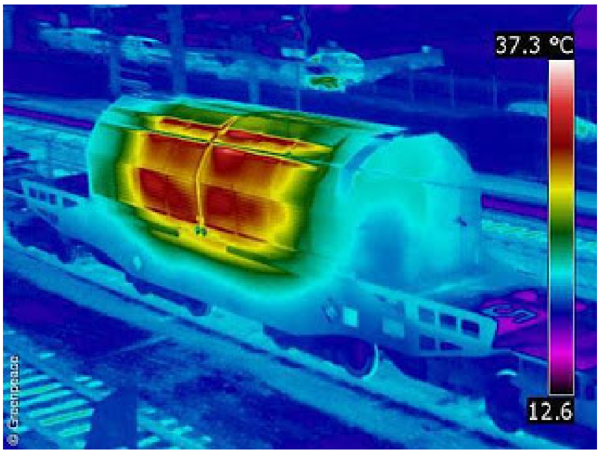
Pseudocolour Representation of Greyscale Images (3)



Thermography allows to measure the temperature of objects by their infrared radiation. Typically one depicts low temperatures in blue and high temperatures in red. **Left:** Pseudocolour representation of the transport of a nuclear waste container on a train. Source: Greenpeace. **Right:** The pseudocolour representation proves that the woman freezes at her hands and her nose. Source: R. Reischuk.

Pseudocolour Representation of Greyscale Images (4)

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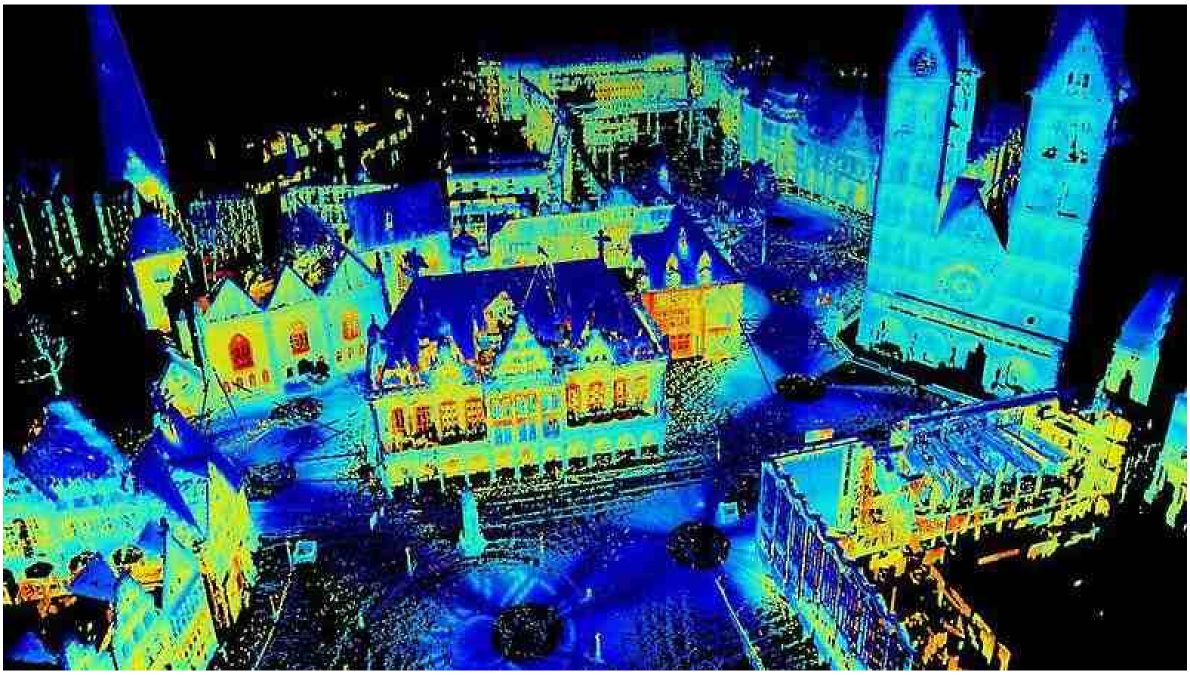

A pseudocolour representation of a thermographic measurement of the market square of Bremen. One can see that the facades of many historical buildings and in particular their windows lose too much heat. Source: A. Nüchter.

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False Colour Representation of Vectorial Images

- transforms a multichannel image to a colour image
- often used in remote sensing and astronomical imaging:
  frequencies outside the visible spectrum are mapped to colours

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False Colour Representation of Vectorial Images (2)

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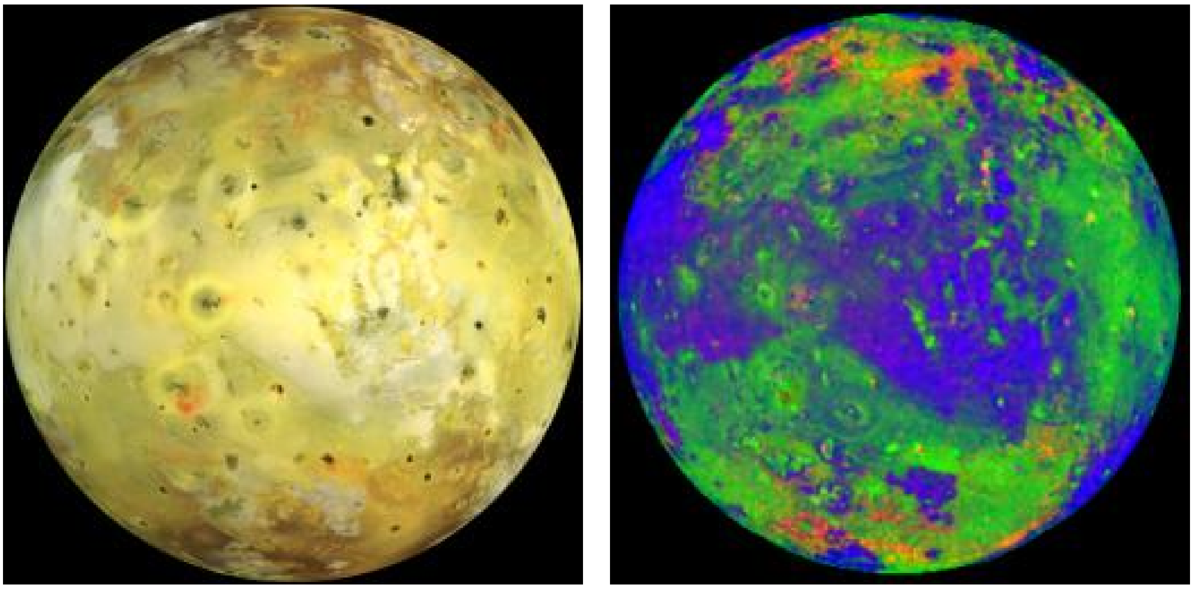

Left: True colour representation of Jupiter's moon Io. **Right:** False colour representation that combines information from two visible and two infrared frequency bands. The depicted colours red, green and blue show the quotient between two of the four channels. They allow a better interpretation of the surface structure: Red depicts hot vulcanoes, green presumably characterises regions with much sulphur, and blue indicates frozen sulphur dioxide. Source: NASA, `http://www.jpl.nasa.gov/galileo/images/io/iocolor.html`.

False Colour Representation of Vectorial Images (3)

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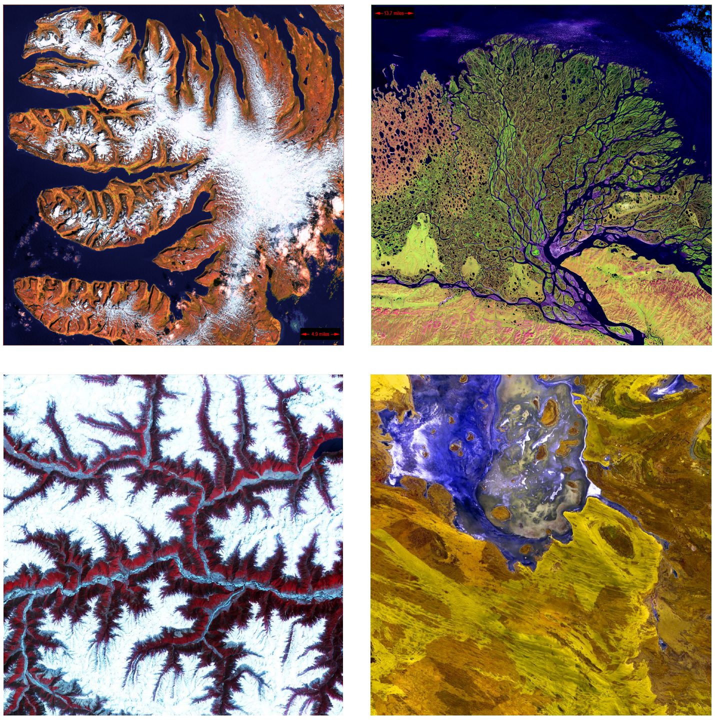

Remote sensing images in false colour representation, such that they look like art. **Top left:** West Fjords, Iceland. **Top right:** Lena delta, Russia. **Bottom middle:** Himalaya region. **Bottom right:** Lake Disappointment, Australia. Source: NASA, `http://earthasart.gsfc.nasa.gov/`.

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Adding Images

Problem

- Some imaging methods (e.g. electron microscopy) create very noisy images:

$$\underbrace{f(x,y)}_{\text{noisy}} = \underbrace{v(x,y)}_{\text{no noise}} + \underbrace{n(x,y)}_{\text{noise}}$$

- This operation is pointwise, but depends on the position $(x,y)^\top$.

Solution

- If (!) images of the object can be taken multiple times under the same conditions, one can average these images to reduce noise.
- Let us assume that the noise has mean 0 and is uncorrelated to the image. Then averaging M images $f_i : \Omega \to \mathbb{R}$ with noise variance

$$\sigma^2 = \frac{1}{|\Omega|} \int_\Omega |n(x,y)|^2 \, dx \, dy$$

creates a reduced noise variance $\bar{\sigma}^2 = \sigma^2/M$.

- Thus, if one wants to reduce the standard deviation $\bar{\sigma}$ of the averaged image to $1/10$ of its original value σ, one needs $10^2 = 100$ images !

Adding Images (2)

original, 323×279 pixels — Gaussian noise, $\sigma^2 = 52.29$ — averaged 4 times

averaged 16 times — averaged 64 times — averaged 256 times

Denoising by averaging. Author: J. Weickert.

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Subtracting Images

Example: Digital Subtraction Angiography (DSA)

- medical imaging method for visualising the blood flow through the vessels
- X-ray images are taken before and after injecting a fluorescent contrast agent
- difference image makes active vessels visible

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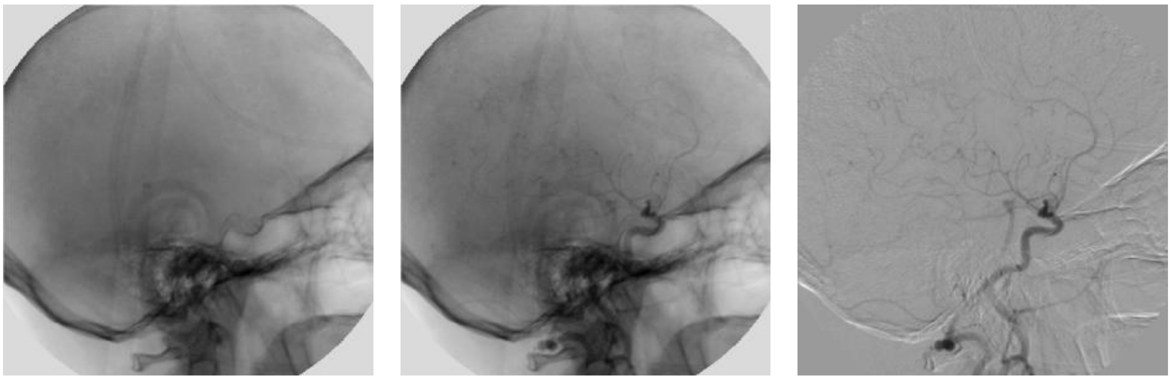

Digital subtraction angiography. **Left:** Initial image (so-called mask). **Middle:** After injecting a contrast agent. **Right:** The difference image removes the background and visualises vessel structures with blood flow. Source: `http://www.isi.uu.nl/Research/Gallery/DSA/`

Summary

- A point operation performs a global transformation of the greyscales.
- typical application:
  new representation of the grey values with improved human perception
- The most important point transforms include:
  – affine rescaling
  – thresholding
  – logarithmic dynamic compression
  – gamma correction
  – histogram equalisation
- Pseudo and false colour representations further improve the visible information content for humans.
- Pixelwise averaging of images reduces noise.
- Subtraction of images allows background elimination.

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References

- R. C. Gonzalez, R. E. Woods: *Digital Image Processing*. Pearson, Upper Saddle River, Global Edition, 2017.
  (Sections 3.2 to 3.3 describe point transformations.)
- R. Jain, R. Kasturi, B. G. Schunck: *Machine Vision*. McGraw-Hill, New York, 1995.
  (see in particular Chapter 4)
- Wikipedia webpage on histogram equalisation:
  `https://en.wikipedia.org/wiki/Histogram_equalization`
  (provides more details using cumulative histograms)

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