4.13

a)

Let us see Step response 1. It has oscillation. Hence, it goes with closed loop system a, and poles II. It is close to the stability limit, then it goes with opem loop system B. Thus, A-b-2-I and B-a-1-II.

b)

Let us see Step response 1. It has faster response than 2. Hence, it goes with closed loop system b (check band width), and poles I (check dominant pole), and open loop system B (higher crossover frequency). Thus, A-a-2-II and B-b-1-I.

Lead-lag Compensation

Lead-lag compensation is a controller design method which uses the frequency response of the system. A controller is given by the form of

$$F(s) = F_{\text{lead}}(s)F_{\text{lag}}(s)$$

where

$$F_{\text{lead}}(s) = K\frac{\tau_D s + 1}{\beta\tau_D s + 1}, \quad F_{\text{lag}}(s) = \frac{\tau_I s + 1}{\tau_I s + \gamma}.$$

Lead compensation

The term $F_{\text{lead}}(s)$ improves the transient response of the system such as overshoot. The design procedure is usually as following:

1. Decide desired crossover frequency ω_c and phase margin φ_m.
2. Calculate phase to be recovered by $F_{\text{lead}}(s)$.
3. Determine β by Figure 1.

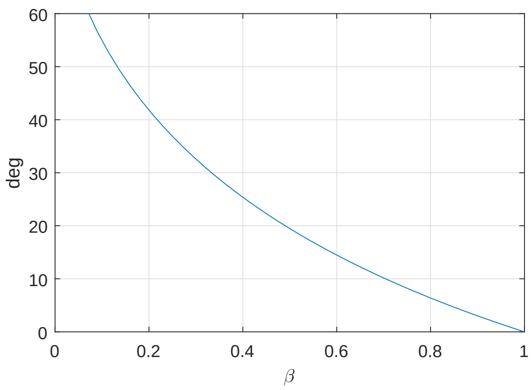

Figure 1: Maximum phase recovery versus β (Figure 5.13 in the textbook)

4. Determine τ_D by
$$\tau_D = \frac{1}{\omega_c\sqrt{\beta}}.$$
5. Determine K by $|F_{\text{lead}}(j\omega_c)G(j\omega_c)| = 1$.

Lag compensation

The term $F_{\text{lag}}(s)$ improves the steady-state response of the system. The design procedure is usually as following:

1. Decide desired low frequency gain $F(0)$.
2. Determine γ by $F(0)$.
3. Determine τ_I by $\tau_I = 10/\omega_c$.

5.20

a)

$$G_c(s) = \frac{F(s)G(s)}{1 + F(s)G(s)}$$

b)

By definition, when $\omega = \omega_c$, we have $|F(j\omega_c)G(j\omega_c)| = 1$. Hence, $|KG(j\omega_c)| = |G(j\omega_c)| = 1$. From Figure 5.20a, $\omega_c = 1$[rad/s]. Phase margin is the difference of phase between the one at gain crossover frequency and $-180°$. Thus, $\varphi_m = -130 - (-180) = 50°$. Since $\varphi_c > 0$, the closed loop system is stable.

c)

Here, "twice as fast" means that the gain crossover frequency ω_c' with new value of K is doubled by the one with $K = 1$, i.e., $\omega_c' = 2\omega_c$. Hence,

$$|KG(j\omega_c')| = |KG(j2\omega_c)| = 1 \Rightarrow K = \frac{1}{0.3} = 3.33.$$

d)

Let us check new phase margin. $\varphi_m' = -175 + 180 = 5°$, which means that the closed loop system is closer to instability.

e)

We have to determine K, τ_D, and β such that

- "twice as fast" $\iff$ gain crossover frequency ω_c' is twice as high as ω_c
- "same overshoot" $\iff$ same phase margin as φ_m

Thus,

$$|F_{\text{lead}}(j\omega_c')G(j\omega_c')| = 1 \iff |F_{\text{lead}}(j\omega_c') \cdot 0.3| = 1$$
$$\varphi_m' = 50°.$$

The phase margin can be written as

$$\begin{aligned}\varphi_m' &= \arg(F_{\text{lead}}(j\omega_c')G(j\omega_c')) + 180° \\ &= \arg(F_{\text{lead}}(j\omega_c')) + \arg(G(j\omega_c')) + 180° \\ &= \arg(F_{\text{lead}}(j\omega_c')) - 175° + 180°.\end{aligned}$$