

MFE 405; Project 2

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```
In [128... import numpy as np
import matplotlib.pyplot as plt
```

Problem 1.

Compare the convergence rates of the four methods below by doing the following: Use the Binomial Method to price a 6-month American Put option with the following information: the risk-free interest rate is 5.5% per annum; the volatility is 25% per annum; the current stock price is \$180; and the strike price is \$170. Divide the time interval into n parts to estimate the price of this option. Use $n = 20, 40, 80, 100, 200, 500$, to estimate the price and draw all resulting prices in one graph, where the horizontal axis measures n , and the vertical one the price of the option.

(a) Use the binomial method in which: $u = \frac{1}{d}$; $d = c - \sqrt{c^2 - 1}$; $c = \frac{1}{2}(e^{-r\Delta t} + e^{(r + \frac{1}{2}\sigma^2)\Delta t})$; $p = \frac{e^{-r\Delta t} - d}{u - d}$

```
In [129... # define problem parameters
n_array = [20, 40, 80, 100, 200, 500]
T = .5
r = .055
sigma = .25
S0 = 180
K = 170
```

```
In [131... # define problem (a) function
def binomial_method_a(S0, K, T, r, sigma, n_array):
    # initiate array to store values
    values = []
    # iterate through array
    for interval in n_array:
        # set dt, c, d, u, and p
        dt = T/interval
        c = 1/2*(np.exp(-r*dt) + np.exp((r+sigma**2)*dt))
        d = c - np.sqrt(c**2 - 1)
        u = 1/d
        p = (np.exp(r*dt) - d)/(u-d)
        # initiate matrices for stock and option values
        S_values = np.zeros((interval+1, interval+1))
        P_values = np.zeros((interval+1, interval+1))
        # set initial stock value to zero
        S_values[0, 0] = S0
        # generate stock prices, we go step by step in interval range
        for step in range(1, interval+1):
            S_values[step, 0] = S_values[step-1, 0] * u
            # make sure there are 2 states for each step
```

```

        for state in range(1, step+1):
            S_values[step, state] = S_values[step-1, state-1] * d
            # set option value to max between strike and stock value
            P_values[step, state] = max(K-S_values[step, state], 0)
            # work backwards, we need to see whether EV or HV is bigger
        for step in range(interval-1, -1, -1):
            for state in range(step+1):
                exercise_value = K - S_values[step, state]
                # calculate hold value
                hold_value = (p*P_values[step+1, state] + (1-p)*P_values[step, state])
                P_values[step, state] = max(exercise_value, hold_value)

            values.append(P_values[0, 0])
    return values

```

(b) Use the binomial method in which: $u = e^{(r - .5\sigma^2)dt + \sigma\sqrt{dt}}$; $d = e^{(r - .5\sigma^2)dt - \sigma\sqrt{dt}}$; $p = \frac{1}{2}$

```

In [132]: # define problem (b) function. same as a the only difference is u, d, and p
def binomial_method_b(S0, K, T, r, sigma, n_array):
    # initiate array to store values
    values = []
    # iterate through array
    for interval in n_array:
        # set dt, c, d, u, and p
        dt = T/interval
        d = np.exp((r - .5*sigma**2)*dt - sigma*np.sqrt(dt))
        u = np.exp((r - .5*sigma**2)*dt + sigma*np.sqrt(dt))
        p = 1/2
        # initiate matrices for stock and option values
        S_values = np.zeros((interval+1, interval+1))
        P_values = np.zeros((interval+1, interval+1))
        # set initial stock value to zero
        S_values[0, 0] = S0
        # generate stock prices, we go step by step in interval range
        for step in range(1, interval+1):
            S_values[step, 0] = S_values[step-1, 0] * u
            # make sure there are 2 states for each step
            for state in range(1, step+1):
                S_values[step, state] = S_values[step-1, state-1] * d
            # set option value to max between strike and stock value
            P_values[step, state] = max(K-S_values[step, state], 0)
            # work backwards, we need to see whether EV or HV is bigger
        for step in range(interval-1, -1, -1):
            for state in range(step+1):
                exercise_value = K - S_values[step, state]
                # calculate hold value
                hold_value = (p*P_values[step+1, state] + (1-p)*P_values[step, state])
                P_values[step, state] = max(exercise_value, hold_value)

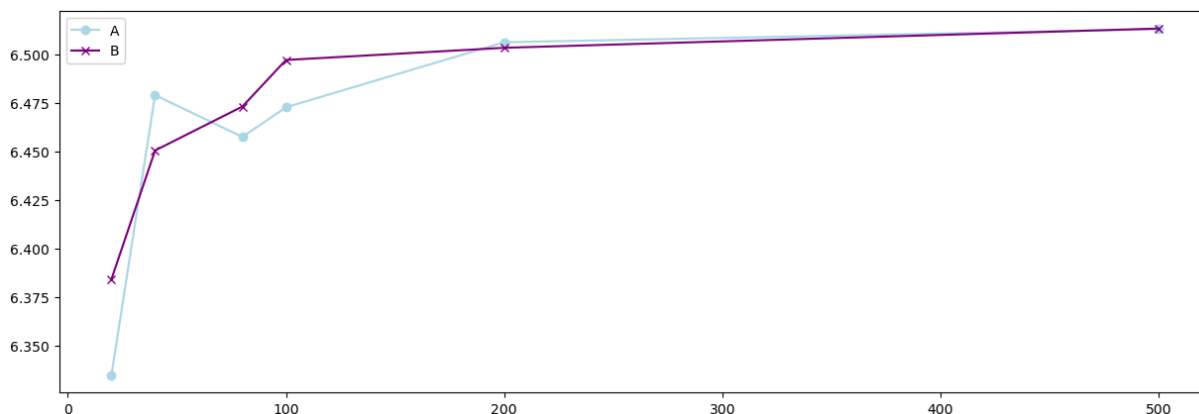
            values.append(P_values[0, 0])
    return values

```

Outputs: Graphs: Two plots in one graph.

```
In [133... # get values
value_a = binomial_method_a(S0, K, T, r, sigma, n_array)
value_b = binomial_method_b(S0, K, T, r, sigma, n_array)

# plot the figure
plt.figure(figsize=(15, 5))
plt.plot(n_array, value_a, label='A', color='lightblue', marker='o')
plt.plot(n_array, value_b, label='B', color='purple', marker='x')
plt.legend()
plt.show()
```



Problem 2.

Consider the following information on the stock of a company and American put options on it: $S_0 = \$180$, $X = \$170$, $r = 0.055$, $\sigma = 0.25$, $T = 6m$, $\mu = 0.15$ Using the CRR Binomial tree method, estimate the following and draw their graphs:

(i) Delta of the put option as a function of S_0 , for S_0 ranging from \$170 to \$190, in increments of \$2.

```
In [134... # define crr model
def crr(range_start, range_end, K, T, r, sigma):
    deltas = []
    for S0 in range(range_start, range_end+1, 2):
        # assume 500 time steps
        n = 500
        dt = T/n
        d = np.exp(-sigma * np.sqrt(dt))
        u = np.exp(sigma * np.sqrt(dt))
        p = 1/2*(1 + ((r - .5*sigma**2)*dt)/sigma)
        # initiate matrices for stock and option values
        S_values = np.zeros((n+1, n+1))
        P_values = np.zeros((n+1, n+1))
        # set initial stock value to zero
        S_values[0, 0] = S0
        # generate stock prices, we go step by step in interval range
        for step in range(1, n+1):
            S_values[step, 0] = S_values[step-1, 0] * u
            # make sure there are 2 states for each step
            for state in range(1, step+1):
```

```

        S_values[step, state] = S_values[step-1, state-1] * d
        # set option value to max between strike and stock value
        P_values[step, state] = max(K-S_values[step, state], 0)
        # work backwards, we need to see whether EV or HV is bigger
        for step in range(n-1, -1, -1):
            for state in range(step+1):
                exercise_value = K - S_values[step, state]
                # calculate hold value
                hold_value = (p*P_values[step+1, state] + (1-p)*P_values[step, state])
                P_values[step, state] = max(exercise_value, hold_value)

        # in theory I only need two nodes but
        delta = (P_values[1, 0] - P_values[1, 1])/(S_values[1, 0] - S_values[1, 1])
        deltas.append(delta)
    return deltas

```

(ii) Delta of the put option, as a function of T (time to expiration), T ranging from 0 to 0.18 in increments of 0.003.

```

In [135]: # copy crr, make edits
def crr_time(S0, K, tstart, tend, tincrement, r, sigma):
    deltas = []
    T = tstart + tincrement
    while T <= tend:
        # assume 500 time steps
        n = 500
        dt = T/n
        d = np.exp(-sigma * np.sqrt(dt))
        u = np.exp(sigma * np.sqrt(dt))
        p = 1/2*(1 + ((r - .5*sigma**2)*dt)/sigma)
        # initiate matrices for stock and option values
        S_values = np.zeros((n+1, n+1))
        P_values = np.zeros((n+1, n+1))
        # set initial stock value to zero
        S_values[0, 0] = S0
        # generate stock prices, we go step by step in interval range
        for step in range(1, n+1):
            S_values[step, 0] = S_values[step-1, 0] * u
            # make sure there are 2 states for each step
            for state in range(1, step+1):
                S_values[step, state] = S_values[step-1, state-1] * d
            # set option value to max between strike and stock value
            P_values[step, state] = max(K-S_values[step, state], 0)
            # work backwards, we need to see whether EV or HV is bigger
            for step in range(n-1, -1, -1):
                for state in range(step+1):
                    exercise_value = K - S_values[step, state]
                    # calculate hold value
                    hold_value = (p*P_values[step+1, state] + (1-p)*P_values[step, state])
                    P_values[step, state] = max(exercise_value, hold_value)

        # in theory I only need two nodes but
        delta = (P_values[1, 0] - P_values[1, 1])/(S_values[1, 0] - S_values[1, 1])
        deltas.append(delta)
        T += tincrement

```

```
return deltas
```

(iii) Theta of the put option, as a function of T (time to expiration), T ranging from 0 to 0.18 in increments of 0.003.

```
In [136... # copy crr, make edits
def crr_theta(S0, K, tstart, tend, tincrement, r, sigma):
    thetas = []
    T = tstart + tincrement
    while T <= tend:
        # assume 500 time steps
        n = 500
        dt = T/n
        d = np.exp(-sigma * np.sqrt(dt))
        u = np.exp(sigma * np.sqrt(dt))
        p = 1/2*(1 + ((r - .5*sigma**2)*dt)/sigma)
        # initiate matrices for stock and option values
        S_values = np.zeros((n+1, n+1))
        P_values = np.zeros((n+1, n+1))
        # set initial stock value to zero
        S_values[0, 0] = S0
        # generate stock prices, we go step by step in interval range
        for step in range(1, n+1):
            S_values[step, 0] = S_values[step-1, 0] * u
            # make sure there are 2 states for each step
            for state in range(1, step+1):
                S_values[step, state] = S_values[step-1, state-1] * d
            # set option value to max between strike and stock value
            P_values[step, state] = max(K-S_values[step, state], 0)
            # work backwards, we need to see whether EV or HV is bigger
            for step in range(n-1, -1, -1):
                for state in range(step+1):
                    exercise_value = K - S_values[step, state]
                    # calculate hold value
                    hold_value = (p*P_values[step+1, state] + (1-p)*P_values[step, state])
                    P_values[step, state] = max(exercise_value, hold_value)

        # in theory I only need two nodes but
        theta = (P_values[2, 1] - P_values[0, 0])/2*dt
        thetas.append(theta)
        T += tincrement

    return thetas
```

(iv) Vega of the put option, as a function of S0, for S0 ranging from \$170 to \$190, in increments of \$2.

```
In [137... def crr_vegas(range_start, range_end, K, T, r, sigma):
    x = crr(range_start, range_end, K, T, r, sigma)
    y = crr(range_start, range_end, K, T, r, sigma+.05)
    vegas = []
    for pricex, pricey in zip(x, y):
        vega = (pricey - pricex)/.05
```

```

    vegas.append(vega)
    return vegas

```

Outputs: Graphs: 4 separate graphs.

```

In [55]: fig, ax = plt.subplots(2, 2, figsize=(10, 5))
ax[0, 0].plot(crr(170, 190, 170, 0.5, 0.055, 0.25))
ax[0, 0].set_title('Delta S0')

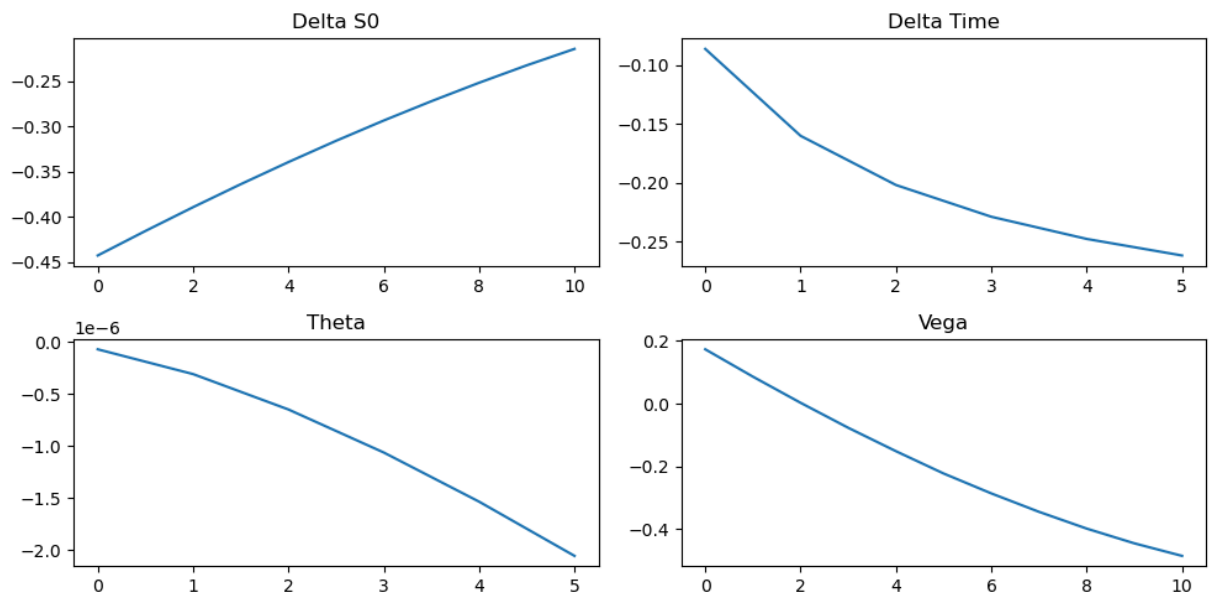
ax[0, 1].plot(crr_time(180, 170, 0, 0.18, 0.03, 0.055, 0.25))
ax[0, 1].set_title('Delta Time')

ax[1, 0].plot(crr_theta(180, 170, 0, 0.18, 0.03, 0.055, 0.25))
ax[1, 0].set_title('Theta')

ax[1, 1].plot(crr_vegas(170, 190, 170, 0.5, 0.055, 0.25))
ax[1, 1].set_title('Vega')

plt.tight_layout()
plt.show()

```



Problem 3.

Compare the convergence rates of the two methods, (a) and (b), described below, by doing the following: Use the Trinomial-tree method to price a 6-month American put option with the following information: the risk-free interest rate is 5.5% per annum, the volatility is 25% per annum, the current stock price is \$180, and the strike price is \$170. Divide the time interval into n equal parts to estimate the option price. Use $n = 20, 40, 70, 80, 100, 200, 500$; to estimate option prices and draw them all in one graph, where the horizontal axis measures n , and the vertical one measures option price. The two methods are in (a) and (b) below:

(a) Use the Trinomial-tree method applied to the stock price-process (S_t) in which:

$$dS_t = \frac{1}{d} S_t dt; d = e^{-\sigma \sqrt{3dt}}; p_u = \frac{rdt(1-u) + (rdt)^2 + \sigma^2 dt}{2}$$

$d)(1-d)\} p_d = \frac{rdt(1-d) + (rdt)^2 + \sigma^2 dt}{(u-d)(u-1)}; p_m = 1 - p_u - p_d$

```
In [283... # define problem (a) function
def trinomial_method_a(S0, K, T, r, sigma, n_array):
    # initiate array to store values
    values = []
    # iterate through array
    for n in n_array:
        dt = T / n
        d = np.exp(-sigma * np.sqrt(3 * dt))
        u = 1 / d
        p_u = ((r * dt * (1 - d) + (r * dt)**2 + sigma**2 * dt) / ((u - d) *
        p_d = ((r * dt * (1 - u) + (r * dt)**2 + sigma**2 * dt) / ((u - d) *
        p_m = 1 - p_u - p_d

        # stock, option prices
        S = np.zeros((2 * n + 1, n + 1))
        P = np.zeros((2 * n + 1, n + 1))

        # initial condition
        S[n, 0] = S0

        # generate stock prices
        for j in range(1, n + 1):
            for i in range(n - j + 1, n + j):
                S[i + 1, j] = S[i, j - 1] * u
                S[i, j] = S[i, j - 1]
                S[i - 1, j] = S[i, j - 1] * d

        for i in range(2 * n + 1):
            P[i, n] = max(K - S[i, n], 0)

        # backward induction
        for j in range(n - 1, -1, -1):
            for i in range(n - j, n + j + 1):
                P[i, j] = (p_u * P[i + 1, j + 1] + p_m * P[i, j + 1] + p_d *
                if j <= n:
                    P[i, j] = max(P[i, j], K - S[i, j])

        # store values
        values.append(P[n, 0])

    return values
```

(b) Use the Trinomial-tree method applied to the Log-stock price-process (X_T) in which: $\Delta X_u = \sigma \sqrt{3dt}$; $\Delta X_d = -\sigma \sqrt{3dt}$ $p_d = \frac{1}{2}(\frac{\sigma^2 dt + (r - 0.5\sigma^2)^2 dt^2}{\Delta X_u^2} - \frac{(r - 0.5\sigma^2)^2 dt}{\Delta X_u})$; $p_u = \frac{1}{2}(\frac{\sigma^2 dt + (r - 0.5\sigma^2)^2 dt^2}{\Delta X_u^2} + \frac{(r - 0.5\sigma^2)^2 dt}{\Delta X_u})$ $p_m = 1 - p_u - p_d$

```
In [288... # define problem (a) function
def trinomial_method_b(S0, K, T, r, sigma, n_array):
```

```

# initiate array to store values
values = []
# iterate through array
for n in n_array:
    # set dt, c, d, u, and p
    dt = T/n
    X_u = sigma * np.sqrt(3*dt)
    X_d = -sigma * np.sqrt(3*dt)
    p_u = .5 * (((sigma**2*dt + (r - .5*sigma**2)**2 * dt**2)/(X_u**2)) +
    p_d = .5 * (((sigma**2*dt + (r - .5*sigma**2)**2 * dt**2)/(X_u**2)) -
    p_m = 1 - p_u - p_d
    # stock, option prices
    S = np.zeros((2 * n + 1, n + 1))
    P = np.zeros((2 * n + 1, n + 1))

    # initial condition
    S[n, 0] = S0

    # generate stock prices
    for j in range(1, n + 1):
        for i in range(n - j + 1, n + j):
            S[i + 1, j] = S[i, j - 1] * np.exp(X_u)
            S[i, j] = S[i, j - 1]
            S[i - 1, j] = S[i, j - 1] * np.exp(X_d)

    for i in range(2 * n + 1):
        P[i, n] = max(K - S[i, n], 0)

    # backward induction
    for j in range(n - 1, -1, -1):
        for i in range(n - j, n + j + 1):
            P[i, j] = (p_u * P[i + 1, j + 1] + p_m * P[i, j + 1] + p_d *
            P[i, j] = max(P[i, j], K - S[i, j])

    # store values
    values.append(P[n, 0])

return values

```

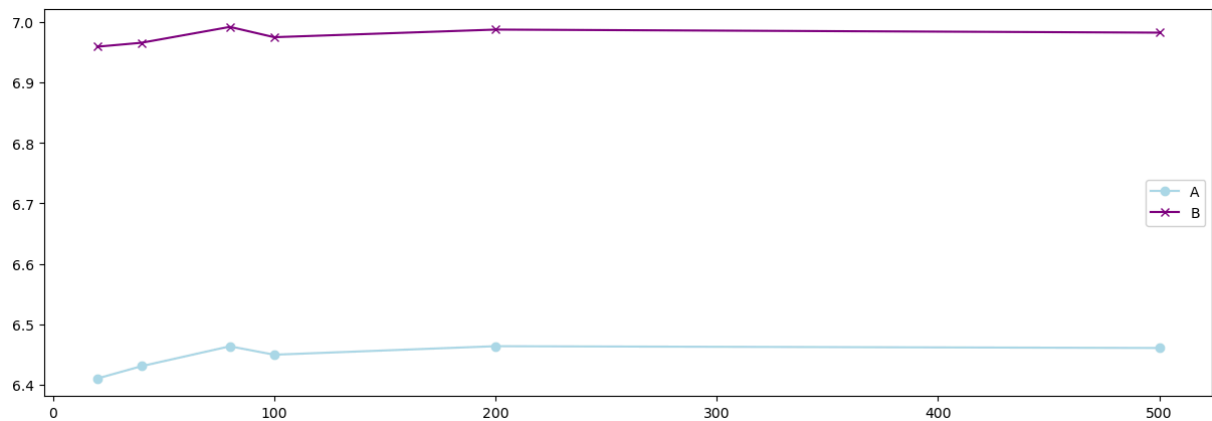
Outputs: Graphs: plot in a graph.

```

In [289... # get values
value_a_trinomial = trinomial_method_a(S0, K, T, r, sigma, n_array)
value_b_trinomial = trinomial_method_b(S0, K, T, r, sigma, n_array)

# plot the figure
plt.figure(figsize=(15, 5))
plt.plot(n_array, value_a_trinomial, label='A', color='lightblue', marker='c')
plt.plot(n_array, value_b_trinomial, label='B', color='purple', marker='x')
plt.legend()
plt.show()

```

Problem 4

Consider the following information on the stock of company XYZ: The current stock price is \$180, and the volatility of the stock price is $\sigma = 25\%$ per annum. Assume the prevailing risk-free rate is $r = 5.5\%$ per annum. Use the following method to price the specified option:

(a) Use the LSMC method with $N=100,000$ paths simulations (50,000 plus 50,000 antithetic variates) and a time step of $\Delta = 1/\sqrt{N}$ to price an American Put option with strike price of $X = \$170$ and maturity of 0.5-years and 1.5-years. Use the first k of the Laguerre Polynomials for $k = 2, 3, 4, 5$. (That is, you will compute 8 prices here). Compare the prices for the 4 cases, $k = 2, 3, 4, 5$ and comment on the choice of k .

Define regression function that will solve for $\beta = (X^T X)^{-1} X^T y$

```
In [141... def regression(X, y):
    beta = np.dot((np.dot(np.linalg.inv(np.dot(X.T, X))), X.T)), y)
    return beta
```

```
In [143... # define function that will generate Laguerre polynomials
def laguerre(x, k):
    # get zeros array
    L = np.zeros((len(x), k))
    # set first one at one
    L[:, 0] = 1
    # handle first k
    if k > 1:
        # set first one to 1-x
        L[:, 1] = 1 - x
    # handle other k's
    for n in range(2, k):
        L[:, n] = ((2 * n - 1 - x) * L[:, n - 1] - (n - 1) * L[:, n - 2]) / n
    return L
```

```
In [144... # to have a clear visual on the result
import pandas as pd
```

```

In [271... def lsmc(S0, X, T, r, sigma, N, k):
    # set time increment and number of steps
    dt = 1/np.sqrt(N)
    steps = int(T/dt)
    df = np.exp(-r * dt)

    # simulate brownian motion
    W = np.random.normal(0, 1, 50000)
    # normal simulations
    S_normal = S0 * np.exp(np.cumsum(r - .5*sigma**2)*dt + sigma * np.sqrt(c
    # antithetic
    S_antithetic = S0 * np.exp(np.cumsum(r - .5* sigma**2)*dt - sigma * np.s
    # combine
    S = np.vstack((S_normal, S_antithetic))
    # get exercise values (payoffs)
    ev = np.maximum(X-S, 0)
    # store them for each step iterating backwards
    evs = ev[:, -1]

    for time in range(steps - 1, 0, -1):
        # for each time get paths that are in the money
        itm = S[:, time] < X
        # store them separately
        S_itm = S[itm, time]
        # discount values
        dcf = evs[itm] * df

        L = laguerre(S_itm, k)
        A = regression(L, dcf)

        ecv = L.dot(A)
        immediate_exercise = np.maximum(X - S_itm, 0)
        exercise = immediate_exercise > ecv
        evs[itm] = np.where(exercise, immediate_exercise, dcf)
    price = evs.mean() * df
    return price

p4_df = pd.DataFrame(index = ['.5', '1.5'],
                      columns = ['k=2', 'k=3', 'k=4', 'k=5'])
p4_df.at['.5', 'k=2'] = lsmc(180, 170, .5, .055, .25, .25, 2)
p4_df.at['.5', 'k=3'] = lsmc(180, 170, .5, .055, .25, .25, 3)
p4_df.at['.5', 'k=4'] = lsmc(180, 170, .5, .055, .25, .25, 4)
p4_df.at['.5', 'k=5'] = lsmc(180, 170, .5, .055, .25, .25, 5)
p4_df.at['1.5', 'k=2'] = lsmc(180, 170, 1.5, .055, .25, .25, 2)
p4_df.at['1.5', 'k=3'] = lsmc(180, 170, 1.5, .055, .25, .25, 3)
p4_df.at['1.5', 'k=4'] = lsmc(180, 170, 1.5, .055, .25, .25, 4)
p4_df.at['1.5', 'k=5'] = lsmc(180, 170, 1.5, .055, .25, .25, 5)
print(p4_df)

# calculate payoffs (ev) it will be maximum between X-S
# work backwards to figure out how many path cross exercise value
# need to get continuation value for each step which is option value at that
# based on that fact we know whether we exercise or not

```

	k=2	k=3	k=4	k=5
.5	16.702439	28.79679	12.131152	0.0
1.5	32.173035	19.597963	25.678233	2.376152

(b) Use the LSMC method with $N=100,000$ paths simulations (50,000 plus 50,000 antithetic variates) and a time step of $\Delta = 1/\sqrt{N}$ to price an American Put option with strike price of $X = \$170$ and maturity of 0.5-years and 1.5-years. Use the first k of the Hermite Polynomials for $k = 2, 3, 4, 5$. (That is, you will compute 8 prices here). Compare the prices for the 4 cases, $k = 2, 3, 4, 5$ and comment on the choice of k .

```
In [146... # define hermite polynomial function generator
def hermite(x, k):
    # length
    H = np.zeros(len(x), k)
    H[:, 0] = 1
    # handle first k
    for n in range(1, k):
        H[:, n] = (-1)**n * np.exp(.5*x**2) * np.exp(.5*(-x)**2)
    return H
```

```
In [272... def lsmc_h(S0, X, T, r, sigma, N, k):
    # set time increment and number of steps
    dt = 1/np.sqrt(N)
    steps = int(T/dt)
    df = np.exp(-r * dt)

    # simulate brownian motion
    W = np.random.normal(0, 1, 50000)
    # normal simulations
    S_normal = S0 * np.exp(np.cumsum(.5 * r * sigma**2)*dt + sigma * np.sqrt
    # antithetic
    S_antithetic = S0 * np.exp(np.cumsum(.5 * r * sigma**2)*dt - sigma * np.
    # combine
    S = np.vstack((S_normal, S_antithetic))
    # get exercise values (payoffs)
    ev = np.maximum(X-S, 0)
    # store them for each step iterating backwards
    evs = ev[:, -1]

    for time in range(steps - 1, 0, -1):
        # for each time get paths that are in the money
        itm = S[:, time] < X
        # store them separately
        S_itm = S[itm, time]
        # discount values
        dcf = evs[itm] * df

        H = hermite(S_itm, k)
        A = regression(H, dcf)

        ecv = H.dot(A)
        immediate_exercise = np.maximum(X - S_itm, 0)
        exercise = immediate_exercise > ecv
        evs[itm] = np.where(exercise, immediate_exercise, dcf)
```

```

    price = evs.mean() * df
    return price

p4_df_2 = pd.DataFrame(index = ['.5', '1.5'],
                        columns = ['k=2', 'k=3', 'k=4', 'k=5'])
p4_df_2.at['.5', 'k=2'] = lsmc_h(180, 170, .5, .055, .25, .25, 2)
p4_df_2.at['.5', 'k=3'] = lsmc_h(180, 170, .5, .055, .25, .25, 3)
p4_df_2.at['.5', 'k=4'] = lsmc_h(180, 170, .5, .055, .25, .25, 4)
p4_df_2.at['.5', 'k=5'] = lsmc_h(180, 170, .5, .055, .25, .25, 5)
p4_df_2.at['1.5', 'k=2'] = lsmc_h(180, 170, 1.5, .055, .25, .25, 2)
p4_df_2.at['1.5', 'k=3'] = lsmc_h(180, 170, 1.5, .055, .25, .25, 3)
p4_df_2.at['1.5', 'k=4'] = lsmc_h(180, 170, 1.5, .055, .25, .25, 4)
p4_df_2.at['1.5', 'k=5'] = lsmc_h(180, 170, 1.5, .055, .25, .25, 5)
print(p4_df_2)

```

	k=2	k=3	k=4	k=5
.5	0.877032	13.121164	10.45334	30.20704
1.5	9.711041	26.182219	0.0	10.04102

(c) Use the LSMC method with $N=100,000$ paths simulations (50,000 plus 50,000 antithetic variates) and a time step of $\Delta = 1/\sqrt{N}$ to price an American Put option with strike price of $X = \$170$ and maturity of 0.5-years and 1.5-years. Use the first k of the Simple Monomials for $k = 2, 3, 4, 5$. (That is, you will compute 8 prices here). Compare the prices for the 4 cases, $k = 2, 3, 4, 5$ and comment on the choice of k .

```

In [148...] def monomials(x, k):
    return x**k

```

```

In [273...] def lsmc_m(S0, X, T, r, sigma, N, k):
    # set time increment and number of steps
    dt = 1/np.sqrt(N)
    steps = int(T/dt)
    df = np.exp(-r * dt)

    # simulate brownian motion
    W = np.random.normal(0, 1, 50000)
    # normal simulations
    S_normal = S0 * np.exp(np.cumsum(.5 * r * sigma**2)*dt + sigma * np.sqrt
    # antithetic
    S_antithetic = S0 * np.exp(np.cumsum(.5 * r * sigma**2)*dt - sigma * np.
    # combine
    S = np.vstack((S_normal, S_antithetic))
    # get exercise values (payoffs)
    ev = np.maximum(X-S, 0)
    # store them for each step iterating backwards
    evs = ev[:, -1]

    for time in range(steps - 1, 0, -1):
        # for each time get paths that are in the money
        itm = S[:, time] < X
        # store them separately
        S_itm = S[itm, time]
        # discount values
        dcf = evs[itm] * df

```

```

M = monomials(S_itm, k)
A = regression(M, dcf)

ecv = M.dot(A)
immediate_exercise = np.maximum(X - S_itm, 0)
exercise = immediate_exercise > ecv
evs[itm] = np.where(exercise, immediate_exercise, dcf)
price = evs.mean() * df
return price

p4_df_3 = pd.DataFrame(index = ['.5', '1.5'],
                        columns = ['k=2', 'k=3', 'k=4', 'k=5'])
p4_df_3.at['.5', 'k=2'] = lsmc_m(180, 170, .5, .055, .25, .25, 2)
p4_df_3.at['.5', 'k=3'] = lsmc_m(180, 170, .5, .055, .25, .25, 3)
p4_df_3.at['.5', 'k=4'] = lsmc_m(180, 170, .5, .055, .25, .25, 4)
p4_df_3.at['.5', 'k=5'] = lsmc_m(180, 170, .5, .055, .25, .25, 5)
p4_df_3.at['1.5', 'k=2'] = lsmc_m(180, 170, 1.5, .055, .25, .25, 2)
p4_df_3.at['1.5', 'k=3'] = lsmc_m(180, 170, 1.5, .055, .25, .25, 3)
p4_df_3.at['1.5', 'k=4'] = lsmc_m(180, 170, 1.5, .055, .25, .25, 4)
p4_df_3.at['1.5', 'k=5'] = lsmc_m(180, 170, 1.5, .055, .25, .25, 5)
print(p4_df_3)

```

	k=2	k=3	k=4	k=5
.5	23.672941	10.287958	4.769637	8.776895
1.5	32.545743	9.981765	21.547401	2.986268

(d) Compare all your findings above and comment. Note: You will need to use the weighted-polynomials as it was done by the authors of the method.

- 1 and 3 seem similar, 2 is off probably because I made a mistake in defining Hermite polynomials but I can't find it.

Problem 5.

Consider the following information on the stock of company XYZ: The volatility of the stock price is $\sigma = 25\%$ per annum. Assume the prevailing risk-free rate is $r = 5.5\%$ per annum. Use the $X = \ln(S)$ transformation of the Black-Scholes PDE, and $\Delta t = 0.002$, with $\Delta X = \sigma\sqrt{\Delta t}$; then with $\Delta X = \sigma\sqrt{3\Delta t}$; then with $\Delta X = \sigma\sqrt{4\Delta t}$, and a uniform grid (on X) to price an American Put option with strike price of $X = \$170$, expiration of 6 months and current stock prices ranging from \$170 to \$190; using the specified methods below:

(a) Explicit Finite-Difference method

```

In [211...] # since according to the propt we need price for $1 increments:
from scipy.interpolate import interp1d

```

```

In [235...] # get explicit 1
def explicit_1(S0_start, S0_end, K, r, sigma, T, dt):
    dx = sigma * np.sqrt(dt)
    X_start = np.log(S0_start)
    X_end = np.log(S0_end)
    # define prices and increment
    N = int((X_end - X_start) / dx)

```

```

log_grid = np.linspace(X_start, X_end, N + 1)
S = np.exp(log_grid) # Actual stock price grid
# Time steps
M = int(T / dt)

# put option value matrix
P = np.zeros((N + 1, M + 1))
P[:, -1] = np.maximum(K - S, 0)

# set probabilities
P_u = dt * (((sigma**2)/(2*dx**2)) + ((r - .5 * sigma**2)/(2*dx)))
P_m = 1 - dt * (sigma**2/dx**2) - r*dt
P_d = dt * (((sigma**2)/(2*dx**2)) - ((r - .5 * sigma**2)/(2*dx)))

# implement efd
for j in range(M - 1, -1, -1):
    for i in range(1, N):
        P[i, j] = P_u * P[i+1, j+1] + P_m * P[i+1, j] + P_d * P[i+1, j-1]
    # boundary conditions
    P[0, j] = max(K - S[0], P[0, j+1]*np.exp(-r*dt))
    P[N, j] = 0
    P[:, j] = np.maximum(P[:, j], K - S)
# interpolate to get the increments
target_S = np.arange(170, 191)
interp_func = interp1d(S, P[:, 0], kind='linear', fill_value="extrapolat
target_P = interp_func(target_S)

return target_P, target_S

```

```

In [239... # same as the function above with different dx
def explicit_2(S0_start, S0_end, K, r, sigma, T, dt):
    dx = sigma * np.sqrt(3*dt)
    X_start = np.log(S0_start)
    X_end = np.log(S0_end)
    # define prices and increment
    N = int((X_end - X_start) / dx)
    log_grid = np.linspace(X_start, X_end, N + 1)
    S = np.exp(log_grid) # Actual stock price grid
    # Time steps
    M = int(T / dt)

    # put option value matrix
    P = np.zeros((N + 1, M + 1))
    P[:, -1] = np.maximum(K - S, 0)

    # set probabilities
    P_u = dt * (((sigma**2)/(2*dx**2)) + ((r - .5 * sigma**2)/(2*dx)))
    P_m = 1 - dt * (sigma**2/dx**2) - r*dt
    P_d = dt * (((sigma**2)/(2*dx**2)) - ((r - .5 * sigma**2)/(2*dx)))

    # implement efd
    for j in range(M - 1, -1, -1):
        for i in range(1, N):
            P[i, j] = P_u * P[i+1, j+1] + P_m * P[i+1, j] + P_d * P[i+1, j-1]
        # boundary conditions
        P[0, j] = max(K - S[0], P[0, j+1]*np.exp(-r*dt))

```

```

        P[N, j] = 0
        P[:, j] = np.maximum(P[:, j], K - S)
    # interpolate to get the increments
    target_S = np.arange(170, 191)
    interp_func = interp1d(S, P[:, 0], kind='linear', fill_value="extrapolate")
    target_P = interp_func(target_S)

    return target_P, target_S

```

```

In [244... # same as the function above with different dx
def explicit_3(S0_start, S0_end, K, r, sigma, T, dt):
    dx = sigma * np.sqrt(4*dt)
    X_start = np.log(S0_start)
    X_end = np.log(S0_end)
    # define prices and increment
    N = int((X_end - X_start) / dx)
    log_grid = np.linspace(X_start, X_end, N + 1)
    S = np.exp(log_grid) # Actual stock price grid
    # Time steps
    M = int(T / dt)

    # put option value matrix
    P = np.zeros((N + 1, M + 1))
    P[:, -1] = np.maximum(K - S, 0)

    # set probabilities
    P_u = dt * (((sigma**2)/(2*dx**2)) + ((r - .5 * sigma**2)/(2*dx)))
    P_m = 1 - dt * (sigma**2/dx**2) - r*dt
    P_d = dt * (((sigma**2)/(2*dx**2)) - ((r - .5 * sigma**2)/(2*dx)))

    # implement efd
    for j in range(M - 1, -1, -1):
        for i in range(1, N):
            P[i, j] = P_u * P[i+1, j+1] + P_m * P[i+1, j] + P_d * P[i+1, j-1]
        # boundary conditions
        P[0, j] = K - S[0] * np.exp(-r*(M-j)*dt)
        P[N, j] = 0
        P[:, j] = np.maximum(P[:, j], K - S)
    # interpolate to get the increments
    target_S = np.arange(170, 191)
    interp_func = interp1d(S, P[:, 0], kind='linear', fill_value="extrapolate")
    target_P = interp_func(target_S)

    return target_P, target_S

```

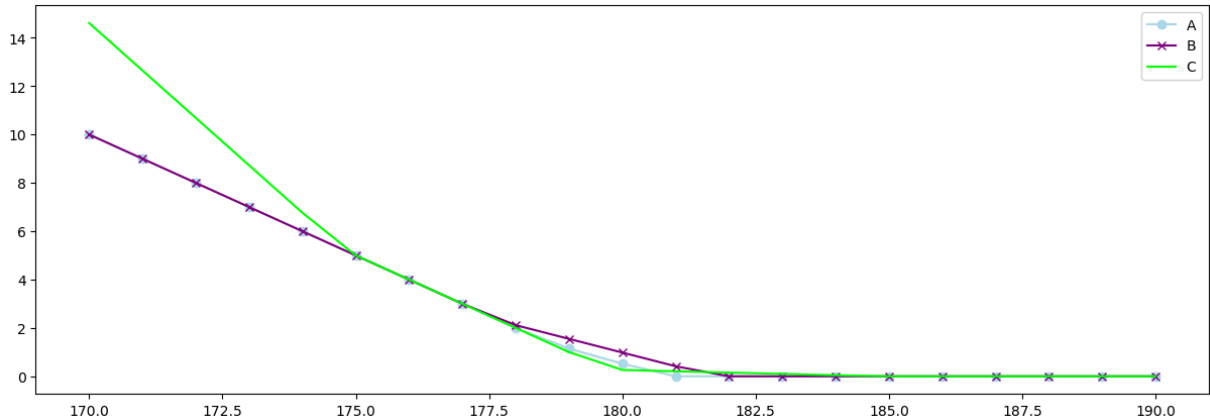
```

In [245... # get values
put_a, grid_a = explicit_1(170, 190, 180, .055, .25, .5, .002)
put_b, grid_b = explicit_2(170, 190, 180, .055, .25, .5, .002)
put_c, grid_c = explicit_3(170, 190, 180, .055, .25, .5, .002)

# plot the figure
plt.figure(figsize=(15, 5))
plt.plot(grid_a, put_a, label='A', color='lightblue', marker='o')
plt.plot(grid_b, put_b, label='B', color='purple', marker='x')
plt.plot(grid_c, put_c, label='C', color='lime')

```

```
plt.legend()
plt.show()
```



```
In [255... # implicit
def implicit_1(S0_start, S0_end, K, r, sigma, T, dt):
    dx = sigma * np.sqrt(dt)
    X_start = np.log(S0_start)
    X_end = np.log(S0_end)
    # define prices and increment
    N = int((X_end - X_start) / dx)
    log_grid = np.linspace(X_start, X_end, N + 1)
    S = np.exp(log_grid) # Actual stock price grid
    # Time steps
    M = int(T / dt)

    # put option value matrix
    P = np.zeros((N + 1, M + 1))
    P[:, -1] = np.maximum(K - S, 0)

    P_u = -.5*dt*(((sigma**2)/(dx**2)) + ((r - .5 * sigma**2)/dx))
    P_m = 1 + dt*(sigma**2/dx**2) + r*dt
    P_d = -.5*dt*(((sigma**2)/(dx**2)) - ((r - .5 * sigma**2)/dx))

    # implement efd
    for j in range(M - 1, -1, -1):
        for i in range(1, N):
            P[i, j] = P_u * P[i+1, j+1] + P_m * P[i+1, j] + P_d * P[i+1, j-1]
        # boundary conditions
        P[0, j] = K - S[0] * np.exp(-r*(M-j)*dt)
        P[N, j] = 0
        P[:, j] = np.maximum(P[:, j], K - S)
    # interpolate to get the increments
    target_S = np.arange(170, 191)
    interp_func = interp1d(S, P[:, 0], kind='linear', fill_value="extrapolat
    target_P = interp_func(target_S)

    return target_P, target_S
```

```
In [256... def implicit_2(S0_start, S0_end, K, r, sigma, T, dt):
    dx = sigma * np.sqrt(3*dt)
    X_start = np.log(S0_start)
    X_end = np.log(S0_end)
```



```

# define prices and increment
N = int((X_end - X_start) / dx)
log_grid = np.linspace(X_start, X_end, N + 1)
S = np.exp(log_grid) # Actual stock price grid
# Time steps
M = int(T / dt)

# put option value matrix
P = np.zeros((N + 1, M + 1))
P[:, -1] = np.maximum(K - S, 0)

P_u = -.5*dt*(((sigma**2)/(dx**2)) + ((r - .5 * sigma**2)/dx))
P_m = 1 + dt*(sigma**2/dx**2) + r*dt
P_d = -.5*dt*(((sigma**2)/(dx**2)) - ((r - .5 * sigma**2)/dx))

# implement efd
for j in range(M - 1, -1, -1):
    for i in range(1, N):
        P[i, j] = P_u * P[i+1, j+1] + P_m * P[i+1, j] + P_d * P[i+1, j-1]

# boundary conditions
P[0, j] = K - S[0] * np.exp(-r*(M-j)*dt)
P[N, j] = 0
P[:, j] = np.maximum(P[:, j], K - S)
# interpolate to get the increments
target_S = np.arange(170, 191)
interp_func = interp1d(S, P[:, 0], kind='linear', fill_value="extrapolate")
target_P = interp_func(target_S)

return target_P, target_S

```

```

In [257... def implicit_3(S0_start, S0_end, K, r, sigma, T, dt):
    dx = sigma * np.sqrt(4*dt)
    X_start = np.log(S0_start)
    X_end = np.log(S0_end)
    # define prices and increment
    N = int((X_end - X_start) / dx)
    log_grid = np.linspace(X_start, X_end, N + 1)
    S = np.exp(log_grid) # Actual stock price grid
    # Time steps
    M = int(T / dt)

    # put option value matrix
    P = np.zeros((N + 1, M + 1))
    P[:, -1] = np.maximum(K - S, 0)

    P_u = -.5*dt*(((sigma**2)/(dx**2)) + ((r - .5 * sigma**2)/dx))
    P_m = 1 + dt*(sigma**2/dx**2) + r*dt
    P_d = -.5*dt*(((sigma**2)/(dx**2)) - ((r - .5 * sigma**2)/dx))

    # implement efd
    for j in range(M - 1, -1, -1):
        for i in range(1, N):
            P[i, j] = P_u * P[i+1, j+1] + P_m * P[i+1, j] + P_d * P[i+1, j-1]

    # boundary conditions

```

```

P[0, j] = K - S[0] * np.exp(-r*(M-j)*dt)
P[N, j] = 0
P[:, j] = np.maximum(P[:, j], K - S)
# interpolate to get the increments
target_S = np.arange(170, 191)
interp_func = interp1d(S, P[:, 0], kind='linear', fill_value="extrapolate")
target_P = interp_func(target_S)

return target_P, target_S

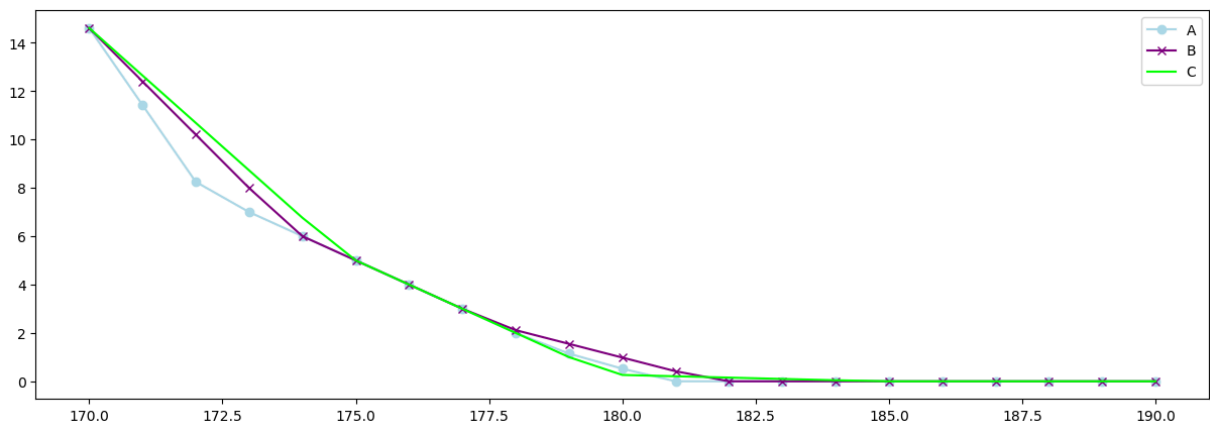
```

```

In [258]: # get values
put_2a, grid_2a = implicit_1(170, 190, 180, .055, .25, .5, .002)
put_2b, grid_2b = implicit_2(170, 190, 180, .055, .25, .5, .002)
put_2c, grid_2c = implicit_3(170, 190, 180, .055, .25, .5, .002)

# plot the figure
plt.figure(figsize=(15, 5))
plt.plot(grid_2a, put_2a, label='A', color='lightblue', marker='o')
plt.plot(grid_2b, put_2b, label='B', color='purple', marker='x')
plt.plot(grid_2c, put_2c, label='C', color='lime')
plt.legend()
plt.show()

```



```

In [298]: # crank-nikolson
def cn_1(S0_start, S0_end, K, r, sigma, T, dt):
    dx = sigma * np.sqrt(dt)
    X_start = np.log(S0_start)
    X_end = np.log(S0_end)
    # define prices and increment
    N = int((X_end - X_start) / dx)
    log_grid = np.linspace(X_start, X_end, N + 1)
    S = np.exp(log_grid)
    # time steps
    M = int(T / dt)

    # put option value matrix
    P = np.zeros((N + 1, M + 1))
    P[:, -1] = np.maximum(K - S, 0)

    P_u = -.25*dt*(((sigma**2)/(dx**2)) + ((r - .5 * sigma**2)/dx))
    P_m = 1 + dt*(sigma**2/dx**2) + r*dt
    P_d = -.25*dt*(((sigma**2)/(dx**2)) - ((r - .5 * sigma**2)/dx))

```

```

# implement efd
for j in range(M - 1, -1, -1):
    for i in range(1, N):
        P[i, j] = (-P_u * P[i+1, j+1] - (P_m - 1) * P[i+1, j] - P_d * P[i, j+1])

# boundary conditions
P[0, j] = K - S[0] * np.exp(-r*(M-j)*dt)
P[N, j] = 0
P[:, j] = np.maximum(P[:, j], K - S)
# interpolate to get the increments
target_S = np.arange(170, 191)
interp_func = interp1d(S, P[:, 0], kind='linear', fill_value="extrapolate")
target_P = interp_func(target_S)

return target_P, target_S

```

In [262]...

```

# crank-nikolson
def cn_2(S0_start, S0_end, K, r, sigma, T, dt):
    dx = sigma * np.sqrt(3*dt)
    X_start = np.log(S0_start)
    X_end = np.log(S0_end)
    # define prices and increment
    N = int((X_end - X_start) / dx)
    log_grid = np.linspace(X_start, X_end, N + 1)
    S = np.exp(log_grid) # Actual stock price grid
    # Time steps
    M = int(T / dt)

    # put option value matrix
    P = np.zeros((N + 1, M + 1))
    P[:, -1] = np.maximum(K - S, 0)

    P_u = -.25*dt*(((sigma**2)/(dx**2)) + ((r - .5 * sigma**2)/dx))
    P_m = 1 + dt*(sigma**2/dx**2) + r*dt
    P_d = -.25*dt*(((sigma**2)/(dx**2)) - ((r - .5 * sigma**2)/dx))

    # implement efd
    for j in range(M - 1, -1, -1):
        for i in range(1, N):
            P[i, j] = (-P_u * P[i+1, j+1] - (P_m - 1) * P[i+1, j] - P_d * P[i, j+1])

# boundary conditions
P[0, j] = K - S[0] * np.exp(-r*(M-j)*dt)
P[N, j] = 0
P[:, j] = np.maximum(P[:, j], K - S)
# interpolate to get the increments
target_S = np.arange(170, 191)
interp_func = interp1d(S, P[:, 0], kind='linear', fill_value="extrapolate")
target_P = interp_func(target_S)

return target_P, target_S

```

In [263]...

```

# crank-nikolson
def cn_3(S0_start, S0_end, K, r, sigma, T, dt):

```

```

dx = sigma * np.sqrt(4*dt)
X_start = np.log(S0_start)
X_end = np.log(S0_end)
# define prices and increment
N = int((X_end - X_start) / dx)
log_grid = np.linspace(X_start, X_end, N + 1)
S = np.exp(log_grid) # Actual stock price grid
# Time steps
M = int(T / dt)

# put option value matrix
P = np.zeros((N + 1, M + 1))
P[:, -1] = np.maximum(K - S, 0)

P_u = -.25*dt*(((sigma**2)/(dx**2)) + ((r - .5 * sigma**2)/dx))
P_m = 1 + dt*(sigma**2/dx**2) + r*dt
P_d = -.25*dt*(((sigma**2)/(dx**2)) - ((r - .5 * sigma**2)/dx))

# implement efd
for j in range(M - 1, -1, -1):
    for i in range(1, N):
        P[i, j] = (-P_u * P[i+1, j+1] - (P_m - 1) * P[i+1, j] - P_d * P[i, j+1])

# boundary conditions
P[0, j] = K - S[0] * np.exp(-r*(M-j)*dt)
P[N, j] = 0
P[:, j] = np.maximum(P[:, j], K - S)
# interpolate to get the increments
target_S = np.arange(170, 191)
interp_func = interp1d(S, P[:, 0], kind='linear', fill_value="extrapolate")
target_P = interp_func(target_S)

return target_P, target_S

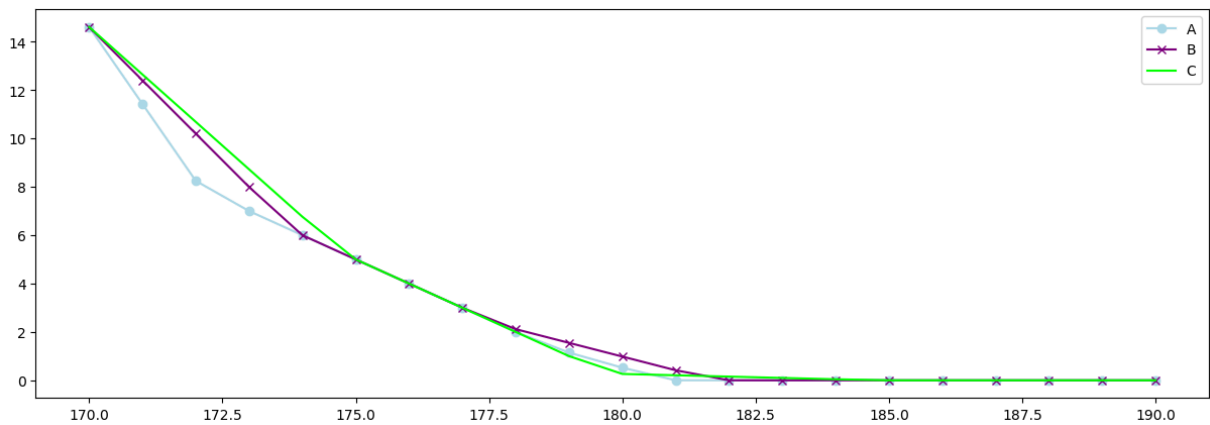
```

```

In [299... # get values
put_3a, grid_3a = cn_1(170, 190, 180, .055, .25, .5, .002)
put_3b, grid_3b = cn_2(170, 190, 180, .055, .25, .5, .002)
put_3c, grid_3c = cn_3(170, 190, 180, .055, .25, .5, .002)

# plot the figure
plt.figure(figsize=(15, 5))
plt.plot(grid_3a, put_3a, label='A', color='lightblue', marker='o')
plt.plot(grid_3b, put_3b, label='B', color='purple', marker='x')
plt.plot(grid_3c, put_3c, label='C', color='lime')
plt.legend()
plt.show()

```



In [266... *# not working LOL*

```
write_up = pd.DataFrame(columns = ['Explicit dx(a)', 'Explicit dx(b)', 'Explicit dx(c)', 'Crank-Nikolson dx(a)', 'Crank-Nikolson dx(b)', 'Crank-Nikolson dx(c)'],
                        index= ['$170', '$171', '$172', '$173', '$174', '$175', '$176', '$177', '$178', '$179', '$180', '$181', '$182', '$183', '$184', '$185', '$186', '$187', '$188', '$189', '$190'])

explicit_da, grid_da = explicit_1(170, 190, 180, .055, .25, .5, .002)
explicit_db, grid_db = explicit_2(170, 190, 180, .055, .25, .5, .002)
explicit_dc, grid_dc = explicit_3(170, 190, 180, .055, .25, .5, .002)
implicit_da, grid_2da = implicit_1(170, 190, 180, .055, .25, .5, .002)
implicit_db, grid_2db = implicit_2(170, 190, 180, .055, .25, .5, .002)
implicit_dc, grid_2dc = implicit_3(170, 190, 180, .055, .25, .5, .002)
crank_da, grid_3da = cn_1(170, 190, 180, .055, .25, .5, .002)
crank_db, grid_3db = cn_2(170, 190, 180, .055, .25, .5, .002)
crank_dc, grid_3dc = cn_3(170, 190, 180, .055, .25, .5, .002)
write_up['Explicit dx(a)'] = explicit_da
write_up['Explicit dx(b)'] = explicit_db
write_up['Explicit dx(c)'] = explicit_dc
write_up['Implicit dx(a)'] = implicit_da
write_up['Implicit dx(b)'] = implicit_db
write_up['Implicit dx(c)'] = implicit_dc
write_up['Crank-Nikolson dx(a)'] = crank_da
write_up['Crank-Nikolson dx(b)'] = crank_db
write_up['Crank-Nikolson dx(c)'] = crank_dc
write_up
```

Out [266...

	Explicit dx(a)	Explicit dx(b)	Explicit dx(c)	Implicit dx(a)	Implicit dx(b)	Implicit dx(c)	Crank- Nikolson dx(a)
\$170	10.000000	10.000000	14.611304	14.611304	14.611304	14.611304	14.611304
\$171	9.000000	9.000000	12.649298	11.429950	12.405434	12.649298	11.429950
\$172	8.000000	8.000000	10.687292	8.248597	10.199564	10.687292	8.248597
\$173	7.000000	7.000000	8.725286	7.000000	7.993694	8.725286	7.000000
\$174	6.000000	6.000000	6.763279	6.000000	6.000000	6.763279	6.000000
\$175	5.000000	5.000000	5.000000	5.000000	5.000000	5.000000	5.000000
\$176	4.000000	4.000000	4.000000	4.000000	4.000000	4.000000	4.000000
\$177	3.000000	3.000000	3.000000	3.000000	3.000000	3.000000	3.000000
\$178	2.000000	2.115195	2.000000	2.000000	2.115195	2.000000	2.000000
\$179	1.144948	1.548443	1.000000	1.144948	1.548443	1.000000	1.144948
\$180	0.521333	0.981691	0.262743	0.521333	0.981691	0.262743	0.521333
\$181	0.000000	0.414939	0.207885	0.000000	0.414939	0.207885	0.000000
\$182	0.000000	0.000000	0.153028	0.000000	0.000000	0.153028	0.000000
\$183	0.000000	0.000000	0.098170	0.000000	0.000000	0.098170	0.000000
\$184	0.000000	0.000000	0.043313	0.000000	0.000000	0.043313	0.000000
\$185	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
\$186	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
\$187	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
\$188	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
\$189	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000
\$190	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000

Problem 6.

Consider the following information on the stock of company XYZ: The volatility of the stock price is $\sigma = 25\%$ per annum. Assume the prevailing risk-free rate is $r = 5.5\%$ per annum. Use the Black-Scholes PDE (for S) to price American Put options with strike prices of $K = \$170$, expiration of 6 months and current stock prices for a range from \$170 to \$190; using the specified methods below: (a) Explicit Finite-Difference method:

In [295...

[illegible]

[illegible]

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[illegible]

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[illegible]

[illegible]

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[illegible]

[illegible]

34/36

[illegible]

```

0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
Option values at step 243: [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
Option values at step 244: [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
Option values at step 245: [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
Option values at step 246: [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
Option values at step 247: [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
Option values at step 248: [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
Option values at step 249: [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]
Option values at step 250: [0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0.
0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0. 0.]

```

```

Out[295... array([0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0., 0.,
0., 0., 0., 0., 0., 0., 0., 0.])

```