

# Dynamic Economic Dispatch for Microgrids Including Battery Energy Storage

X.P. Liu<sup>1</sup>, M. Ding<sup>1</sup>, J.H. Han<sup>2</sup> and P.P. Han<sup>1</sup>

1. School of Electrical Engineering and Automation, Hefei University of Technology, Hefei 230009, China

2. School of Computer and Information, Hefei University of Technology, Hefei 230009, China

Received: September 09, 2010 / Accepted: December 29, 2010 / Published: May 31, 2011.

**Abstract:** Microgrids integrate distributed renewable energy resources, controllable loads and energy storage in a more economic and reliable fashion. Battery energy storage units are essential for microgrid operation, which make microgrid become a strong coupling system in the time domain. Hence, the traditional methods of static dispatch are no longer suitable for microgrids. This paper proposes a dynamic economic dispatch method for microgrids. Considering microgrid as a discrete time system, the dynamic economic dispatch is to find the optimal control strategy for the system in finite time period. Based on this idea, the dynamic economic dispatch model for microgrids is established, and then the corresponding dynamic programming algorithm is designed. Finally, an example of microgrid is given, and the dynamic economic dispatch results are compared with that of the static dispatch. The comparison confirms the effectiveness of the proposed dynamic dispatch method.

**Key words:** Battery energy storage, dynamic systems, economic dispatch, microgrids.

## 1. Introduction

The microgrid concept provides a new paradigm for defining the operation of distributed generation [1-4]. It consists of distributed generation units such as photovoltaic panels, fuel cells, microturbines, small wind generators, together with energy storage devices and controllable loads that operate in low voltage networks. Microgrid is regarded as a single controllable system that provides both power and heat to its local area. These systems can be connected to the main power grid or be operated autonomously in an islanded mode, if disconnected from the main grid.

Power system dispatch can be classified into two categories. One is static dispatch, which searches the optimal solution in each separated time period, without taking the relationships among different periods into account; the other is dynamic dispatch [5, 6], which

considers the coupling in the time domain, such as ramp rate constraints for generators. The results of dynamic economic dispatch are more effective. Integrated with battery energy storage units [7, 8], microgrid becomes a strong coupling system in the time domain. Accordingly, dynamic dispatch is more significant for microgrids.

This paper proposes a dynamic economic dispatch method for microgrids. Considering microgrid as a discrete time system, dynamic dispatch is to find the optimal control strategy for the system in finite time period. Based on this idea, the dynamic economic dispatch model for microgrids is established, and then the corresponding dynamic programming algorithm is designed. Finally, an example proves the effectiveness of the proposed model and algorithm.

## 2. Formulation of Dynamic Economic Dispatch for Microgrids

Dynamic economic dispatch for microgrids is to determine the power output of controllable generating

---

**Corresponding author:** X.P. Liu, Ph.D., assistant professor, research fields: distributed generation, smart grid. E-mail: lxp.hfut@gmail.com.

units, as well as the power output (or input) of energy storage units and main grid in each time period, in order to minimize the total operating costs and meet some constraints.

The objective function is defined in the form:

$$\min C = \sum_{k=0}^{N-1} [\sum_{h=1}^{m-1} F_h(P_{hk}) + F_{\text{grid}}(P_{\text{grid } k}) + F_{\text{batt}}(P_{\text{batt } k})] \quad (1)$$

where a cycle includes  $N$  time periods. Connected with main power grid, the microgrid includes  $m-1$  controllable generating units and an energy storage unit. In Eq. (1),  $P_{hk}$  is the power output of generating unit  $h$  in period  $k$ ,  $P_{\text{grid } k}$  and  $P_{\text{batt } k}$  are respectively the power output of main grid and battery energy storage in period  $k$  (where negative value means input), and  $F_h$ ,  $F_{\text{grid}}$  and  $F_{\text{batt}}$  are respectively the cost function of generating unit  $h$ , main grid and battery energy storage (where negative function value of  $F_{\text{grid}}$  means income from power sale).

Constraints are as follows:

$$\sum_{h=1}^{m-1} P_{hk} + P_{\text{grid } k} + P_{\text{batt } k} = P_{\text{load } k} - P_{\text{unctrl } k} \quad (2)$$

$$P_h^{\min} \leq P_{hk} \leq P_h^{\max} \quad (3)$$

$$|P_{h,k+1} - P_{hk}| \leq r_h^{\max} \Delta t \quad (4)$$

$$P_{\text{grid}}^{\min} \leq P_{\text{grid } k} \leq P_{\text{grid}}^{\max} \quad (5)$$

$$-P_{\text{charge}}^{\max} \leq P_{\text{batt } k} \leq P_{\text{discharge}}^{\max} \quad (6)$$

$$E_{\min} \leq E_0 + \sum_{l=0}^k P_{\text{batt } l} \Delta t \leq E_{\max} \quad (7)$$

where  $k=0,1,\dots,N-1$ ,  $h=1,2,\dots,m-1$ . Eq. (2) is power balance constraint, where  $P_{\text{load } k}$  is total active load in period  $k$ , and  $P_{\text{unctrl } k}$  is total power output of uncontrollable resources in period  $k$ . Inequality (3) and (4) are respectively constraints of power limit and ramp rate for generators, where  $P_h^{\max}$ ,  $P_h^{\min}$  and  $r_h^{\max}$  are respectively upper limit, lower limit and maximal ramp rate of generating unit  $h$ . Inequality (5) is power transmission constraint between microgrid and main grid, where  $P_{\text{grid}}^{\max}$  and  $P_{\text{grid}}^{\min}$  are respectively upper and lower power limits. Inequality (6) is power constraint for energy storage unit, where  $P_{\text{charge}}^{\max}$  and  $P_{\text{discharge}}^{\max}$  are respectively maximal charging and discharging power. Inequality (7) is energy constraint for energy storage

unit, where  $E_{\max}$  and  $E_{\min}$  are respectively the upper and lower energy limits,  $E_0$  is the stored energy of battery unit at the beginning of a cycle, and  $\Delta t$  is the length of time period.

Considering that dynamic economic dispatch scheme for microgrids is executed in cycles, it may be assumed that the stored energy of battery unit at the beginning and the end of a cycle are equal. Accordingly, add the following constraint:

$$E_0 + \sum_{k=0}^{N-1} P_{\text{batt } k} \Delta t = E_0 \quad (8)$$

where notations mean the same as in inequality (7).

### 3. Model Based on Dynamic System Theory

Based on dynamic system theory, microgrid can be regarded as a discrete time system with finite time period. Thereby, dynamic economic dispatch for microgrids is to find the optimal control strategy of the system.

#### 3.1 Optimal Control for Dynamic System

The optimal control for dynamic system refers to a discrete time system with finite time period, as well as a cost functional summed up in the time domain. Discrete time dynamic system with  $N$  periods is defined in the form:

$$x_{k+1} = f_k(x_k, u_k, w_k) \quad (9)$$

where  $k=0,1,\dots,N-1$ . In Eq. (9),  $x_k$  is system state,  $u_k$  is decision variable (specifically, action at state  $x_k$  in period  $k$ ), and  $w_k$  is random variable.

Define  $g_k(x_k, u_k, w_k)$  as cost in period  $k$ , and cost of the dynamic system is as follows:

$$g_N(x_N) + \sum_{k=0}^{N-1} g_k(x_k, u_k, w_k) \quad (10)$$

where  $g_N(x_N)$  is terminal cost of the system. Considering random variable  $w_k$ , the solution is to determine decision variables  $u_0, u_1, \dots, u_{N-1}$ , in order to minimize expectation of expression (10).

There is functional relation between  $u_k$  and  $x_k$ , which is defined as  $u_k = \mu_k(x_k)$ . Hence the solution essentially determines an optimal function sequence, i.e.  $\pi = \{\mu_0, \mu_1, \dots, \mu_{N-1}\}$ , named "strategy".

Given initial system state  $x_0$ , the cost functional is defined as

$$J_{\pi}(x_0) = E\{g_N(x_N) + \sum_{k=0}^{N-1} g_k[x_k, \mu_k(x_k), w_k]\} \quad (11)$$

where notations mean the same as in (10). Thereby, the optimal control for dynamic system is to search for strategy  $\pi^*$ , so as to make  $J_{\pi^*}(x_0) = \min_{\pi \in \Pi} J_{\pi}(x_0)$  where  $\Pi$  is set of strategies.

### 3.2 Dynamic Economic Dispatch Model for Microgrids

The microgrid dispatch cycle includes  $N$  time periods. In each period, the stored energy of battery unit is regarded as system state; the energy output of each generating unit is decision variable; and the loads and uncontrollable resources are random factors. Taking the form of formula (9), state changing equation of microgrid is built as follows:

$$x_{k+1} = x_k + \sum_{h=1}^m u_{hk} - w_k, \quad k = 0, 1, \dots, N-1 \quad (12)$$

where  $k = 0, 1, \dots, N-1$ . In Eq. (12),  $x_k$  is stored energy at the beginning of period  $k$ , i.e.  $x_k = E_k$ ;  $w_k$  is energy consumption minus output of uncontrollable resources in period  $k$ , i.e.  $w_k = (P_{loadk} - P_{unctrlk})\Delta t$ . Since microgrid includes more than one generating unit, decision variables in period  $k$  may be expressed as a vector, i.e.  $\mathbf{u}_k = (u_{1k}, u_{2k}, \dots, u_{mk})^T$ . The element  $u_{hk}$  is energy output of controllable generator  $h$  in period  $k$ , i.e.  $u_{hk} = P_{hk}\Delta t$ ,  $h = 1, 2, \dots, m$  where main grid is regarded as generating unit  $m$ .

Given initial system state  $x_0$ , the cost function is defined as:

$$J_{\pi}(x_0) = E\left\{\sum_{k=0}^{N-1} \left[\sum_{h=1}^{m-1} F_h\left(\frac{u_{hk}}{\Delta t}\right) + F_{\text{grid}}\left(\frac{u_{mk}}{\Delta t}\right) + F_{\text{batt}}\left(\frac{x_{k+1} - x_k}{\Delta t}\right)\right]\right\} \quad (13)$$

where cost in period  $k$  includes operating costs of all controllable generation units and energy storage unit. The dynamic economic dispatch for microgrids is to search for an optimal strategy, i.e.  $\pi^* = \{\mu_0^*, \mu_1^*, \dots, \mu_{N-1}^*\}$  where  $\mathbf{u}_k = \mu_k^*(x_k)$ , so as to make Eq. (13) minimum.

In the search process, constraints of dynamic economic dispatch in formula (2)-(8) should be

satisfied. Specifically, formula (2) is met by the system state changing equation; formula (3) and (5) can be satisfied by limiting value range of decision vector; formula (7) can be satisfied by limiting variation range of system state; formula (8) can be satisfied by making terminal state the same as initial state, i.e.  $x_N = x_0 = E_0$ ; and formula (4) and (6) will be considered in the following algorithm.

## 4. Dynamic Programming Algorithm for Solution

Dynamic programming algorithms are generally used for solving optimal control problems for dynamic system, which can be realized by forward or backward iteration. This paper selects forward iteration to search for the optimal strategy.

Define  $J_k(x_k)$  as the sum of expected cost before period  $k$  when adopting optimal strategy. The decision vector in period  $k$  is determined by  $\mathbf{u}_k = \mu_k^*(x_k)$ , in order to minimize the sum of expected cost before period  $k+1$ , namely

$$J_{k+1}(x_{k+1}) = \min_{\mathbf{u}_k} E\left\{\sum_{h=1}^{m-1} F_h\left(\frac{u_{hk}}{\Delta t}\right) + F_{\text{grid}}\left(\frac{u_{mk}}{\Delta t}\right) + F_{\text{batt}}\left(\frac{x_{k+1} - x_k}{\Delta t}\right) + J_k(x_k)\right\} \quad (14)$$

which is the forward iterative equation.

For designing iterative algorithm, system state needs discretization. Let  $s_n$  and  $s_0$  be respectively upper and lower energy limits of battery unit, i.e.  $s_n = E_{\max}$ ,  $s_0 = E_{\min}$ . Set the sampling interval by  $\Delta s = \frac{s_n - s_0}{n}$ ,

and define the discrete state space by  $S = \{s_0, s_1, \dots, s_n\}$  where  $s_i = s_0 + i\Delta s$ . Without considering the randomness (namely, taking  $w_k$  as fixed variables), minimal cost for state  $s_i$  in period  $k$  is calculated as follows:

$$J_{k+1}(s_i) = \min_{s_j \in S} \left\{ F\left(\frac{s_i - s_j + w_k}{\Delta t}\right) + F_{\text{batt}}\left(\frac{s_i - s_j}{\Delta t}\right) + J_k(s_j) \right\} \quad (15)$$

where  $k = 0, 1, \dots, N-1$ ,  $i = 0, 1, \dots, n$ . If the constraint in inequality (6) is considered, the value range  $s_j \in S$  in formula (15) will be replaced by the inequality as

$$-P_{\text{charge}}^{\max} \leq \frac{S_j - S_i}{\Delta t} \leq P_{\text{discharge}}^{\max}.$$

In formula (15),  $F(\frac{S_i - S_j + w_k}{\Delta t})$  means the minimal total cost of controllable generating units and main grid, which is formulized as follows:

$$F(\frac{S_i - S_j + w_k}{\Delta t}) = \min \left\{ \sum_{h=1}^{m-1} F_h(\frac{u_{hk}}{\Delta t}) + F_{\text{grid}}(\frac{u_{mk}}{\Delta t}) \right\} \quad (16)$$

s.t.

$$\sum_{h=1}^m u_{hk} = S_i - S_j + w_k \quad (17)$$

According to equal incremental cost principle, the subproblem above may be solved by using the priority list method. Since the decision vector in period  $k-1$  is known, the range of  $u_{hk}$  is further limited by inequality (4), if considering ramp rate constraints.

The dynamic programming algorithm is designed as follows:

Step 1: Set  $k=0$ , and  $J_0(s_i) = \begin{cases} 0, & s_i = E_0 \\ +\infty, & s_i \neq E_0 \end{cases}$ .

Step 2: For each  $s_i \in S$ ,

(a) use priority list method to solve the subproblem in (16), (17), and get the results as  $F(\frac{S_i - S_j + w_k}{\Delta t})$ ,  $j=1,2,\dots,n$ ;

(b) considering the range of  $s_j$  in (6), calculate  $J_{k+1}(s_i)$  by formula (15).

Step 3:  $k \leftarrow k+1$ .

Step 4: If  $k < N$ , go to Step 2; otherwise, go to the next step.

Step 5: Output  $J_N(E_0)$  and the corresponding strategy as results.

## 5. Examples

In this paper, an example of microgrid is selected from Ref. [9]. Connected with main power grid, the microgrid includes wind turbines, photovoltaic panels, fuel cells and diesel engine. The photovoltaic and wind resources are uncontrollable. All the source data used in calculation are the same as Ref. [9]. A battery energy storage unit is appended, of which the cost coefficient is 0.0043 \$/kWh, the lower and upper energy limits are

60 kWh and 300 kWh, and the maximal charge and discharge power are both 30 kW. The length of time period is 1 hour, and a cycle contains 24 periods. The energy stored in batteries at the beginning and the end is 100 kWh. To the microgrid with and without energy storage unit, dynamic and static economic dispatch results are respectively obtained for comparison.

The energy is transmitted between microgrid and main grid in two ways: unidirectional power flow to microgrid and bidirectional power exchange. In calculation, average price or spot price may be adopted. Fig. 1 and Fig. 2 are respectively the results of static and dynamic dispatch for microgrid, where unidirectional power flow and average price are selected. Fig. 3 and Fig. 4 are respectively the results of static and dynamic dispatch for microgrid, where bidirectional power flow and spot price are selected.

The operating costs of the case above are given in Table 1. It can be seen that the dynamic economic dispatch for microgrids with battery energy storage

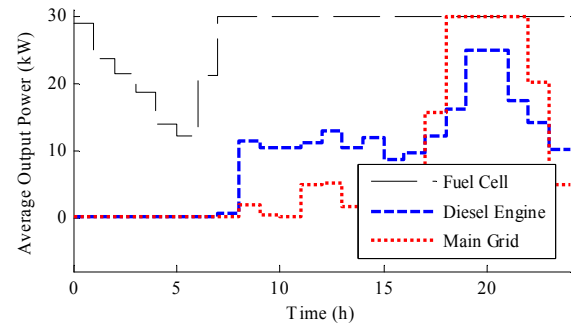


Fig. 1 Results of static dispatch for microgrid by unidirectional power flow and average price.

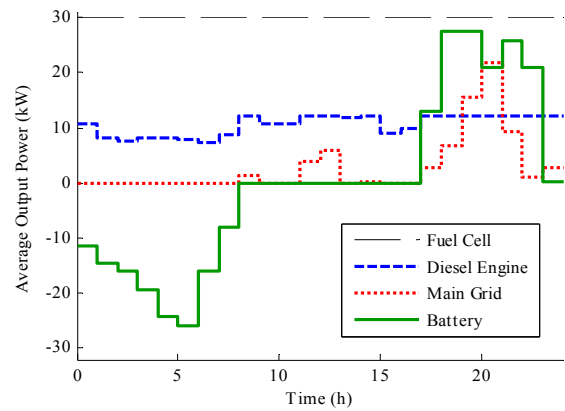
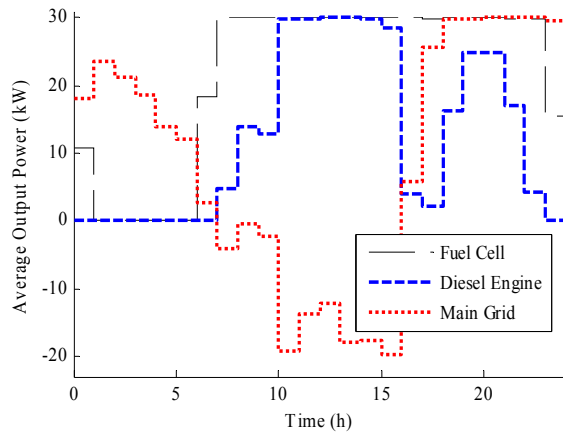
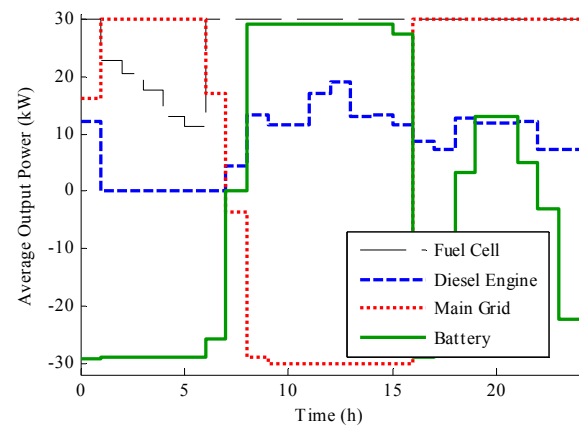


Fig. 2 Results of dynamic dispatch for microgrid by unidirectional power flow and average price.



**Fig. 3** Results of static dispatch for microgrid by bidirectional power flow and spot price.



**Fig. 4** Results of dynamic dispatch for microgrid by bidirectional power flow and spot price.

**Table 1** Operating costs of above dispatch cases.

Precondition	Daily operating costs of static dispatch/\$	Daily operating costs of dynamic dispatch/\$	Reduce/%
Unidirectional power flow and average price	52.60	49.73	5.45
Bidirectional power flow and spot price	41.68	26.01	37.59

effectively reduce the operating costs.

## 6. Conclusions

As battery energy storage units are integrated, microgrid becomes a strong coupling system in the time domain. Hence, the traditional methods of static dispatch are no longer suitable for microgrids. In this paper, from the view of optimal control for dynamic system in finite time period, a dynamic economic dispatch model for microgrids with battery energy storage is established. Then the corresponding dynamic programming algorithm is proposed. Finally, comparison results of an example prove that the proposed model and algorithm are effective.

## References

- [1] R.H. Lasseter, Microgrids and distributed generation, *Journal of Energy Engineering* 133 (3) (2007) 144-149.
- [2] N. Hatziargyriou, H. Asano, R. Iravani, C. Marnay, Microgrids, *IEEE Power and Energy Magazine* 5 (4) (2007) 78-94.
- [3] B. Kroposki, R. Lasseter, T. Ise, S. Morozumi, S. Papathanassiou, N. Hatziargyriou, Making microgrids work, *IEEE Power and Energy Magazine* 6 (3) (2008) 40-53.
- [4] F. Katiraei, R. Iravani, N. Hatziargyriou, A. Dimeas, Microgrids management, *IEEE Power and Energy Magazine* 6 (3) (2008) 54-65.
- [5] M. Basu, Dynamic economic emission dispatch using nondominated sorting genetic algorithm-II, *International Journal of Electrical Power and Energy Systems* 30 (2) (2008) 140-149.
- [6] S.K. Shaili, A. Natarajan, Constrained optimization using evolutionary programming for dynamic economic dispatch, in: *Proceedings of International Conference on Intelligent Sensing and Information Processing*, Bangalore, India, Jan., 2005.
- [7] F. Carastro, M. Sumner, P. Zanchetta, An enhanced shunt active filter with energy storage for microgrids, in: *Proceedings of IEEE Industry Applications Society Annual Meeting*, Edmonton, Canada, Oct., 2008.
- [8] Y.Q. Zhu, Z.D. Yin, J. Tian, Microgrids based on DC energy pool, in: *Proceedings of IEEE Energy 2030 Conference*, Atlanta, USA, Nov., 2008.
- [9] M. Ding, Y.Y. Zhang, M.Q. Mao, W. Yang, X.P. Liu, Steady model and operation optimization for microgrids under centralized control, *Automation of Electric Power Systems* 33 (24) (2009) 78-82.