A Review of Unit Commitment

ELENE4511

Brittany Wright

May 28, 2013

Table of Contents

1. Introduction

Every day regional electricity networks deliver hundreds of GWh of energy from generating units to consumers. Demand varies rather predictably throughout the day, but it can also fluctuate significantly in real time. To ensure that the system remains secure and that power is reliably delivered requires careful planning. The daily on/off scheduling of the system's energy resources is known as unit commitment. It is a large-scale optimization problem that determines the operating status of hundreds of generating units based on a set of complicated constraints. Due to the scale of the problem and the frequency at which it must be solved, unit commitment has become a major research area in the past few decades. This paper aims to provide a better understanding of unit commitment through past and present research.

The second section defines unit commitment through an explanation of its optimization problem and its use in power markets. The third section focuses on optimization with explanations and a comparison of two solving methods. It also includes a brief subsection on current research.

2. Unit Commitment

Unit commitment (UC) aims to schedule the most cost-effective combination of generating units to meet forecasted load and reserve requirements, while adhering to generator and transmission constraints. Generally, UC is completed for a time horizon of one day to one week and determines which generators will be operating during which hours. This commitment schedule takes into account the inter-temporal parameters of each generator (minimum run time, minimum down time, notification time, etc.) but does not specify production levels, which are determined five minutes before delivery. The determination of these levels is known as economic dispatch and it is "the least-cost usage of the committed assets during a single period to meet the demand" [1].

2.1 Problem Definition

The objective is to minimize the total system cost, *F*, of generating power from *N* units over a specific time horizon, *T*. The total cost from each generator at a given period of time is the fuel cost, C_i, plus the any start-up costs, S_i, that may be incurred during the period. (All equations are based on [2].)

$$F = \sum_{t=1}^{T} \sum_{i=1}^{N} [C_i(P_i(t)) + S_i(x_i(t), u_i(t))]$$

The fuel costs, C_i, are dependent on the level of power generation P_i(t). The start-up costs, S_i, are dependent on the state of the unit, x_i, which indicate the number of hours the unit has been on (positive) or off (negative), and the discrete decision variable, u_i, which denotes if power generation of the unit at time *t* is up (1) or down (-1) from the unit at time *t+1*.

2.1.1 System-wide Constraints

The objective function is to be minimized subject to a series of system and generator constraints. First, the system demand, $P_d(t)$, must be met.

$$\sum_{i=1}^{N} P_i(t) = P_d(t)$$

There also must be sufficient spinning reserve, r_i, to ensure reliability. The required system spinning reserve is designated as P_r. The actual spinning reserve for a unit *i* is zero if the unit is off or $r_i = min\{P_i^{max}(t) - P_i(t), r_i^{max}\}$.

$$\sum_{i=1}^{N} r_i(x_i(t), P_i(t)) \geq P_r(t)$$

2.1.2 Generator Constraints

Thus far, the generators have been referred to as a general collection of power producing units, but in actuality the system is comprised of several types of power generators (CCGT, coal-fired, nuclear, etc.) that each operate differently. Even within these types, each individual unit will have its own physical characteristics. Several of the parameters to be considered as generator constraints are minimum run time, minimum down time, ramp rates, and notification time. The notification time is the hours or minutes needed to start the unit from a hot, cold or intermediate state [1].

The state of the unit changes if a start-up or shut-down occurs. This is described in this model by the variables mentioned above, x_i and u_i.

$$x_i(t+1) = x_i(t) + u_i(t) \text{ if } (x_i(t))(u_i(t)) > 0$$

$$x_i(t+1) = u_i(t) \text{ if } (x_i(t))(u_i(t)) < 0$$

If no change of state occurs, the hours accumulate. If a shut-down or start-up does take place, the count starts over again.

Generators have a certain range of operation between a minimum and maximum load. Below the minimum load level the unit cannot stably produce electricity.

$$P_i^{min}(t) \leq P_i(t) \leq P_i^{max}(t) \quad \text{if } x_i(t) > 0$$

$$P_i(t) = 0 \quad \text{if } x_i(t) < 0$$

Certain types of generators have the ability to ramp up or down to higher or lower outputs. The ramp rate, which is generally expressed in MW/hr, is here used as the maximum change in load levels between two consecutive hours, Δ_i.

$$[P_i(t+1) - \Delta_i] \leq P_i(t) \leq [P_i(t+1) + \Delta_i] \quad \text{if } x_i(t) \geq 1 \text{ and } x_i(t+1) \geq 1$$

Because of physical characteristics generators cannot be turned off and on immediately. They have a minimum run time $\tau_{run}$and a minimum down time τ_{down}.

$$u_i(t) = 1 \text{ if } 1 \leq x_i(t) \leq \tau_{run}$$

$$u_i(t) = -1 \text{ if } -\tau_{down} \leq x_i(t) \leq -1$$

The above equations outline the very basic UC problem. Actual systems must consider additional factors, such as emission, fuel and transmission constraints. Security-constrained unit commitment (SCUC) determines an optimal schedule, and also ensures that delivery of that schedule is physically feasible based on the constraints of the transmission network. Methods of solving the UC problem will be discussed in section 3.

2.2 Role of UC in Markets

The least-cost generation function described in the previous section takes into account only the fuel and start-up costs of individual generators. In reality, the commitment schedule may be embedded in several competitive markets (energy, various ancillary services, and capacity) and owners will bid a price that accounts for their costs [3].

2.2.1 Electricity Markets

The electricity markets in the US are relatively young. The Federal Energy Regulatory Commission (FERC), which regulates the interstate transmission of electricity, introduced reforms in 1996 to establish regional, competitive power markets [3]. Several regions, shown in figure 1, opted to establish an Independent System Operator (ISO), separating control and ownership of the grid [4]. The ISO is also able to perform market functions like establishing day-ahead and real-time markets for different products. Those regions not identified in figure 1 continue to trade bilaterally with much of the generation and transmission system being owned by vertically integrated utilities [3].

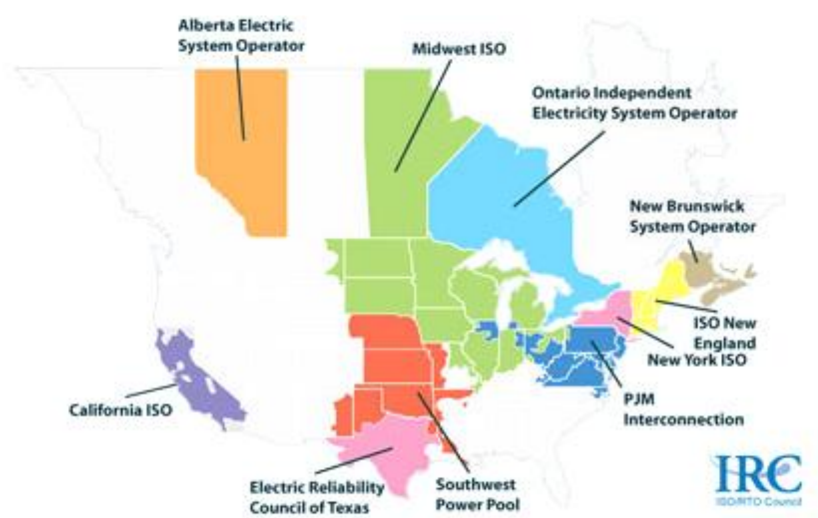

Figure 1: Regional electricity markets in the USA [3]

The market design of each region has developed differently. The major and most mature markets are California ISO (CAISO), Midwest ISO (MISO), PJM Interconnection (PJM), New York ISO (NYISO), ISO New England (ISO-NE). All of these include day-ahead, hour-ahead, and real-time markets for energy and ancillary services. PJM, ISO-NE, and NYISO also have capacity markets [4]. Because of differences in market structure and system requirements, each ISO tackles the UC problem in its own way. (This will be discussed further in section 3.) Although the UC objective function was previously defined as the minimization of total production cost, the aim of ISOs is always to maximize social welfare [1].

Some regions have Regional Transmission Organizations (RTOs), which are similar to ISOs but hold responsibilities beyond the scope of the ISOs, primarily pertaining to the transmission network. RTOs maintain the short-term reliability of the grid. This includes real-time energy balancing and securing residual reserve capabilities. Residual Unit Commitment (RUC) is described by the FERC as a procedure "that strips out virtual bids, inserts RTO's/ISO's forecasts of demand and variable energy resources, and commits any additional units needed for reliability" [4].

Short-term unit commitment (STUC) may also be carried out in the hours prior to delivery. At this time, the ISO could schedule on units that have fast response times. These generators may include some gas and oil-fired generators, which can ramp up much faster than some base-load suppliers, such as coal-fired plants. This ability, however, comes at a price and these units are generally only called upon to meet peak loads.

2.2.2 Bidding

Shortly after restructuring, the markets were divided on their approach to supply-side bidding. CAISO and NE-ISO used "one-part" incremental energy bids that took account of all costs, while PJM and NYISO employed "three-part" bids [3]. The three components are based on the start-up costs, minimum load costs and energy bids [5]. By breaking down the bid, generator owners were able to better define their costs, allowing ISOs to better define the UC problem. CAISO and NE-ISO now also utilize "three-part" bids.

Start-up costs ($/start) are based on the current cooling state of the unit. If a generator was recently turned off, it would still be in a hot state and would require less cost to return to operation. Figure 2 shows how the cost increases as the unit cools.

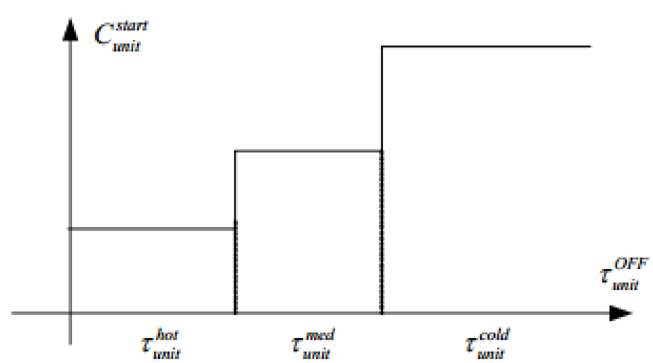

Figure 2: Start-up costs based on cooling state [5]

The minimum load cost ($/hr) is a fixed cost incurred whenever the unit is online [1]. This is also referred to as the no load cost because at this minimum level the unit is running but not actually supplying electricity to the grid.

The energy bid ($/MWh) curve is a monotonically increasing function of the incremental costs to produce the next MWh of energy. The curve is bound by the minimum and maximum loads of the generator [5].

Generators can also bid on the provision of ancillary services, which include regulation up/down, spinning reserves, and non-spinning reserves [5]. Although these bids are made separately, the energy, ancillary services, and transmission markets cannot be considered entirely independent as they all have a basis in the same physical electricity network. There are two general methods of market-clearing: sequential and simultaneous. The sequential method, clearing each product in turn, may yield less than optimal results and, by its nature, creates arbitrage opportunities [3]. The simultaneous method focuses on the co-optimization of related markets; all markets are not included because of computational limitations. In 2011, ISO-NE and PJM were simultaneously clearing energy and reserve markets while NYISO, MISO, and CAISO also included regulation in their co-optimizations [4].

2.2.3 Beyond Security and Reliability

The UC problem is also important to other aspects of the power market and network. Market participants use Price-Based UC (PBUC) to maximize their payoffs. The model for scheduling is not constrained by the security of the system but driven by price signals [6]. In the longer-term horizon, this is significant as a generator will schedule a planned outage or maintenance for a time when profit losses will be minimal [7].

SCUC solutions can also provide information that is helpful in assessing potential for market power abuse. The preventative (ex ante) market power mitigation measures aim to identify market power abuse early through structural or conduct-and-impact approaches [8]. The structural approach analyzes the number and distribution of generators and concentration of relevant markets. The conduct-and-impact method assesses an individual firm's conduct (bid prices) and potential market impact. Authorities also must be aware of situations like transmission line congestion in which temporary isolation of a zone could lead to the abuse of market power [8].

3. Optimization

Unit commitment is a large-scale problem, often dealing with hundreds of generating units in a region, making it difficult to find the optimal solution in an acceptable amount of time. Several optimization methods are currently used and many more are being researched. A recent literature review identified nine of these methodologies: priority list method, dynamic programming, Lagrangian relaxation, genetic algorithms, simulated annealing, particle swarm optimization, tabu-search method, fuzzy logic algorithm, and evolutionary programming [9]. This list of approaches is not exhaustive and does not highlight that much of the present research includes a combination of methods. In this section, the primary focus will be on the two approaches most used in the US markets: mixed integer programming (MIP) and Lagrangian relaxation (LR).

In 1999, unit commitment was solved primarily by linear programming (LP) and Lagrangian relaxation, but due to approximations these yielded suboptimal solutions. At the time, the use of mixed integer programming had been dismissed because of the unacceptable solving time. Improvements to the method eventually led to PJM testing MIP against its present software in its day-ahead market. The annual bid cost savings were estimated at $60 million. PJM adopted the method into its day-ahead market in 2004 and its real-time market in 2006 [4]. CAISO implemented the MIP technique during its market redesign in 2009 [5]. Many markets have since implemented MIP in their markets.

3.1 Mixed Integer Programming

To better understand the capabilities of mixed integer programming it is important to recognize that MIP is a special class of linear programming. Momoh [10] defines a linear programming problem as "the problem of allocating a number *m* of resources among 1,2,…,*n* activities in such a way as to maximize the worth from all the activities." In such a problem, all constraints and all relationships between decision variables and activities are linear. There are four implicit assumptions to the linear programming approach: proportionality, additivity, divisibility and certainty [10]. The assumption of divisibility states that decision variables do not have to be discrete values. Integer programming (IP) and mixed integer programming are the special cases in which all or some of the decision variables are discrete values. If the integer values can only be zero or one, as is the case in on/off scheduling, they are called binary decision variables.

The three main approaches to solving LP problems are the graphical approach, variations of the simplex method and the tableau approach.

3.1.1 Formulation of MIP and Methods of solving

The basic MIP formulation in the case of UC, aims to minimize an objective function that consists of continuous, *y*, and discrete, *z*, decision variables.

$$\min cy + dz$$

$$\text{subject to: } Ay + Ez \leq b \text{ where } y \geq 0 \text{ and } z \epsilon \{0,1\}$$

There are several methods of solving MIP problems, but the two considered here are the most used in power systems.

The cutting-plane technique views the MIP problem as a LP relaxation and iteratively narrows down its feasible region until the constraints are satisfied and an optimal solution is found. The first step is to consider the MIP as a non-integer LP problem, which has a feasible region that includes all the feasible solutions of the MIP problem [11]. This LP relaxation can be solved by the simplex method. The resulting optimal solution of the relaxation can only be the optimal solution of the MIP if the constraints are met. If they are not met, an additional linear inequality constraint (a cut) is considered, refining the feasible region of the LP relaxation [11]. The process is then repeated until a solution is reached.

As its name implies, the branch-and-bound technique is made up of two major steps. Branching is the task of breaking down the master problem to smaller sub-problems. This is done by considering the vector *z*, in the MIP problem presented above, as having binary variables, 0 or 1. The resulting sub-problems can be formulated in a tree structure and solved using straight-forward LP methods to find the optimal solution [11]. Solving all sub-problems would require a considerable amount of time so a second step known as bounding is applied. The bounds of a sub-problem are defined by its optimal solution and objective (minimization or maximization). If a feasible solution exists for the first sub-problem encountered it is considered the best solution and is referred to as the first incumbent. The bounds of subsequent solutions are compared with the incumbent. The incumbent is replaced if a better bound is found, otherwise the branch is fathomed [10].

The efficiency of the cutting-plane technique is dependent on cuts made, while the efficiency of the branch-and-bound algorithm depends on how the bounds are found and how the tree is searched [11]. The current leading approach to solving MIP problems is a combination of these two techniques known as the branch-and-cut [6].

3.2 Lagrangian Relaxation

Momoh [10] describes Lagrangian relaxation as a way of incorporating the complicated constraints of an optimization problem into the objective function by using a penalty term.

3.2.1 Formulation of the Lagrangian and Dual Functions

Based on the problem definition presented in section 2.1, the corresponding Lagrangian, *L*, is given as:

$$L = \sum_{t=1}^{T} \left[\sum_{i=1}^{N} \left[C_i\big(P_i(t)\big) + S_i\big(x_i(t), u_i(t)\big) \right] + \lambda(t) \left[P_d(t) - \sum_{i=1}^{N} P_i(t) \right] \right.$$
$$\left. + \mu(t) \left[P_r(t) - \sum_{i=1}^{N} r_i(x_i(t), P_i(t)) \right] \right]$$

In the above equation, only the system-wide constraints, not the generator constraints, are relaxed by using Lagrangian multipliers, $\lambda(t)$ and $\mu(t)$, which correspond to the demand and reserve requirements, respectively. By using the duality theorem, the above can be broken down into a “two-level maximum-minimum optimization problem” [2].

The low level problem focuses on the minimization of the Lagrangian. If the Lagrangian multipliers are fixed, the minimization of *L* can be decomposed into a set of *N* simpler minimization problems; one problem for each generating unit, i=1,2,...N [12]. The low-level sub-problem can be written for unit *i* as:

min *L*, with

$$L = \sum_{t=1}^{T} \left\{ \left[C_i\big(P_i(t)\big) + S_i\big(x_i(t), u_i(t)\big) \right] - \lambda(t) P_i(t) - \mu(t) r_i(x_i(t), P_i(t)) \right\}$$

The minimization sub-problems are subject to their corresponding generator constraints and can be solved using dynamic programming [2], [12].

The high level problem focuses on the maximization of the dual function, θ. The dual function includes the optimal Lagrangian, $L_i^*(\lambda,\mu)$.

max $\theta(\lambda, \mu)$, with

$$\theta(\lambda, \mu) = \sum_{i=1}^{N} L_i^* (\lambda, \mu) + \sum_{t=1}^{T} [\lambda(t)P_d(t) + \mu(t)P_r(t)]$$

Because discrete decision variables are used in the low level optimization, the dual function may not be differentiable at all points. For this reason, a subgradient algorithm is used for updating the multipliers [2].

The sub-problems are solved first and the multipliers are updated in the high level optimization. The values of the Lagrangian form a lower bound for feasible solutions of the original problem. The duality gap is the difference between the optimal solution to the original (primal) problem and the optimal solution of the dual problem [13].

3.3 Comparison of MIP and LR

The underlying push to use the MIP method is because of its ability to reach a globally optimal solution [6]. In competitive markets, slight deviations from the optimum can drastically change the commitment schedule. For the LR approach the duality gap is small (1-2%), but not optimal [11]. Another disadvantage of Lagrangian Relaxation is that the dual solution may be infeasible (does not satisfy system-wide constraints) and heuristics[1] are required to reach a feasible schedule [2], [11].

3.3.1 Modeling

Another advantage of mixed integer programming is it has “more flexible and accurate modeling capabilities” than LR [6]. For example, ISO-NE identified in 2006 that prices could be improved by reducing out-of-merit commitment. ISOs make contingency decisions based on information provided by market participants. However, due to modeling limitations, combined cycle plants[2] could only submit one economic minimum, which was often based on a multiple-turbine configuration [15]. (The actual minimum could be achieved through an alternative configuration of one turbine.) MIP has the ability to model multiple configurations for one unit. This allows ISOs to better characterize each unit and more effectively schedule units in contingency situations. By 2011, only PJM had adopted the capabilities of including combined-cycle modeling. CAISO models each configuration as a separate resource, but also takes into account temporal and financial elements of shifting between configurations. MISO continues to use single submissions [4].

More complicated constraints result in additional complexity for the LR method. The two-level framework discussed in section 3.2.1 becomes three levels when generator ramp rates are considered [2]. See figure 3.

[1] Heuristics are iterative algorithms that are used when no acceptable converging algorithm exits or to reduce the computation time of a converging algorithm. Converging algorithms can also include heuristic components [14].
[2] An example of a combined cycle plant is a combined cycle gas turbine (CCGT), which uses waste heat from the gas turbine to generate steam and turn a steam turbine.

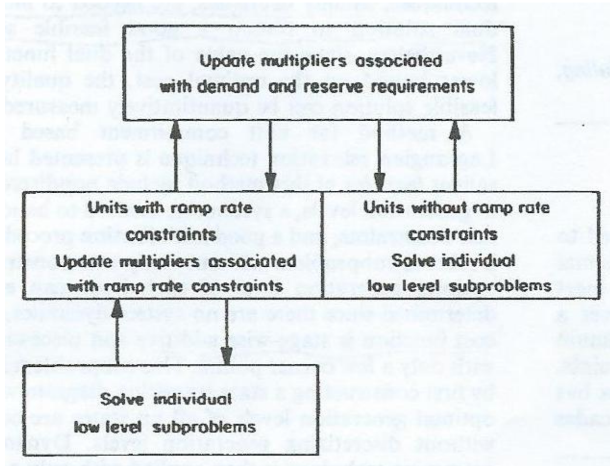

Figure 3: Three-level framework of LR with additional constraints [2]

3.3.2 Complexity

The major disadvantage of using MIP is the computational complexity. Even the most efficient technique of branch-and-cut is NP-hard[3] [6]. If vector *z* has *n* binary variables, there would be a total of 2^n LP problems to solve in the worst case [11]. Li and Shahidehpour [6] highlighted several areas of improvement that could make the MIP methodology more efficient, including "tightening of the LP relaxation" and identifying user-defined cutting-planes and branch selections based on knowledge of the problem structure.

Lagrangian relaxation is more computationally efficient. The high level iterations required to update multipliers contribute minimally to the overall computation time [11]. The complexity is on the order of the product of the number of units and commitment horizon; it varies almost linearly [6], [11].

3.4 Optimization Research

3.4.1 Methods and Constraints

Researchers continue to refine the UC problem by investigating new combinations of algorithms. The primary focus is still to achieve better and faster optimization while incorporating a large and often complicated set of constraints. For example, Duo *et al.* [12] examined the use of evolutionary programming techniques to improve on the initial LR solution. Other researchers focus on including more realistic constraints, such as considering the restricted operating zones of a generator's input-output curve [17]. For a complete understanding of methodological developments see the recent literature review [9].

---

[3] When discussing algorithm complexity, the class P includes only problems that can be solved by algorithms in polynomial time. By definition, the class NP (non-deterministic polynomial) includes "all decision problems for which there is a polynomial-time algorithm that can check whether a purported solution to the problem is actually a solution." The subclass of NP Complete states that if a polynomial-time algorithm exists for one problem, then such an algorithm exists for all problems in the subclass. NP-hard refers to a broader range of non-decision problems that "can be shown to be at least as hard to solve as a certain NP complete decision problem" [16].

Security-constrained unit commitment is an extension of the basic UC problem that includes transmission constraints, such as those pertaining to branch-flow[4] and bus voltages.[5] Fu *et al.* [20] propose using Benders decomposition to yield a UC problem with a transmission security check (sub-problem). In the iterative process, security violations in the network are identified, additional constraints are added to the UC problem, and then the UC problem is solved again. This continues until the constraints are met.

3.4.2 Uncertainty

As the generation mix shifts to include more renewable energy and as demand becomes more price sensitive[6], there is increased uncertainty in real-time resource commitment and dispatch. For example, the UC solution reached the day before would not have taken into account significant changes in wind power generation on the day. Bertsimas *et al.* [21] described the current research of uncertainty challenges as being divided into two approaches. The first approach, which is widely used in the power industry, is called the reserve adjustment method. It involves obtaining a commitment schedule based on the forecasted demand but requiring a much larger reserve capacity – economically inefficient solution. The second approach uses stochastic programming [21]. This technique is best illustrated by an example from wind power generation. The wind forecast and corresponding error distribution are used to produce a single “predicted scenario and large numbers of error scenarios” for wind power generation [22]. A large number of scenarios is necessary to have confidence in the approach, but this significantly increased the complexity of an already large-scale optimization problem.

4. Conclusions

Unit commitment is a process that receives inputs from and itself affects all stages of the power system (generation, transmission, market). The two main challenges of the UC optimization continue to be the large scale of the problem, which increases complexity, and determining a globally optimal solution, which would best satisfy the competitive markets. The current approach used in most of the US markets is currently mixed integer programming, but many researchers have been developing new methods that combine a variety of optimization techniques. The UC problem is far from fully solved and additional challenges and areas of research will emerge with time, as the way we generate and consume electricity changes.

[4] The power flow through a line cannot exceed thermal limitations [18].

[5] The voltage magnitude bounds are 0.95 and 1.05 per unit. The range is defined from above by the capabilities of circuit breakers and below by the operational requirements of generators [19].

[6] Historically, near-term demand has not been responsive to changes in price, as most consumers do not take an active and direct role in the markets [8].

5. References

1. A. Lelic, I. "Unit Commitment and Dispatch." Introduction to Wholesale Electricity Markets (WEM 101), Northampton, MA, November 5-9, 2012. Available: http://www.iso-ne.com/support/training/courses/wem101/07_unit_commitment_dispatch.pdf
2. X. Guan *et al.*, "An Optimization-Based Method for Unit Commitment." *Electric Power and Energy Systems* 14, no. 1 (February 1992): 9-17.
3. B.F. Hobbs *et al., The Next Generation of Electric Power Unit Commitment Models*. Hingham, MA: Kluwer Academic Publishers, 2001.
4. "Recent ISO Software Enhancements and Future Software and Modeling Plans." Washington, D.C.: Federal Energy Regulatory Commission, 2011.
5. "Market Optimization Details." In *Technical Bulletin*: California ISO, 2009. Available: http://www.caiso.com/Documents/TechnicalBulletin-MarketOptimizationDetails.pdf
6. T. Li and M. Shahidehpour. "Price-Based Unit Commitment: A Case of Lagrangian Relaxation Versus Mixed Integer Programming." *IEEE Transactions on Power Systems* 20, no. 4 (November 2005).
7. A.J. Conejo *et al.*, "Generation Maintenance Scheduling in Restructured Power Markets." *IEEE Transactions on Power Systems* 20, no. 2 (May 2005).
8. J.D. Reitzes *et al.*, "Review of PJM's Marke Power Mitigation Practices in Comparison to Other Organized Electricity Markets." The Brattle Group, 2007.
9. A. Bhardwaj *et al.*, "Unit Commitment in Electrical Power System - a Literature Review." Paper presented at the 2012 IEEE International Power Engineering and Optimization Conference, Melaka, Malaysia, 6-7 June 2012.
10. Momoh, J. A. *Electrical Power System Applications of Optimization*. Power Engineering. edited by H. Lee Willis New York: Marcel Dekker, Inc., 2001.
11. X. Guan *et al.*, "Optimization Based Methods for Unit Commitment: Lagrangian Relaxation Versus General Mixed Integer Programming." 2003 IEEE Power Engineering Society General Meeting, v. 2, 2003.
12. H. Duo *et al.*, "A Solution for Unit Commitment Using Lagrangian Relaxation Combined with Evolutionary Programming." *Electric Power Systems Research* 51, no. 1 (July 1999): 71-77.
13. A. Merlin and P. Sandrin. "A New Method for Unit Commitment at Electricite De France." *IEEE Transactions on Power Apparatus and Systems* 102, no. 5 (May 1983).
14. Muller-Merbach, H. "Heursitics and Their Design: A Survey." *European Journal of Operational Research* 8, no. 1 (September 1981): 1-23.
15. "2006 Wholesale Markets Plan." ISO New England, September 2005. Available: http://www.iso-ne.com/pubs/whlsle_mkt_pln/archives/2006_wmp.pdf
16. A. Stanoyevitch, *Discrete Structures with Contemporary Applications*. Boca Raton, Florida: CRC Press, 2011.
17. H. Daneshi *et al.*, "Mixed Integer Programming Method to Solve Security Constrained Unit Commitment with Restricted Operating Zone Limits." edited by 2008 IEEE International Conference on Electro/Information Technology, 187-92. Ames, IA, 2008.
18. A.I. Cohen *et al.*, "Security Constrained Unit Commitment for Open Markets." Paper presented at the Proceedings of the 21st International Conference on Power Industry Computer Applications (PICA), 1999.
19. M.B. Cain *et al.,* "History of Optimal Power Flow and Formulations." Federal Energy Regulatory Commission. Available: http://www.iso-ne.com/committees/comm_wkgrps/prtcpnts_comm/pac/mtrls/2013/mar202013/a2_planning_technical_guide.pdf

20. Y. Fu *et al.*, "Security-Constrained Unit Commitment with Ac Constraints." *IEEE Transactions on Power Systems* 20, no. 3 (November 2005).
21. D. Bertsimas *et al.*, "Adaptive Robust Optimization for the Security Constrained Unit Commitment Problem." *IEEE Transactions on Power Systems* 28, no. 1 (February 2013): 52-63.
22. R. Ye *et al.*, "Security Constrained Unit Commitment with Multiple Wind Farms Integrated." In *2010 9th International Power and Energy Conference*, 116-21. Singapore.