Steady periodic Memristor oscillator with transient chaotic behaviours

Article *in* Electronics Letters · March 2010

DOI: 10.1049/el.2010.3114 · Source: IEEE Xplore

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Steady periodic memristor oscillator with transient chaotic behaviours

B.C. Bao, Z. Liu and J.P. Xu

By replacing Chua's diode in the canonical Chua's oscillator with a smooth flux-controlled memristor, a memristor based oscillator is presented. The memristor oscillator generates a steady periodic orbit and has a transition from transient chaotic to steady periodic behaviour. The complicated dynamical behaviour is extremely dependent on the initial condition of the memristor.

Introduction: The memristor, characterised by a relation of the type $f(\varphi, q) = 0$, is a missing circuit element studied by Chua in 1971 [1] and realised by Williams's group of HP Labs in 2008 [2]. The flux-controlled memristor described by $i = W(\varphi)v$ is a passive two-terminal electronic device characterised by a nonlinear constitutive relation between the current flowing through the device and the voltage across the device terminals. The nonlinear function $W(\varphi)$, defined as $W(\varphi) = \mathrm{d}q(\varphi)/\mathrm{d}\varphi$, is called the memductance. It represents the slope of a scalar function $q = q(\varphi)$ and is called the memristor constitutive relation.

Memristor based applications have attracted much attention recently. Itoh and Chua derived several oscillators from Chua's oscillators by replacing Chua's diodes with memristors characterised by a monotone-increasing and piecewise-linear nonlinearity [3]. In the present Letter, we assume that the memristor is flux-controlled and characterised by a smooth continuous cubic monotone-increasing nonlinearity as follows

$$q(\varphi) = a\varphi + b\varphi^3 \tag{1}$$

where $a, b > 0$. From (1), the memductance $W(\varphi)$ is obtained as

$$W(\varphi) = \mathrm{d}q(\varphi)/\mathrm{d}\varphi = a + 3b\varphi^2 \tag{2}$$

Thus, by replacing the Chua's diode in the canonical Chua's oscillator with a memristor characterised by (1), a new memristor based oscillator can be designed, upon which its dynamical behaviours are investigated in detail.

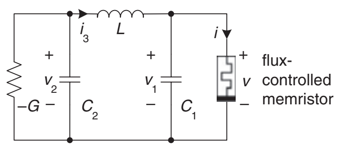

Fig. 1 *Canonical Chua's oscillator with flux-controlled memristor*

Smooth memristor oscillator: Fig. 1 shows a smooth flux-controlled memristor based oscillator. From Fig. 1, we can obtain a set of four first-order differential equations, which define the relation among the four circuit variables (v_1, v_2, i_3, φ) [3]. By letting $x = v_1$, $y = i_3$, $z = v_2$, $w = \varphi$, $\alpha = 1/C_1$, $\beta = 1/C_2$, $\gamma = G/C_2$, and $L = 1$, and defining the nonlinear functions $q(w)$ and $W(w)$ as

$$\begin{cases} q(w) = aw + bw^3, \\ W(w) = \mathrm{d}q(w)/\mathrm{d}w = a + 3bw^2 \end{cases} \tag{3}$$

The differential equations of Fig. 1 are transformed into the form with a time scale factor k as follows

$$\begin{cases} \dot{x} = k\alpha(y - W(w)x), \\ \dot{y} = k(z - x), \\ \dot{z} = k(-\beta y + \gamma z), \\ \dot{w} = kx \end{cases} \tag{4}$$

where $k > 0$.

The equilibrium state of (4) is given by set $A = \{(x, y, z, w) | \; x = y = z = 0, \; w = c\}$, which corresponds to the w-axis. Here, c is a real constant. The Jacobian matrix J at this equilibrium set is given by

$$J = \left. \begin{bmatrix} -k\alpha W(w) & k\alpha & 0 & -6k\alpha bxw \\ -k & 0 & k & 0 \\ 0 & -k\beta & k\gamma & 0 \\ k & 0 & 0 & 0 \end{bmatrix} \right|_A \tag{5}$$

with its characteristic equation given by

$$\lambda[\lambda^3 + k(\alpha W(c) - \gamma)\lambda^2 + k^2(\alpha + \beta - \alpha\gamma W(c))\lambda + k^3\alpha(\beta W(c) - \gamma)] = 0 \tag{6}$$

where $W(c) = a + 3bc^2$. The coefficients of the cubic polynomial equation in brackets are all nonzero. Then according to the Routh-Hurwitz condition, for $\alpha = 1$, $\beta = 0.65$, $\gamma = 0.65$, $a = 0.2$, $b = 1$, and $k = 100$, the real parts of the roots of this cubic polynomial equation are negative if and only if

$$0.5164 < |c| < 0.6902 \tag{7}$$

To make the equilibrium set A unstable, the constant c in the cubic polynomial equation of (6) should be chosen to satisfy the following conditions

$$|c| < 0.5164 \text{ and } |c| > 0.6902 \tag{8}$$

The four eigenvalues λ_i ($i = 1, 2, 3, 4$) of the equilibrium set A for several typical constant c are listed in Table 1. They are characterised by an unstable or stable saddle-focus except for the zero eigenvalue in the constant c range. Thus we can know that the dynamical behaviours of the canonical Chua's oscillator with the memristor are closely dependent on the initial values of state variable w. Such characteristics well reflect the unique properties of the memristor element.

Table 1: Four eigenvalues λ_i ($i = 1, 2, 3, 4$) of equilibrium set A

$\lvert c\rvert$	λ_1	λ_2	λ_3	λ_4
0	49.2196	$-57.1098 + j255.7544$	$-57.1098 - j255.7544$	0
0.5164	0	-55.2106	-529.7894	0
0.6	-745.7686	$-10.6157 + j38.3874$	$-10.6157 - j38.3874$	0
0.6902	-993.8606	$j51.7108$	$-j51.7108$	0
1	-2048.3142	$16.6571 + j65.2719$	$16.6571 - j65.2719$	0

Steady period with transient chaos: When $\alpha = 1$, $\beta = 0.65$, $\gamma = 0.65$, $a = 0.2$, $b = 1$, and $k = 100$, two typical stable periodic orbits with different initial conditions $(0, 10^{-10}, 0, 0)$ and $(0, 10^{-10}, 0, 1)$ are shown in Figs. 2*a* and *b*, respectively, in which the nonlinear constitutive relations $i = W(\varphi)v$ of this memristor oscillator on $v - i$ plane are depicted, where $v = v_1 = x$ and $\varphi = w$. Obviously, Fig. 2 verifies that the memristor has a pinched hysteresis loop characteristic. If the initial conditions $(0, 10^{-10}, 0, 0.6)$ with the same parameters are selected, it can be verified from numerical simulation that the trajectory of system (4) converges to the position (0, 0, 0, 0.6) rapidly.

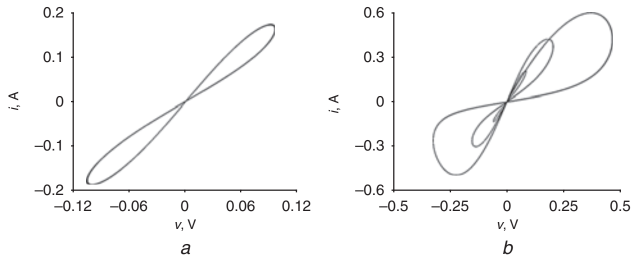

Fig. 2 *Steady periodic orbits of smooth memristor oscillator with different initial values*

a $(0, 10^{-10}, 0, 0)$
b $(0, 10^{-10}, 0, 1)$

The appearance of chaos on finite time scales is known as transient chaos. The phenomenon of transient chaos accompanied with boundary crisis is frequently encountered in dynamical systems [4]. In a boundary crisis, with the increase of control parameter p, the distance between a strange attractor and the boundary of the basin of attraction in the phase space decreases until they touch each other at a critical value ($p = p_c$). At this point the attractor also touches an unstable periodic orbit, and the chaotic attractor exhibits a crisis. When $p = p_c$, the chaotic attractor no longer exists and will be replaced by a chaotic transient. In the initial stage of this regime, the system behaviour is virtually indistinguishable from chaotic, but after then the system rapidly passes to another stable state that can be stationary, periodic, or chaotic as well.

As an example, when $\alpha = 1$, $\beta = 0.65$, $\gamma = 0.65$, $a = 0.2$, $b = 1$, $k = 100$, the phenomenon of stable period with complex transient chaos occurs in system (4), i.e. the orbit of system (4) has a transition from chaotic to periodic behaviour with the time evolutions. Let us consider the case for initial values (0, 10^{-10}, 0, 0). Fig. 3 shows the time series of state variable v (or x) in system (4), where the steady period 1 shown in Fig. 2*a* will happen after $t > 153$ s. Obviously, this time series is different from one generated by a chaotic system. The transient state in the time interval $t \in [0s, 47s]$ is so disordered and chaotic, the steady state after $t = 153$ s is ordered and periodic, and the transition state in the time interval $t \in [47s, 153s]$ is quasi-periodic. The projection of the phase portrait on $v - i$ plane of the transient chaotic orbit in the transient interval is shown in Fig. 4, in which illustrates a 2-scroll chaotic attractor reflecting the basic dynamical behaviour on a finite time scale in the time interval $t \in [0s, 20s]$. The phenomenon can also be illustrated in the Lyapunov exponent spectra against time to show the changes of dynamical behaviours of system (4) with initial values (0, 10^{-10}, 0, 0) under the conditions of fixed parameters, as shown in Fig. 5.

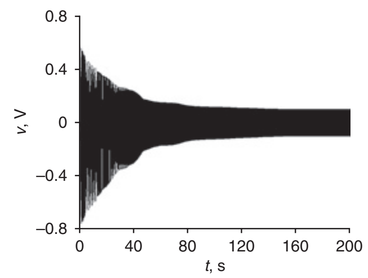

Fig. 3 *Plot of variable v against time for initial values (0, 10^{-10}, 0, 0)*

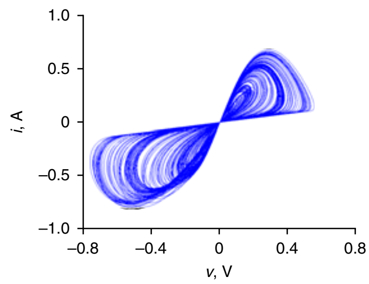

Fig. 4 *Transient chaotic attractor with initial values (0, 10^{-10}, 0, 0)*

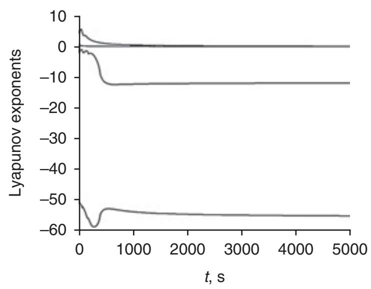

Fig. 5 *Lyapunov exponents against time with initial values (0, 10^{-10}, 0, 0)*

Conclusions: By replacing Chua's diode with a flux-controlled memristor, we have derived a smooth memristor-based oscillator from the canonical Chua's oscillator. The novel oscillator has complex transient nonlinear dynamics and can oscillate in steady periodic orbits. Depending on the initial value of the memristor, a transition from transient chaotic to steady periodic behaviour with the time evolutions exists in the memristor oscillator. The research results demonstrate that the introduction of the flux-controlled memristor makes the dynamical behaviours more complicated, and completely different from the existing chaotic system.

Acknowledgments: This work was supported by the National Natural Science Foundation of China under grant no. 60971090 and the Natural Science Foundation of Jiangsu Province, China under grant no. BK2009105.

5 November 2009
doi: 10.1049/el.2010.3114
One or more of the Figures in this Letter are available in colour online.

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