

Dynamical responses in a new neuron model subjected to electromagnetic induction and phase noise



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HIGHLIGHTS

- New neuron model is presented with electromagnetic induction being considered.
- Magnetic flux is used to describe the effect of electromagnetic induction.
- Memristor is used to realize feedback and coupling between membrane potential and electromagnetic field.
- Double coherence resonance is detected and multiple modes in electrical activities are observed.

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ABSTRACT

Complex electrical activities in neuron can induce time-varying electromagnetic field and the effect of various electromagnetic inductions should be considered in dealing with electrical activities of neuron. Based on an improved neuron model, the effect of electromagnetic induction is described by using magnetic flux, and the modulation of magnetic flux on membrane potential is realized by using memristor coupling. Furthermore, additive phase noise is imposed on the neuron to detect the dynamical response of neuron and phase transition in modes. The dynamical properties of electrical activities are detected and discussed, and double coherence resonance behavior is observed, respectively. Furthermore, multiple modes of electrical activities can be observed in the sampled time series for membrane potential of the neuron model.

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1. Introduction

The neurodynamics [1–9] on biological system has been paid much attention since the breakthrough on electrical activities of isolate neuron model in 1950s. The Hodgkin–Huxley neuron model is thought as a reliable neuron model because the effect of ion channels can be described. Indeed, some simplified neuron models can also be helpful to understand the dynamical properties of neuron, for example, the mathematical Hindmarsh–Rose neuron model [2] is effective to reproduce main properties of neuronal activities and can be available for bifurcation analysis. Readers can find detailed description for other neuron models in Ref. [4], as mentioned in Ref. [3], reliable neuron circuits can be set up to detect the response of neuron to external stimuli. Based on most of the neuron models, stochastic resonance [10–16] can be found by applying appropriate noise and periodical forcing on the isolate neuron and even neuronal network [17–19]. Stochastic resonance and coherence resonance on neuron and neuronal network can induce distinct regularity in sampled time series for membrane

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potential, and spatial regular distribution [20–24] under applying optimal intensity of different kinds of noise, such as Gaussian white noise [24], Lévy noise [14] and channel noise [23]. For stochastic resonance, external periodical forcing or intrinsic autaptic driving are important for generating continuous pulses or wave fronts like pacemaker [13,25] in presence of noise. As a result, bifurcation parameters [26,27] such as time delay and conductance for ion channels can be adjusted to enhance coherence and also induce decoherence of network [28].

By now, dynamical analysis and synchronization transition has been extensively discussed on many neuron models, and it is confirmed that the external forcing current and bifurcation parameters can change the modes of electrical activities [29–35]. In fact, some realistic factors should be considered by dealing with these neuron models. For example, autapse, a specific synapse connected to the body of neuron via a close loop. As reported in Refs. [34–36], autapse plays important biological function in regulating the electrical activities of neuron and network. In the case of neuronal network [37,38], autapse driving can regulate the collective behaviors of neurons like a pacemaker and even generate regular spatial patterns such as spiral waves or continuous pulses. Furthermore, the effect of electromagnetic induction in neuron should be considered during the changing of concentration of ions in the cell. According to the physical law of electromagnetic induction, time-varying electromagnetic field can be induced when different ion currents across the channels embedded into the membrane, and magnetic flux across the membrane is also changed. As a result, Lv et al. [39,40] suggested that magnetic flux across the membrane can be used to describe the effect of electromagnetic induction, and it is confirmed that electromagnetic radiation can also be imposed the model to investigate the transition of electrical activities in neuron. However, these results presented in Refs. [39,40] have been carried out on the Hindmarsh–Rose and the effect of noise and ion channels is out of consideration. Besides the Lévy noise in Ref. [14], it is interesting to investigate the response of the improved biological neuron model driven by phase noise [41–44]. Readers can explore the previous review [45] and references therein for neurodynamics. In this paper, the effect of electromagnetic induction is considered on the Hodgkin–Huxley neuron model, and then phase noise is considered to detect the possible emergence of stochastic resonance, and the emergence of multiple modes in electrical activities.

2. Model description

Magnetic flux φ is used to describe the effect of electromagnetic induction, and the dynamical equations developed from the original Hodgkin–Huxley neuron model are described as follows

$$\begin{cases} C_m \frac{dV}{dt} = -(I_K + I_{Na} + I_L + AC_m \cos \omega t) + I_{ext} + k\rho(\varphi)(V + V_e); \\ \frac{dy}{dt} = \alpha_y(V)(1 - y) - \beta_y(V)y; \quad (y = m, h, n) \\ \frac{d\varphi}{dt} = k_1 V - k_2 \varphi; \\ \frac{dQ}{dt} = \omega_1 + \sqrt{2D}\xi(t) \end{cases} \quad (1)$$

$$\begin{cases} \rho(\varphi) = (\alpha + 3\beta\varphi^2); \quad V_e = A \sin \omega t / \omega; \quad I_{ext} = A_1 \sin(Q(t)); \\ I_K = 36n^4(V + V_e + 12); \quad \alpha_n = 0.01 \frac{10 - V}{\exp[(10 - V)/10] - 1}, \quad \beta_n = 0.125 \exp[-V/80]; \\ I_{Na} = 120n^3h(V + V_e - 115); \quad \alpha_m = 0.1 \frac{25 - V}{\exp[(25 - V)/10] - 1}, \quad \beta_m = 4 \exp[-V/18] \\ I_L = 0.3(V + V_e - 10.6); \quad \alpha_h = 0.07 \exp[-V/20], \quad \beta_h = \frac{1}{\exp[(30 - V)/10] + 1} \end{cases} \quad (2)$$

where the variable V , φ represents the membrane potential and magnetic flux across the membrane, respectively. V_e is the additive induction membrane induced by external electric stimuli, C_m , m , n , h is the membrane capacitance, and the gate variable for channels, the function $\rho(\varphi)$ is the conductance developed from memristor and used for memory associated with magnetic field. A , A_1 , ω is the amplitude and angular frequency for external forcing currents, $\xi(t)$ is Gaussian white noise, ω_1 is the angular frequency of phase noise $Q(t)$ [41–44]. For detailed description about the parameters α , β , k , k_1 , k_2 , readers can find in Ref. [40]. The schematic diagram for the neuronal circuit is plotted in Fig. 1.

3. Numerical results and discussion

In this section, the fourth order Runge–Kutta algorithm is used for dynamical equations, time step $h = 0.01$, the initial values for the variables are selected as $V_0 = -64.999801$ mV, $m_0 = 0.052938$, $h_0 = 0.5916$, $n_0 = 0.317726$, $\varphi_0 = 1$, the parameters are set as $\alpha = \beta = 0.1$, $k_1 = 0.1$, $k_2 = 1$, $A = 2.5 \mu\text{A}/\text{cm}^2$, the membrane capacitance is set as $C_m = 1 \mu\text{F}/\text{cm}^2$. At first, the dependence of magnetic flux on the membrane potential is investigated without additive

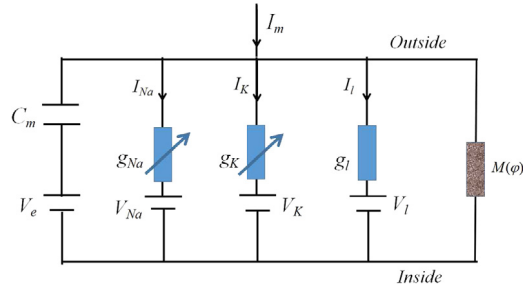


Fig. 1. Schematic diagram for neuronal circuit under electromagnetic induction, I_m is the external forcing current, C_m denotes the capacitance of membrane, V_e , V_{Na} , V_K , V_l the additive voltage on membrane and the gate voltage for ion channels under electromagnetic radiation, $M(\varphi)$ is the memristor.

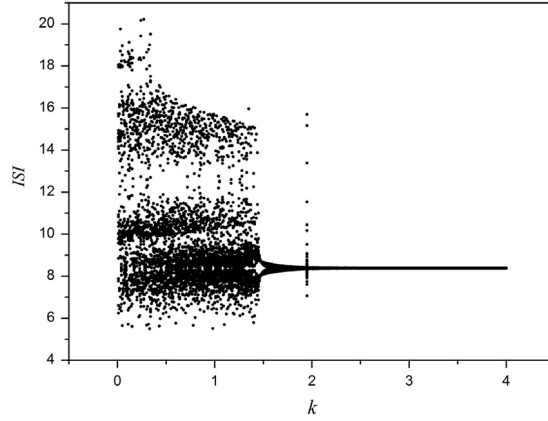


Fig. 2. Bifurcation diagram for ISI via feedback gain k , the amplitude and angular frequency of the external forcing is selected at $A = 2.5 \mu\text{A}/\text{cm}^2$, $\omega = 0.75$, and no noise is considered.

Gaussian white noise and phase noise being considered, the ISI (interspike interval) is estimated under different feedback gain k , and the bifurcation diagram is plotted in Fig. 2.

The results in Fig. 2 confirmed that the electrical activities can be controlled and selected as isolate mode with increasing the feedback gain k , which describes the coupling and adjusting on membrane potential by magnetic flux, it indicates that the memory effect is enhanced thus the electrical mode is selected. It also finds that the membrane potential is much dependent on the magnetic flux, and sampled time series for membrane potential of neuron are calculated in Fig. 3.

Indeed, multiple modes of electrical activities in neuron are found when the effect of electromagnetic induction, which is described by magnetic flux and coupling with membrane potential, is considered. As a result, appropriate mode in electrical activities can be selected by applying appropriate feedback gain on the membrane potential. In the following, the phase noise is imposed on the improved model, and the dynamical properties in electric activities are detected. The phase noise is generated from Gaussian white noise and the sampled time series for phase noise are calculated in Fig. 4.

It is confirmed that phase noise can be triggered from Gaussian white noise and the noise intensity can be adjusted by the Gaussian noise completely. Furthermore, the phase noise under different intensities is imposed on the neuron, and the sampled time series for membrane potential are detected, the results are plotted in Fig. 5.

It is interesting to find that the oscillating behavior of neuron is enhanced with increasing the noise intensity, particularly; multiple modes of electrical activities can be detected from the time series for membrane potentials. That is to say, appropriate noise is effective to trigger various modes of electrical activities of isolate neuron. For excitable neuron model, the SNR (signal to noise ratio) is often calculated to detect the stochastic resonance. The SNR is often defined by $\text{SNR} = 10 \log_{10}(S/B)$ where S and B represent the values of the output power spectrum density (PSD) at the peak (height of the signal peak) and the base of the signal feature (the amplitude of the background noise measured at the base of the signal peak), respectively [46–48]. In fact, the calculation for SNR provides an effective method for statistical and nonlinear analysis in signals. On the other hand, the coefficient variability (CV) of ISI series is often calculated to discern the coherence degree, the ISI is marked as T , and then the CV [49,41] is approached by

$$\text{CV} = \sqrt{(\langle T^2 \rangle - \langle T \rangle^2) / \langle T \rangle}. \quad (3)$$

It indicates that a smaller value for CV can be associated with a better coherence. The results for SNR and CV from output series of membrane potentials are plotted in Fig. 6.

It is similar with the previous works about stochastic resonance, particularly, double distinct peaks can be found in the curve with changing the intensity of noise, and it is called double coherence resonance [41]. The SNR is approached with large

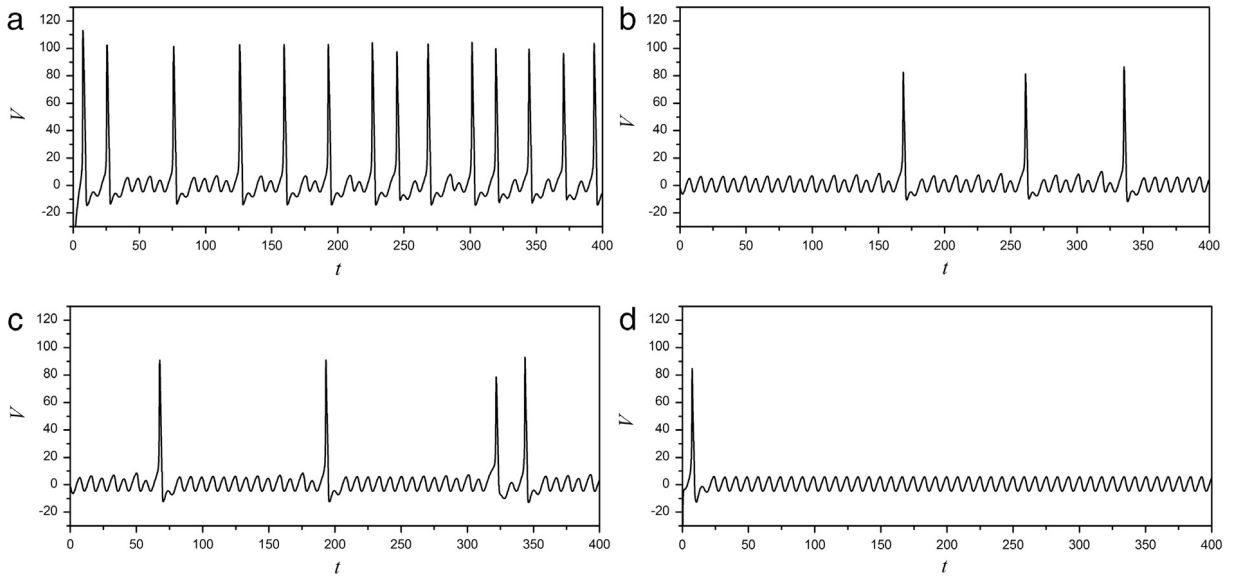


Fig. 3. Sampled time series for membrane potential by applying different feedback gains, for (a) $k = 0$, (b) $k = 1.37$, (c) $k = 1.95$, (d) $k = 3$, the amplitude and angular frequency of the external forcing is selected as $A = 2.5 \mu\text{A}/\text{cm}^2$, $\omega = 0.75$, respectively.

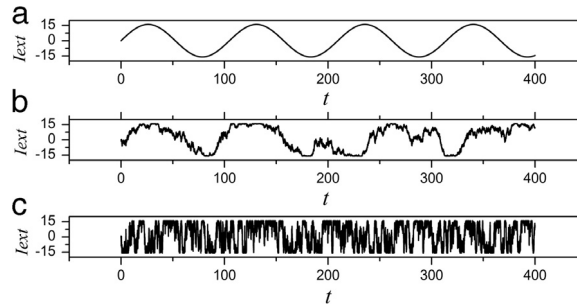


Fig. 4. Sampled time series for external forcing current under different noise intensity, for (a) $D = 0$, (b) $D = 10$, (c) $D = 60$, the amplitude and angular frequency of the phase noise is selected as $A_1 = 16 \mu\text{A}/\text{cm}^2$, $\omega_1 = 0.06$, $k = 3$.

value, which means that the regular oscillating behavior is distinct over noise; indeed, it is the phase noise that enhances the oscillating behavior with certain regular rhythm. Furthermore, the noise is removed to investigate the effect of magnetic flux on the membrane potential by changing the feedback gain k under different values, and the bifurcation analysis is presented in Fig. 7. And the time series for membrane potentials are calculated in Fig. 8.

The results in Fig. 7 find that ISI of neuron can be stabilized with increase of the feedback gain k , which the magnetic flux can adjust the membrane potential to regular rhythm. To confirm the dynamical properties in electrical activities, the sampled time series for membrane potentials are calculated in Fig. 8.

It is found in Fig. 8 that mode of electrical activities in neuron depends on the selection of feedback gain k , indeed, multiple modes of electrical activities emerge under appropriate feedback gain and it could be associated with some realistic biological properties, which neuron can present multiple modes at fixed parameters. With increase of the feedback gain, the modulation of magnetic flux on membrane potential can be approached completely thus the electrical activities select the most suitable mode under the fixed parameters. Furthermore, the phase noise is considered, and the results are plotted in Fig. 9.

The results in Fig. 9 confirmed that multiple modes in electrical activities can be triggered under appropriate phase noise, and the electrical activities of neuron show certain sensitivity to external phase noise. It is interesting to detect the SNR with the same parameters being selected; the results are plotted in Fig. 10.

The diagram for SNR in Fig. 10 shows some differences from the results in Fig. 6 though two peaks are still observed in the curve for SNR by increasing the noise intensity at a larger feedback gain. The two peaks in Fig. 10 are not distinct and their values are smaller than the two peaks of SNR in Fig. 6. And all the SNR are approached without distinct diversity, that is to say, larger feedback gain can induce larger SNR and keep robust to the noise background, the potential mechanism could be that multiple modes in electrical activities of neuron are generated under noise and electromagnetic induction, and the

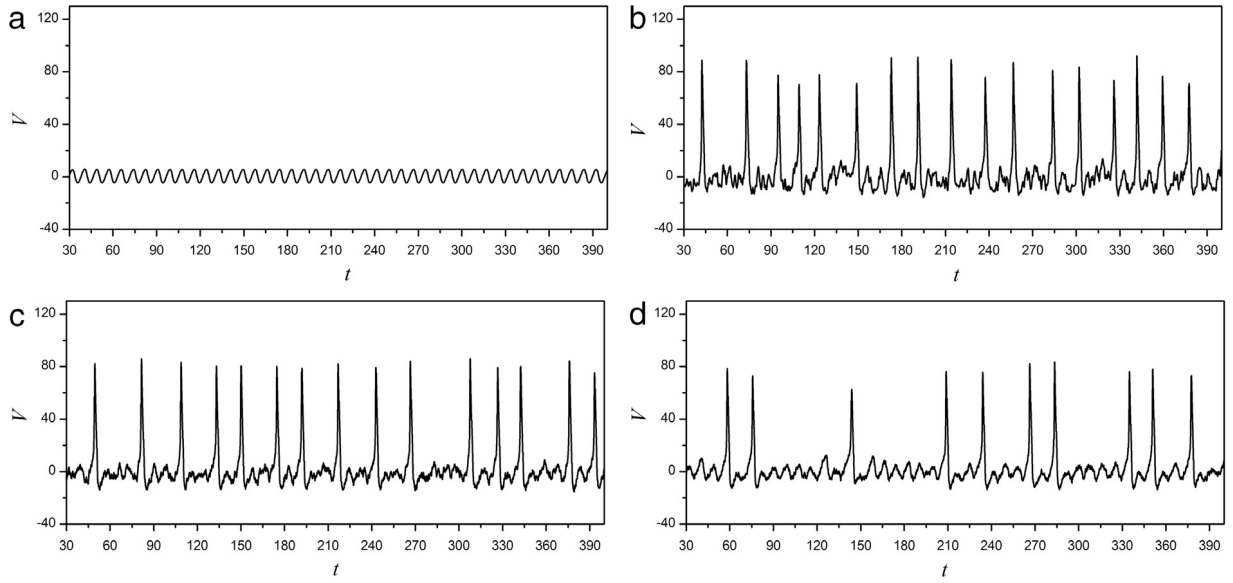


Fig. 5. Sampled time series for membrane potential under different noise intensity, for (a) $D = 0$, (b) $D = 100$, (c) $D = 300$, (d) $D = 500$, the amplitude and angular frequency of the phase noise is selected as $A_1 = 16 \mu\text{A}/\text{cm}^2$, $\omega_1 = 0.06$, $k = 3$.

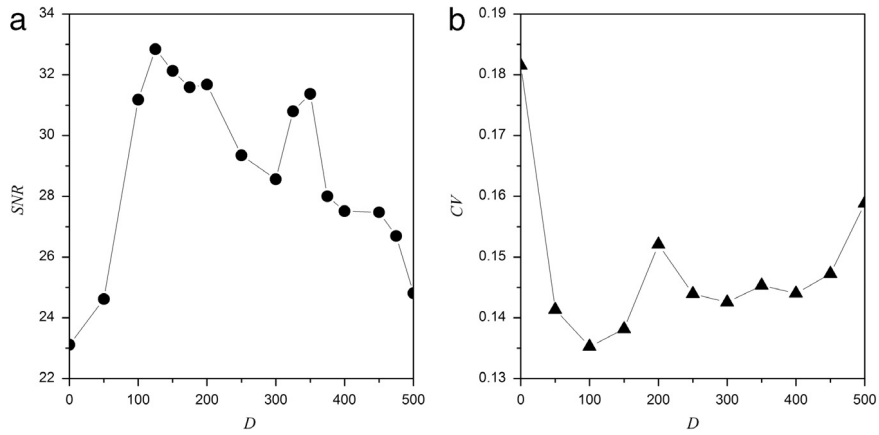


Fig. 6. SNR and CV for output series of membrane potential is calculated under different noise intensities, $A_1 = 16 \mu\text{A}/\text{cm}^2$, $\omega_1 = 0.06$, $k = 3$.

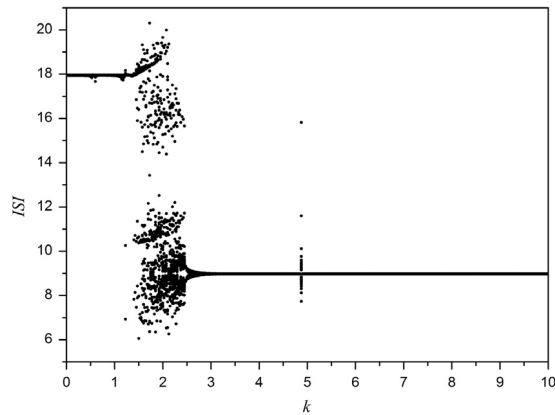


Fig. 7. Bifurcation diagram for ISI via feedback gain k at $\omega = 0.7$, $A = 2.5 \mu\text{A}/\text{cm}^2$, and no noise is considered.

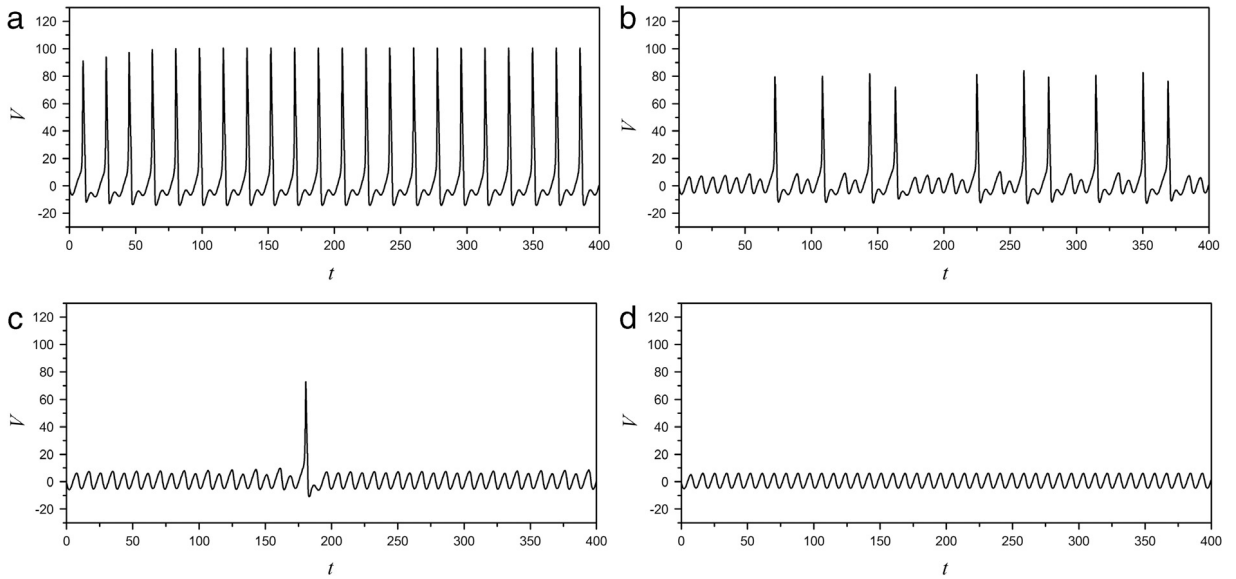


Fig. 8. Sampled time series for membrane potential under different feedback gains, for (a) $k = 0.5$, (b) $k = 1.9$, (c) $k = 4.875$, (d) $k = 6$, the amplitude and angular frequency of the I_{ext} is selected as $A = 2.5 \mu\text{A}/\text{cm}^2$, $\omega = 0.7$.

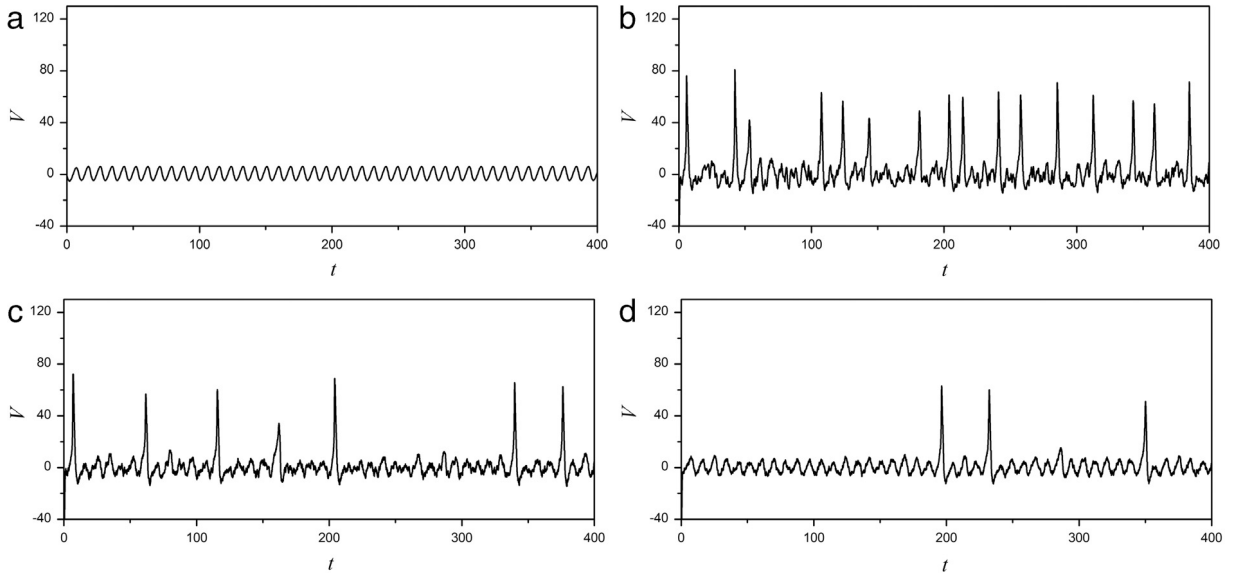


Fig. 9. Sampled time series for membrane potential under different noise intensity, for (a) $D = 0$, (b) $D = 100$, (c) $D = 300$, (d) $D = 500$, the amplitude and angular frequency of the phase noise is selected as $A_1 = 16 \mu\text{A}/\text{cm}^2$, $\omega_1 = 0.06$, $k = 6$.

electrical modes are dependent on magnetic flux greatly. Furthermore, additive Gaussian white noise is also considered, the numerical results found that multiple modes of electrical activities can be observed in the improved model.

Above all, phase noise is imposed on our improved neuron model, which the effect of electromagnetic induction is described by using magnetic flux, stochastic resonance-like behavior can be observed when the feedback gain between magnetic flux and membrane potential is weak. Furthermore, multiple modes in electrical activities can be observed in electrical activities and stochastic resonance is suppressed with the increase of feedback gain on membrane potential.

4. Conclusions

In this paper, the effect of electromagnetic induction is considered on the original Hodgkin–Huxley neuron model by introducing an additive magnetic flux variable, which is coupled the membrane potential across a memristor. SNR and CV are calculated to detect the stochastic resonance behavior under phase noise, multiple modes of electrical activities

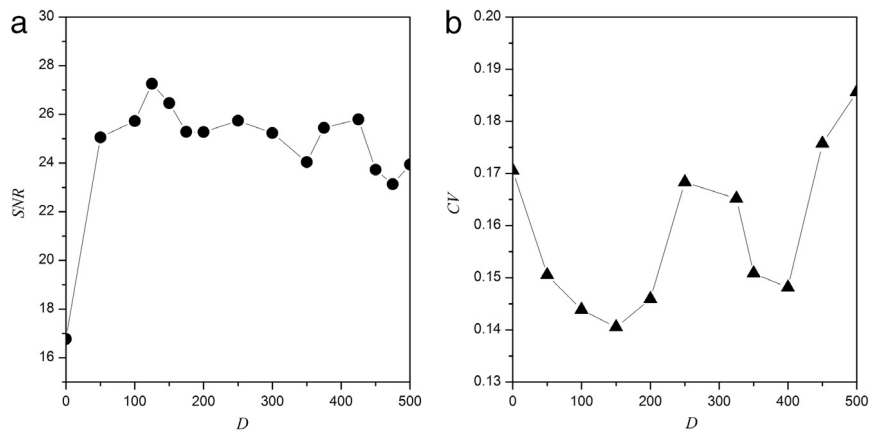


Fig. 10. SNR and CV for output series of membrane potential under different noise intensities, $A_1 = 16 \mu\text{A}/\text{cm}^2$, $\omega_1 = 0.06$, $k = 6$.

are observed. It is interesting to observe the occurrence of double coherence resonance which was ever detected in another neuron model [41]. With increasing the feedback gain of magnetic flux on membrane potential, the stochastic resonance is suppressed (two peaks in SNR curve are not distinct) because of emergence of multiple modes in electrical activities of neuron. The biological neuron model presents more complex dynamical behaviors, particularly; the emergence of multiple modes in electrical activities can throw light on further investigation on dynamical response of neuron under electromagnetic radiation. Furthermore, the modulation of astrocytes on neuronal transmission [50,51] can be considered by adding the magnetic flux on the neuron-coupled-astrocytes model, and this topic could be carried out by readers in this field for their interests.

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