Nonlinear Dyn (2020) 102:2739–2757
https://doi.org/10.1007/s11071-020-06076-6

ORIGINAL PAPER

Pattern control of external electromagnetic stimulation to neuronal networks

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Received: 19 June 2020 / Accepted: 4 November 2020 / Published online: 18 November 2020

Abstract The application of external electromagnetic stimulation to regulate the electrophysiological activities of specific brain regions can provide an ideal control or treatment scheme for some non-organic mental diseases. To further explore the effectiveness of electromagnetic stimulation in the treatment of mental illnesses, the regulatory abilities of external electromagnetic stimulation on the pattern dynamics of Newman–Watts small-world neuronal networks are systematically studied. The main stability function is used to construct four periodic or chaotic synchronous networks. Also, the average discharge frequency and the consistency coefficient are selected to measure the regulatory effects of external electromagnetic stimulation on the network dynamics. Numerical experiments show that electromagnetic stimulation can inhibit the electrophysiological activities of neuronal networks. Periodic electromagnetic stimulation with large amplitude or stochastic electromagnetic stimulation with large deviation has a more significant inhibitory effect on the discharge activities, not only effectively desynchronizing the discharge activities, but also controlling the evolution of spatiotemporal patterns, and even inducing the synchronization transition. Additionally, the number of stimulated neurons in neuronal networks also plays an important role in the evolution of spatiotemporal patterns. This study could provide theoretical guidance for the physiological application of electromagnetic stimulation in the treatment of certain mental diseases.

Keywords Neuronal network · Electromagnetic stimulation · Main stability function · Synchronization transition · Pattern control

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1 Introduction

The human brain composed of nearly 100 billion neurons and a large number of glial cells has a very complex physiological structure, which is mainly responsible for the collection, integration, processing and transmission of neural information. Generally, different brain regions correspond to different physiological functions. Each neuron in the same brain region not only has relatively independent electrophysiological activities, but also interacts with each other to achieve common physiological function. Up to now, many practical mathematical models of neurons have been established [1–3], and the effects of some factors such as time delay [4,5], autapse [6,7] and noise [8] on the neurophysiological activities have been analyzed in computational neuroscience. At the same time, the relationship between the dynamic behavior of neurons and the coding of neural information has been discussed [9–12].

The application of electronic devices makes the human brain in a complex and changeable electromagnetic environment. Therefore, the effect of external electromagnetic fields on the nervous system has aroused widespread concern [13–18]. Theoretically, it is necessary to study the influence of changeable electromagnetic fields inside and outside the neuronal system on the dynamic behavior of the nervous system [19–23]. At present, in addition to the extra transmembrane voltage induced by electromagnetic field [24–28], there are two more popular research methods for achieving electromagnetic effect. One is to introduce the radiating membrane current according to the energy conversion theory to obtain an improved neuron model under the action of electromagnetic radiation. At this time, electromagnetic radiation not only suppresses the spike discharge activities of neurons, but also destroys the wave propagation in the two-dimensional neuronal network with nearest-neighbor connection [29,30]. The other is to introduce the induced transmembrane current based on the working principle of memristor to explore the influence of different electromagnetic stimulation on the discharge behavior of neurons or neuronal networks [31]. Under the effect of electromagnetic induction, electromagnetic stimulation can not only affect the discharge activity of a single neuron, induce the transition of more complex discharge modes and the phenomenon of stochastic resonance [32–36], but also effectively modulate the dynamic behavior of neuronal networks, such as synchronous discharge, pattern evolution, and local signal transmission [37–39]. In particular, the mode of field coupling between neurons can also be realized with the help of magnetic flux variables [40]. In addition, physiological experiments have shown that, in addition to promoting synaptic vesicle endocytosis at the axon terminals, external electromagnetic stimulation can also change the brain's tissue structure by reducing abnormal synaptic connections, and even help improve the brain's associative memory ability [41–43].

When applying external electromagnetic stimulation to regulate the electrophysiological activities of the nervous system, it is not only necessary to consider what method or equipment is used to achieve the regulatory effect of electromagnetic stimulation, but also to quantify the relationship between the parameters of external electromagnetic stimulation and the regulatory effect. For example, in the treatment of neuropsychiatric patients, wearable transcranial electrical or magnetic stimulation equipment can be designed to apply electromagnetic stimulation to a specific brain part, which may be determined by the lesion location. According to the characteristics of neuropsychiatric patients, the relevant parameters of electromagnetic stimulation can be adjusted to achieve the desired effect of regulation or treatment. Currently, the main ways of applying electromagnetic stimulation to the cranial nervous system are transcranial electrical stimulation and transcranial magnetic stimulation. Transcranial magnetic stimulation can produce an induced electric field in the brain, which can directly depolarize the neuronal somata in the brain nervous system and initialize action potentials in the initial segment of axons [13]. Transcranial electrical stimulation can significantly regulate the electrophysiological activities of neurons in some brain areas [14], which can be used to weaken or eliminate some abnormal discharge patterns, such as epilepsy or other pathological rhythms [44–46]. At present, the electromagnetic stimulation technology acting on the brain nervous system is still immature, and there are many problems to be solved in theory and practice. Although transcranial electromagnetic stimulation has been used clinically to assist the treatment of mental illnesses, it is necessary to further explore the underlying mechanism and improve the treatment effect [47–49].

As a potential non-invasive treatment, how can electromagnetic effect be used more effectively to treat certain non-organic mental diseases? For this purpose, this paper selects the FitzHugh–Nagumo (FHN) neurons as the research object, and quantitatively investigates the regulatory function of external electromagnetic stimulation on the pattern evolution of Newman–Watts small-world neuronal networks. First, the small-world network model under electromagnetic effect is established based on electromagnetic induction. Secondly, the main stability function is used to discuss the stability of neuronal network synchronization under different periodic excitations. Finally, the influence of the oscillation of electromagnetic stimulation on the dynamics of Newman–Watts small-world neuronal networks is quantitatively analyzed, and the regulatory mechanism of electromagnetic stimulation on the evolution of network patterns is explored.

2 Mathematical description

Considering the working principle of magnetic-controlled memristor, the memductance can be expressed as:

$$\mu(\varphi) = \frac{\mathrm{d}q(\varphi)}{\mathrm{d}t} = \alpha + 3\beta\varphi^2, \tag{1}$$

where φ and $q(\varphi)$ indicates the magnetic flux and the amount of charge through the memristor, respectively, and α and β are two structural parameters. Then, the FHN neuron model under electromagnetic effect can be obtained as follows [50,51]:

$$\begin{cases} \frac{\mathrm{d}V}{\mathrm{d}t} = V(V-a)(1-V) - W + I_{\mathrm{ind}}, \\ \frac{\mathrm{d}W}{\mathrm{d}t} = \epsilon(V - cW - I_{\mathrm{ext}}), \\ \frac{\mathrm{d}\varphi}{\mathrm{d}t} = k_1 V - k_2\varphi + \varphi_{\mathrm{ext}}, \end{cases} \tag{2}$$

subject to

$$I_{\mathrm{ind}} = -k(\alpha + 3\beta\varphi^2)V \tag{3}$$

and

$$I_{\mathrm{ext}} = A\cos(2\pi f t), \tag{4}$$

where V and W denote membrane potential and recovery variable, respectively. I_{ind} represents the induced current generated by electromagnetic induction, and k, k_1 and k_2 are three system parameters determined by memristor [51]. The additional signal excitation corresponding to the regulatory effect of factors such as neurological medications or environmental temperature on the recovery variable can be characterized as $I_{\mathrm{ext}} = A\cos(2\pi f t)$, where A and f are the amplitude and the frequency of the periodic excitation, respectively. φ_{ext} represents the external electromagnetic stimulation. Two kinds of electromagnetic stimulation with physiological significance are considered in this paper. One is periodic electromagnetic stimulation, that is, $\varphi_{\mathrm{ext}} = B\cos(2\pi f_0 t)$, where B and f_0 are the oscillating amplitude and the oscillating frequency, respectively. The other is stochastic electromagnetic stimulation in the form of white noise. Its statistical characteristics satisfy $< \varphi_{\mathrm{ext}}(t) >= 0$ and $< \varphi_{\mathrm{ext}}(t)\cdot\varphi_{\mathrm{ext}}(t') >= \sigma^2\cdot\delta(t-t')$, where σ is the standard deviation and $\delta(\cdot)$ denotes the Dirac delta function.

Because the Newman–Watts small-world network can better simulate the structural characteristics of real neuronal networks [52–55], the Newman–Watts small-world network consisting of N FHN neurons is selected for discussion, which can be described as follows:

$$\begin{cases} \frac{\mathrm{d}V_i}{\mathrm{d}t} = V_i(V_i-a)(1-V_i) - W_i + I_{\mathrm{ind}i} + \tilde{I}_i, \\ \frac{\mathrm{d}W_i}{\mathrm{d}t} = \epsilon(V_i - cW_i - I_{\mathrm{ext}i}), \\ \frac{\mathrm{d}\varphi_i}{\mathrm{d}t} = k_1 V_i - k_2\varphi_i + \zeta_i\varphi_{\mathrm{ext}i}, \end{cases} \tag{5}$$

In this paper, the synaptic coupling currents are defined as:

$$\tilde{I}_i = g_0 \sum_j \gamma_{ij}(V_j(t) - V_i(t)), \quad i = 1, 2, \ldots, N, \tag{6}$$

where g_0 is the coupling strength and γ_{ij} represents the coupling relationship between neurons i and j. If the ith neuron and the jth neuron are coupled to each other, then $\gamma_{ij} = \gamma_{ji} = 1$, and otherwise, $\gamma_{ij} = \gamma_{ji} = 0$. Here, the Newman–Watts small-world network is implemented by randomly adding some edges to a ring network, that is, the coupling connection between two non-adjacent neurons is randomly established in terms of a linking probability p_1. In addition, in order to investigate the influence of the dynamic behavior of neurons subjected to electromagnetic stimulation on the electrophysiological activities of the entire network, the index values of neurons subjected to electromagnetic stimulation are randomly generated in light of the stimulated probability p_2, that is, the probability distribution of the random variable ζ_i satisfies

$$P\{\zeta_i\} = \begin{cases} p_2, & \zeta_i = 1; \\ 1 - p_2, & \zeta_i = 0. \end{cases} \tag{7}$$

Suppose that $f(V, W, \varphi) = V(V-a)(1-V) - W + I_{\mathrm{ind}}$, $g(V, W, \varphi) = \epsilon(V - cW - I_{\mathrm{ext}})$, $h(V, W, \varphi) = k_1 V - k_2\varphi + \zeta\varphi_{\mathrm{ext}}$, the above Newman–Watts small-world neuronal network model can be transformed as follows:

$$\begin{cases} \frac{\mathrm{d}V_i}{\mathrm{d}t} = f(V_i, W_i, \varphi_i) + \tilde{I}_i, \\ \frac{\mathrm{d}W_i}{\mathrm{d}t} = g(V_i, W_i, \varphi_i), \\ \frac{\mathrm{d}\varphi_i}{\mathrm{d}t} = h(V_i, W_i, \varphi_i), \end{cases} \tag{8}$$

where the coupling currents can be expressed as:

$$\tilde{I}_i = g_0 \sum_{j \neq i} \gamma_{ij} V_j(t) - g_0 (\sum_{j \neq i} \gamma_{ij}) V_i(t), \quad i = 1, 2, \ldots, N. \tag{9}$$

Let

$$G_{ij} = \begin{cases} \gamma_{ij}, & j \neq i; \\ -\sum\limits_{j \neq i} \gamma_{ij}, & j = i, \end{cases} \tag{10}$$

then

$$\begin{cases} \frac{\mathrm{d}V_i}{\mathrm{d}t} = f(V_i, W_i, \varphi_i) + g_0 \sum\limits_j G_{ij} V_j, \\ \frac{\mathrm{d}W_i}{\mathrm{d}t} = g(V_i, W_i, \varphi_i), \\ \frac{\mathrm{d}\varphi_i}{\mathrm{d}t} = h(V_i, W_i, \varphi_i), \end{cases} \tag{11}$$

where the electrical coupling matrix is

$$G = G^{\mathrm{T}} = \begin{pmatrix} -\sum\limits_{j \neq 1} \gamma_{1j} & \gamma_{12} & \cdots & \gamma_{1N} \\ \gamma_{21} & -\sum\limits_{j \neq 2} \gamma_{2j} & \cdots & \gamma_{2N} \\ \cdots & \cdots & \cdots & \cdots \\ \gamma_{N1} & \gamma_{N2} & \cdots & -\sum\limits_{j \neq N} \gamma_{Nj} \end{pmatrix}. \tag{12}$$

Let $x_i = (V_i, W_i, \varphi_i)^{\mathrm{T}}$, $L(x_i) = (V_i, 0, 0)^{\mathrm{T}}$, $F(x_i) = (f(V_i, W_i, \varphi_i), g(V_i, W_i, \varphi_i), h(V_i, W_i, \varphi_i))^{\mathrm{T}}$, then the FHN neuronal network system can be expressed as:

$$\dot{x}_i = F(x_i) + g_0 \sum_j G_{ij} L(x_i), \quad i = 1, 2, \ldots, N. \tag{13}$$

3 Synchronization stability analysis of neuronal networks

The main stability function can be used to investigate the synchronization stability of neuronal networks [56–58]. Assuming that the FHN neuronal network is synchronized, the dynamic equation in the synchronous manifold can be simplified as:

$$\dot{x} = F(x) + g_0 \sum_j G_{ij} L(x) = F(x), \tag{14}$$

where the synchronous manifold can be defined by $x_1 = x_2 = \cdots = x_N$, and be guaranteed to be invariant by $\sum_j G_{ij} = 0$.

Suppose that $X = (x_1, x_2, \ldots, x_N)$, $F(X) = (F(x_1), F(x_2), \ldots, F(x_N))$, $L(X) = (L(x_1), L(x_2), \ldots, L(x_N))$, then

$$\dot{X} = F(X) + g_0 G \otimes L(X), \tag{15}$$

where $G \otimes L(X)$ represents the direct product of G and $L(X)$.

Let $\xi = (\xi_1, \xi_2, \ldots, \xi_N)$, where ξ_i is the variation of x_i, then

$$\dot{\xi} = (1_N \otimes \mathrm{D}F + g_0 G \otimes \mathrm{D}L)\, \xi, \tag{16}$$

where $\mathrm{D}F = \begin{pmatrix} f_V & f_W & f_\varphi \\ g_V & g_W & g_\varphi \\ h_V & h_W & h_\varphi \end{pmatrix}$, $\mathrm{D}L = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. By diagonalizing G, a block diagonal variational equation can be obtained as follows:

$$\dot{\xi}_i = (\mathrm{D}F + g_0 \lambda_i \cdot \mathrm{D}L) \xi_i, \tag{17}$$

where $\lambda_i (i = 1, 2, \ldots, N)$ is the ith eigenvalue of G.

Thus, the general variational equation can be gotten as follows:

$$\dot{\xi}_0 = (\mathrm{D}F + \rho \cdot \mathrm{D}L)\, \xi_0, \tag{18}$$

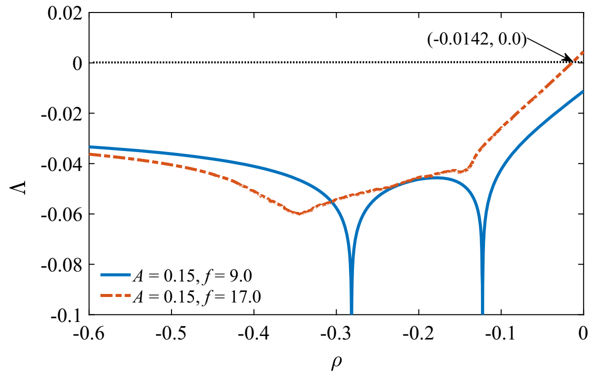
i.e.,

$$\dot{\xi}_0 = \left. \begin{pmatrix} f_V + \rho & f_W & f_\varphi \\ g_V & g_W & g_\varphi \\ h_V & h_W & h_\varphi \end{pmatrix} \right|_{x_0} \cdot \xi_0, \tag{19}$$

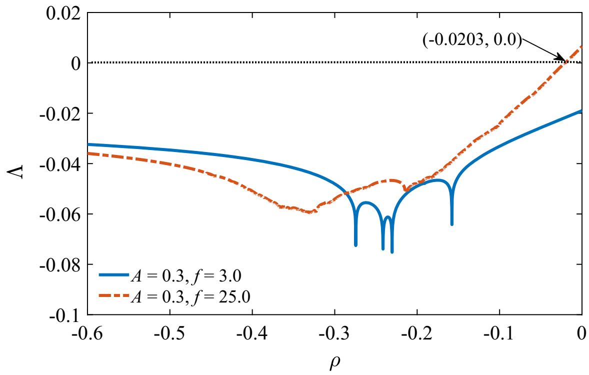
where $\rho = g_0 \lambda$, and x_0 is determined by $\dot{x} = F(x)$.

In numerical experiments, first, it is necessary to calculate the maximum Lyapunov exponent $\Lambda = \Lambda(\rho)$ as a function of parameter ρ according to the dynamic system (18), i.e., the main stability function of the network system [58]. Furthermore, the value range Ω of parameter ρ corresponding to the network synchronization can be determined by $\Lambda < 0$. Secondly, since the eigenvalues of the above coupling matrix G satisfy $\lambda_N \leq \lambda_{N-1} \leq \cdots \leq \lambda_2 < \lambda_1 = 0$, the necessary condition for the synchronization stability of the neuronal network can be reduced to $g_0 \lambda_2 \in \Omega$. It can be seen that when the dynamic equations corresponding to network nodes are selected, the synchronization stability of neuronal networks is mainly determined by the coupling matrix and the coupling strength, more specifically, the product of the second largest eigenvalue of the coupling matrix and the coupling strength [58,59].

For all numerical experiments, the values of system parameters are set to $a = 0.1$, $\epsilon = 0.01$, $c = 2.0$, $k = 0.1$, $k_1 = 0.2$, $k_2 = 0.9$, $\alpha = 0.1$, $\beta = 0.02$. The default initial values of three system variables are $V(0) = -0.2$, $W(0) = -0.2$, and $\varphi(0) = 0.5$, respectively. The time step is set to $\Delta t = 0.01$, and the number of FHN neurons in Newman–Watts small-world networks is set to $N = 50$.



(a)



(b)

Fig. 1 Dependence of maximum Lyapunov exponent Λ of dynamic system (18) on parameter ρ under different periodic excitations, where $p_2 = 0.0$. The maximum Lyapunov exponent is always less than zero for $A = 0.15$, $f = 9.0$, and $A = 0.3$, $f = 3.0$, while the maximum Lyapunov exponent is greater than zero in a very small range of parameter ρ for $A = 0.15$, $f = 17.0$, and $A = 0.3$, $f = 25.0$. Specifically, for $A = 0.15$, $f = 17.0$, and $A = 0.3$, $f = 25.0$, the maximum Lyapunov exponent Λ is approximately zero when the parameter ρ is equal to $\rho_1 = -0.0142$ and $\rho_2 = -0.0203$, respectively. (Color figure online)

To investigate the influence of the network coupling structure on the synchronization activity, four different periodic signal excitations are selected for comparison and analysis. For a single FHN neuron, $A = 0.15$, $f = 9.0$ and $A = 0.3$, $f = 3.0$ correspond to the periodic discharge state, and $A = 0.15$, $f = 17.0$ and $A = 0.3$, $f = 25.0$ correspond to the chaotic discharge state. Applying the synchronous manifold analysis method of the main stability function, the main stability curves are calculated according to Eq. (18), corresponding to the function relation $\Lambda(\rho)$ between the maximum Lyapunov exponent Λ and the network structure parameter ρ, as shown in Fig. 1. For $A = 0.15$, $f = 9.0$ or $A = 0.3$, $f = 3.0$, as long as $\rho < 0$, the maximum Lyapunov exponent of the neuronal network system without external electromagnetic stimulation is always less than zero, which indicates that the synchronization activity of the neuronal network composed of periodically moving neurons has inherent stability and is not affected by the coupling matrix and the coupling strength.

However, for $A = 0.15$, $f = 17.0$ or $A = 0.3$, $f = 25.0$, with the decrease of ρ, the value of Λ can change from positive to negative at $\rho = \rho_1 = -0.0142$ or $\rho = \rho_2 = -0.0203$, respectively. This reflects that the synchronization of the neuronal network consisting of chaotic neurons is not always stable, and usually depends on the network coupling structure determined by the coupling matrix and the coupling strength. Given the second largest eigenvalue λ_2 of the coupling matrix, for $A = 0.15$ and $f = 17.0$, the neuronal network may be in a stable synchronous motion state when the coupling strength satisfies $g_0 > \rho_1/\lambda_2$. Comparatively speaking, for $A = 0.3$ and $f = 25.0$, the synchronization of the network may be stable only when $g_0 > \rho_2/\lambda_2$.

Table 1 shows the average second largest eigenvalues of Newman–Watts small-world coupling matrix of 1000 calculations and the corresponding critical coupling strengths of synchronization for five different linking probabilities, where $p_2 = 0.0$. For example, when the linking probability p_1 is equal to 0.08, the average second largest eigenvalues of the coupling matrix is about $\lambda_2 = -1.3255$. At this time, for the periodic excitation with $A = 0.15$, $f = 17.0$ or $A = 0.3$, $f = 25.0$, the critical coupling strength required for network synchronization is approximately 0.0107 or 0.0153, respectively.

In the following discussion, considering the randomness of the Newman–Watts small-world coupling matrix generation, the selected coupling strength is $g_0 = 0.02$ under the premise of $p_1 = 0.08$. This can ensure the stability of the synchronization activity under the additional excitation with $A = 0.15$, $f = 17.0$ or $A = 0.3$, $f = 25.0$. It can be seen from Fig. 2 that when $p_2 = 0.0$, $p_1 = 0.08$ and $g_0 = 0.02$, four different periodic excitations can

Table 1 Dependence of average second largest eigenvalue λ_2 of 1000 Newman–Watts small-world coupling matrices and corresponding critical coupling strengths g_{01} and g_{02} on linking probability p_1, where $g_{01} = \rho_1/\lambda_2$, $g_{02} = \rho_2/\lambda_2$, $\rho_1 = -0.0142$, and $\rho_2 = -0.0203$

p_1	0.02	0.04	0.06	0.08	0.1
λ_2	−0.2145	−0.5307	−0.9155	−1.3255	−1.7764
g_{01}	0.0662	0.0268	0.0155	0.0107	0.0080
g_{02}	0.0946	0.0383	0.0222	0.0153	0.0114

finally synchronize the Newman–Watts small-world neuronal network with initial disturbance. In particular, for the periodic excitation with $A = 0.3$ and $f = 25.0$, the entire neuronal network will be in a stable synchronous motion state after about 2600 time units. It not only reveals the sensitivity of the network composed of chaotic neurons to initial values, but also indicates the effect of the small-world coupling between neurons on the network synchronization activity, and verifies the effectiveness of $g_0 = 0.02$.

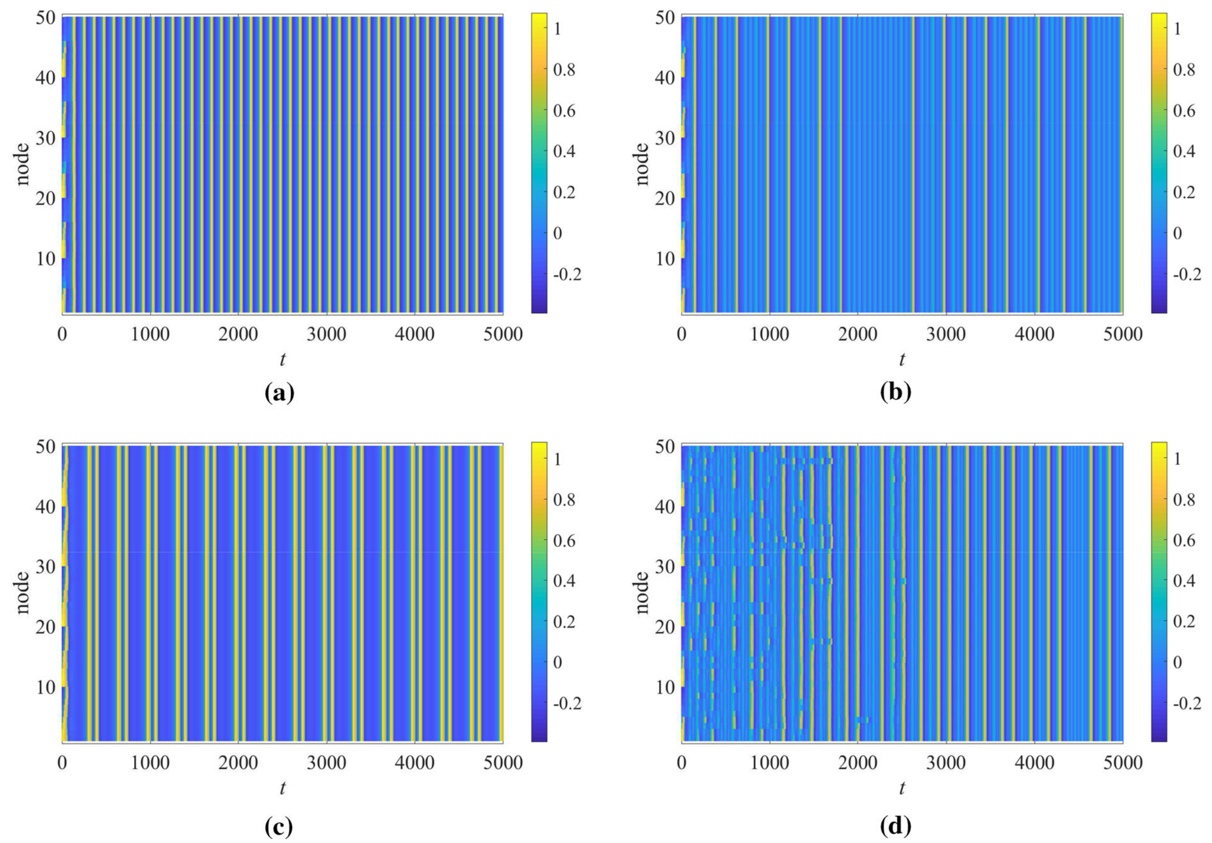


Fig. 2 Spatiotemporal pattern evolution of Newman–Watts small-world neuronal networks with initial disturbance, where $p_2 = 0.0$, $p_1 = 0.08$ and $g_0 = 0.02$. The amplitude and the frequency of periodic excitation are set to **a** $A = 0.15$, $f = 9.0$, **b** $A = 0.15$, $f = 17.0$, **c** $A = 0.3$, $f = 3.0$, and **d** $A = 0.3$, $f = 25.0$. Regardless of whether the individual neurons are in a periodic state or a chaotic state, the entire network can eventually evolve into a synchronized motion state under the conditions of $p_1 = 0.08$ and $g_0 = 0.02$. (Color figure online)

4 Pattern regulation of electromagnetic stimulation

The dynamics of neuronal networks is mainly characterized by spatiotemporal pattern and synchronization [60–65]. Therefore, based on the above research, the regulatory effect of external electromagnetic stimulation on spatiotemporal pattern and synchronization of Newman–Watts small-world networks will be deeply explored. To quantify the influence of external electromagnetic stimulation on the network electrophysiological activities, the average discharge frequency of all neurons and the consistency coefficient are selected for measurement. Here, the consistency coefficient is used to measure the network synchronization. Usually, after the time interval to be studied is divided into m small time windows, the electrophysiological state sequence can be characterized by 0 and 1, respectively, corresponding to the resting and spiking state, that is, $Y(l) = 0$ or 1, $l = 1, 2, \ldots, m$. In terms of the standardized method of cross-correlation coefficient, the consistency coefficient between two neurons can be expressed as follows:

$$K_{ij} = \left(\sum_{l=1}^{m} Y_i(l) Y_j(l) \right) \Bigg/ \sqrt{\sum_{l=1}^{m} Y_i(l) \sum_{l=1}^{m} Y_j(l)}. \tag{20}$$

Thus, the consistency coefficient of the N-neuron coupled network is defined as follows:

$$K = \left(2 \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} K_{ij} \right) \Bigg/ (N(N-1)). \tag{21}$$

Generally, the larger the consistency coefficient, the better the network synchronization [66]. Furthermore, the average membrane potential of the neuronal network is defined as follows:

$$\overline{V}(t) = \frac{1}{N} \sum_{i=1}^{N} V_i(t). \tag{22}$$

In addition, owing to the randomness of electromagnetic stimulation, network structure and distribution of stimulated neurons, the second-order stochastic Runge–Kutta algorithm is used to simulate the dynamics of individual neurons, and an average of 40 calculations is required to calculate the two metrics [67].

4.1 Regulation of periodic electromagnetic stimulation

For the aforementioned four different additional periodic signals, Fig. 3 shows the dependence of average discharge frequency and consistency coefficient on the amplitude of external periodic electromagnetic stimulation, where two frequencies of periodic electromagnetic stimulation are selected for comparison, i.e., $f_0 = 2.0$ and $f_0 = 10.0$. For the periodic synchronous network corresponding to $A = 0.15$, $f = 9.0$ or $A = 0.3$, $f = 3.0$, the average discharge frequency usually begins to decreases only after the amplitude of periodic electromagnetic stimulation exceeds a certain threshold. In contrast, the chaotic synchronous network corresponding to $A = 0.15$, $f = 17.0$ or $A = 0.3$, $f = 25.0$ is more susceptible to external electromagnetic stimulation. As the amplitude B increases, the whole network quickly tends to a state of sub-threshold oscillation, resulting in the rapid decrease in two network metrics. This is mainly because the stability of chaotic synchronous neuronal networks is weaker than that of periodic synchronous neuronal networks, and the corresponding coupling relation between neurons is more likely to be destroyed. Meanwhile, the consistency coefficient almost presents a positive correlation with the average discharge frequency. It can be inferred that with the increase of B, the weakening of the synchronization performance is mainly due to the suppression of electromagnetic stimulation to the network firing activity.

Additionally, there is no essential difference in the effect of two different frequencies of periodic electromagnetic stimulation on the network firing activity. However, for the periodic excitation with $A = 0.15$ and $f = 17.0$, when $f_0 = 2.0$, the average discharge frequency and the consistency coefficient rebound abnormally around $B = 6.0$, and the spatiotemporal pattern presents a relatively regular spike discharge activity with a period of 1000 time units, as shown in Fig. 4. Correspondingly, as the amplitude of the periodic electromagnetic stimulation increases, the overall discharge state of the neuronal network changes, and transitions from a chaotic state to a periodic discharge state through sub-threshold oscillation, as shown in Fig. 5. This not only embodies the sensitivity of the neuronal network to periodic electromagnetic stimulation with certain amplitude and frequency, but also reveals the specific fil-

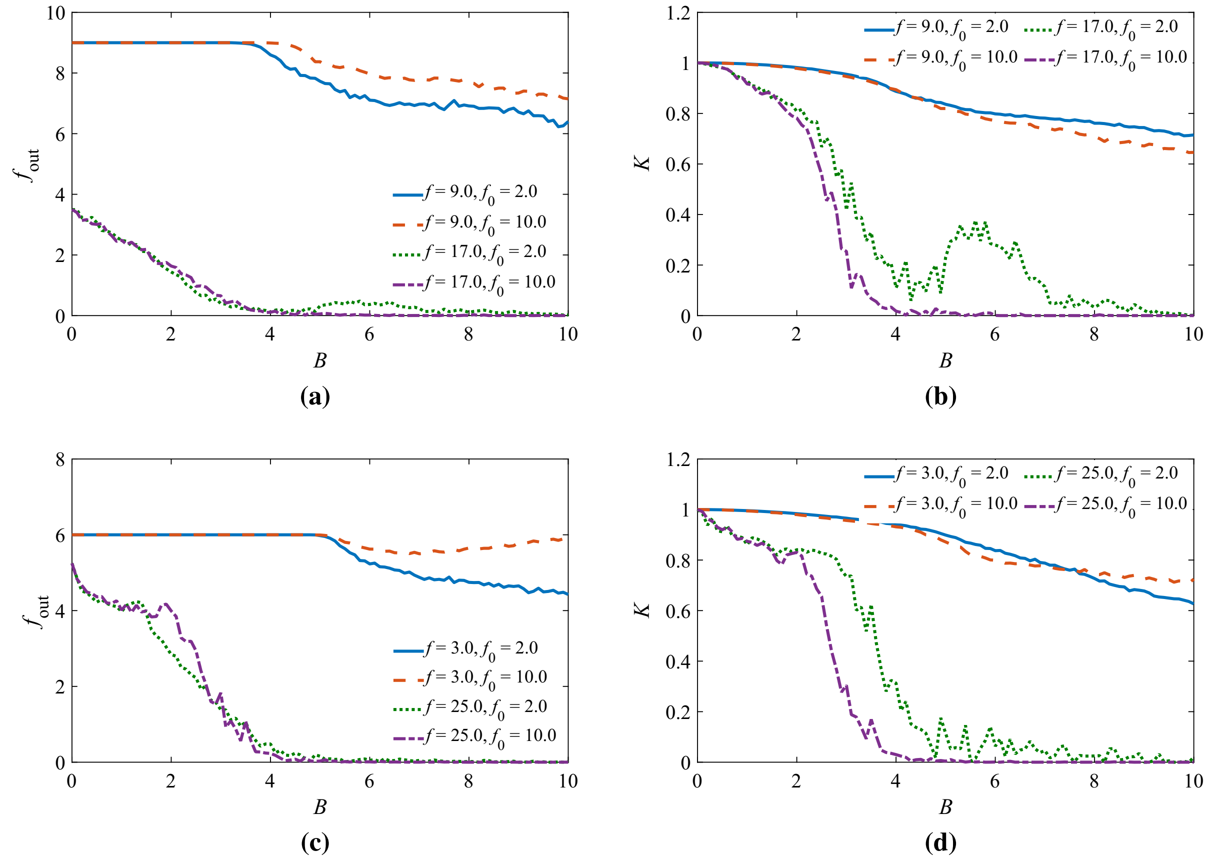


Fig. 3 Dependence of average discharge frequency and consistency coefficient of Newman–Watts small-world neuronal networks on amplitude of periodic electromagnetic stimulation, where $p_1 = 0.08$, $g_0 = 0.02$ and $p_2 = 0.4$. Two amplitudes of additional periodic excitations are **a** and **b** $A = 0.15$, **c** and **d** $A = 0.3$, respectively. With the increase in the amplitude of periodic electromagnetic stimulation, the two metrics exhibit a decreasing trend, indicating that the network synchronization activity is inhibited. In particular, the spike discharge of chaotic synchronous networks is almost completely suppressed and transits to sub-threshold oscillation when the amplitude of periodic electromagnetic stimulation exceeds a certain value. (Color figure online)

tering function of periodic electromagnetic stimulation.

Furthermore, under the periodic electromagnetic stimulation with selected amplitude and frequency, the influence of the dynamic behavior of some neurons on the network synchronization activity is discussed by changing the stimulated probability, determining the number of neurons directly driven by electromagnetic stimulation in the network. As the stimulated probability p_2 increases, Fig. 6 shows the trend of average discharge frequency and consistency coefficient, where $f_0 = 2.0$, $p_1 = 0.08$ and $g_0 = 0.02$. For the periodic synchronous neuronal network, when $B = 3.0$, the two metrics are almost unchanged, which indicates that the dynamics of the network is very weakly affected. When $B = 8.0$, the average discharge frequency almost keeps a linear decreasing trend, and the consistency coefficient shows a trend of decreasing first and then increasing. The underlying mechanism is that under the periodic electromagnetic stimulation with large amplitude, the greater the number of stimulated neurons, the stronger the inhibition of the network firing activity, and therefore the more likely the synchronization transition is to occur after the network activity is desynchronized.

As can be seen from Fig. 7, as the stimulated probability p_2 increases, the spatiotemporal pattern retains a specific discharge sequence after phase tran-

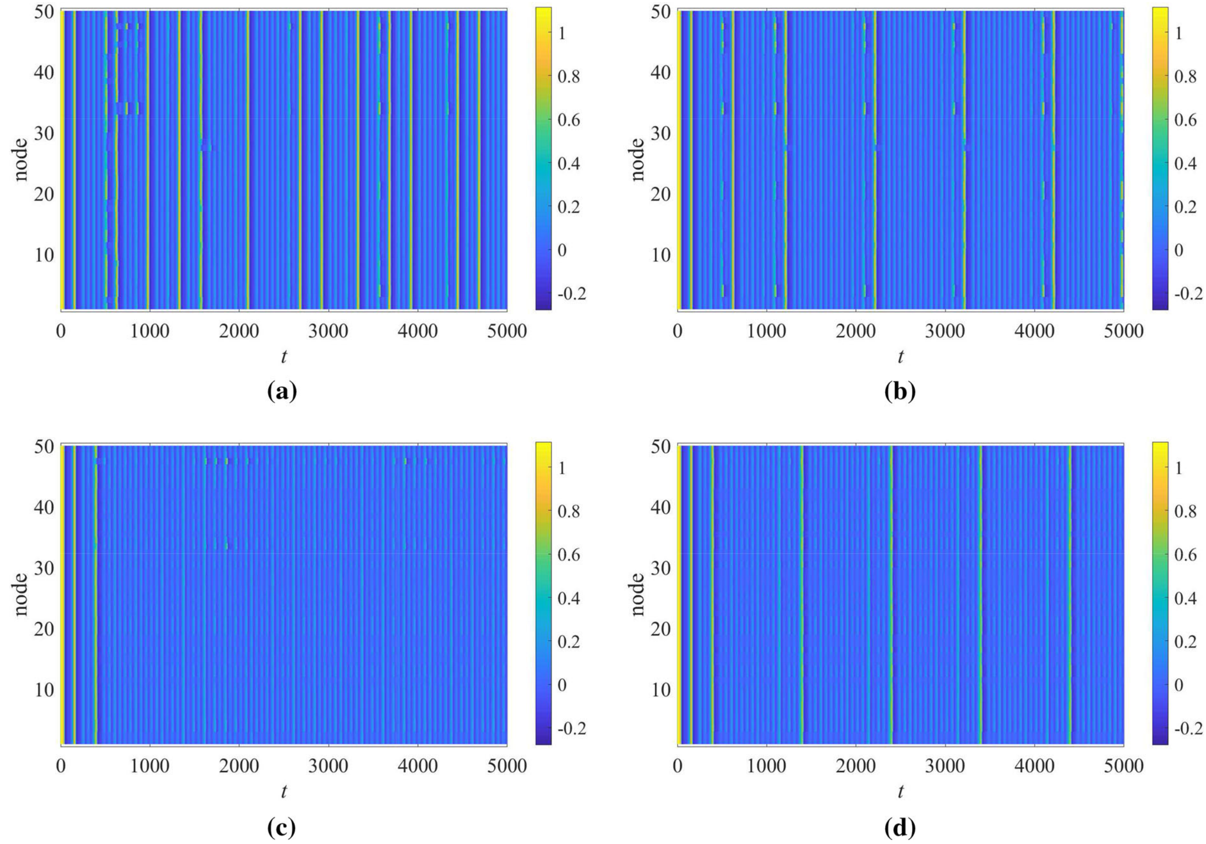


Fig. 4 Spatiotemporal pattern evolution of Newman–Watts small-world neuronal networks under periodic excitation with $A = 0.15$ and $f = 17.0$, where $p_1 = 0.08$, $g_0 = 0.02$, $p_2 = 0.4$ and $f_0 = 2.0$. The amplitude of periodic electromagnetic stimulation activated at $t = 300$ is **a** $B = 1.0$, **b** $B = 2.0$, **c** $B = 4.0$ and **d** $B = 6.0$, respectively. Under the periodic electromagnetic stimulation applied to some neurons, the spatiotemporal pattern of the network is sensitive to the amplitude of periodic electromagnetic stimulation, and the discharge activities of all neurons are almost equivalently regulated. (Color figure online)

sition, and achieves the mode transition from fast-spiking synchronization to quasi-bursting synchronization, fully reflecting the filtering function of electromagnetic stimulation. Here, Fig. 8 shows the sampled time series of average membrane potentials corresponding to Fig. 7. For the chaotic synchronous neuronal network, the two network metrics are easily affected by the stimulated probability to exhibit different trends. For example, for the periodic excitation with $A = 0.15$ and $f = 17.0$, even when $B = 3.0$, the average discharge frequency and the consistency coefficient show a trend of decreasing first and then increasing as the stimulated probability increases, which not only reflects the regulatory ability of the dynamics of some neurons to the network activity, but also reveals the filtering function of electromagnetic stimulation. In particular, the consistency coefficient increases to 1.0 when $p_2 = 1.0$, realizing the pattern transition from chaotic synchronization to subthreshold oscillation and finally to periodic synchronization.

In short, the external periodic electromagnetic stimulation can effectively control the spatiotemporal pattern evolution, and even induce the synchronization transition. For a periodic synchronous neuronal network, when the amplitude of periodic electromagnetic stimulation is greater than a certain threshold, the electromagnetic stimulation can cause the desynchronization of the network by effectively suppressing the neuronal discharge activity, and even realize the synchronization transition in the process of pattern phase transition, thereby revealing the filtering function of electro-

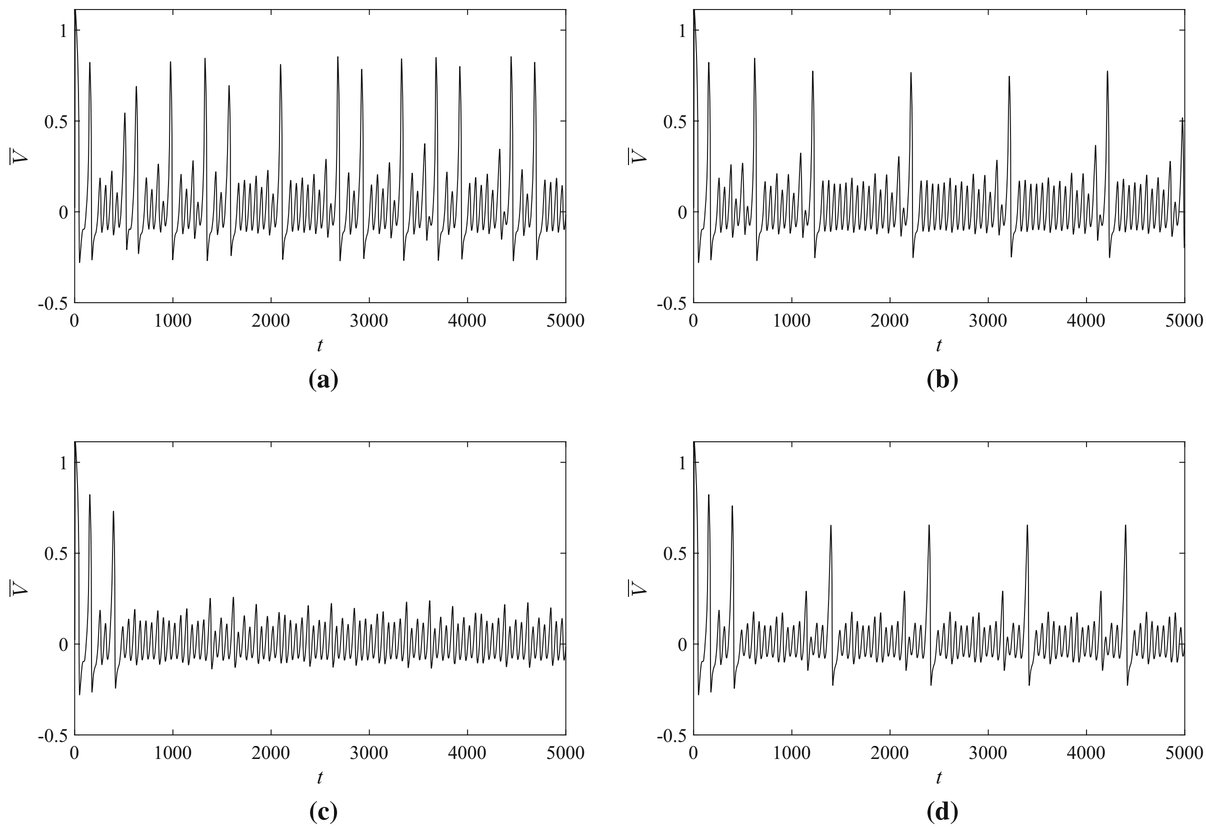


Fig. 5 Sampled time series of average membrane potentials corresponding to Fig. 4, where **a** $B = 1.0$, **b** $B = 2.0$, **c** $B = 4.0$ and **d** $B = 6.0$. With the increase in the amplitude of the periodic electromagnetic stimulation, the entire neuronal network can realize periodic discharge from chaotic state through subthreshold oscillation

magnetic stimulation. For a chaotic synchronous network, due to the instability of the synchronization state and the filtering function of electromagnetic stimulation, the periodic electromagnetic stimulation with smaller amplitude can also exert a stronger suppression effect, thereby destroying the chaotic synchronization state and resulting in the synchronization transition. Furthermore, changing the stimulated probability of neurons can also effectively modulate the dynamics of the network, and even induce more abundant dynamic phenomena. This indicates that the dynamic behavior of partial neurons can alter the network electrophysiological activities, which can provide a reference for regulating the electrophysiological activities of neuronal networks by electromagnetic stimulation.

4.2 Regulation of stochastic electromagnetic stimulation

The electromagnetic stimulation in the form of noise can be applied physiologically by adjusting its standard deviation. Therefore, the regulatory ability of stochastic electromagnetic stimulation on the dynamics of neuronal networks can be quantitatively studied by establishing the functional dependence of average discharge frequency and consistency coefficient on standard deviation. As can be seen from Fig. 9, the stochastic electromagnetic stimulation with large standard deviation usually has a strong inhibition on the network firing activity, thereby leading to a relatively obvious desynchronization phenomenon. As the

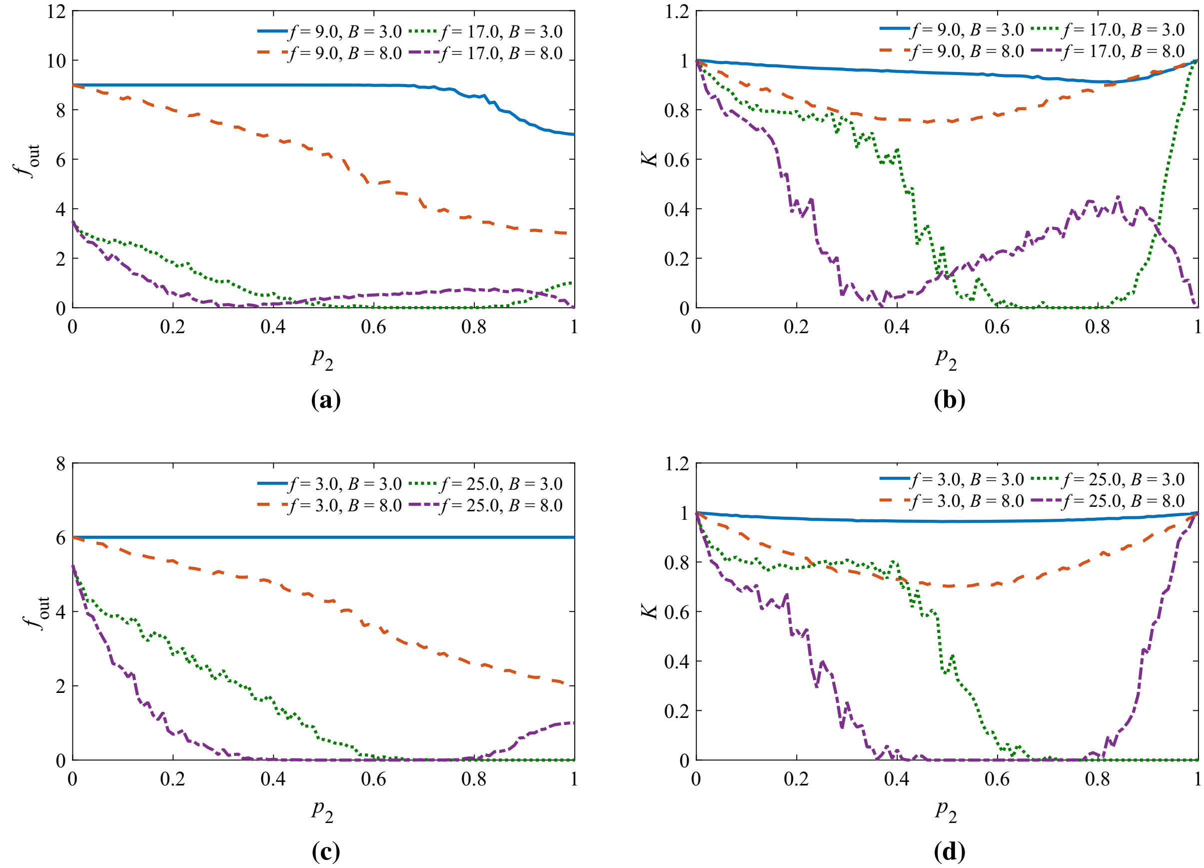


Fig. 6 Dependence of average discharge frequency and consistency coefficient of Newman–Watts small-world neuronal networks on stimulated probability, where $p_1 = 0.08$, $g_0 = 0.02$ and $f_0 = 2.0$. Two amplitudes of additional periodic excitations are **a** and **b** $A = 0.15$, **c** and **d** $A = 0.3$, respectively. With the increase in the stimulated probability, periodic electromagnetic stimulation with small amplitude cannot effectively regulate the periodic synchronous network. However, it can significantly regulate the chaotic synchronous network, which average discharge frequency and consistency coefficient can exhibit a rich nonlinear trend. (Color figure online)

standard deviation σ increases, the discharge activity of the chaotic synchronous neuronal network is easily suppressed and tends to the state of sub-threshold oscillation, as shown in Figs. 10 and 11. In contrast, the periodic synchronous neuronal network can be regulated to a certain extent only when the standard deviation exceeds a certain threshold, embodying the strong stability of the periodic synchronous network.

To explore the influence of the dynamics of some neurons on the whole network activity, Fig. 12 gives the functional dependence of two network metrics on the stimulated probability of neurons when the standard deviation is set to $\sigma = 3.0$ and $\sigma = 8.0$. With the increase in the stimulated probability, the two network metrics almost present a monotonic decreasing trend, which is similar to the trend shown in Fig. 9, indicating that the number of stimulated neurons and the standard deviation are approximately equivalent in regulating the network firing activity. However, when $\sigma = 3.0$, the dynamic behavior of the periodic synchronous network remains almost unchanged, thus confirming the existence of the stimulation threshold. Compared with the periodic electromagnetic stimulation, although the stochastic electromagnetic stimulation can also effectively control the evolution of network patterns, it does not have the filtering func-

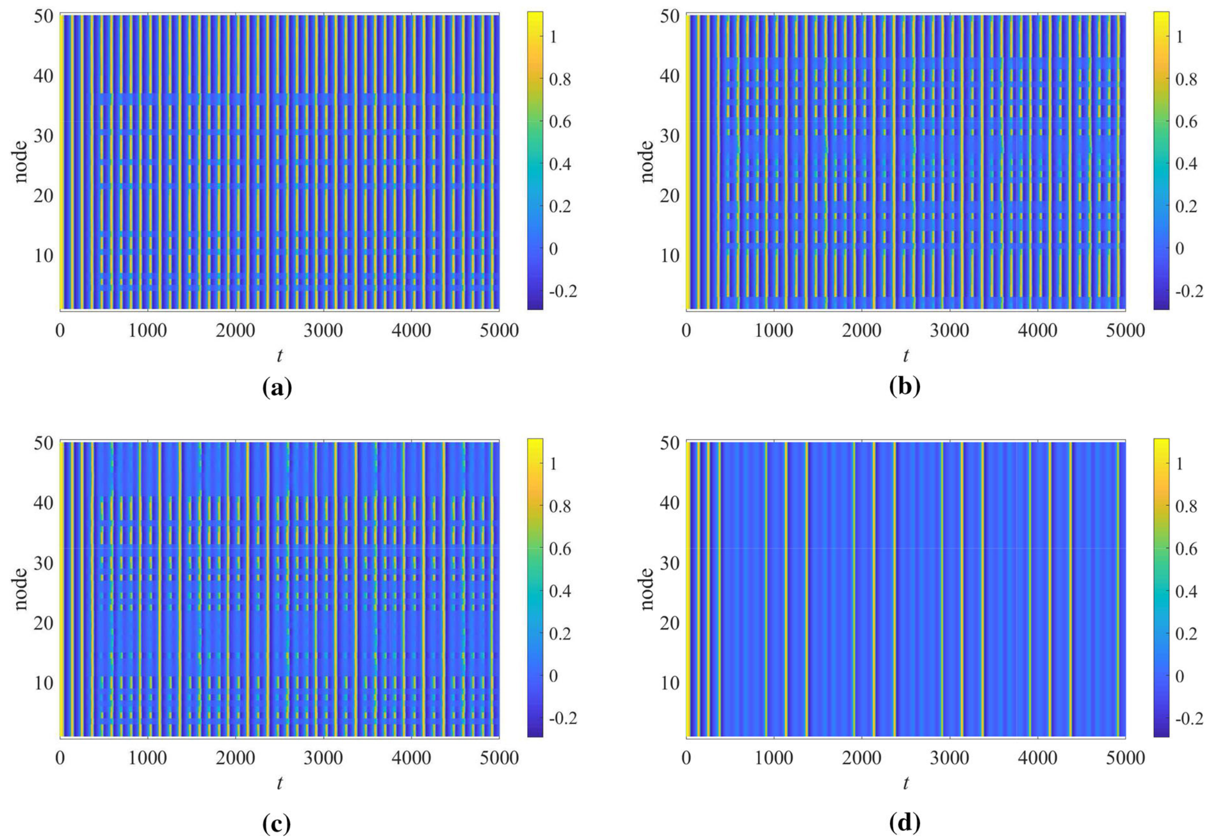


Fig. 7 Spatiotemporal pattern evolution of Newman–Watts small-world neuronal networks under periodic excitation with $A = 0.15$ and $f = 9.0$, where $p_1 = 0.08$ and $g_0 = 0.02$. The periodic electromagnetic stimulation with $B = 8.0$ and $f_0 = 2.0$ is activated at $t = 300$, and the stimulated probability is set to **a** $p_2 = 0.2$, **b** $p_2 = 0.4$, **c** $p_2 = 0.6$ and **d** $p_2 = 1.0$, respectively. As the stimulated probability increases, the periodic electromagnetic stimulation effectively inhibits the high-frequency spike discharge activity, and eventually induces the transition of synchronous mode. (Color figure online)

tion and cannot induce the synchronization transition.

In conclusion, the external electromagnetic effect can not only effectively desynchronize the discharge activity of neuronal networks, but also control the evolution of spatiotemporal patterns. Generally speaking, the electromagnetic stimulation can inhibit the electrophysiological activities of neuronal networks, and the inhibitory effect is more pronounced when the amplitude of periodic electromagnetic stimulation or the standard deviation of stochastic electromagnetic stimulation is larger. It is worth noting that the regulatory effect of external electromagnetic stimulation on neuronal networks discussed in this paper is negative feedback, which can inhibit the firing activities of neuronal networks, as shown in Figs. 7 and 10. Although the above conclusion coincides with that in literature [30,68], it is inconsistent with that in literature [69,70]. At present, it is preliminarily inferred that these conflicting dynamic phenomena may be related to the selected neuron model and its dynamical properties. In the following research work, we will pay attention to this problem and try to reveal its dynamic mechanism by bifurcation analysis. In addition, changing the stimulated probability of neurons can also effectively modulate the network dynamics, which reflects that the number of stimulated neurons plays a pronounced role similar to the amplitude of periodic electromagnetic stimulation or the standard deviation of stochastic electromagnetic stimulation. Therefore, when electromagnetic stimulation is used to regulate the electrophysiological activities of neuronal networks, on

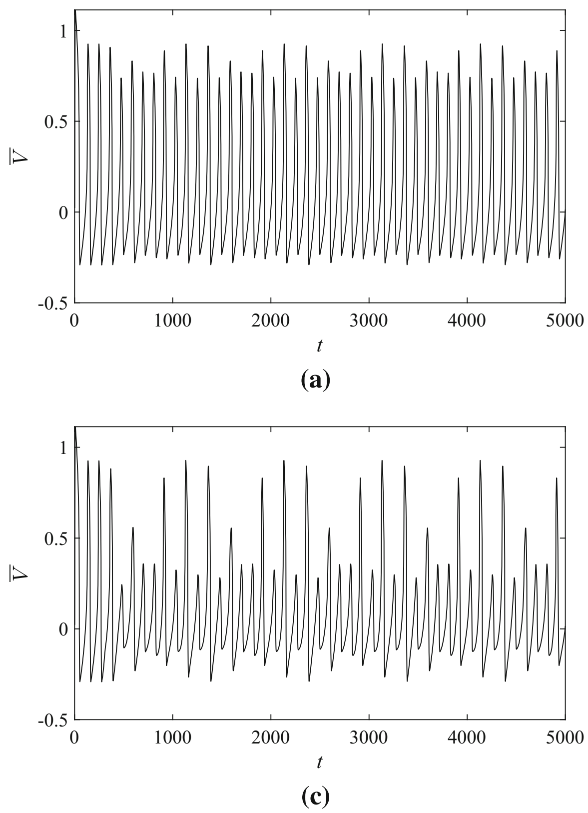


Fig. 8 Sampled time series of average membrane potentials corresponding to Fig. 7, where **a** $p_2 = 0.2$, **b** $p_2 = 0.4$, **c** $p_2 = 0.6$ and **d** $p_2 = 1.0$. With the increase in the stimulated probability, the discharge activity of the neuronal network is inhibited, and can change from high-frequency discharge state to low-frequency quasi-bursting state

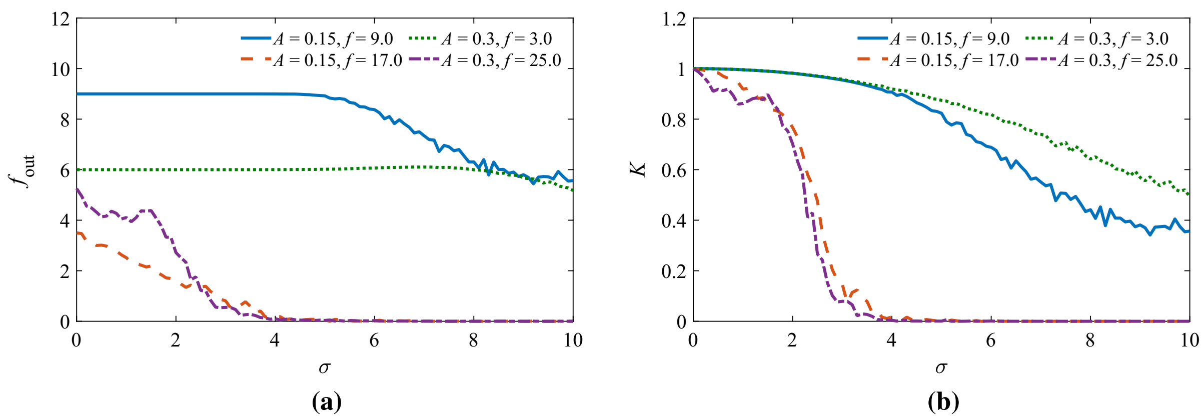


Fig. 9 Dependence of **a** average discharge frequency and **b** consistency coefficient of Newman–Watts small-world neuronal networks on standard deviation, where $p_1 = 0.08$, $g_0 = 0.02$ and $p_2 = 0.4$. As the standard deviation increases, the chaotic synchronous network is thoroughly suppressed, while the periodic synchronous network can only be regulated to a certain extent when the standard deviation exceeds a certain threshold. (Color figure online)

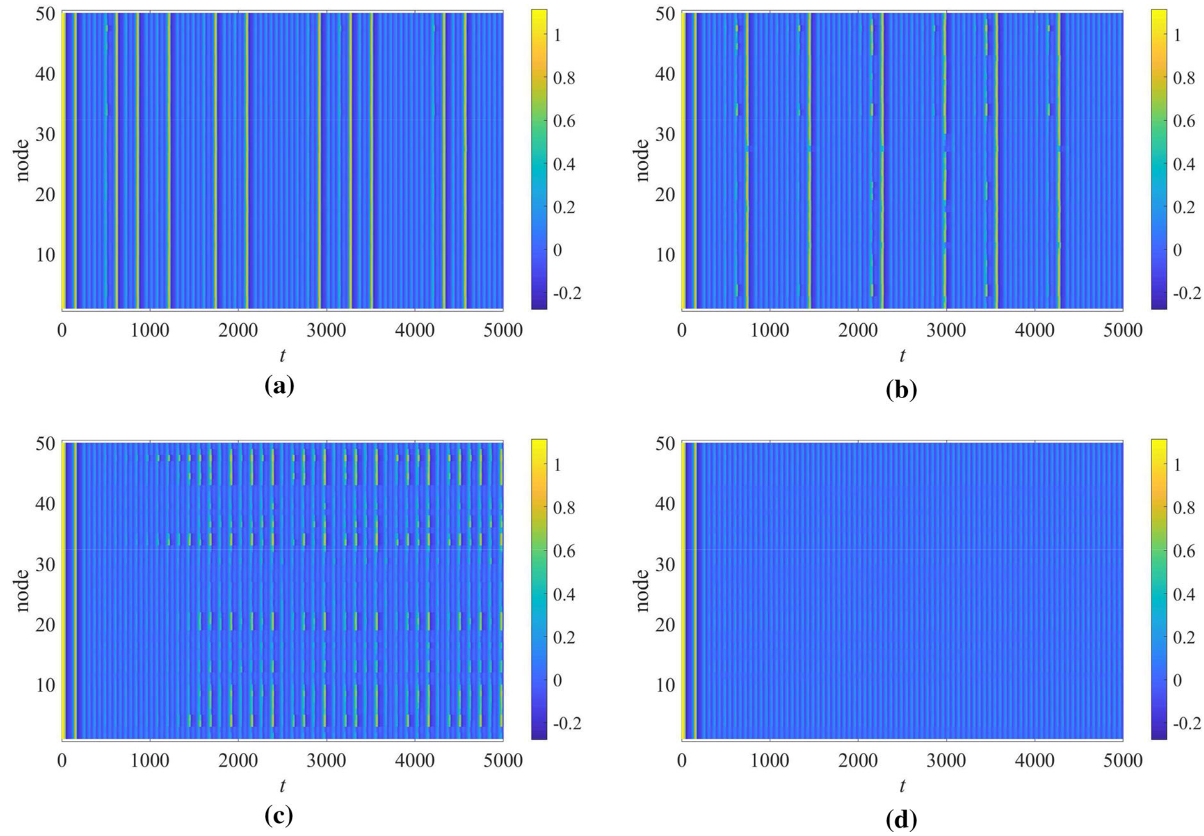


Fig. 10 Spatiotemporal pattern evolution of Newman–Watts small-world neuronal networks under periodic excitation with $A = 0.15$ and $f = 17.0$, where $p_1 = 0.08$, $g_0 = 0.02$ and $p_2 = 0.4$. The standard deviation of stochastic electromagnetic stimulation triggered at $t = 300$ is **a** $\sigma = 1.0$, **b** $\sigma = 2.0$, **c** $\sigma = 3.0$ and **d** $\sigma = 4.0$, respectively. With the increase in the standard deviation, the stochastic electromagnetic stimulation can completely suppress the discharge activity of the chaotic synchronous network, and realize the transition from chaotic synchronization to sub-threshold oscillation. (Color figure online)

the one hand, the evolution of network patterns can be controlled by adjusting the oscillation parameters of electromagnetic stimulation; on the other hand, the dynamic behavior of neuronal networks can be changed by adjusting the number of stimulated neurons. In particular, the periodic electromagnetic stimulation has the filtering function, which can induce the synchronization transition. These results can provide guidance for the physiological application of electromagnetic stimulation in the treatment of certain mental diseases.

5 Conclusion

Due to the influence of environmental, physiological and psychological factors, some brain regions are dysfunctional and have abnormal electrophysiological activities, such as synchronous discharge of all neurons, resulting in abnormal mental states. The two main ways of applying electromagnetic stimulation to the brain nervous system are transcranial electrical stimulation and transcranial magnetic stimulation. Although the technologies related to transcranial electromagnetic stimulation have been applied commercially, they are still immature. There are many problems in theory and practice that need to be solved. It is necessary to explore the physiological mechanism and improve the effect of electromagnetic stimulation. Based on the working principle of memristor and the introduction of magnetic flux variable, a mathematical model reflecting the effect of external electromagnetic stimulation on neurons and neuronal networks can be established,

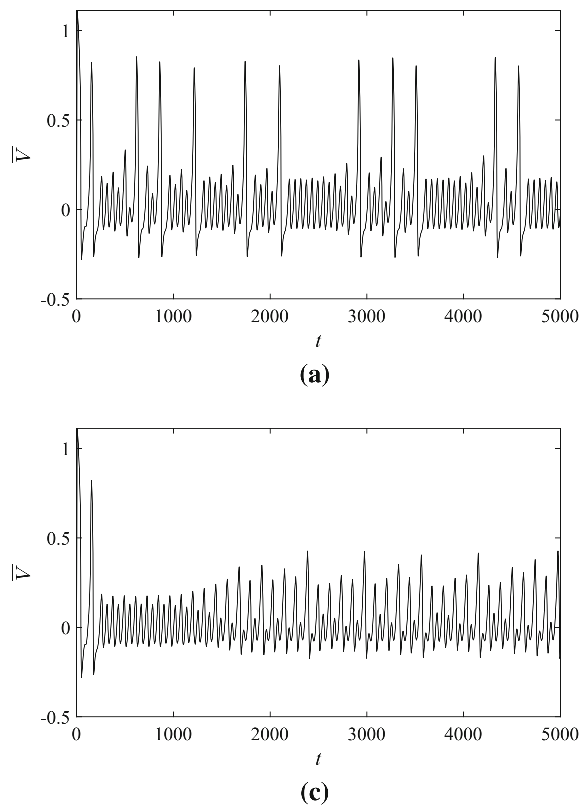


Fig. 11 Sampled time series of average membrane potentials corresponding to Fig. 10, where **a** $\sigma = 1.0$, **b** $\sigma = 2.0$, **c** $\sigma = 3.0$ and **d** $\sigma = 4.0$. As the standard deviation of stochastic electromagnetic stimulation increases, the discharge state of the neuronal network can transition from chaos to sub-threshold oscillation

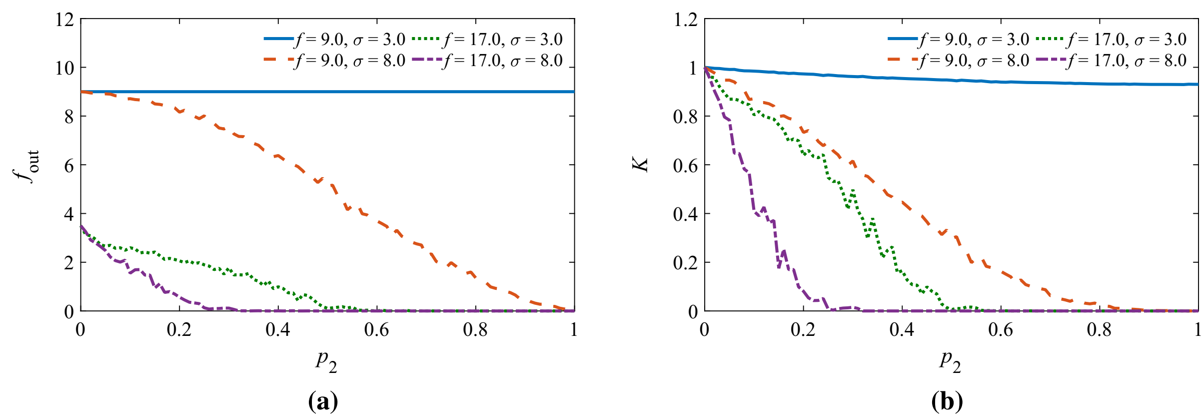


Fig. 12 Dependence of **a** average discharge frequency and **b** consistency coefficient of Newman–Watts small-world neuronal networks on stimulated probability, where $p_1 = 0.08$, $g_0 = 0.02$ and $A = 0.15$. As the stimulated probability increases, even the stochastic electromagnetic stimulation with small deviation can significantly modulate the chaotic synchronous network, which two metrics almost show a monotonic decreasing trend. (Color figure online)

which provides the possibility to study theoretically the regulation of electromagnetic stimulation on the electrophysiological activities of the brain nervous system.

In recent years, two aspects of research have been concerned, one is the regulation of electromagnetic induction current on neuronal discharge activity [31, 32,34–36,48,69,70], and the other is the effect of electromagnetic induction current on neuronal network discharge activity [21,37–39]. Currently, there are few research papers on the latter aspect, especially involving the neuronal network with complex coupling. To provide theoretical reference for the application of electromagnetic stimulation in the treatment of non-organic mental diseases, we choose the more physiologically significant small-world neuronal network as the research object, innovatively investigate the influence of two different types of external electromagnetic stimulation on the network dynamics, and try to quantify the relationship between the parameters of electromagnetic stimulation and the network dynamic changes. This study helps to address the problem of regulating the electrophysiological activities of specific brain regions through external electromagnetic stimulation. To this end, the pattern regulation of external electromagnetic stimulation on neuronal networks is systematically explored by using two network metrics.

First, under the effect of external electromagnetic stimulation, the Newman–Watts small-world network consisting of FHN neurons is established according to the working principle of memristor. Usually, the stability of network synchronization activity is mainly determined by the coupling matrix and the coupling strength when the dynamic equations of network nodes are selected. Using the analysis method related to the main stability function, the stability of network synchronization activity under four additional periodic excitations is analyzed, and the critical coupling strengths of synchronization determined by the linking probability are obtained. Furthermore, for the linking probability of $p_1 = 0.08$, four representative periodic or chaotic synchronous neuronal networks corresponding to the four different periodic excitations are constructed when the coupling strength is set to $g_0 = 0.02$.

Secondly, the average discharge frequency and the consistency coefficient are exploited to quantitatively investigate the ability of external electromagnetic stimulation to regulate the pattern evolution of Newman–Watts small-world neuronal networks. Numerical results show that electromagnetic stimulation can inhibit the electrophysiological activities of neuronal networks. The larger the amplitude of periodic electromagnetic stimulation or the larger the standard deviation of stochastic electromagnetic stimulation, the more significant the inhibitory effect, which can not only effectively desynchronize the network discharge activity, but can also control the evolution of spatiotemporal patterns. In general, electromagnetic stimulation is easier to modulate the spatiotemporal pattern evolution of chaotic synchronous neuronal networks, and even to induce the synchronization transition. Specifically, it can not only induce the transition from chaotic state through subthreshold oscillation to periodic discharge state, but also make the network discharge activity change from high-frequency spiking state to low-frequency quasi-bursting state, which enriches the nonlinear dynamics of neuronal networks. Moreover, changing the stimulated probability of neurons in the networks can also effectively regulate the network dynamics, reflecting the equivalent regulatory ability of the number of stimulated neurons and the oscillation parameters of electromagnetic stimulation in the process of controlling the pattern evolution. In order to more effectively regulate the evolution of network patterns or induce a specific synchronization mode, intermittent or discontinuous electromagnetic stimulation can be considered and needs to be studied in the future.

The above study fully demonstrates the feasibility of external electromagnetic stimulation to control the evolution of neuronal network patterns, which could contribute to applying electromagnetic stimulation to treat certain mental illnesses physiologically.

Acknowledgements This work was supported by the National Natural Science Foundation of China (Nos. 11672233 and 11972292), 111 Project (No. BP0719007), the foundation of National Key Laboratory of Science and Technology on Aerodynamic Design and Research (No. 614220119040101) and the Innovation Foundation for Doctor Dissertation of Northwestern Polytechnical University (No. CX201925).

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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