A Note on the Calculation of the Power Flow along a Straight Thin Wire Scatterer Horizontally Located above a Lossy Ground

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Abstract— **The paper deals with the calculation of active, reactive and apparent power along a horizontal straight thin wire scatterer of finite length. Once the current distribution and voltage distribution along the straight scatterer are determined by solving the Pocklington integro-differential equation and generalized telegrapher's equation respectively, the active power, reactive power and apparent power are calculated from the well-known formulas arising from the circuit theory. Some illustrative computational examples are presented in the paper.**

Keywords—Wire scatterer; current distribution; voltage distribution; active power, reactive power, apparent power

I. INTRODUCTION

Power transferred between antennas in the near field and widely used quantity power transfer efficiency (PTE- a ratio between received and transmitted power) in the analysis of wireless power system (WPT) can be studied in terms of circuit theory, by using transmission line (TL), or antenna theory (AT), respectively [1-3]. In, particular, electromagnetic field coupling to horizontal thin wire scatterers of finite length above a real ground can be analyzed by using the thin wire antenna or transmission line model [4]. The strengths and weaknesses of both approaches could be found in a number of books [4,5] and papers, e.g. [6-9]. Antenna theory (AT), is more rigorous, as transmission line (TL) approximation fails to account radiation effects at higher frequencies. On the other hand, TL approach whose main advantage is simplicity of formulation and computational efficiency is quite popular in many applications providing rapid estimation of the phenomena in an engineering sense [4]. However, as the TL model fails to predict resonances, shows difficulty when presence of a lossy ground is appreciably pronounced and when wires of arbitrary shape need to be studied [10] antenna theory approach has been shown to be more appropriate choice in modeling finite length wires. Note that a trade-off between the antenna theory and TL approach in both frequency and time domain has been discussed in series of papers [6-9]. Some limits on the use of TL approximation have been also examined in [6-9].

A generalized form of telegrapher's equations for the finite length scatterer above a lossy half-space has been derived in [6]. Such a form of equations, as a matter of fact, represents antenna integro-differential equation written in the form of telegraphers equations. Moreover, standard transmission line equations can be deduced as a special, simplified form, of generalized telegrapher's equations under certain set of approximations. Finally, using generalized telegrapher's equations provides one to calculate the scattered voltage along the scatterer, as derived in [6]. Consequently, one ends up with the knowledge of current and voltage distribution along the scatterer, respectively.

This paper extends the work undertaken in [6] aiming to take the advantage of availability of current and voltage distribution, respectively, and to calculate the active, reactive and apparent power, respectively, stemming for the standard circuit theory definitions. It is worth emphasizing that the knowledge of power flow along the scatterer could be useful in the analysis of power transfer efficiency (PTE) for the WPT applications. Future work is likely to deal with near field coupling of the dipole antennas above a real ground pertaining to more realistic scenarios for WPT applications. Some illustrative computational examples pertaining to active, reactive and apparent power, respectively, along the wire are presented in the paper.

II. THEORETICAL BACKGROUND

A straight thin wire scatterer of a finite length L and radius a located at height h above a lossy ground and illuminated by a normally incident electromagnetic field, Fig 1, is of interest.

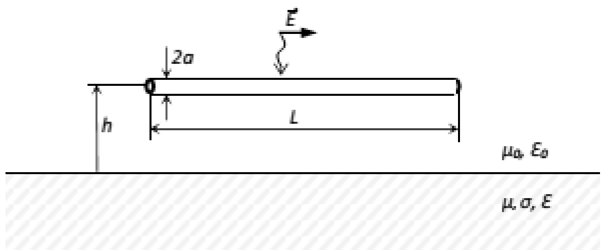

Fig 1. Finite length line above a lossy ground

The current induced along the wire is governed by the Pocklington integro-differential equation [6]

$$E_x^{exc} = j\omega\frac{\mu}{4\pi}\int_0^L I(x')g(x,x')\,dx' - \frac{1}{j4\pi\omega\varepsilon}\frac{\partial}{\partial x}\int_0^L \frac{\partial I(x')}{\partial x'}g(x,x')\,dx' \quad (1)$$

where E_x^{exc} is the excitation field, $I(x')$ is the unknown current distribution along the wire, and $g(x,x')$ is the total Green's function given by

$$g(x,x') = g_0(x,x') - R_{TM}\,g_i(x,x') \quad (2)$$

where $g_0(x, x')$ is the free space-Green function

$$g_o(x, x') = \frac{e^{-jk_o R_o}}{R_o} \tag{3}$$

while $g_i(x, x')$ arises from the image theory and is given by

$$g_i(x, x') = \frac{e^{-jk_o R_i}}{R_i} \tag{4}$$

and R_o and R_i is the corresponding distance from the source to the observation point, respectively.

Furthermore R_{TM} is the reflection coefficient for the transverse polarization by the presence of a lossy ground is taken into account

$$R_{TM} = \frac{n\cos\Theta - \sqrt{n - \sin^2\Theta}}{n\cos\Theta + \sqrt{n - \sin^2\Theta}} \tag{5}$$

where the refraction index n is

$$n = \varepsilon_r - j\frac{\sigma}{\omega\varepsilon_0} \tag{6}$$

and argument θ is defined as:

$$\Theta = \operatorname{arctg}\frac{|x - x'|}{2h} \tag{7}$$

The spatial distribution of the scattered voltage along the wire is governed by following telegrapher's equation [6]

$$V^{sct}(x) = -\frac{1}{j4\pi\omega\varepsilon}\int_0^L \frac{\partial I(x')}{\partial x'} g(x, x')dx' \tag{8}$$

Integro-differential equation (1) and telegrapher's equation (8) can be solved numerically using boundary element approach and mathematical details can be found elsewhere, e.g. in [5], or more recently in [10].

Provided the relations for current and voltage distribution along the straight wire are known the power generated within the system can be readily calculated using the simple expression arising from the circuit theory.

Thus, according to the circuit theory the active power Pa is given by:

$$P_a(x) = \frac{1}{2}\mathrm{Re}\left[V(x)I(x)^*\right] \tag{9}$$

while the reactive power P_r is

$$P_r(x) = \frac{1}{2}\mathrm{Im}\left[V(x)I(x)^*\right] \tag{10}$$

and the apparent power P_s is given by

$$P_s(x) = \frac{1}{2}\left[V(x)I(x)^*\right] \tag{11}$$

where (*) stands for a complex conjugate.

Of particular interest regarding PTE assessment for various WPT applications is the power flow along the wire defined with (9).

III. Numerical Results

Illustrative computational examples deal with 1m long full wave scatterer of radius a=2mm, operating at f=300MHz horizontally located above a lossy half-space (σ=10mS/m, ε_r=10) at height h=0.25m and excited with unitary plane wave. Real part, imaginary part and absolute value of current and voltage along the wire are shown in Fig 2 and 3, respectively.

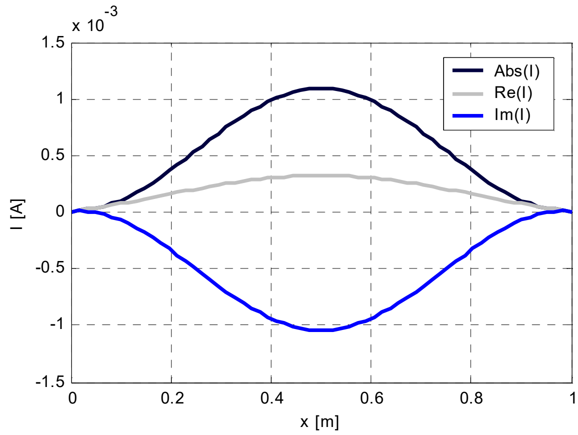

Fig 2 Current distribution along the wire (L=1m, a=2mm, σ=10mS/m, ε_r=10, h=0.25m)

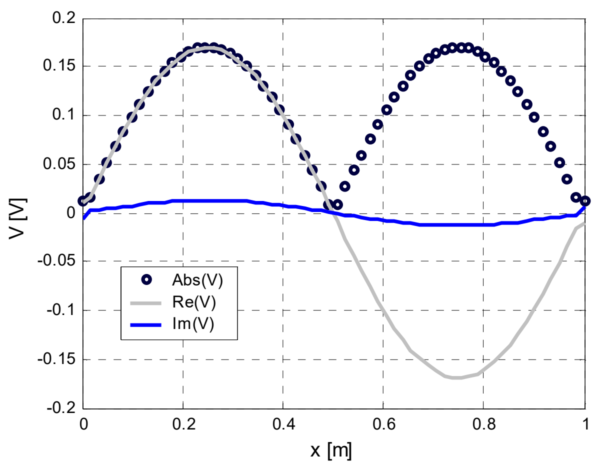

Fig 3 Voltage distribution along the wire (L=1m, a=2mm, σ=10mS/m, ε_r=10, h=0.25m)

These smooth waveforms presented in Figs 2 and 3, respectively are in accordance to the current and voltage waveforms that could be obtained from the TL, theory , as well [4]. Finally, Figs 4 to 6 depict active power flow, reactive power and apparent power, respectively.

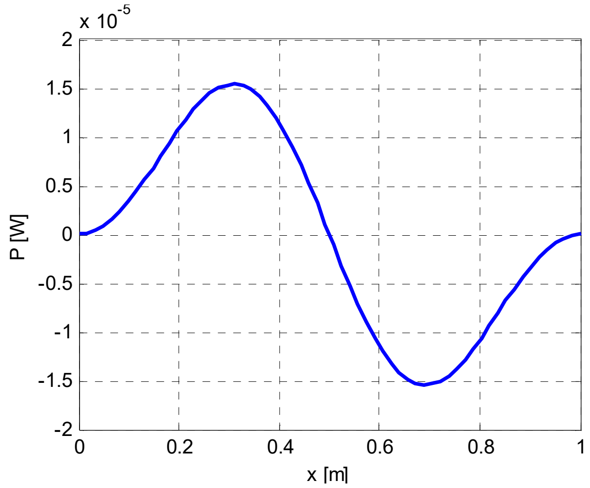

Fig 4 Active power flow along the wire (L=1m, a=2mm, σ=10mS/m, ε_r=10, h=0.25m)

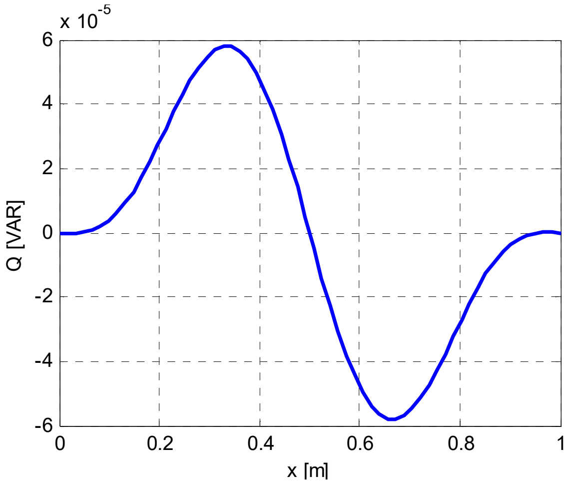

Fig 5 Reactive power (L=1m, a=2mm, σ=10mS/m, ε_r=10, h=0.25m)

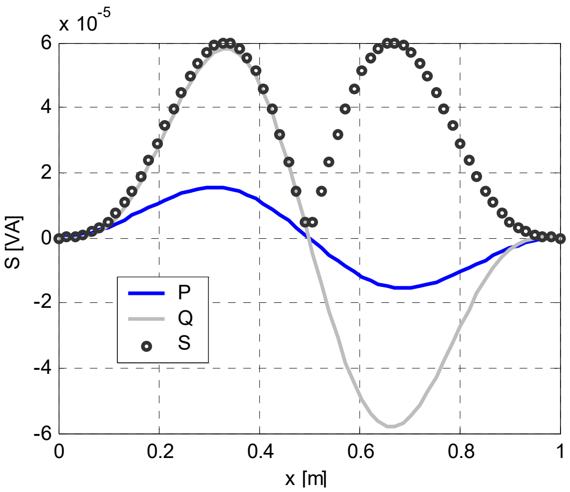

Fig 6 Apparent power (L=1m, a=2mm, σ=10mS/m, ε_r=10, h=0.25m)

The waveforms depicted in Figs 4 to 6 are also in accordance to the current and voltage waveforms that could be obtained from the TL theory [4].

IV. Closure

The active, reactive and apparent power along a horizontal straight thin wire scatterer of finite length have been determined in this work. Once the current distribution and voltage distribution along the straight thin wire scatterer are determined by solving the Pocklington integro-differential equation and generalized telegrapher's equation respectively, the active, reactive and apparent power, respectively have been calculated from the well-known formulas stemming from the circuit theory. Some illustrative computational examples are presented in the paper.

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