Aim: The aim is to develop a solution approach to the Vehicle Routing Problem with Time Window.

Model: Before modeling this problem, all the decision variables, parameters and sets are defined, then the model is developed.

<u>Decision variables:</u>

$$X_{ijk} \begin{cases} 1 & \text{if vehicle } k \text{ goes from vertex } i \text{ to vertex } j \\ 0 & \text{otherwise} \end{cases}$$

W_{ik}: Arriving time of vehicle k to vertex i

<u>Parameters:</u>

C_{ij}: Cost / distance / time of going from vertex i to j

f_k: Fixed cost of vehicle $k = 2 \times n \times C_{max}$

C_{max} → max distance connecting any pair of locations

n → number of customers

Q: Capacity of vehicles

d_i: Demand of customer i

t_{ik}: Service time of customer i

m_{ij}: $\max\{b_i + C_{ij} - a_j\}$, $(i,j) \in E$

a_i: Ready time of customer i

b_i: Due date of customer i

<u>Sets</u>

$G = (N, E)$: Graph with vertices N and edges E

C = Set of customers: $\{1, \dots, n\}$

N = Source + set of customers + sink: $\{0, 1, \dots, n, n+1\}$

K = Set of vehicles: $\{1, \dots, k_{max}\}$

E = Set of edges

<u>Model</u>

$$\min \sum_{k \in K} \sum_{i \in N} \sum_{j \in N} c_{ij} X_{ijk} + \sum_{k \in K} f_k$$

s.t.

$$\sum_{k \in K} \sum_{j \in N} X_{ijk} = 1 \quad \forall i \in C \quad (1)$$

$$\sum_{i \in C} d_i \sum_{j \in N} X_{ijk} \leq Q \quad \forall k \in K \quad (2)$$

$$\sum_{j \in N} X_{0jk} = 1 \quad \forall k \in K \quad (3)$$

$$\sum_{i \in N} X_{ipk} - \sum_{j \in N} X_{pjk} = 0 \quad \forall p \in C, \forall k \in K \quad (4)$$

$$\sum_{i \in N} X_{i,n+1,k} = 1 \quad \forall k \in K \quad (5)$$

$$W_{ik} + t_i + C_{ij} - W_{jk} \leq m_{ij}(1 - X_{ijk}) \quad \forall i,j \in N \quad \forall k \in V \quad (6)$$

$$a_i \leq W_{ik} \leq b_i \quad \forall i \in N, \forall k \in K \quad (7)$$

$$X_{ijk} \in \{0,1\} \quad \forall i,j \in N \quad \forall k \in K \quad (8)$$

$$W_{ik} \geq 0 \quad \forall i \in N \quad \forall k \in K \quad (9)$$

Objective function: Objective is to minimize the fixed cost of a vehicle and the variable cost of each vehicle's route.

Constraints:
1) Ensure that each customer is visited exactly once
2) A vehicle can only be loaded up to its capacity
3) Each vehicle must leave the depot 0
4) After a vehicle arrives at a customer it has to leave for another destination
5) All vehicles must arrive at the depot n + 1
6) Establish the relationship between the vehicle departure time from a customer and its immediate successor
7) Affirm that the time windows are observed
8) Binary constraints
9) Decision variable continuous constraint

Improvements: In order to develop a logical approach, it's decided to limit the number of edges in the input graph G. This is achieved by creating sets by following this procedure:

$$a_i + t_{ik} + c_{ij} \leq b_j \quad \forall i,j \in C \quad \forall k \in K$$

This means that the due date of customer j must be greater than the sum of the ready time of customer i, service time of customer i and the travel time from i to j if there is an edge between vertices i and j. If this is not satisfied between i and j, it is impossible to serve i and j with the same vehicle, therefore there is no edge between these two vertices. This graph modification decreases the number of Xijk decision variables tremendously and makes the solution computationally much easier.

Another improvement that we carried was the following: The number of vehicles were not given and the minimum number of vehicles required was a decision to be made. As the fixed cost of a new vehicle is really big, the solution with the minimum number of vehicles would be definitely the best solution. In order to find the best K value we iterated through Kmin and Kmax which are respectively as shown which means number of vehicles cannot be lower than the sum of demands divided by the capacity and it cannot be higher than the number of customers:

$$K_{min} = \frac{\sum_{i \in C} d_i}{Q} \qquad K_{max} = \#\text{ of customers}$$

After these lower and upper bounds, the problem is solved starting from Kmin. If the solution is infeasible k is increased by 1 and the problem is solved again. This process is repeated until the first feasible solution is found. We know that this is the minimum number of vehicles since the previous solution was infeasible. Since the fixed cost of a vehicle is two times the number of customers times the maximum euclidean distance between any two customer points, it can be interpreted that the smaller amount of vehicle quantity will be preferred to any possible smaller distanced routing. So the optimal solution lies within this number of vehicles. This is also a huge improvement as there is no need to introduce another decision variable which would be:

$$K_k = \begin{cases} 1 & \text{if vehicle } k \text{ is used} \\ 0 & \text{otherwise} \end{cases}$$

Besides not introducing the provided binary variable, binary vehicle assignment constraints are not needed anymore. And these decision variables are also removed from balance constraints, which decreases the hardness of our problem.

By using the pre-processing techniques and solution methodologies provided above, the optimal values have been obtained for the Vehicle Routing Problem with Time Window. It can be observed that our improved model performs in strictly less computation time compared to the general Vehicle Routing Problem with Time Window.

Used software and hardware:

Gurobi Optimizer version 11.0.0 build v11.0.0rc2 (mac64[x86] - Darwin 21.6.0 21G651)

CPU model: Intel(R) Core(TM) i5-5250U CPU @ 1.60GHz
Thread count: 2 physical cores, 4 logical processors, using up to 4 threads