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COMP 303 – Analysis of Algorithms

Project : Maximum Subarray Sum

1. Pseudocode of the brute-force algorithm to find the subarray with a running time complexity of $O(n^2)$:

| | cost | time |
|-------------------------------------|------|-----------|
| declare array | c0 | 1 |
| size(array) <- n | c1 | 1 |
| max <- a[0] | c2 | 1 |
| for i <- 0 to n | c3 | n + 1 |
| currentSum <- 0 | c4 | n |
| for j <- i to n | c5 | $n*(n+1)$ |
| currentSum <- currentSum + array[j] | c6 | $n*n$ |
| do if currentSum > max | c7 | $n*n$ |
| then max <- currentSum | c8 | $n*n$ |
| left <- i | c9 | $n*n$ |
| right <- j | c10 | $n*n$ |
| maxsubarray <- array[left to right] | c11 | $n*n$ |
| print max value | c12 | 1 |

Total Cost :

$(c0*1)+(c1*1)+(c2*1)+(c3*(n+1))+(c4*n)+(c5*(n(n+1)))+(c6*n*n)+(c7*n*n)+(c8*n*n)+(c9*n*n)+(c10*n*n)+(c11*n*n)(c12*1)$

$= O(n^2)$

Graph of the code's execution time :

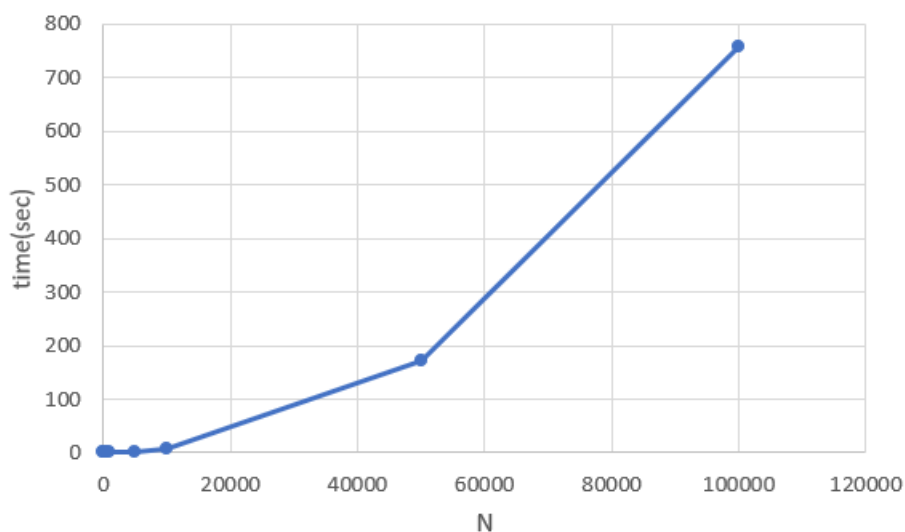


figure 1. $O(n^2)$ execution time graph

$O(N^2)$'s runtime grows faster than our input size. As shown in the figure2, execution time graph is similar to $O(n^2)$. When finding the execution time of the algorithm we've used the `time()` function and calculate the difference of starting and ending time.

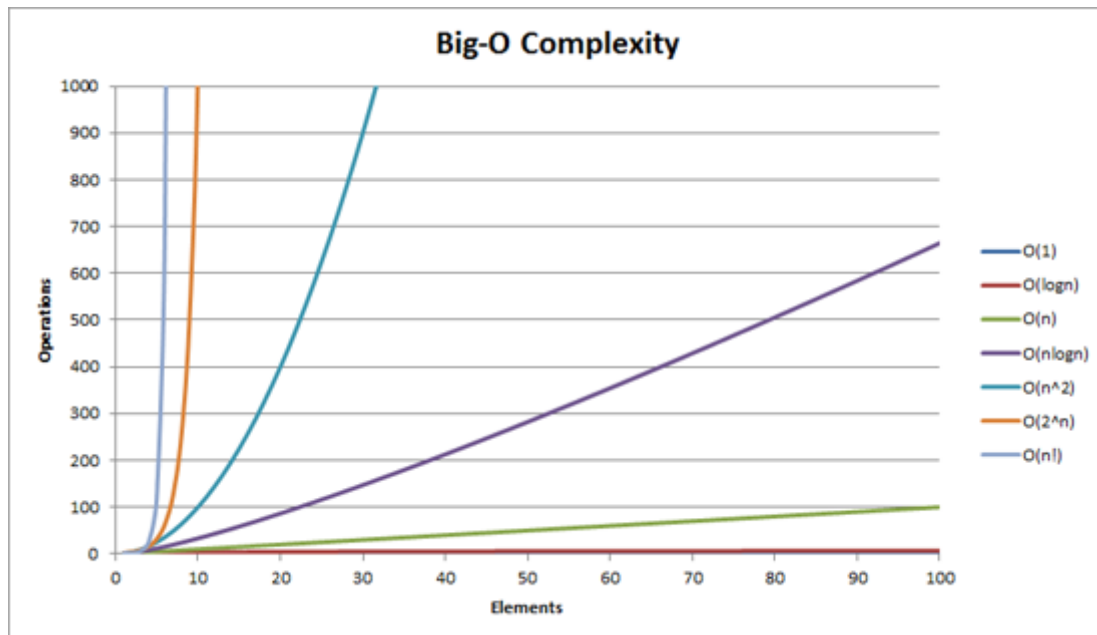


figure 2. big o notation

2. Pseudocode of the brute-force algorithm to find the subarray with a running time complexity of $O(n \lg n)$:

declare array

```
function maxCrossSum(array, l, m, h)
```

```
for i<-m to l-1
```

size : $n/2$

```
i <- i - 1
```

```
sum <- sum + array[i]
```

```
do if sum > left sum
```

```
then left_sum <- sum
```

```
sum <- 0
```

```
for i<-m+1 to h+1
```

size : $n/2$

```
sum <- sum + array[i]
```

```
do if sum > right sum
```

```
then right_sum <- sum
```

```
return left_sum + right_sum
```

```
function maxsubarray(array, l, h)
```

size : n

do if $l = h$

```

then return array[l]

```

```
m <- (1+h)/2
```

```
return max(maxsubarray(array, l, m),
```

```
maxsubarray(array,m+1,h),maxCrossSum(Array, l, m, h))
```

Total Cost : $T(n) = 2T(n/2) + O(n) = O(n \log n)$

Finding maxsubarray's size is n and that means its run time is $O(n)$ in worst case. We are making two recursive calls with using input $n/2$. With using the iteration method we reached the $O(n \log n)$.

Graph of the code's execution time :

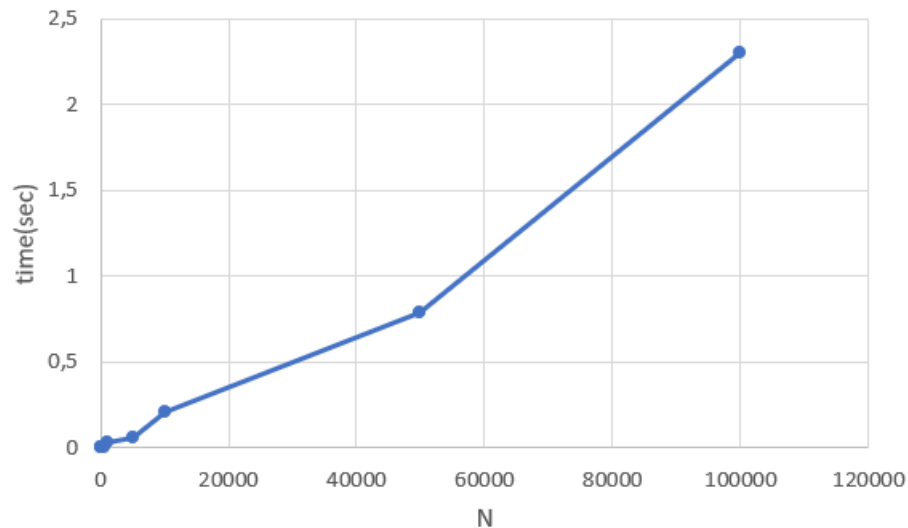


figure 3. $O(n \log n)$ execution time graph

3. Pseudocode of the brute-force algorithm to find the subarray with a running time complexity of $O(n)$:

| | cost | time |
|---|------|-------|
| declare array | c1 | 1 |
| define size as N | c2 | 1 |
| max_value = a[0] | c3 | 1 |
| temp_max = a[0] | c4 | 1 |
| for i<-1 to N | c5 | N |
| temp_max <- MAX(a[i] , temp_max + a[i]) | c6 | N - 1 |
| do if temp_max > max_value | c7 | N - 1 |
| then max_value <- temp_value | c8 | N - 1 |

Total Cost : $(c1*1)+(c2*1)+(c3*1)+(c4*1)+(c5*N)+(c6*(N-1))+(c7*(N-1))+(c8*(N-1))$
 $=O(n)$

Graph of the code's execution time :

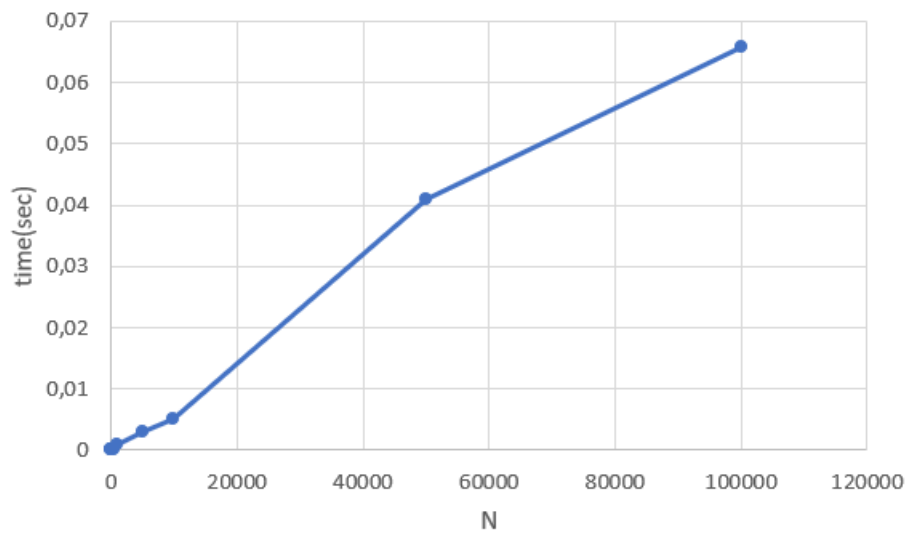


figure4. O(n) execution time graph

4. According to the values we've obtained, it moves from fast to slow in the order of $O(n)$, $O(n \log n)$, $O(n^2)$. When we compare the results with the graphs in figure 2, we saw that we have reached the right results. The problem size n_0 which gives the crossover point at which algorithm with the running time $O(n \log n)$ beats the algorithm with the running time $O(n^2)$ is 50000. We have learned how to analyze the algorithms we use with this project. When choosing the algorithm, we have to make the choice according to the available memory and time constraints for finding the efficient one. We've used the number of data in execution time and loops to analyze algorithms. So we've found the most efficient algorithm.