

$$(1) x^2 u'' + x u' + u = x \quad u'(1) = 2 \quad u'(2) = 0$$

$$x_0 = 1$$

$$u'(x_i) = \frac{u(x_{i+1}) - u(x_{i-1}))}{2h}$$

$$u''(x_i) = \frac{u(x_{i+1}) - 2u(x_i) + u(x_{i-1}))}{h^2}$$

$$z(1): x_i^2 \cdot \frac{u(x_{i+1}) - 2u(x_i) + u(x_{i-1}))}{h^2} + x_i \cdot \frac{u(x_{i+1}) - u(x_{i-1}))}{2h} + u(x_i) = x_i$$

$$u(x_{i+1}) \cdot \left[\frac{x_i^2}{h^2} + \frac{x_i}{2h} \right] + u(x_i) \cdot \left[\frac{-2x_i^2}{h^2} + 1 \right] + u(x_{i-1}) \cdot \left[\frac{x_i^2}{h^2} - \frac{x_i}{2h} \right] = x_i$$

$$\text{dla } x_0 = 1 \quad u'(1) = 2$$

$$2 = \frac{u(x_{i+1}) - u(x_{i-1}))}{2h} \quad \leftarrow \text{wychodzi za przedział}$$

$$u(x_{i-1}) = u(x_{i+1}) - 4h$$

$$z(1): x_i^2 \cdot \frac{2u(x_{i+1}) - 2u(x_i) - 4h}{h^2} + x_i \cdot \frac{4h}{2h} + u(x_i) = x_i$$

$$u(x_{i+1}) \cdot \left[\frac{2x_i^2}{h^2} \right] + u(x_i) \cdot \left[\frac{-2x_i^2}{h^2} + 1 \right] = -x_i + \frac{4x_i^2}{h}$$

$$\text{dla } x_n = 2 \quad u'(2) = 0$$

$$0 = \frac{u(x_{i+1}) - u(x_{i-1}))}{2h} \quad \leftarrow \text{wychodzi za przedział}$$

$$u(x_{i+1}) = u(x_{i-1})$$

$$A \cdot B = C$$

$$B = A^{-1} \cdot C$$

$$z(1): u(x_{i+1}) \cdot \left[\frac{-2x_i^2}{h^2} + 1 \right] + u(x_{i-1}) \cdot \left[\frac{2x_i^2}{h^2} \right] = x_i$$

$$\begin{bmatrix} \frac{-2x_1^2}{h^2} + 1 & \frac{2x_1^2}{h^2} & 0 & 0 & \vdots & 0 & 0 \\ \frac{x_1^2}{h^2} - \frac{x_1}{2h} & \frac{-2x_1^2}{h^2} + 1 & \frac{x_1^2}{h^2} + \frac{x_1}{2h} & 0 & \dots & 0 & 0 \\ 0 & \frac{x_1^2}{h^2} - \frac{x_1}{2h} & \frac{-2x_1^2}{h^2} + 1 & \frac{x_1^2}{h^2} + \frac{x_1}{2h} & \ddots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \frac{2x_n^2}{h^2} & \frac{-2x_n^2}{h^2} + 1 \end{bmatrix} \cdot B = \begin{bmatrix} -x_0 + \frac{4x_0^2}{h} \\ f(x_1) \\ f(x_2) \\ \vdots \\ \end{bmatrix}$$