Introduction to Quantitative Finance

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Agenda

1. Basic Information
2. Course Structure
3. Possible Quants Career Paths
4. Basic Information About Financial Markets
5. Basic Financial Instruments
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    - Types
    - Usage
7. Options
    - Definition
    - Greeks
    - Replication & Stochastic Calculus
    - Options Usage
    - Strategies
    - Exotics
    - Structured Products

Basic Information

The goal of this course is to familiarize you with the basic concepts and theories of modern finance and derivatives.

What to expect?

- The course begins easy but gets tougher as we go along.
- Not very heavy on math, serious theoretical concepts will be explained in an accessible and (hopefully) intuitive way, but **requires regular, individual work.**
- In order to minimize the chance of being lost/disengaged, always try to **ask as soon as you realize you're not following.**
- Come prepared and regularly review your notes from the past weeks.
- There's no such thing as a stupid question!

Contact to coordinator:

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Basic Information

Grading criteria:

- Final exam - 60%
- Quiz – 30%
- Class activities – 10%

Grades scale:

[0%, 50%) = 2	[80%, 90%) = 4.5
[50%, 60%) = 3	[90%, 95%) = 5
[60%, 70%) = 3.5	[95%, 100%) = 5!
[70%, 80%) = 4	

To Pass the course you need to get:

- at least 50% of final exam;
- pass at least 5 out of 10 quizzes.

Class attendance is **not mandatory**: however there are going to be 10 quizes, absences on 6 result in FAILing the course.

All materials can be found on Moodle course webpage - https://elearning.wne.uw.edu.pl/course/view.php?id=3994

Course Plan

1. Introduction to Options and Other Financial Instruments
2. Future – Present Value Lemma Pricing of Basic Financial Derivatives
3. Martingale Theory & Risk Neutral Valuation
4. Binomial Option Pricing Model
5. Brownian Motion
6. Stochastic Calculus & Ito’s Lemma
7. Change of Numeraire & Girsanov’s Theorem
8. Derivations of Black-Scholes Formula
9. Black-Scholes Formula - Dive In
10. Other Stochastic Differential Equations
11. Exam

Possible Quants Career Paths

Quant types:

- **Quant Researchers**: focus on analysing data, recognize patterns and building new models for the business. Researches must be experienced in coding and understand the business in which they operate.

- **Quant Developers**: responsible for building production models, applications and managing the data used by the models. Developers must be experts in programming, but also have a solid knowledge of the underlying financial theory.

- **Quant Validators**: assess and challenge models independently from developers, making sure that the developed models perform as expected and within the firm risk appetite. Validators must have good coding skills and solid understanding of financial as well as math theory.

- **Quant Trader/Portfolio Managers**: a quant asset manager will have a large emphasis placed on portfolio construction and portfolio optimization techniques. Quant traders focus on improving automated market making and execution algorithms. These roles require a strong background in finance in order to understand the dynamics of the market.

Typical Institutions for Quants:

- **Investment Banks/Insurance Companies**: mainly sell-side firms which offer products and financial services to retail/institutional clients.

- **Investment Funds**: mainly buy-side firms which purchase investment products and profit from investing activities.

Time Value of Money

Money has time value because of the opportunity to invest it at some interest rate.

Time Value of Money investigates the following questions:

1) What is the future value, at time n, of any sum of money invested today at an interest rate r?
2) What is the present value, at time t_0, of any sum of money received in the future at time n?

Future Value

The future value of a sum of money P_0 invested today at rate r for n period is: $P_n = P_0 * (1 + r)^n$

Where:

n = number of periods,

P_n = future value of money

P_0 = principal or notional at time zero

r = interest rate

$(1 + r)^n$ = compounding factor

Present Value

Today's value of a sum of money P_n received in the future is: $P_0 = \frac{P_n}{(1+r)^n}$

Note that $1/(1 + r)^n$ is called discounting factor

Time Value of Money

Interest rate compounding (few remarks):

- Each interest rate has a frequency associated, i.e. annual/semi-annual/monthly/etc.
- The future/present value depends on how often the interest earned is added to the principal (**compounding** effect).
- If interest is compounded:

- Annually: $P_n = P_0 * (1+r)^n$
- Semi-annually: $P_n = P_0 * \left(1+\frac{r_{freq}}{2}\right)^{2*n}$
- m times: $P_n = P_0 * \left(1+\frac{r_{freq}}{m}\right)^{m*n}$
- Continuously: $P_n = P_0 \lim_{m\to\infty} \left(1+\frac{r_{freq}}{m}\right)^{m*n} = P_0 e^{r*n}$

- Interest rate can be translated from one frequency to another by using the formula: $(1+r) = \left(1+\frac{r_{freq}}{m}\right)^{m*n}$

1. Assume an annually compounded rate $r = 10\%$. Compute the semi-annual equivalent interest rate:

$(1+0.1) = \left(1+\frac{r}{2}\right)^{2*1}$ -> $r_{semi-annual} = 2 * \left(\sqrt[2]{(1+0.1)} - 1\right) = 9.762\%$ - **Sanity check**: $\$100 * (1.1) = \$110 = \$100 * \left(1+\frac{9.762\%}{2}\right)^2$

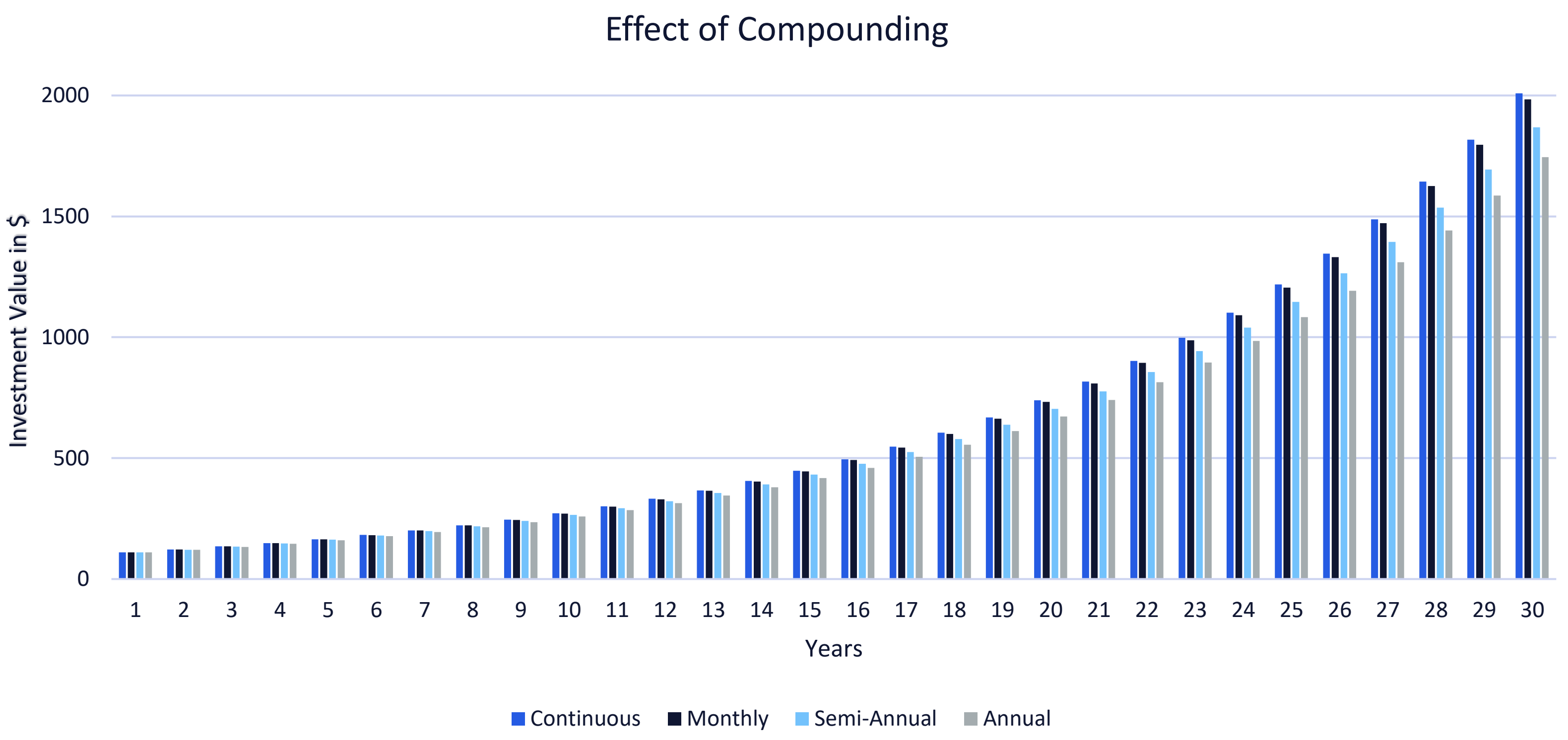
2. Assume a monthly compounded rate $r_{monthly} = 2\%$. Compute the annual equivalent interest rate:

$(1+r) = \left(1+\frac{2\%}{12}\right)^{12*1}$ **->** $r = \left(1+\frac{2\%}{12}\right)^{12*1} - 1 = 2.018\%$ - **Sanity check:** $\$100 * (1.02018) = \$102.018 = \$100 * \left(1+\frac{2\%}{12}\right)^{12}$

Time Value of Money

- The higher the compounding frequency the higher the future value.
- Example: impact of different compounding frequencies on $100 investment. Interest rate = 10%

	Compounding Frequency			
Time	Continuous	Monthly	Semi-Annual	Annual
1	110.52	110.47	110.25	110.00
2	122.14	122.04	121.55	121.00
3	134.99	134.82	134.01	133.10
4	149.18	148.94	147.75	146.41
5	164.87	164.53	162.89	161.05
6	182.21	181.76	179.59	177.16
7	201.38	200.79	197.99	194.87
8	222.55	221.82	218.29	214.36
9	245.96	245.04	240.66	235.79
10	271.83	270.70	265.33	259.37
11	300.42	299.05	292.53	285.31
12	332.01	330.36	322.51	313.84
13	366.93	364.96	355.57	345.23
14	405.52	403.17	392.01	379.75
15	448.17	445.39	432.19	417.72
16	495.30	492.03	476.49	459.50
17	547.39	543.55	525.33	505.45
18	604.96	600.47	579.18	555.99
19	668.59	663.35	638.55	611.59
20	738.91	732.81	704.00	672.75
21	816.62	809.54	776.16	740.02
22	902.50	894.31	855.72	814.03
23	997.42	987.96	943.43	895.43
24	1102.32	1091.41	1040.13	984.97
25	1218.25	1205.69	1146.74	1083.47
26	1346.37	1331.95	1264.28	1191.82
27	1487.97	1471.42	1393.87	1311.00
28	1644.46	1625.50	1536.74	1442.10
29	1817.41	1795.71	1694.26	1586.31
30	2008.55	1983.74	1867.92	1744.94



Time Value of Money

Futures Value Examples

1. Assume you have $10,000 to invest in *Investment Gurus of Giggles,* your pension fund. Assuming that *Investment Gurus of Giggles* achieves a stunning annual return of 10% (they are Gurus by the way...). What is the future value of the invested sum in 20 years?

$$P_0 = ?$$

2. Using data from the first exercise, assume that now the 10% interest rate is a) compounded semi-annually, b) continuously compounded. What is the future value of the invest sum?

   a. $P_0 = ?$

   b. $P_0 = ?$

Present Value Examples

1. *Please-Bail-Out Bank* will receive $1,000 in 1 year from now and $1,200 in 2 years from now. Assuming an annual interest rate r = 5 %, what is the present value of your future cashflow?

$$P_0 = ?$$

2. Assume now interest rates of 3% and 5% annually compounded for the first and second year respectively. What is the present value of future cashflow?

$$P_0 = ?$$

Time Value of Money

Futures Value Examples

1. Assume you have $10,000 to invest in *Investment Gurus of Giggles,* your pension fund. Assuming that *Investment Gurus of Giggles* achieves a stunning annual return of 10% (they are Gurus by the way...). What is the future value of the invested sum in 20 years?

$$\$10000 * (1.1)^{20} \cong \$67275$$

2. Using data from the first exercise, assume that now the 10% interest rate is a) compounded semi-annually, b) continuously compounded. What is the future value of the invest sum?

   a. $\$10000 * \left(1 + \frac{10\%}{2}\right)^{40} \cong \$70400$

   b. $\$10000 * e^{0.1*20} \cong \$73890$

Present Value Examples

1. *Please-Bail-Out Bank* will receive $1,000 in 1 year from now and $1,200 in 2 years from now. Assuming an annual interest rate r = 5 %, what is the present value of your future cashflow?

$$PV = \frac{\$1000}{(1.05)^1} + \frac{\$1200}{(1.05)^2} = \$952.381 + \$1088.435 \cong \$2040.816$$

2. Assume now interest rates of 3% and 5% annually compounded for the first and second year respectively. What is the present value of future cashflow?

$$PV = \frac{\$1000}{(1.03)^1} + \frac{\$1200}{(1.05)^2} = \$970.874 + \$1088.435 \cong \$2059.309$$

Time Value of Money – Alternative Way

Another way of deriving the continuously compounded result is via a differential equation. Suppose I have an amount M(t) in the bank at time t, how much does this increase in value from one day to the next? If I look at my bank account at time t and then again a short while later, time t + dt, the amount will have increased by:

$$M(t + dt) - M(t) \approx \frac{dM}{dt} dt + \cdots$$

where the right-hand side comes from a Taylor series expansion of *M(t + dt).* But I also know that the interest I receive must be proportional to the amount I have, *M*, the interest rate, *r*, and the time step, *dt*. Thus,

$$\frac{dM}{dt} dt = rM(t)\, dt$$

Dividing by *dt* gives the ordinary differential equation

$$\frac{dM}{dt} = rM(t)$$

the solution of which is

$$M(t) = M(0)e^{rt}$$

Time Value of Money – Alternative Way

This equation relates the value of the money I have now to the value in the future. Conversely, if I know I will get one dollar at time *T* in the future, its value at an earlier time *t* is simply

$$e^{-r(T-t)}$$

I can relate cashflows in the future to their **present value** by multiplying by this factor. As an example, suppose that *r* is 5% i.e. r = 0.05, then the present value of $1,000,000 to be received in two years *(T=2)* is

$$M(T) ** e^{-r*(T-t)} = \$1{,}000{,}000 * e^{-0.05*(2-0)} = \$904{,}837$$

The present value is clearly less than the future value. Interest rates are a very important factor determining the present value of future cashflows.

For the moment we assume constant interest rate, later we will generalize.

Financial Markets

A marketplace is intended to bring buyers and sellers together, facilitating them to trade. Supply and demand are not always balanced, markets have to rely on intermediaries to facilitate trading (brokers, dealers, liquidity providers).

Financial Markets Categories:

- **Primary market** - dedicated market for the issuance of new securities.
- **Secondary market** - already issued securities can be traded.

Secondary Markets:

Market Exchange:

- Centralized platform where buyers and sellers can trade securities according to pre-established rules and regulations.
- When two parties reach make a trade, the price at which the transaction is executed is communicated throughout the market.
- Market makers are obliged to provide liquidity.

Over-the-Counter (OTC):

- Decentralized market where securities are traded directly between two parties. Less regulated, transparent and liquid markets.
- The price at which securities a trade occurred may not be posted to the public.
- Market makers (dealers) are not obliged to provide liquidity.

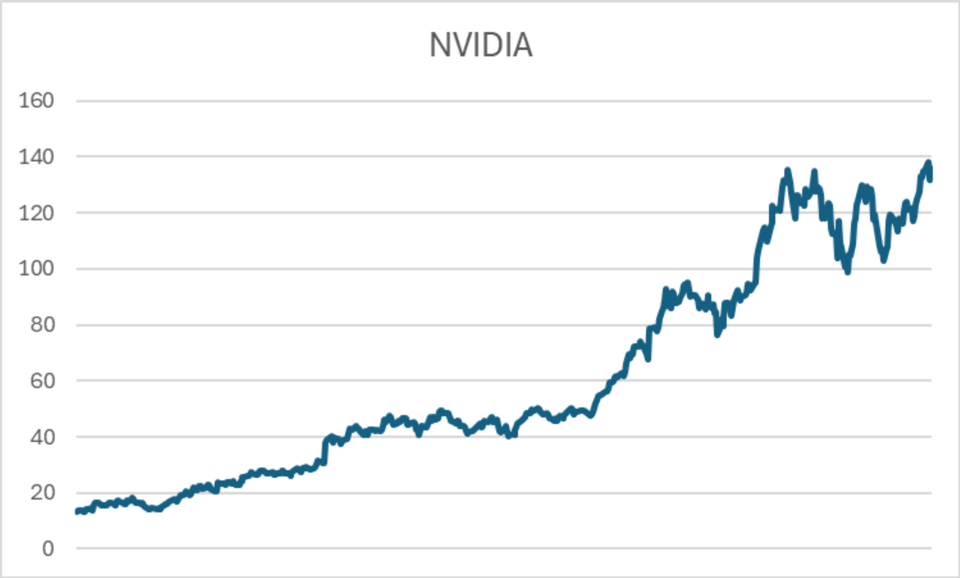
Financial Instruments - Equity

- **The equity** of a corporation represents the value of the remaining assets once all liabilities have been deducted. A share in a company (also known as **stock** or **share**) represents a portion of the total equity.
- Stocks allow companies to **finance themselves** by making their ownership public.
- A **finite number of shares** are issued by public corporations and traded in the markets.
- The owner of a stock is entitled of **voting** and **property rights**.
- Investors can profit from investing in a stock via **capital gain** and periodic payments known as **dividends**.
- The total value of outstanding shares is referred as the corporation's **market capitalization** (computed as price times total shares number).
- In U.S. stocks are grouped based on the following market cap:

  1) Large-cap if market cap. > $10 billion
  2) Mid-cap if $1 billion < market cap. < $10 billion
  3) Small-cap if market cap.< $1 billion
  4) Micro-caps if market cap. < $100 million

Financial Instruments - Equity

The behaviour of the quoted prices of stocks is far from being predictable.

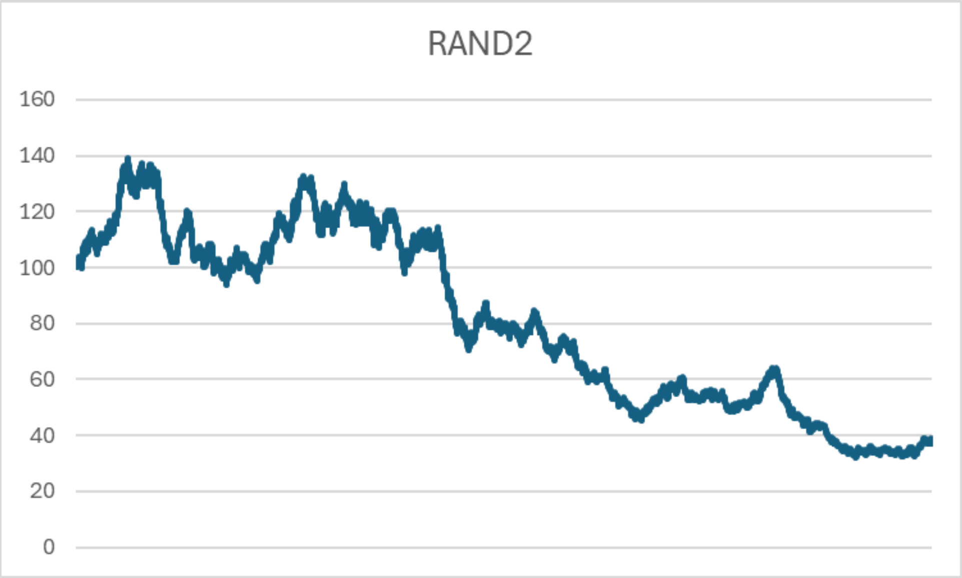


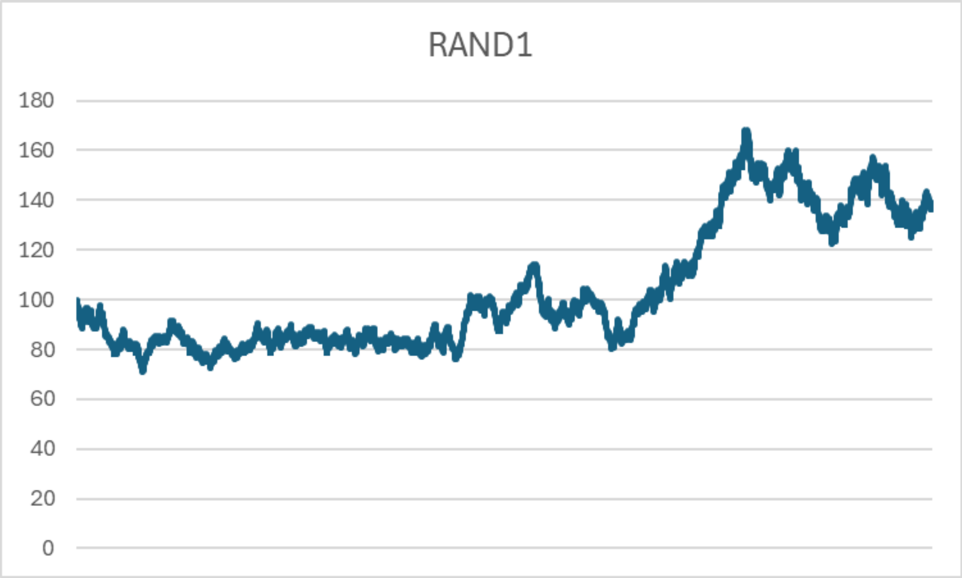
It does not mean we can not model the stock prices. It does mean we need to model equities` prices in **probabilistic sense**. No doubt the reality of the situation lies somewhere between complete predictability and perfect randomness, not least because there have been many cases of market manipulation where large trades have moved stock prices in a direction that was favorable to the person doing the moving.

Financial Instruments - Equity

We will go to proper mathematical modeling later. Nevertheless, the simplest way to simulate a random walk that looks something like a stock price is one of the simplest random processes **- tossing of a coin**.

Let`s assume initial price of stock (S_0) equal to \$100. If you throw a head multiply the number by 1.01, if you throw a tail multiply by 0.99. Continue this process and plot your value on a graph each time you throw the coin. Results of two particular experiments are shown below.





Financial Instruments – Fixed Income

- A fixed-income asset represents a debt for a specific amount (the **principal**), whereby the issuer is obliged to repay the holder at some future date (the **maturity date**).

- Debt can be issued by **governments**, **companies** and **agencies**.

- Fixed-Income securities (generally referred as **Bonds**) are traded in the markets (mostly **OTC**).

- Bondholders have **capital rights.**

- Investors can profit from buying a bond via **capital gain** and (possible) periodic payments known as **coupon.**

Define:

Zero-Coupon Bond: a bond that does not pay coupons but only the principal (at maturity).

Coupon Bond: a bond that does pay coupons and the principal (at maturity).

Financial Instruments – Fixed Income

The **price of a bond** is **the present value** of future **coupons and principal payment** (also called face value).

$$P_0 = \sum_{t=1}^{T} \frac{C_t}{(1+r)^t} + \frac{M}{(1+r)^T}$$

P_0 is today's price of the bond,

C_t is the coupon payment at time t,

M is the principal/face value paid at time T,

r is the interest rate.

The price and coupons of a bond are expressed as a percentage of its face value.

Coupon rates are expressed in annual terms.

Accrued Interest:

When an investor purchases a bond between coupon payments, the investor must compensate the seller of the bond for the coupon interest earned from the time of the last coupon payment to the settlement date of the bond.

Dirty Price:

The amount the buyer pays the seller is the agreed price plus accrued interest.

Clean Price:

The price of a bond without accrued. Bonds are generally quoted in terms of clean price.

Financial Instruments – Fixed Income – Pricing

Zero-Coupon Bond Pricing

The price of a ZCB (zero-coupon bond) is the present value of the principal repaid at maturity.

1. Consider a ZCB that pays $100 in 3 years. Compute the price of the bond assuming an annual interest rate of 5%.

$$P_0 = ?$$

Coupon Bond Pricing

1. Consider a 3-year bond that pays every six months a coupon of 2%. What is the price of the bond using a semi-annual interest rate of 5% and a face value of $ 100?

$$P_0 = ?$$

2. Consider a 3 year bond that pays 2% coupon annually. The interest rates are 3%, 5%, and 10% for the first, second and third year respectively. What is the price of the bond?

$$P_0 = ?$$

Financial Instruments – Fixed Income – Pricing

Zero-Coupon Bond Pricing

The price of a ZCB (zero-coupon bond) is the present value of the principal repaid at maturity.

1. Consider a ZCB that pays \$100 in 3 years. Compute the price of the bond assuming an annual interest rate of 5%.

$$P_0 = \frac{\$100}{(1+0.05)^3} = \$86.38$$

Coupon Bond Pricing

1. Consider a 3 year bond that pays every six months a coupon of 2%. What is the price of the bond using a semi-annual interest rate of 5% and a face value of \$ 100?

$$P_0 = \frac{100*0.01}{(1+0.025)^1} + \frac{100*0.01}{(1+0.025)^2} + \frac{100*0.01}{(1+0.025)^3} + \frac{100*0.01}{(1+0.025)^4} + \frac{100*0.01}{(1+0.025)^5} + \frac{100+100*0.01}{(1+0.025)^6} = \$91.74$$

2. Consider a 3 year bond that pays 2% coupon annually. The interest rates are 3%, 5%, and 10% for the first, second and third year respectively. What is the price of the bond?

$$P_0 = \frac{2}{(1+0.03)^1} + \frac{2}{(1+0.05)^2} + \frac{102}{(1+0.1)^3} = \$80.39$$

Financial Instruments – Fixed Income – Pricing

Dirty vs Clean Price

Consider the following bond:

Face Value = $100

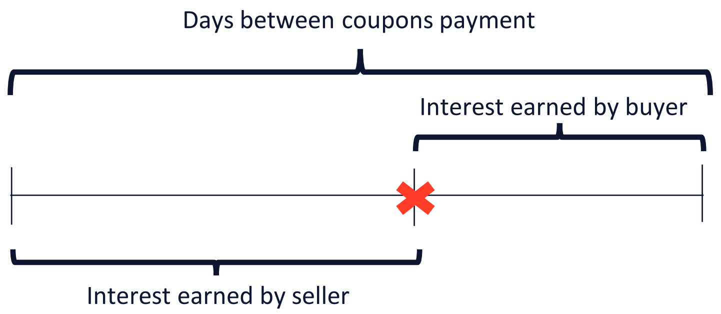
Interest Rate = 8% annually compounded

Coupon Rate = 10%

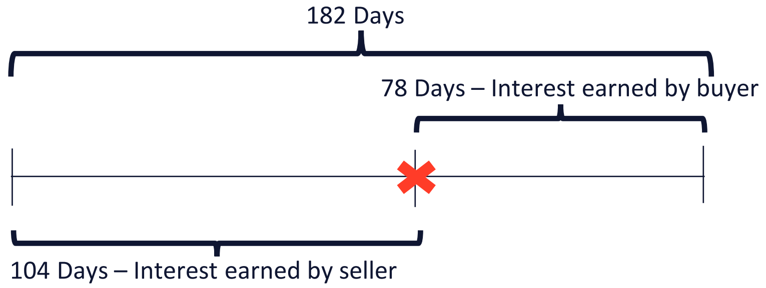
Coupon Frequency = semi-annual (182 days)

Residual Maturity = 2.5 years

Assuming that *Cash Cow Chuckles Llc.* wants to buy the bond between the two coupon payment, compute the accrued interest, dirty and clean price.



Financial Instruments – Fixed Income – Pricing



$$a)\ w\ periods = \frac{days\ left\ before\ next\ coupon\ payment}{days\ in\ a\ coupon\ period}$$

$$b)\ Present\ Value = \frac{Expected\ Cashflow}{(1+r)^{t-1+w}}$$

- Compute the Dirty Price:

$$P_0 = ?$$

- Compute the Accrued Interest:

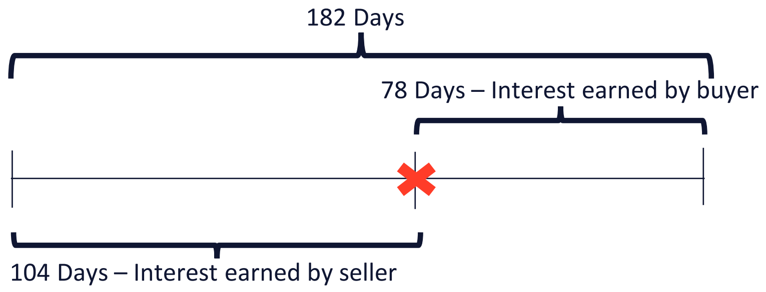
$$Days\ in\ accrued\ interest\ period = days\ in\ coupon\ period\ - days\ left\ before\ next\ coupon\ payment$$

$$Accrued\ Interest = Coupon\ * (1 - w)$$

- Compute the Clean Price:

$$Clean\ price = Dirty\ price\ - Accrued\ Interest$$

Financial Instruments – Fixed Income – Pricing



$$a)\ w\ periods = \frac{days\ left\ before\ next\ coupon\ payment}{days\ in\ a\ coupon\ period} = \frac{78}{182} = 0.4286$$

$$b)\ Present\ Value = \frac{Expected\ Cashflow}{(1+r)^{t-1+w}}$$

- Compute the Dirty Price:

$$P_0 = \frac{5}{(1+0.04)^{0.4286}} + \frac{5}{(1+0.04)^{1.4286}} + \frac{5}{(1+0.04)^{2.4286}} + \frac{5}{(1+0.04)^{3.4286}} + \frac{105}{(1+0.04)^{4.4286}} = \$106.819$$

- Compute the Accrued Interest:

$$Days\ in\ accrued\ interest\ period = days\ in\ coupon\ period\ - days\ left\ before\ next\ coupon\ payment = 182 - 78 = 104$$

$$Accrued\ Interest = Coupon\ * (1 - w) = \$5\ * (1 - 0.4286) = \$2.8570$$

- Compute the Clean Price:

$$Clean\ price = Dirty\ price\ - Accrued\ Interest = \$106.8192 - \$2.8570 = \$103.9622$$

Financial Instruments – Fixed Income – Pricing

Yield-to-Maturity (YTM)

The yield to maturity is the interest rate that will make the present value of a bond's cash flows equal to its market price plus accrued interest.

To find the yield to maturity, we first determine the expected cash flows and then search, by trial and error, for the interest rate that will make the present value of cash flows equal to the market price plus accrued interest.

Consider the following bond: $P_0 = \$98.50, Face\ Value = \$100, Coupon\ Rate = 10\%, Coupon\ Freq. = Semi - Annual, Maturity = 3\ years.$

Compute the YTM (y):

$$P_0 = \frac{5}{(1+y)^1} + \frac{5}{(1+y)^2} + \frac{5}{(1+y)^3} + \frac{5}{(1+y)^4} + \frac{5}{(1+y)^5} + \frac{105}{(1+y)^6} = \$98.50$$

The semi-annual YTM is equal to ?

Assumptions/Limitations of Yield-to-Maturity

1. Reinvestment risk: the coupon payments can be reinvested at the yield to maturity.
2. Interest rate risk: the bond is held to maturity.

Financial Instruments – Fixed Income – Pricing

Yield-to-Maturity (YTM)

The yield to maturity is the interest rate that will make the present value of a bond's cash flows equal to its market price plus accrued interest.

To find the yield to maturity, we first determine the expected cash flows and then search, by trial and error, for the interest rate that will make the present value of cash flows equal to the market price plus accrued interest.

Consider the following bond: $P_0 = \$98.50, Face\ Value = \$100, Coupon\ Rate = 10\%, Coupon\ Freq. = Semi - Annual, Maturity = 3\ years.$

Compute the YTM (y):

$$P_0 = \frac{5}{(1+y)^1} + \frac{5}{(1+y)^2} + \frac{5}{(1+y)^3} + \frac{5}{(1+y)^4} + \frac{5}{(1+y)^5} + \frac{105}{(1+y)^6} = \$98.50$$

The semi-annual YTM is equal to 5.298%. To express it in annual terms,

the convention used is to multiply it by the frequency,

in our example by 2. Thus, the YTM is 10.6%.

		Discounted Cashflows		
Time	**Cashflows**	**r=3%**	**r=5%**	**r=5.29835188%**
1	5	4.85	4.76	4.75
2	5	4.71	4.54	4.51
3	5	4.58	4.32	4.28
4	5	4.44	4.11	4.07
5	5	4.31	3.92	3.86
6	105	87.94	78.35	77.03
Bond Price		**110.83**	**100**	**98.5**

Remember! As the yield goes up, price goes down.

Assumptions/Limitations of Yield-to-Maturity

1. Reinvestment risk: the coupon payments can be reinvested at the yield to maturity.
2. Interest rate risk: the bond is held to maturity.

Financial Instruments – Fixed Income – Pricing

Par Yield (Par)

The par yield for a certain bond maturity is the coupon rate that causes the bond price to equal its par value. The par value is basicaly the principal value of the bond.

Consider the following bond: $Face\ Value = \$100,\ Coupon\ Rate = 15\%,\ Coupon\ Freq. = Semi - Annual,\ Maturity = 2\ years.$

To compute the Par Yield (y) we need to solve below equation:

$$Par\ Value = \sum_{i=1}^{n} \frac{c_i}{(1+y)^i}$$

Where, the bond have n cash flows (c_i) paid at time t_i, and yield of y.

Financial Instruments – Fixed Income – Pricing

Par Yield (Par)

The par yield for a certain bond maturity is the coupon rate that causes the bond price to equal its par value. The par value is basicaly the principal value of the bond.

Consider the following bond: $Face\ Value = \$100$, $Coupon\ Rate = 15\%$, $Coupon\ Freq. = Semi - Annual$, $Maturity = 2\ years$.

To compute the Par Yield (y) we need to solve below equation:

$$Par\ Value = \frac{7.5}{(1+y)^1} + \frac{7.5}{(1+y)^2} + \frac{7.5}{(1+y)^3} + \frac{107.5}{(1+y)^4} = \$100$$

The par yield is equal to 7.5%. So if YTM of this bond would be equal to 7.5%, it would be ***traded at Par***.

Financial Instruments – Fixed Income – Duration

Duration

The duration (or Macaulay duration) of a bond, as its name implies, is a measure of how long on average the holder of the bond has to wait before receiving cash payments. A zero-coupon bond that lasts n years has a duration of n years. However, a coupon-bearing bond lasting *n* years has a duration of less than *n* years, because the holder receives some of the cash payments prior to year n.

Consider the bond with *n* cash flows (c_i) paid at time t_i, price P_0 and yield of *y. We know the price of the bond, assuming continuous compounding, is equal to:*

$$P_0 = \sum_{i=1}^{n} c_i \, e^{-yt_i}$$

Then the duration is defined as:

$$D = \frac{\sum_{i=1}^{n} t_i c_i \, e^{-yt_i}}{P_0} = \sum_{i=1}^{n} t_i \left[\frac{c_i e^{-yt_i}}{P_0} \right]$$

The duration is therefore a weighted average of the times when payments are made, with the weight applied to time t_i being equal to the proportion of the bond's total present value provided by the cash flow at time t_i. The sum of the weights is 1. Note that for the purposes of the definition of duration all discounting is done at the bond yield rate of interest, y.

Financial Instruments – Fixed Income – Duration

Duration

Consider the following bond: $Face\ Value = \$100$, $Coupon\ Rate = 10\%$, $Coupon\ Freq. = Semi - Annual$, $Maturity = 3\ years$, $yield = 12\%$ per annum with continuous compounding.

Compute the duration of this bond.

$$D = ?$$

Financial Instruments – Fixed Income – Duration

Duration

Consider the following bond: $Face\ Value = \$100, Coupon\ Rate = 10\%, Coupon\ Freq. = Semi - Annual, Maturity = 3\ years,\ \ yield = 12\%$ per annum with continuous compounding.

Compute the duration of this bond.

$$D = \sum_{i=1}^{n} t_i \left[\frac{c_i e^{-yt_i}}{P_0} \right] = 0.5 \frac{5 * e^{-0.12*0.5}}{94.213} + 1 \frac{5 * e^{-0.12*1}}{94.213} + 1.5 \frac{5 * e^{-0.12*1.5}}{94.213} + 2 \frac{5 * e^{-0.12*2}}{94.213} + 2.5 \frac{5 * e^{-0.12*2.5}}{94.213} + 3 \frac{105 * e^{-0.12*3}}{94.213} = 2.653$$

Time (years)	Cash Flow ($)	Present Value	Weight	Time x Weight
0.5	5	4.709	0.050	0.025
1.0	5	4.435	0.047	0.047
1.5	5	4.176	0.044	0.066
2.0	5	3.933	0.042	0.083
2.5	5	3.704	0.039	0.098
3.0	105	73.256	0.778	2.333
Total:	130	94.213	1.000	**2.653**

Financial Instruments – Fixed Income – Duration

Duration Usage

When a small change Δy in the yield is considered, it is approximately true that:

$$\Delta P_0 = \frac{dP_0}{dy}\Delta y$$

From the previous slide we know:

$$P_0 = \sum_{i=1}^{n} c_i\, e^{-yt_i}$$

And so:

$$\Delta P_0 = \frac{d\left(\sum_{i=1}^{n} c_i\, e^{-yt_i}\right)}{dy}\Delta y = \Delta y \sum_{i=1}^{n} -t_i c_i\, e^{-yt_i} = -\Delta y \sum_{i=1}^{n} t_i c_i\, e^{-yt_i} = -\Delta y \sum_{i=1}^{n} t_i c_i\, e^{-yt_i}\frac{P_0}{P_0} = -\Delta y \sum_{i=1}^{n} \frac{t_i c_i e^{-yt_i}}{P_0} P_0$$

Having that, we can conclude

$$\Delta P_0 = -D\Delta y P_0$$

or

$$\frac{\Delta P_0}{P_0} = -D\Delta y$$

Financial Instruments – Fixed Income – Duration

Duration Usage

Relationship we derived on the previous slide: $\frac{\Delta P_0}{P_0} = -D\Delta y$ can be modified to so cold Dolar Duration.

$$\frac{\Delta P_0}{P_0} = -D\Delta y \Leftrightarrow -\frac{\Delta P_0}{\Delta y} = \underbrace{DP_0}_{\textbf{Dollar Duration}}$$

Dollar duration can be used for a 1st order approximation (in the Taylor expansion) of bond price change w.r.t yield change. For a more precise approximation we have to take account of convexity, i.e., go to a 2nd order.

Bond price sensitivity w.r.t 1 bps (0.01%) yield change is called DV01.

Financial Instruments – Fixed Income – Duration

Duration Usage

Consider the bond we used before: $P_0 = \$94.213, Face\ Value = \$100,\ Coupon\ Rate = 10\%,\ Coupon\ Freq. = Semi - Annual,\ Maturity = 3\ years,\ yield = 12\%$ per annum with continuous compounding and duration $= 2.653$.

1. What is new approximated bond price if the yield on the bond increases by 10 basis points (0.1%)?

$$\Delta_1 P_0 = ?$$

$$P_{\Delta y_1} = ?$$

2. What is new approximated bond price if the yield on the bond increases by 220 basis points (2.2%)?

$$\Delta_2 P_0 = ?$$

$$P_{\Delta y_2} = ?$$

3. What is new approximated bond price if the yield on the bond increases by 1220 basis points (12.2%)?

$$\Delta_3 P_0 = ?$$

$$P_{\Delta y_3} = ?$$

Financial Instruments – Fixed Income – Duration

Duration Usage

Consider the bond we used before: $P_0 = \$94.213, Face\ Value = \$100,\ Coupon\ Rate = 10\%,\ Coupon\ Freq. = Semi - Annual,\ Maturity = 3\ years,\ yield = 12\%$ per annum with continuous compounding and duration $= 2.653$.

1. What is new approximated bond price if the yield on the bond increases by 10 basis points (0.1%)?

$$\Delta_1 P_0 = -P_0 D \Delta y = -94.213 * 2.653 * 0.001 = -0.250$$

$$P_{\Delta y_1} = P_0 + \Delta P_0 = 94.213 - 0.250 = 93.963$$

2. What is new approximated bond price if the yield on the bond increases by 220 basis points (2.2%)?

$$\Delta_2 P_0 = -P_0 D \Delta y = -94.213 * 2.653 * 0.022 = -5.499$$

$$P_{\Delta y_2} = P_0 + \Delta P_0 = 94.213 - 5.499 = 88.714$$

3. What is new approximated bond price if the yield on the bond increases by 1220 basis points (12.2%)?

$$\Delta_3 P_0 = -P_0 D \Delta y = -94.213 * 2.653 * 0.122 = -30.494$$

$$P_{\Delta y_3} = P_0 + \Delta P_0 = 94.213 - 30.494 = 63.719$$

Financial Instruments – Fixed Income – Duration

Duration Usage

Consider the bond we used before: $P_0 = \$94.213, Face\ Value = \$100,\ Coupon\ Rate = 10\%,\ Coupon\ Freq. = Semi - Annual,\ Maturity = 3\ years,\ yield = 12\%$ per annum with continuous compounding and duration $= 2.653$.

4. What are the errors of approximations?

$$abs.error_1 = P_1 - P_{\Delta y_1} = 93.963 - 93.963 = 0$$

$$rel.error_1 = \frac{P_1 - P_{\Delta y_1}}{P_1} = \frac{93.963 - 93.963}{93.963} = 0\%$$

$$abs.error_2 = P_2 - P_{\Delta y_2} = 88.883 - 88.714 = 0.169$$

$$rel.error_2 = \frac{P_2 - P_{\Delta y_2}}{P_2} = \frac{88.883 - 88.714}{88.883} = 0.19\%$$

$$abs.error_3 = P_3 - P_{\Delta y_3} = 68.449 - 63.719 = 4.730$$

$$rel.error_3 = \frac{P_3 - P_{\Delta y_3}}{P_3} = \frac{68.449 - 88.714}{68.449} = 6.91\%$$

Financial Instruments – Fixed Income – Modified Duration

Modified Duration

Macaulay duration is based on the assumption that y is expressed with continuous compounding. If y is expressed with m times per year compounding, it can be shown that change in bond price follows approximatelly below equation:

$$\Delta P_0 = \frac{-P_0 D \Delta y_c}{1 + \frac{y_m}{m}}$$

Thus, we can define so called $Modified\ Duration\ (MD)$, relaxing continous compounding assumption. MD is specified as follows:

$$MD = \frac{D}{1 + \frac{y_m}{m}}$$

It allows the duration relationship to be simplified to:

$$\Delta P_0 = -P_0 MD \Delta y$$

Financial Instruments – Fixed Income – Modified Duration

Modified Duration Usage

Consider our favorite bond: $P_0 = \$94.213, Face\ Value = \$100,\ Coupon\ Rate = 10\%,\ Coupon\ Freq. = Semi - Annual,\ Maturity = 3\ years,$ $yield = 12\%$ per annum with continuous compounding and duration $= 2.653$.

1. What is *Modified Duration* of such bond? The yield of the bond, expressed with semiannual compounding is 12.3673%

$$MD = ?$$

2. You are an analyst at *Diversify or Die* hedge fund. You were asked to preapre one of the risk sensitivity, Modified Duration, for a new Profit Approximation tool for fixed income trading desk. You want to first calculate it on an example bond with the following features: a 3-year bond that pays a coupon of 10% and a face value of $100. The interest rates are 3%, 5%, and 10% for the first, second and third year respectively.*

$$DV01 = ?$$

***hint:** The formula for converting a periodically compounded rate to a continuously compounded rate is $r_c = m * ln\left(1 + \frac{r_{freq}}{m}\right)$

The formula for converting a continuously compounded rate to a periodically compounded rate is $r_{freq} = m\left(e^{\frac{r_c}{m}} - 1\right)$

Financial Instruments – Fixed Income – Modified Duration

Modified Duration Usage

Consider our favorite bond: $P_0 = \$94.213, Face\ Value = \$100,\ Coupon\ Rate = 10\%,\ Coupon\ Freq. = Semi - Annual,\ Maturity = 3\ years,$ $yield = 12\%$ per annum with continuous compounding and duration $= 2.653$.

1. What is *Modified Duration* of such bond? The yield of the bond, expressed with semiannual compounding is 12.3673%

$$MD = \frac{D}{1 + \frac{y_m}{m}} = \frac{2.653}{1 + \frac{12.3673}{2}} = 2.498501417$$

Financial Instruments – Fixed Income – Modified Duration

Modified Duration Usage

2. You are an analyst at *Diversify or Die* hedge fund. You were asked to preapre one of the risk sensitivity for a new Profit Approximation tool for fixed income trading desk. The sensitivity should be based on annually coumponued yield. You want to first calculate your sensitivity based on an example bond with the following features: a 3-year bond that pays a coupon of 10% annually and a face value of $100. The spot interest rates are 3%, 5%, and 10% for the first, second and third year respectively.*

$$P_0 = \frac{10}{(1+0.03)^1} + \frac{10}{(1+0.05)^2} + \frac{110}{(1+0.1)^3} = \$101.4237$$

$$\frac{10}{(1+ytm_m)^1} + \frac{10}{(1+ytm_m)^2} + \frac{110}{(1+ytm_m)^3} = \$101.4237 \Rightarrow ytm_m \approx 9.4332\%$$

Our ytm is annually coumpanded. So we need to convert to continously coumpaunded.

$$ytm_c = m * ln\left(1 + \frac{ytm_{annual}}{m}\right) = 1 * ln\left(1 + \frac{0.094332}{1}\right) = 0.090144$$

$$D = \sum_{i=1}^{n} t_i \left[\frac{c_i e^{-ytm_c t_i}}{P_0}\right] = 1\frac{10 * e^{-0.090144*1}}{101.4237} + 2\frac{10 * e^{-0.090144*2}}{101.4237} + 3\frac{110 * e^{-0.090144*3}}{101.4237} = 2.737477$$

$$MD = \frac{D}{1 + \frac{y_m}{m}} = \frac{2.737477}{1 + \frac{0.094332}{1}} = 2.502 \Rightarrow Dollar\ Duration = MD * P_0 = 101.4237 * 2.502 = 253.7118 \Rightarrow DV01 = \frac{253.7118}{10{,}000} = 0.02537118$$

Financial Instruments – FRA

Forward Rates

Forward rate is an interest rate on which money can be lent from one future period to another, implied by the current zero-coupon (spot) rates.

Let`s consider following simple term structure for continously compounded interest rates, the 3% per annum rate for 1 year and the 4% per annum rate for 2 years. It means that a $100 invested today for a year is worth:

$$100 * e^{0.03*1} = 103.05$$

and a $100 invested today for two years is worth:

$$100 * e^{0.04*2} = 108.33$$

What then should be an one-year intrest rate in a year (forward rate)? In ideal world it should make no diffrence if we invest our $100 for two years or for a year and then reinvest for another year. Thus,

$$100 * e^{0.03*1} * e^{fwd_{1,1}*1} = 100 * e^{0.04*2}$$

or

$$103.05 * e^{fwd_{1,1}} = 108.33$$

Financial Instruments – FRA

Forward Rates

What then should be an one-year intrest rate in a year (forward rate)? In ideal world it should make no diffrence if we invest our $100 for two years or for a year and then reinvest for another year. Thus,

$$100 * e^{0.03*1} * e^{fwd_{1,1}*1} = 100 * e^{0.04*2}$$

or

$$103.05 * e^{fwd_{1,1}} = 108.33$$

thus

$$e^{fwd_{1,1}} = \frac{108.33}{103.05} = 1.051237$$

taking logharitm

$$ln\left(e^{fwd_{1,1}}\right) = ln(1.051237)$$

so

$$fwd_{1,1} \approx 0.05$$

Our one-year in a year intrest rate is approximetally equal to 5%.

Financial Instruments – FRA

Forward Rates

In general, if R_1 and R_2 are the zero rates for maturities T_1 and T_2, respectively, and R_{T_1,T_2} is the forward interest rate for the period of time between T_1 and T_2, then

$$R_{T_1,T_2} = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

or

$$R_{T_1,T_2} = R_2 + (R_2 - R_1)\frac{T_1}{T_2 - T_1}$$

1. Consider a following term structure. The interest rates are 3%, 5%, and 10% for the first, second and third year respectively. What is a one-year forward rate in two years?

$$R_{T_1,T_2} = ?$$

Financial Instruments – FRA

Forward Rates

In general, if R_1 and R_2 are the continosly zero rates for maturities T_1 and T_2, respectively, and R_{T_1,T_2} is the forward interest rate for the period of time

between T_1 and T_2, then

$$R_{T_1,T_2} = \frac{R_2 T_2 - R_1 T_1}{T_2 - T_1}$$

or

$$R_{T_1,T_2} = R_2 + (R_2 - R_1)\frac{T_1}{T_2 - T_1}$$

1. Consider a following term structure. The interest rates are 3%, 5%, and 10% for the first, second and third year respectively. What is a one-year forward rate in two years?

$$R_{2,3} = 0.1 + (0.1 - 0.05)\frac{2}{3 - 2} = 0.2$$

Sanity check: $100e^{0.1*3} = 134.9859$ $100e^{0.05*2}e^{0.2*1} = 134.9859$

Financial Instruments – FRA

Forward Rate Agreement

A forward rate agreement (FRA) is an over-the-counter agreement designed to ensure that a certain interest rate will apply to either borrowing or lending a certain principal during a specified future period of time. The assumption underlying the contract is that the borrowing or lending would normally be done at given RFR (Risk Free Rate) e.g. SONIA (Sterling Overnight Index Average).

Note, RFRs replaced IBORs, e.g. SONIA (UK) and SOFR (US [Secured Overnight Financing Rate]) replaced LIBOR.

Consider a forward rate agreement where company X is agreeing to lend money to company Y for the period of time between T_1 and T_2. Define:

R_k- The rate of interest agreed to in the FRA

R_f- The forward RFR interest rate for the period between times T_1 and T_2, calculated today

R_m- The actual RFR interest rate observed in the market at time T_1 for the period between times T_1 and T_2

L- The principal underlying the contract.

We will depart from standard assumption of continuous compounding and assume that the rates R_k, R_f , and R_m are all measured with a compounding frequency reflecting the length of the period to which they apply. This means that if $T_2 - T_1 = 0.5$, they are expressed with semi-annual compounding; if $T_2 - T_1 = 0.25$, they are expressed with quarterly compounding; and so on. (This assumption corresponds to the usual market practice for FRAs.)

Financial Instruments – FRA

Forward Rate Agreement

Consider a forward rate agreement where company X is agreeing to lend money to company Y for the period of time between T_1 and T_2. Define:

R_k- The rate of interest agreed to in the FRA

R_f- The forward RFR interest rate for the period between times T_1 and T_2, calculated today

R_m- The actual RFR interest rate observed in the market at time T_1 for the period between times T_1 and T_2

L - The principal underlying the contract.

Normally company X would earn R_m from the RFR loan. The FRA means that it will earn R_k. The extra interest rate (which may be negative) that it earns as a result of entering into the FRA is $R_k - R_m$. The interest rate is set at time T_1 and paid at time T_2. The extra interest rate therefore leads to a cash flow to company X at time T_2 of

$$L(R_k - R_m)(T_2 - T_1)$$

and for Y

$$L(R_m - R_k)(T_2 - T_1)$$

From above, we see that there is another interpretation of the FRA. It is an agreement where company X will receive interest on the principal between T_2 and T_1 at the fixed rate of R_k and pay interest at the realized RFR rate of R_m. Company Y will pay interest on the principal between T_2 and T_1 at the fixed rate of R_k and receive interest at R_m.

Financial Instruments – FRA

Forward Rate Agreement

Usually, FRAs are settled at time T_1 rather than T_2 . So, the payoff must then be discounted from time T_2 to T_1.

$$\frac{L(R_k - R_m)(T_2 - T_1)}{1 + R_m(T_2 - T_1)}$$

and

$$\frac{L(R_m - R_k)(T_2 - T_1)}{1 + R_m(T_2 - T_1)}$$

Suppose that a *Cashflow Chaos Bank* enters into an FRA that is designed to ensure it will receive a fixed rate of 4% on a principal of $100 million for a 3-month period starting in 3 years. The FRA is an exchange where RFR is paid and 4% is received for the 3-month period. If 3-month RFR proves to be 4.5% for the 3-month period the cash flow to the lender will be?

$$CF_L = ?$$

Financial Instruments – FRA

Forward Rate Agreement

Usually, FRAs are settled at time T_1 rather than T_2 . So, the payoff must then be discounted from time T_2 to T_1.

$$\frac{L(R_k - R_m)(T_2 - T_1)}{1 + R_m(T_2 - T_1)}$$

and

$$\frac{L(R_m - R_k)(T_2 - T_1)}{1 + R_m(T_2 - T_1)}$$

Suppose that a *Cashflow Chaos Bank* enters into an FRA that is designed to ensure it will receive a fixed rate of 4% on a principal of $100 million for a 3-month period starting in 3 years. The FRA is an exchange where RFR is paid and 4% is received for the 3-month period. If 3-month RFR proves to be 4.5% for the 3-month period the cash flow to the lender will be?

$$CF_{L,3.25} = L(R_k - R_m)(T_2 - T_1) = 100{,}000{,}000(0.04 - 0.045)(3.25 - 3) = -125{,}000$$

which means

$$CF_{L,3} = \frac{CF_{L,3.25}}{1 + R_m(T_2 - T_1)} = \frac{-125{,}000}{1 + 0.045(3.25 - 3)} = -123{,}609$$

Note that cash flow received by the counterpart will be exactly opposite so +123,609.

Financial Instruments – FRA

Forward Rate Agreement Valuation

To value an FRA we first note that it is always worth zero when R_k= R_f, which is usually the case when the FRA is first initiated.

Compare two FRAs. The first promises that the RFR forward rate R_f will be received on a principal of L between times T_1 and T_2; the second promises that R_k will be received on the same principal between the same two dates. The two contracts are the same except for the interest payments received at time T_2. The excess of the value of the second contract over the first is, therefore, the present value of the difference between these interest payments, or for FRA, where R_k is received

$$P_0 = L\left(R_k - R_f\right)(T_2 - T_1)e^{-R_2 T_2}$$

where R_2 is the continuously compounded riskfree spot rate for a maturity T_2.

Similarly, the value of an FRA where R_k is paid is

$$P_0 = L\left(R_f - R_k\right)(T_2 - T_1)e^{-R_2 T_2}$$

Financial Instruments – FRA

Forward Rate Agreement Valuation

You are a trader at *Bull Market Busters.* You just received a proposition to enter a following Forward Rate Agreement where your company will receive a rate of 6%, measured with annual compounding, and pay RFR on a principal of $100 million between the end of year 1 and the end of year 2. In this case, the forward rate is 5% with continuous compounding or 5.127% with annual compounding. Your counterpart wants you to pay $835,000 to enter the contract. Should you accept the offer?

$$P_0 = ?$$

Financial Instruments – FRA

Forward Rate Agreement Valuation

You are a trader at *Bull Market Busters.* You just received a proposition to enter a following Forward Rate Agreement where your company will receive a rate of 6%, measured with annual compounding, and pay RFR on a principal of $100 million between the end of year 1 and the end of year 2. In this case, the forward rate is 5% with continuous compounding or 5.127% with annual compounding. Your counterpart wants you to pay $835,000 to enter the contract. Should you accept the offer?

$$P_0 = L\left(R_k - R_f\right)(T_2 - T_1)e^{-R_2 T_2}$$

$$P_0 = 100{,}000{,}000(0.06 - 0.05127)(2 - 1)e^{-0.04*2}$$

$$P_0 = 805{,}800$$

Since the proposed price is higher then present value

$$P_0 = 805{,}800 < 835{,}000$$

The proposed FRA is overpriced by the counterpart, thus you should not enter the trade.

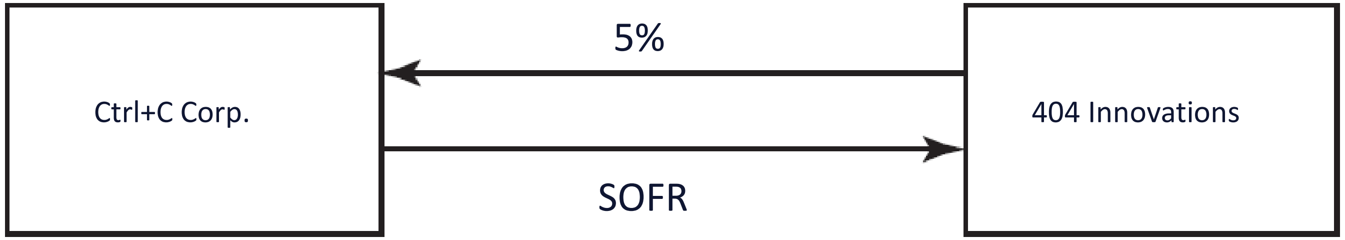
Financial Instruments – SWAP

SWAP

A swap is an over-the-counter agreement between two companies to exchange cash flows in the future. The agreement defines the dates when the cash flows are to be paid and the way in which they are to be calculated. Usually, the calculation of the cash flows involves the future value of an interest rate, an exchange rate, or other market variable.

The most common type of swap is a ‘‘plain vanilla’’ interest rate swap (IRS). In this swap a company agrees to pay cash flows equal to interest at a predetermined fixed rate on a notional principal for a predetermined number of years. In return, it receives interest at a floating rate on the same notional principal for the same period of time.

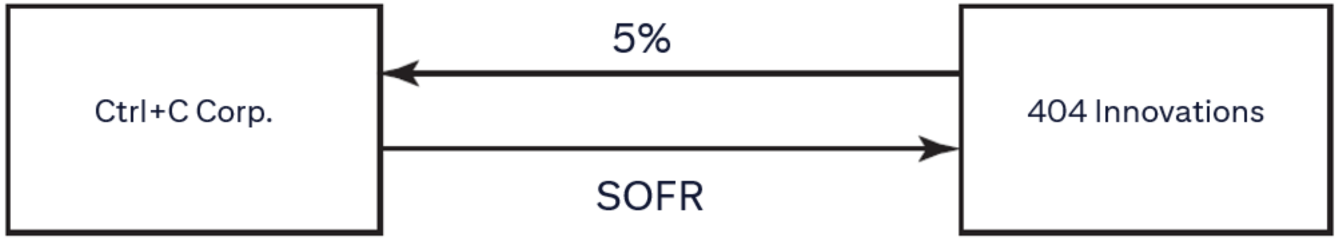
Consider a hypothetical 3-year swap initiated on March 5, 2024, between *404 Innovations* and *Ctrl+C Corp*. We suppose *404 Innovations* agrees to pay *Ctrl+C Corp.* an interest rate of 5% per annum on a principal of $100 million, and in return *Ctrl+C Corp.* agrees to pay *404 Innovations* the 6-month SOFR rate on the same principal. *404 Innovations* is the fixed-rate payer; *Ctrl+C Corp.* is the floating rate payer. We assume the agreement specifies that payments are to be exchanged every 6 months and that the 5% interest rate is quoted with semi-annual compounding.



Financial Instruments – SWAP

SWAP

Consider a hypothetical 3-year swap initiated on March 5, 2024, between two tech companies *404 Innovations* and *Ctrl+C Corp*. We suppose *404 Innovations* agrees to pay *Ctrl+C Corp.* an interest rate of 5% per annum on a principal of \$100 million, and in return *Ctrl+C Corp.* agrees to pay *404 Innovations* the 6-month SOFR rate on the same principal. *404 Innovations* is the fixed-rate payer; *Ctrl+C Corp.* is the floating rate payer. We assume the agreement specifies that payments are to be exchanged every 6 months and that the 5% interest rate is quoted with semi-annual compounding.



So cash flows for *404 Innovations* are as follow:

$$\sum_{i=0}^{n} LR_{k,i}(T_{i+1}-T_i) - LR_{m,i}(T_{i+1}-T_i) = \sum_{i=0}^{n} L\left(R_{k,i}-R_{m,i}\right)(T_{i+1}-T_i)$$

and for *Ctrl+C Corp:*

$$\sum_{i=0}^{n} LR_{m,i}(T_{i+1}-T_i) - LR_{k,i}(T_{i+1}-T_i) = \sum_{i=0}^{n} L\left(R_{m,i}-R_{k,i}\right)(T_{i+1}-T_i)$$

Date	Six-month SOFR	Floating cash flow paid	Fixed cash flow paid	Net cash flow
05/03/2024	4.2			
05/09/2024	4.8	2,100,000	- 2,500,000	- 400,000
05/03/2025	5.3	2,400,000	- 2,500,000	- 100,000
05/09/2025	5.5	2,650,000	- 2,500,000	150,000
05/03/2026	5.6	2,750,000	- 2,500,000	250,000
05/09/2026	5.9	2,800,000	- 2,500,000	300,000
05/03/2027		2,950,000	- 2,500,000	450,000
Sum		15,650,000	-15,000,000	650,000

Financial Instruments – SWAP

SWAP valuation

Now, hom much swap should cost? An interest rate swap is worth close to zero when it is first initiated. After it has been in existence for some time, its value may be positive or negative. There are two valuation approaches. The first regards the swap as the difference between two bonds; the second regards it as a portfolio of FRAs.

For fix leg payer the value is equal to:

$$V_{swap,0} = B_{fix} - B_{flt}$$

or

$$V_{swap,0} = \sum_{i=0}^{n} L\left(R_{f,t_i} - R_{k,t_i}\right)T_{freq}\, e^{-r(T_{i+1}-T_0)}$$

While for float leg payer the value is equal to:

$$V_{swap,0} = B_{flt} - B_{fix}$$

or

$$V_{swap,0} = \sum_{i=0}^{n} L\left(R_{k,t_i} - R_{f,t_i}\right)T_{freq}\, e^{-r(T_{i+1}-T_0)}$$

Financial Instruments – SWAP

SWAP valuation

1. Suppose that a financial institution has agreed to pay 12-month SONIA and receive 12% per annum (with annual compounding) on a notional principal of $100 million. The swap has a remaining life of 2 years. The SONIA rates with continuous compounding for a year and two-years maturities are 9.5% and 13%, respectively. What is the value of such SWAP using bond replication method?

$$V_{swap,0} = ?$$

2. Now what is the value of such SWAP using FRA replication method?

$$V_{swap,0} = ?$$

Financial Instruments – SWAP

SWAP valuation

1. Suppose that a financial institution has agreed to pay 12-month SONIA and receive 12% per annum (with annual compounding) on a notional principal of $100 million. The swap has a remaining life of 2 years. The SONIA rates with continuous compounding for a year and two-years maturities are 9.5% and 13%, respectively. What is the value of such SWAP using bond replication method?

$$B_{fix} = \sum_{t=1}^{T} C_t e^{-rt} + Me^{-rT} = 12{,}000{,}000 * e^{-0.095*1} + 112{,}000{,}000 * e^{-0.13*2} = 97{,}270{,}252.82$$

Because we know floating pays SONIA and we are at coupon payment date so:

$$B_{flt} = 100{,}000{,}000$$

Or we can calculate it manualy: $B_{flt} = \sum_{t=1}^{T} C_t e^{-rt} + Me^{-rT}$ *But we do not know annual compounding rates and forward rate for a year in a year, so:*

First let`s calculate one-year rate with annual compounding: $R_m = m\left(e^{R_C/m} - 1\right) = 1\left(e^{0.095/1} - 1\right) = 0.0996588551$

Now we need to calculate one-year in a year rate: $R_{12M,24M} = \frac{R_{24M}T_{24M} - R_{12M}T_{12M}}{T_{24M} - T_{12M}} = \frac{0.13*2 - 0.095*1}{2-1} = 0.165$

And convert it into annual compuding: $R_m = m\left(e^{R_C/m} - 1\right) = 1\left(e^{0.165/1} - 1\right) = 0.1793931187$

$$B_{flt} = \sum_{t=1}^{T} C_t e^{-rt} + Me^{-rt} = 9{,}965{,}885.51e^{-0.095*1} + 117{,}939{,}311.87e^{-0.13*2} = 100{,}000{,}000$$

Financial Instruments – SWAP

SWAP valuation

1. Suppose that a financial institution has agreed to pay 12-month SONIA and receive 12% per annum (with annual compounding) on a notional principal of $100 million. The swap has a remaining life of 2 years. The SONIA rates with continuous compounding for a year and two-years maturities are 9.5% and 13%, respectively. What is the value of such SWAP using bond replication method?

$$B_{fix} = \sum_{t=1}^{T} C_t e^{-rt} + Me^{-rT} = 12{,}000{,}000 * e^{-0.095*1} + 112{,}000{,}000 * e^{-0.13*2} = 97{,}270{,}252.82$$

Because we know floating pays SONIA and we are at coupon payment date so:

$$B_{flt} = 100{,}000{,}000$$

$$V_{swap,0} = B_{flt} - B_{fix} = 100{,}000{,}000 - 97{,}270{,}252.82 = 2{,}729{,}747.18$$

2. Now what is the value of such SWAP using FRA replication method?

$$V_{swap,0} = ?$$

Financial Instruments – SWAP

SWAP valuation

1. Suppose that a financial institution has agreed to pay 12-month SONIA and receive 12% per annum (with annual compounding) on a notional principal of $100 million. The swap has a remaining life of 2 years. The SONIA rates with continuous compounding for a year and two-years maturities are 9.5% and 13%, respectively. What is the value of such SWAP using bond replication method?

$$V_{swap,0} = B_{flt} - B_{fix} = 100{,}000{,}000 - 97{,}270{,}252.82 = 2{,}729{,}747.18$$

2. Now what is the value of such SWAP using FRA replication method?

$$R_m = m\left(e^{R_c/m} - 1\right) = 1\left(e^{0.095/1} - 1\right) = 0.0996588551$$

$$R_{12M,24M} = \frac{R_{24M}T_{24M} - R_{12M}T_{12M}}{T_{24M} - T_{12M}} = \frac{0.13 * 2 - 0.095 * 1}{2 - 1} = 0.165 \xRightarrow[need\ to\ convert\ from\ cont.\ to\ annual]{}$$

$$R_m = m\left(e^{R_c/m} - 1\right) = 1\left(e^{0.165/1} - 1\right) = 0.1793931187$$

Financial Instruments – SWAP

SWAP valuation

1. Suppose that a financial institution has agreed to pay 12-month SONIA and receive 12% per annum (with annual compounding) on a notional principal of $100 million. The swap has a remaining life of 2 years. The SONIA rates with continuous compounding for a year and two-years maturities are 9.5% and 13%, respectively. What is the value of such SWAP using bond replication method?

$$V_{swap,0} = B_{flt} - B_{fix} = 100{,}000{,}000 - 97{,}270{,}252.82 = 2{,}729{,}747.18$$

2. Now what is the value of such SWAP using FRA replication method?

$$R_{12M} = 0.0996588551 \qquad R_{12M,24M} = 0.1793931187$$

$$V_{swap,0} = \sum_{i=0}^{n} L\left(R_{k,t_i} - R_{f,t_i}\right)T_{freq}\, e^{-r(T_{i+1}-T_0)} =$$

$$= 100{,}000{,}000(0.0996588551 - 0.12) * 1 * e^{-0.095*1} + 100{,}000{,}000(0.1793931187 - 0.12) * 1 * e^{-0.13*1} =$$

$$= -1{,}849{,}768.66 + 4{,}579{,}515.84 = 2{,}729{,}747.17$$

We can observe small differences because of roundings.

Financial Instruments – SWAP

SWAP valuation

1. Suppose that a financial institution has agreed to pay 6-month SONIA+0.2% and receive 11% per annum (with semi-annual compounding) on a notional principal of $50 million. The swap has a remaining life of 3 years. The SONIA rates with continuous compounding for a 6-months, 12-months, 18-months, 24-months, 30-months and 36-months maturities are 2%, 3%, 4%, 5%, 6%, and 7%, respectively. What is the value of such SWAP using bond replication method?

$$B_{fix} = \sum_{t=1}^{T} C_t e^{-rt} + Me^{-rT} = 50{,}000{,}000 * \frac{0.11}{2} * e^{-0.02*0.5} + 2{,}750{,}000 * e^{-0.03*1} + 2{,}750{,}000 * e^{-0.04*1.5} +$$

$$+2{,}750{,}000 * e^{-0.05*2} + 2{,}750{,}000 * e^{-0.06*2.5} + 52{,}750{,}000 * e^{-0.07*3} = 55{,}594{,}783.54$$

For floating bond we need 6-month spot rate with semi-annual compounding and forward rates. We can not use the „payment date" trick in this case as floating pays SONIA+spread (0.2%).

$$R_{6M,semi-annual} = m\left(e^{R_c/m} - 1\right) = 2\left(e^{0.02/2} - 1\right) = 0.0201003341683359$$

$$R_{6M,12M} = \frac{R_{12M}T_{12M} - R_{6M}T_{6M}}{T_{12M} - T_{6M}} = \frac{0.03 * 1 - 0.02 * 0.5}{1 - 0.5} = 0.04 \quad R_{6M,12M\ semi-annual} = 2\left(e^{0.04/2} - 1\right) = 0.0404026800535116$$

Financial Instruments – SWAP

SWAP valuation

1. Suppose that a financial institution has agreed to pay 6-month SONIA+0.2% and receive 11% per annum (with semi-annual compounding) on a notional principal of $50 million. The swap has a remaining life of 3 years. The SONIA rates with continuous compounding for a 6-months, 12-months, 18-months, 24-months, 30-months and 36-months maturities are 2%, 3%, 4%, 5%, 6%, and 7%, respectively. What is the value of such SWAP using bond replication method?

$$R_{6M,12M} = \frac{R_{12M}T_{12M} - R_{6M}T_{6M}}{T_{12M} - T_{6M}} = \frac{0.03 * 1 - 0.02 * 0.5}{1 - 0.5} = 0.04 \quad R_{6M,12M\ semi-annual} = 2\left(e^{0.04/2} - 1\right) = 0.0404026800535116$$

$$R_{12M,18M} = \frac{R_{18M}T_{18M} - R_{12M}T_{12M}}{T_{18M} - T_{12M}} = \frac{0.04 * 1.5 - 0.03 * 1}{1.5 - 1} = 0.06 \quad R_{12M,18M semi-annual} = 2\left(e^{0.06/2} - 1\right) = 0.0609090679070339$$

$$R_{18M,24M} = \frac{R_{24M}T_{24M} - R_{18M}T_{18M}}{T_{24M} - T_{18M}} = \frac{0.05 * 2 - 0.04 * 1.5}{2 - 1.5} = 0.08 \quad R_{18M,24M semi-annual} = 2\left(e^{0.08/2} - 1\right) = 0.0816215483847764$$

$$R_{24M,30M} = \frac{R_{30M}T_{30M} - R_{24M}T_{24M}}{T_{30M} - T_{24M}} = \frac{0.06 * 2.5 - 0.05 * 2}{2.5 - 2} = 0.1 \quad R_{24M,30M semi-annual} = 2\left(e^{0.1/2} - 1\right) = 0.102542192752048$$

$$R_{30M,36M} = \frac{R_{36M}T_{36M} - R_{30M}T_{30M}}{T_{36M} - T_{30M}} = \frac{0.07 * 3 - 0.06 * 2.5}{3 - 2.5} = 0.12 \quad R_{30M,36M semi-annual} = 2\left(e^{0.12/2} - 1\right) = 0.123673093090719$$

Financial Instruments – SWAP

SWAP valuation

1. Suppose that a financial institution has agreed to pay 6-month SONIA+0.2% and receive 11% per annum (with semi-annual compounding) on a notional principal of $50 million. The swap has a remaining life of 3 years. The SONIA rates with continuous compounding for a 6-months, 12-months, 18-months, 24-months, 30-months and 36-months maturities are 2%, 3%, 4%, 5%, 6%, and 7%, respectively. What is the value of such SWAP using bond replication method?

$$B_{fix} = 55{,}594{,}783.54$$

$$R_{6M,semi-annual} = 0.0201003341683359 \quad R_{6M,12M\ semi-annual} = 0.0404026800535116$$

$$R_{12M,18Msemi-annual} = 0.0609090679070339 \quad R_{18M,24Msemi-annual} = 0.0816215483847764$$

$$R_{24M,30Msemi-annual} = 0.102542192752048 \quad R_{30M,36Msemi-annual} = 0.123673093090719$$

$$B_{flt} = \sum_{t=1}^{T} C_t e^{-rt} + Me^{-rt} = 50{,}000{,}000 * \frac{0.0201003341683359 + 0.002}{2} * e^{-0.02*0.5} +$$

$$+50{,}000{,}000 * \frac{0.0404026800535116 + 0.002}{2} * e^{-0.03*1} + 50{,}000{,}000 * \frac{0.0609090679070339 + 0.002}{2} * e^{-0.04*1.5} +$$

$$+50{,}000{,}000 * \frac{0.0816215483847764 + 0.002}{2} * e^{-0.05*2} + 50{,}000{,}000 * \frac{0.102542192752048 + 0.002}{2} * e^{-0.06*2.5} +$$

$$+(50{,}000{,}000 + 50{,}000{,}000 * \frac{0.123673093090719 + 0.002}{2}) * e^{-0.07*3} = 50{,}273{,}919.48$$

Financial Instruments – SWAP

SWAP valuation

1. Suppose that a financial institution has agreed to pay 6-month SONIA+0.2% and receive 11% per annum (with semi-annual compounding) on a notional principal of $50 million. The swap has a remaining life of 3 years. The SONIA rates with continuous compounding for a 6-months, 12-months, 18-months, 24-months, 30-months and 36-months maturities are 2%, 3%, 4%, 5%, 6%, and 7%, respectively. What is the value of such SWAP using bond replication method?

$$B_{fix} = 55{,}594{,}783.54$$

$$R_{6M,semi-annual} = 0.0201003341683359 \quad R_{6M,12M\ semi-annual} = 0.0404026800535116$$

$$R_{12M,18Msemi-annual} = 0.0609090679070339 \quad R_{18M,24Msemi-annual} = 0.0816215483847764$$

$$R_{24M,30Msemi-annual} = 0.102542192752048 \quad R_{30M,36Msemi-annual} = 0.123673093090719$$

$$B_{flt} = 50{,}273{,}919.48$$

$$V_{swap,0} = B_{flt} - B_{fix} = 50{,}273{,}919.48 - 55{,}594{,}783.54 = -5{,}320{,}864.06$$

Financial Instruments – SWAP

SWAP valuation

1. Suppose that a financial institution has agreed to pay 6-month SONIA+0.2% and receive 11% per annum (with semi-annual compounding) on a notional principal of $50 million. The swap has a remaining life of 3 years. The SONIA rates with continuous compounding for a 6-months, 12-months, 18-months, 24-months, 30-months and 36-months maturities are 2%, 3%, 4%, 5%, 6%, and 7%, respectively. What is the value of such SWAP using bond replication method?

$$V_{swap,0} = B_{flt} - B_{fix} = 50{,}273{,}919.48 - 55{,}594{,}783.54 = -5{,}320{,}864.06$$

2. Now what is the value of such SWAP using FRA replication method?

$$R_{6M,semi-annual} = 0.0201003341683359 \quad R_{6M,12M\ semi-annual} = 0.0404026800535116$$
$$R_{12M,18Msemi-annual} = 0.0609090679070339 \quad R_{18M,24Msemi-annual} = 0.0816215483847764$$
$$R_{24M,30Msemi-annual} = 0.102542192752048 \quad R_{30M,36Msemi-annual} = 0.123673093090719$$

$$V_{swap,0} = \sum_{i=0}^{n} L\left(R_{k,t_i} - R_{f,t_i}\right)T_{freq}\, e^{-r(T_{i+1}-T_0)} = 50{,}000{,}000((0.0201003341683359 + 0.002) - 0.11) * 0.5 * e^{-0.02*0.5} +$$
$$+50{,}000{,}000((0.0404026800535116 + 0.002) - 0.11) * 0.5 * e^{-0.03*1} + 50{,}000{,}000((0.0609090679070339 + 0.002) - 0.11) * 0.5 * e^{-0.04*1.5} +$$
$$+50{,}000{,}000((0.0816215483847764 + 0.002) - 0.11) * 0.5 * e^{-0.05*2} + 50{,}000{,}000((0.102542192752048 + 0.002) - 0.11) * 0.5 * e^{-0.06*2.5} +$$
$$+50{,}000{,}000((0.123673093090719 + 0.002) - 0.11) * 0.5 * e^{-0.07*3} = -2{,}175626.24 + (-1{,}639{,}987.93) + (-1{,}108{,}714.24) + (-596{,}705.25) +$$
$$+(-117{,}439.46) + 317{,}609.06 = -5{,}320{,}864.06$$

Financial Instruments – SWAP

SWAP valuation

1. Suppose that a financial institution has agreed to pay 6-month SONIA and receive 8% per annum (with semi-annual compounding) on a notional principal of $100 million. The swap has a remaining life of 1.25 years. The SONIA rates with continuous compounding for 3-month, 9-month, and 15-month maturities are 10%, 10.5%, and 11%, respectively. The 6-month SONIA rate at the last payment date was 10.2% (with semi-annual compounding). What is the value of such SWAP using bond replication method?

$$V_{swap,0} = ?$$

2. Now what is the value of such SWAP using FRA replication method?

$$V_{swap,0} = ?$$

SWAP valuation

1. Suppose that a financial institution has agreed to pay 6-month SONIA and receive 8% per annum (with semi-annual compounding) on a notional principal of $100 million. The swap has a remaining life of 1.25 years. The SONIA rates with continuous compounding for 3-month, 9-month, and 15-month maturities are 10%, 10.5%, and 11%, respectively. The 6-month SONIA rate at the last payment date was 10.2% (with semi-annual compounding). What is the value of such SWAP using bond replication method?

$$B_{fix} = \sum_{t=1}^{T} C_t e^{-rt} + Me^{-rT} = 4{,}000{,}000 * e^{-0.1*0.25} + 4{,}000{,}000 * e^{-0.105*0.75} + 104{,}000{,}000 * e^{-0.11*1.25} = 98{,}237{,}895.90$$

$B_{flt} = \sum_{t=1}^{T} C_t e^{-rt} + Me^{-rT}$ *or in that case, we know the next coupon and that all the rest is equal to principal at the next payment date so:*

$$B_{flt} = C_t e^{-rt} + Me^{-rt} = 5{,}100{,}000e^{-0.1*0.25} + 100{,}000{,}000e^{-0.1*0.25} = 102{,}505{,}071.75$$

$$V_{swap,0} = B_{flt} - B_{fix} = 102{,}505{,}071.75 - 98{,}237{,}895.90 = 4{,}267{,}175.85$$

2. Now what is the value of such SWAP using FRA replication method?

$$V_{swap,0} = ?$$

Financial Instruments – SWAP

SWAP valuation

1. Suppose that a financial institution has agreed to pay 6-month SONIA and receive 8% per annum (with semi-annual compounding) on a notional principal of \$100 million. The swap has a remaining life of 1.25 years. The SONIA rates with continuous compounding for 3-month, 9-month, and 15-month maturities are 10%, 10.5%, and 11%, respectively. The 6-month SONIA rate at the last payment date was 10.2% (with semi-annual compounding). What is the value of such SWAP using bond replication method?

$$V_{swap,0} = 4{,}267{,}175.85$$

2. Now what is the value of such SWAP using FRA replication method?

$$R_{3M,9M} = \frac{R_{9M}T_{9M} - R_{3M}T_{3M}}{T_{9M} - T_{3M}} = \frac{0.105 * 0.75 - 0.1 * 0.25}{0.75 - 0.25} = 0.1075 \xRightarrow[need\ to\ convert\ from\ cont.\ to\ semi-annual]{}$$

$$R_m = m\left(e^{R_c/m} - 1\right) = 2\left(e^{0.1075/2} - 1\right) = 0.11044$$

$$R_{9M,15M} = \frac{R_{15M}T_{15M} - R_{9M}T_{9M}}{T_{15M} - T_{9M}} = \frac{0.11 * 1.25 - 0.105 * 0.75}{1.25 - 0.75} = 0.1175 \xRightarrow[need\ to\ convert\ from\ cont.\ to\ semi-annual]{}$$

$$R_m = m\left(e^{R_c/m} - 1\right) = 2\left(e^{0.1175/2} - 1\right) = 0.12102$$

Financial Instruments – SWAP

SWAP valuation

1. Suppose that a financial institution has agreed to pay 6-month SONIA and receive 8% per annum (with semi-annual compounding) on a notional principal of $100 million. The swap has a remaining life of 1.25 years. The SONIA rates with continuous compounding for 3-month, 9-month, and 15-month maturities are 10%, 10.5%, and 11%, respectively. The 6-month SONIA rate at the last payment date was 10.2% (with semi-annual compounding). What is the value of such SWAP using bond replication method?

$$V_{swap,0} = 4{,}267{,}175.85$$

2. Now what is the value of such SWAP using FRA replication method?

$$R_{3M,9M} = 0.11044 \qquad R_{9M,15M} = 0.12102$$

$$V_{swap,0} = \sum_{i=0}^{n} L\left(R_{k,t_i} - R_{f,t_i}\right) T_{freq}\, e^{-r(T_{i+1}-T_0)} =$$

$$= 100{,}000{,}000(0.102 - 0.08) * 0.5 * e^{-0.1*0.25} + 100{,}000{,}000(0.11044 - 0.08) * 0.5 * e^{-0.105*0.75} +$$

$$+100{,}000{,}000(0.12102 - 0.08) * 0.5 * e^{-0.11*1.25} = 1{,}072{,}840.90 + 1{,}406{,}811.02 + 1{,}787{,}523.93 =$$

$$= 4{,}267{,}175.85$$

Financial Instruments – Commodity and Foreign Exchange (FX)

Commodity:

A good sold for **production** or **consumption** (Metals, Energy, Agricultural products, etc.).

A commodity transaction may be **physical** (delivery of the commodity) or **financial** (a cashflow from one party to the other).

Nowadays, most of the commodity transaction are done through derivative contracts which are traded in exchanges and OTC markets.

Foreign Exchange:

Foreign exchange (FX) represents the trading of **currencies**, essentially by the transfer of ownership of deposits.

FX transaction **lock-in the exchange** rate for cash.

A U.S Dollar/Japanese Yen trade consists of selling U.S. dollars (base currency) to another counterparty in exchange for a specific amount of Yen (quoting/counter currency).

How many Euro can I buy with PLN 10000 today? The €1 = PLN 4.63, thus $\frac{PLN}{PLN/€}\frac{10000}{4.63/1} = €2159.83$

Derivatives

Definition:

Financial derivatives are a type of financial contract whose value is dependent on an underlying asset, group of assets, or benchmark. A derivative is set between two or more parties that can trade on an exchange or over-the-counter (OTC).

Derivatives Underlying:

There are numerous underlyings for derivatives available right now, and new ones are being developed every year.

– **Equities** of companies listed on public exchanges, such as Microsoft or Citigroup.

– **Fixed income instruments**, such as government bonds, corporate bonds, credit spreads, or baskets of mortgages.

– **Commodities**, such as gold, oil, silver, cotton, electricity or weather.

– **Indices**, such as the FTSE 100, Hang Seng of Hong Kong, or Nikkei of Japan.

– **Foreign exchange**.

– **Events**, such as football games or shipping catastrophes.

Derivatives – Definition of Common Terms

The most common terms in finance world:

- **Premium**: The amount paid for the contract initially. How to find this value is the subject of much of this book.
- **Underlying (asset)**: The financial instrument on which the option value depends. Stocks, commodities, currencies and indices are going to be denoted by S. The option payoff is defined as some function of the underlying asset at expiry.
- **Strike (price) or exercise price**: The amount for which the underlying can be bought (call) or sold (put). This will be denoted by E. This definition only really applies to the simple calls and puts. We will see more complicated contracts in later chapters and the definition of strike or exercise price will be extended.
- **Expiration (date) or expiry (date)**: Date on which the option can be exercised or date on which the option ceases to exist or give the holder any rights. This will be denoted by T .
- **Intrinsic value**: The payoff that would be received if the underlying is at its current level when the option expires.
- **Time value**: Any value that the option has above its intrinsic value. The uncertainty surrounding the future value of the underlying asset means that the option value is generally different from the intrinsic value.
- **In the money**: An option with positive intrinsic value. A call option when the asset price is above the strike, a put option when the asset price is below the strike.
- **Out of the money**: An option with no intrinsic value, only time value. A call option when the asset price is below the strike, a put option when the asset price is above the strike.
- **At the money**: A call or put with a strike that is close to the current asset level.
- **Long position**: A positive amount of a quantity, or a positive exposure to a quantity.
- **Short position**: A negative amount of a quantity, or a negative exposure to a quantity. Many assets can be sold short, with some constraints on the length of time before they must be bought back.

Derivatives

The most common types of derivative contracts:

Forwards: a customized OTC derivative contract obligating counterparties to buy (receive) or sell (deliver) an asset at a specified price on a future date.

Futures: exchange-listed financial derivatives contracts that oblige the buyer to purchase some underlying asset (or the seller to sell that asset) at a predetermined future price and date.

Options: OTC or exchange-listed financial derivatives that give buyers the right, but not the obligation, to buy or sell an underlying asset at an agreed-upon price and date. **Call Options** allow the holder to buy the asset at a stated price within a specific timeframe. **Put Options**, on the other hand, allow the holder to sell the asset at a stated price within a specific timeframe.

Swaps: OTC agreement between two parties to exchange cash flows in future. The agreement defines the dates when the cash flows are to be paid and the way in which they are to be computed.

Asset Backed Securities (ABS): bonds or notes backed by financial assets. The asset pool is usually a group of small and illiquid assets that would be difficult to sell individually. Pooling these assets into financial instruments allows them to be sold to investors.

The use of derivatives:

Hedging or mitigating risk in an underlying: by entering into a derivative contract whose value moves in the opposite direction to their underlying position, hedgers aim to reduce their risk.

Speculate and make a profit: if the value of the underlying asset moves the way a trader expects.

Obtain **exposure to an underlying** which cannot be traded directly (e.g. indexes, weather derivatives).

Define the risk: traders can use options to give them quite defined risk exposures, such as setting a maximum loss for a position.

Tailored exposures: derivatives traders can take positions that profit if an underlying moves in a given direction, stays in or out of a specified range, or reaches a certain level.

Arbitrage: derivatives can be used to make a risk-free profit.

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