Project: Bacterial growth

Given the parameters of a model, we can use ODE solvers to predict the behavior of a model, $x(t)$.

Given data, can we find which model parameters are the best fit?

a Write code to simulate the following model

$$\frac{dN}{dt} = \lambda N \left(1 - (N/\theta)^{\alpha}\right) \tag{1}$$

where the user (you) specifies the parameter values $\lambda = 1, \theta = 10^3, \alpha = 2$, and the initial condition $N(0) = 200$. Plot a time series of $N_{\text{sim}}(t)$.

b Write code that loads in bacterial growth experimental data into two arrays: One for the amount of bacteria $N_{\text{exp}}(t_i)$ and the other for measurement time points t_i. Plot the time series $N_{\text{exp}}(t_i)$.

c Plot the solution $N(t)$ and the experimental data $N_{\text{exp}}(t)$ on the same axes. Set $N(0)$ to match the experimental data exactly. Manually explore different parameter values of λ, θ, α to see which values best fit the data.

d Write code that computes the sum of squared error (SSE), which is

$$SSE = \sum_i \left(N_{\text{sim}}(t_i) - N_{\text{exp}}(t_i)\right)^2 \tag{2}$$

Hint: The Matlab solver `ode45` can take a time vector as an input argument. If the time vector has more than two elements, it will return the solution only at those elements. This is useful if you only need the solution at certain times t_i.

e Using the code from the previous part, create a *function* that takes parameters and returns the SSE. Hint: A function can be defined in a separate m-file. See example in Module 3, Part 1.

f Use an automated minimization algorithm like Matlab's `fminsearch` to find the parameters that minimize the sum of squared error. These parameters are the "best fit" or "maximum likelihood" parameters $\hat{\lambda}, \hat{\theta}, \hat{\alpha}$. Plot, on the same axes, the experimental and best-fit model time series, $N_{\text{exp}}(t)$ and $N_{\text{sim}}(t)$.

g Now repeat the above, but use the model with $\alpha = 1$. Did the fit improve, or worsen?

h Now repeat the above, but with

$$\frac{dN}{dt} = \lambda N. \tag{3}$$

Note this is numerically the same as Eq. 1 with $\theta \gg N$. Did the fit improve, or worsen?