CONTINUOUS RANDOM VARIABLES

$X \in S$ $\qquad$ S - CONTINUOUS

EXAMPLES $\qquad S \in [0, 1]$

$S \in (-\infty, \infty)$

$S \in [0, \infty)$

HAS UNITS !

$p_X(x)$

PROBABILITY DENSITY

$$\mathbb{P}(a < x \leq b) = \int_a^b p_X(\tilde{x})\, d\tilde{x}$$

$$\int_S p(x)\, dx = 1$$

CUMULATIVE DISTRIBUTION

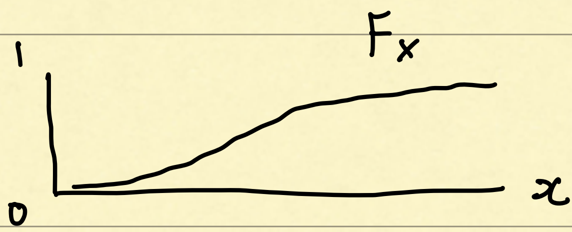
$$F_X(x) = \int_{-\infty}^{x} p_X(\tilde{x})\, d\tilde{x} \quad \Rightarrow \quad p_X(x) = \frac{d}{dx} F_X(x)$$

$$= \mathbb{P}(X \leq x)$$

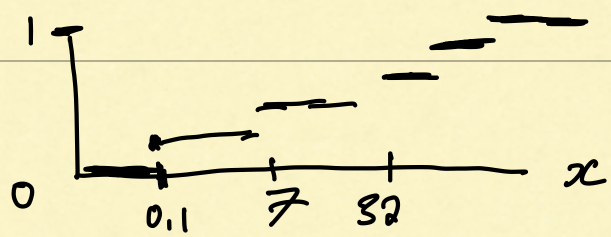
$F_X(x) \to 0$ AS $x \to -\infty$

$F_X(x) \to 1$ AS $x \to +\infty$

F_x IS NON-DECREASING

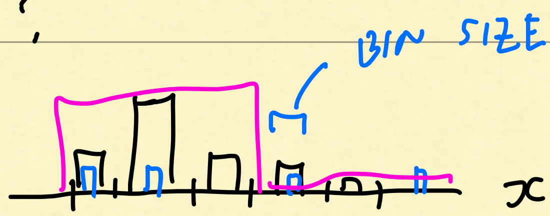


$$\text{DATA} = \begin{bmatrix} 0.1 \\ 7 \\ 32 \\ \vdots \end{bmatrix} \rightarrow$$

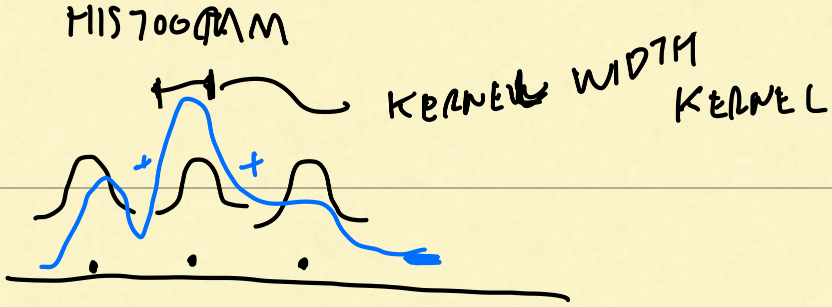


EMPIRICAL CUMULATIVE

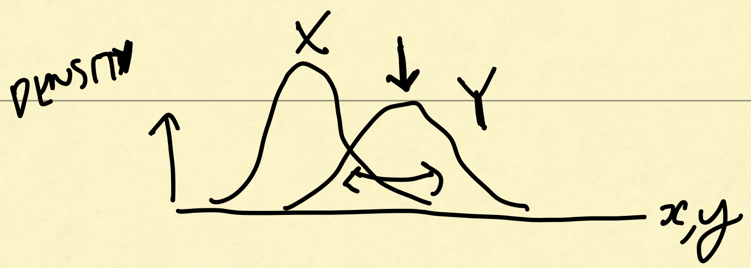
DENSITY ?

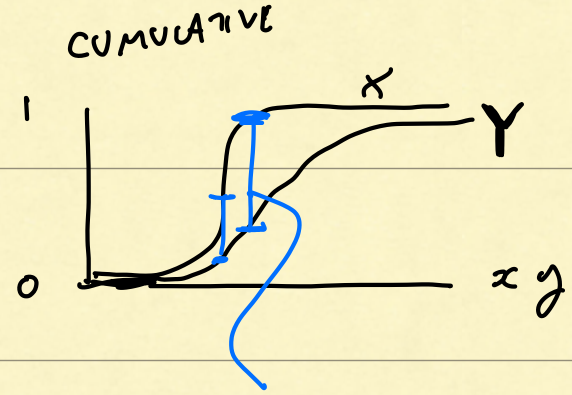


HISTOGRAM



COMPARISON

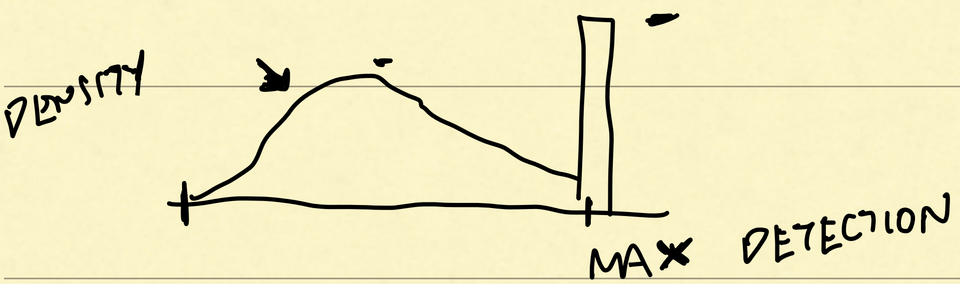


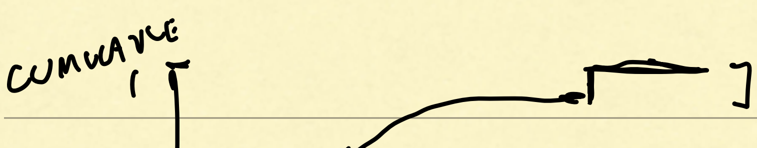


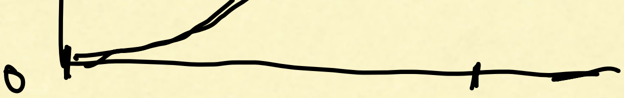
$\mathrm{MAX}\left(F_X(\tilde{x}) - F_Y(\tilde{x})\right)$

KOLMOGOROV-SMIRNOV

- DATA WITH BOTH DISCRETE & CONTINUOUS VALUES

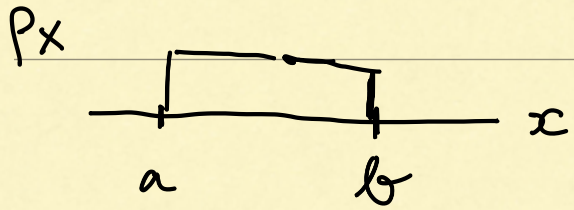




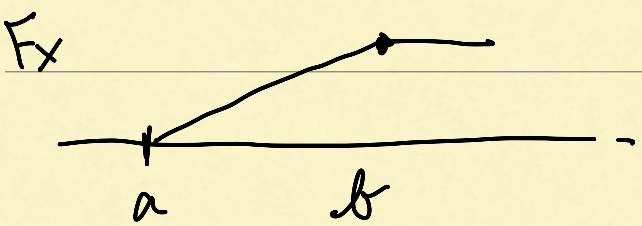


FAMOUS CONTINUOUS RANDOM VARIABLES

- UNIFORM $\quad X \sim \text{UNIF}(a, b)$

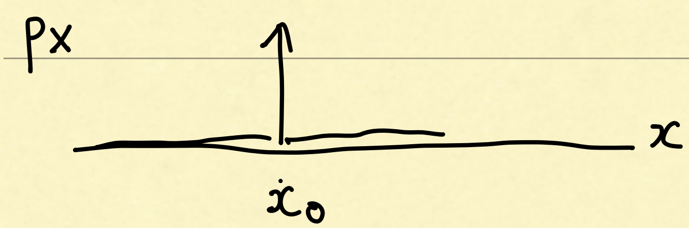


$$p_X(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{ELSE} \end{cases}$$

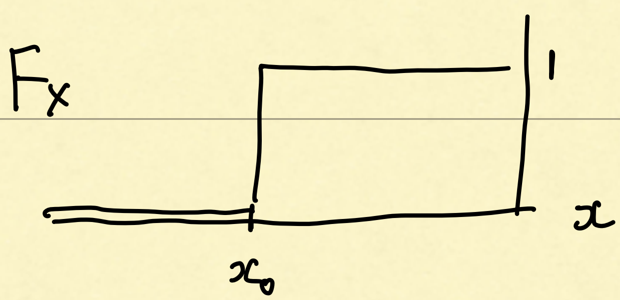


$$F_X(x) = \begin{cases} 0 & x < a \\ \frac{x-a}{b-a} & a < x < b \\ 1 & b < x \end{cases}$$

- DELTA, DIRAC



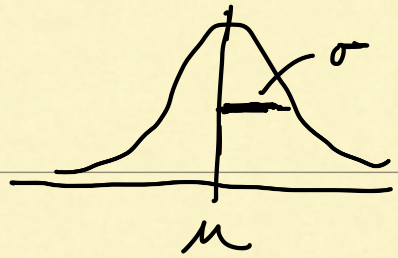
$$p_X(x) = \delta(x - x_0)$$



$$F_X = \begin{cases} 0 & x < x_0 \\ 1 & x_0 < x \end{cases}$$

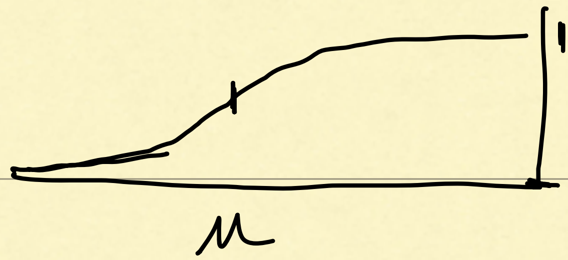
- GAUSSIAN $\quad X \sim \text{NORMAL}(\mu, \sigma)$

p_X



$$p_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

F_X

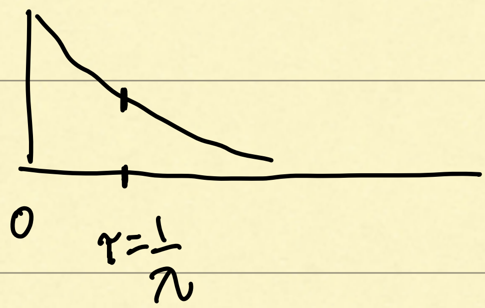


$$F_X(x) = \text{erf}(x)$$

IF $\mu = 0, \; \sigma = 1$

- EXPONENTIAL $\quad T \sim \text{EXP}(\lambda)$

p_T

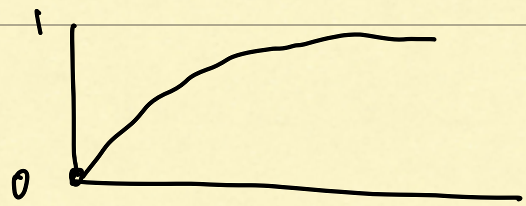


$$p_T(t) = \lambda e^{-\lambda t}$$

$t \geq 0$

$$p_T(t) = \frac{1}{\tau} e^{-t/\tau}$$

F_T

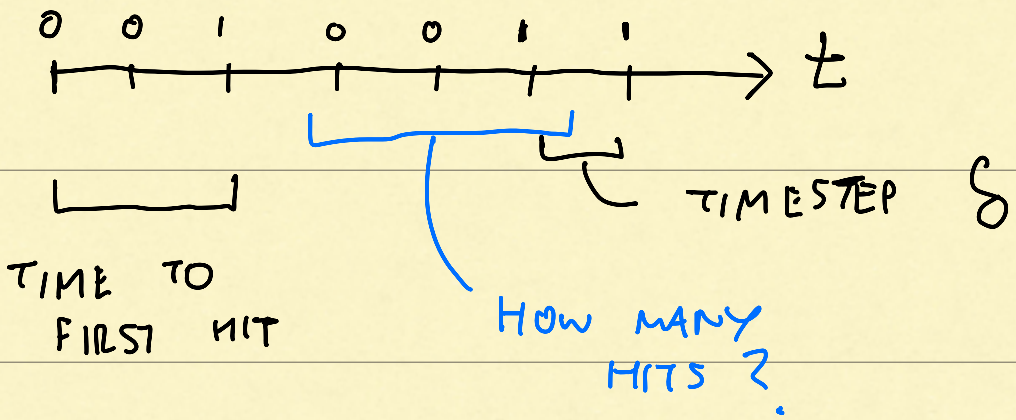


$$F_T(t) = 1 - e^{-\lambda t}$$

$$= 1 - e^{-t/\tau}$$

POISSON PROCESS

RECALL THE (DISCRETE) BERNOULLI PROCESS

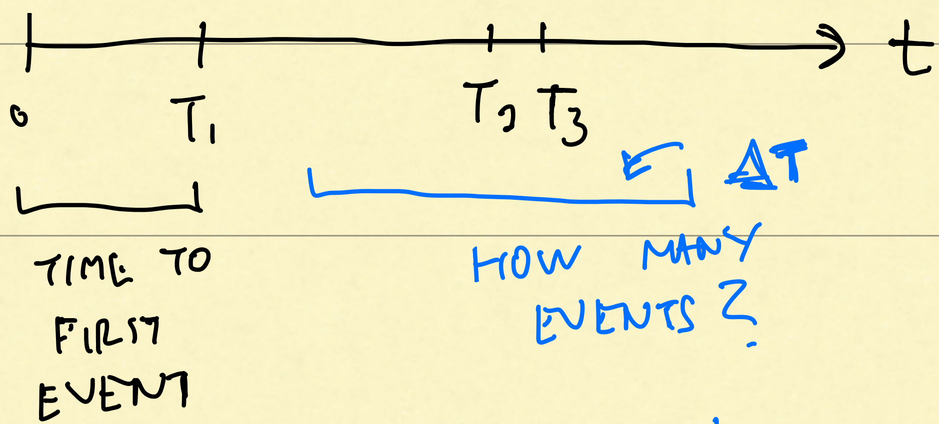


A POISSON PROCESS IS THE CONTINUOUS TIME LIMIT OF A BERNOULLI PROCESS

AS $\delta \to 0$, $p = \lambda \delta$

$\lambda = \dfrac{p}{\delta}$ FIXED

POISSON RATE



$N \sim$ POISSON( )

$T \sim \mathrm{EXP}(\lambda)$

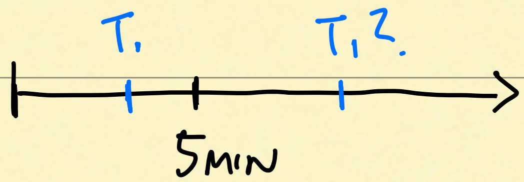
$$p_N(i) = \frac{(\lambda \Delta T)\, e}{i!}$$

$i = 0, 1, 2 \ldots$

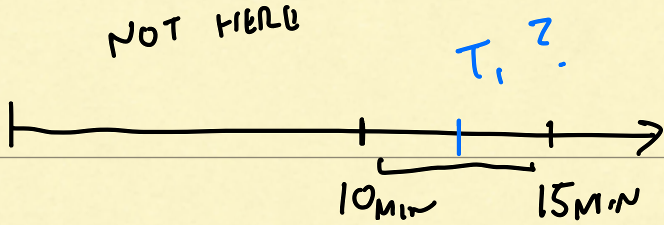
$0! = 1$

PROPERTIES

- WAITING TIME TO EVENT



$$\mathbb{P}(T_1 < 5_{\text{MIN}}) = 1 - e^{-\lambda \cdot 5\text{MIN}}$$



$$\mathbb{P}(T_1 < 15_{\text{MIN}} \mid T_1 > 10_{\text{MIN}})$$

$$= \frac{\mathbb{P}(T_1 < 15_{\text{MIN}} \cap T_1 > 10_{\text{MIN}})}{\mathbb{P}(T_1 > 10\text{ MIN})}$$

$$= \frac{\int_{10\text{MIN}}^{15\text{MIN}} p_T(t)\, dt}{1 - e^{-\lambda \cdot 10\text{MIN}}} = \ldots$$

$$= 1 - e^{-\lambda \cdot 5\text{MIN}} = \mathbb{P}(T_1 < 5_{\text{MIN}})$$

$\Rightarrow$ MEMORYLESS PROPERTY

POISSON PROCESS IS THE UNIQUE CONTINUOUS STOCHASTIC PROCESS WITH THE MEMORYLESS PROPERTY