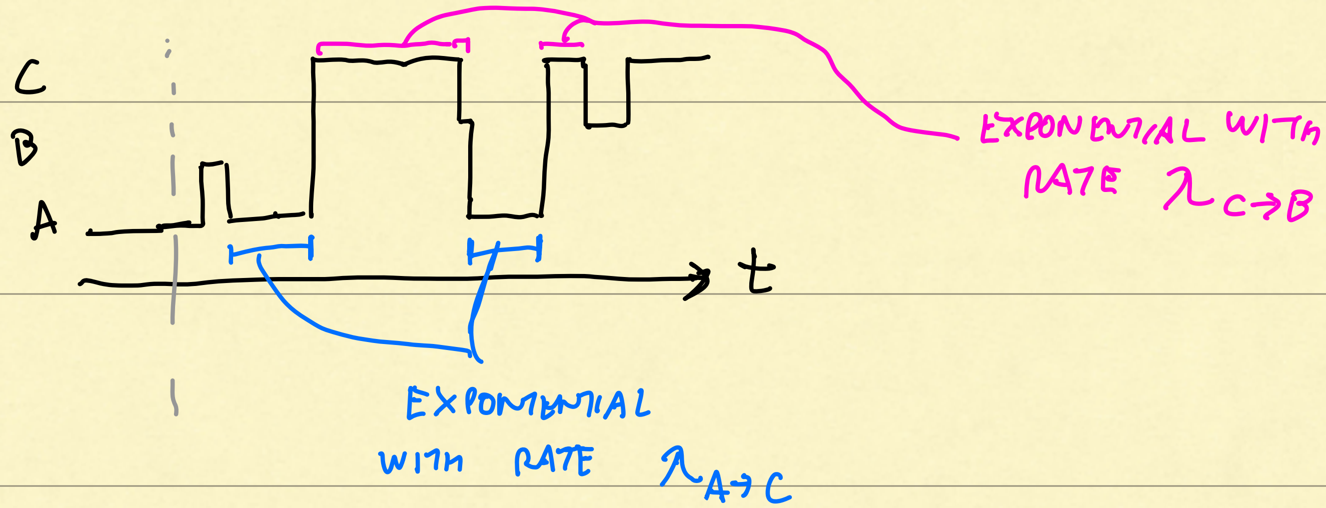
RECALL 1. MARKOV CHAIN

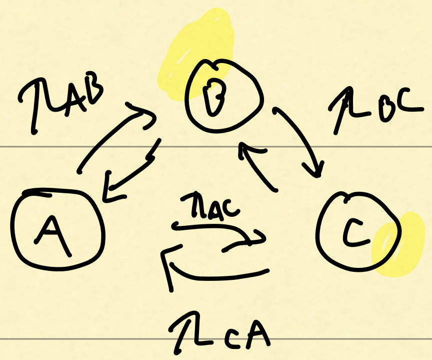
DISCRETE-TIME

2. CONTINUOUS TIME: POISSON

CONTINUOUS-TIME MARKOV CHAIN

A CONTINUOUS-TIME STOCHASTIC PROCESS WHERE STATE SPACE HAS $\underline{N}$ DISCRETE STATES, AND TRANSITIONS ARE POISSON





- THE NEXT EVENT FROM STATE B IS POISSON WITH RATE $\lambda_{BA} + \lambda_{BC}$

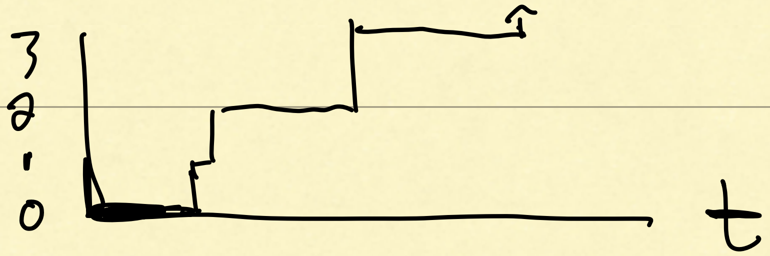
- $$\mathbb{P}(B \to C) = \frac{\lambda_{BC}}{\lambda_{BA} + \lambda_{BC}}$$

$$\vec{\mathbb{P}}(t) = \begin{bmatrix} \mathbb{P}(A;t) \\ \mathbb{P}(B;t) \\ \mathbb{P}(C;t) \end{bmatrix}$$

$$\frac{d}{dt}\vec{\mathbb{P}} = \begin{bmatrix} \boxed{-\sum_{i \neq A} \lambda_{A\to i}} & \lambda_{B\to A} & \lambda_{C\to A} \\ \lambda_{A\to B} & & \\ \lambda_{A\to C} & & \end{bmatrix} \cdot \vec{\mathbb{P}}$$

$\sum$ TO ZERO

EX $N(t)$ - # POISSON EVENTS UP TO TIME t

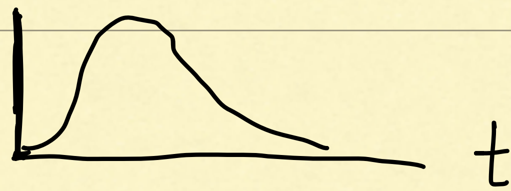


$$\frac{d\vec{\mathbb{P}}}{dt} = \begin{bmatrix} -\lambda & 0 & 0 & 0 \\ \lambda & -\lambda & 0 & 0 \\ 0 & \lambda & -\lambda & 0 \\ 0 & 0 & \lambda & -\lambda \end{bmatrix} \cdot \vec{\mathbb{P}}$$

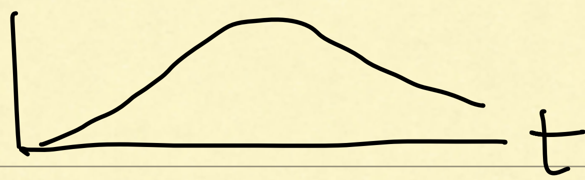
$\vdots$

$$\vec{\mathbb{P}}(t=0) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ \vdots \end{bmatrix}$$

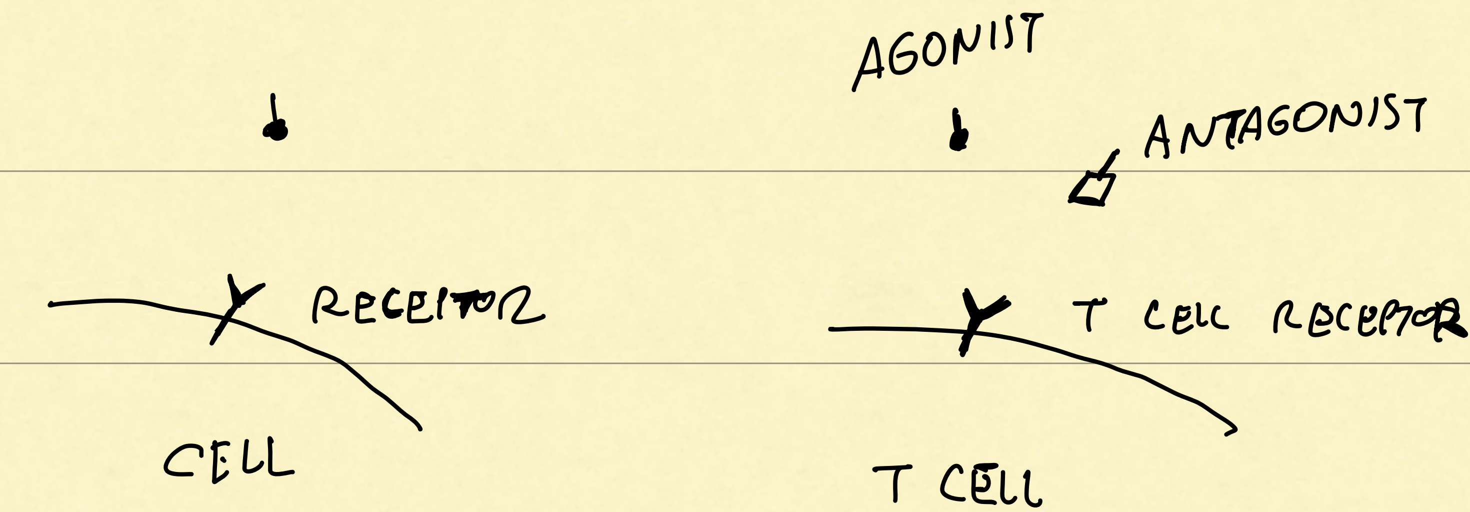
$\mathbb{P}_1(t) =$

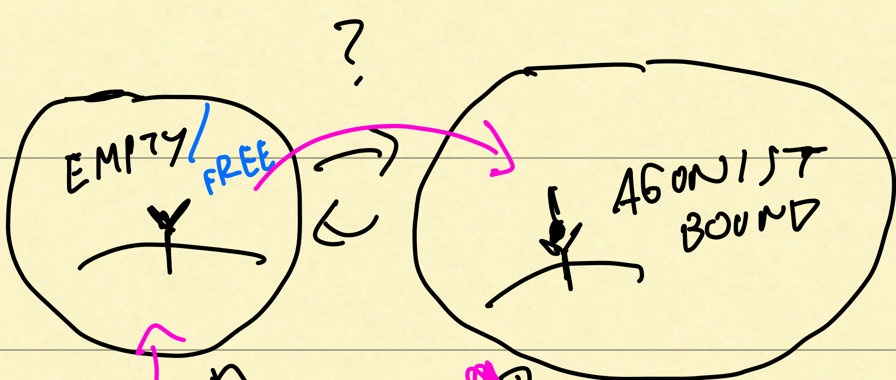


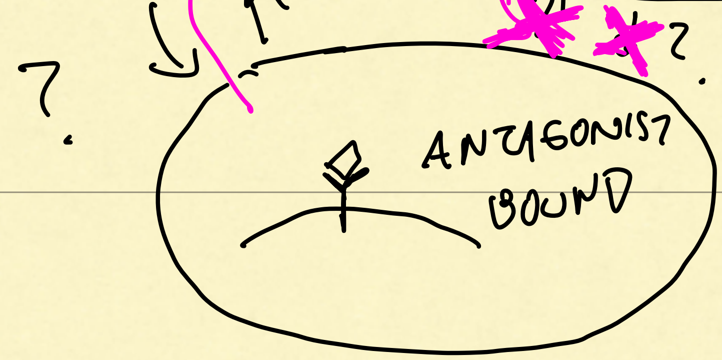
$\mathbb{P}_2(t) =$



PS 5







$$\begin{array}{cc} & \begin{array}{ccc} F & ANT & AG \end{array} \\ \begin{array}{c} F \\ ANT \\ AG \end{array} & \begin{bmatrix} -\lambda & \mu_1 & \mu_2 \\ \lambda(1-\alpha) & -\mu_1 & 0 \\ \lambda\alpha & 0 & -\mu_2 \end{bmatrix} \end{array}$$

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \quad & \quad & \quad \end{bmatrix} \cdot \begin{bmatrix} \mathbb{P}_F \\ \mathbb{P}_{ANT} \\ \mathbb{P}_{AG} \end{bmatrix}$$

$$\mathbb{P}_F + \mathbb{P}_{ANT} + \mathbb{P}_{AG} = 1$$