CONTINUOUS RANDOM VARIABLES

$X \in S \leftarrow$ SAMPLE SPACE

IF S IS CONTINUOUS . . .

EX $S = [0,1]$

$S = (-\infty, \infty)$

$S = [0, \infty)$

DEFINE $p_X(x)$ - PROBABILITY DENSITY

SUCH THAT FOR ANY SUBSET A

HAS UNITS!

$$\int_A p_X(x)\, dx = \mathbb{P}(A)$$

$$\Rightarrow \int_S p_X(x)\, dx = 1$$

CUMULATIVE DISTRIBUTION FUNCTION

$$F_X(x) = \mathbb{P}(X \leq x)$$

$$= \int_{-\infty}^{x} p_X(\tilde{x})\, d\tilde{x}$$

PROPERTIES

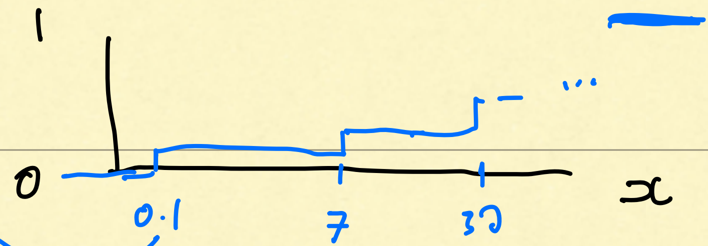
- $F_X(x) \to 0$ AS $x \to -\infty$
- $F_X(x) \to 1$ AS $x \to +\infty$
- F_X IS NON-DECREASING

- $p_X(x) = \frac{d}{dx} F_X(x)$

FEATURES OF CUMULATIVE DISTRIBUTIONS

$$\text{DATA} = \begin{bmatrix} 0.1 \\ 7 \\ 32 \\ \vdots \end{bmatrix}$$

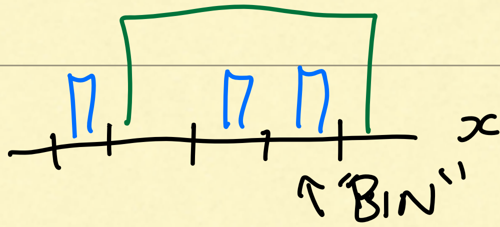
CUMULATIVE = ?



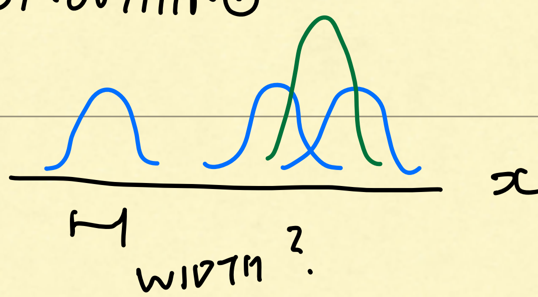
EMPIRICAL CUMULATIVE

DENSITY = ?

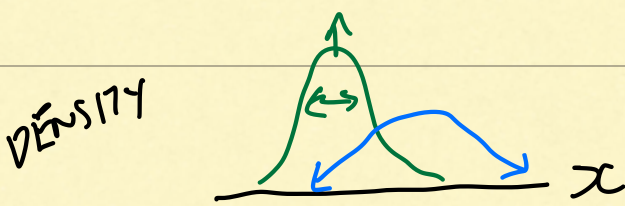
HISTOGRAM

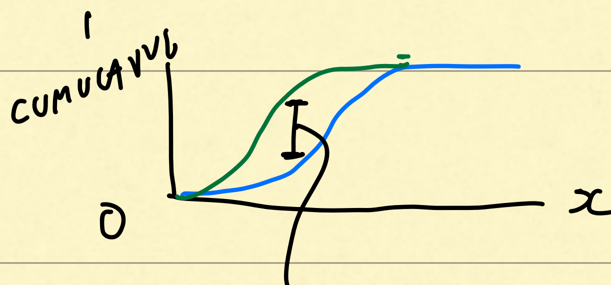


KERNEL SMOOTHING



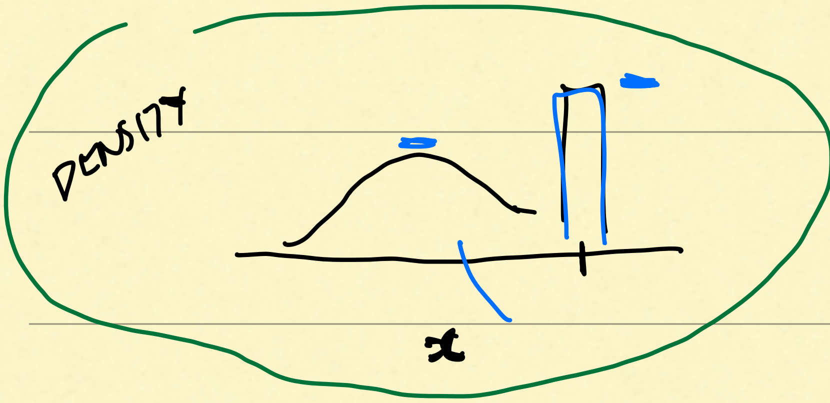
- COMPARING DISTRIBUTIONS



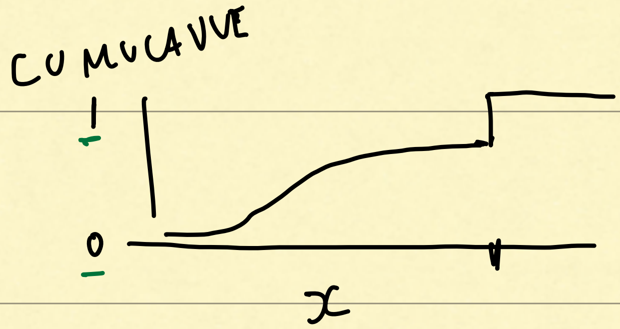


MAX DIFFERENCE

- MIX OF CONTINUOUS & DISCRETE VALUES

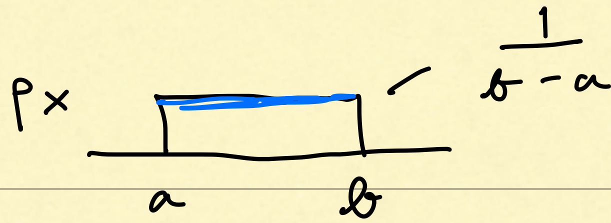


$p_X(x)$

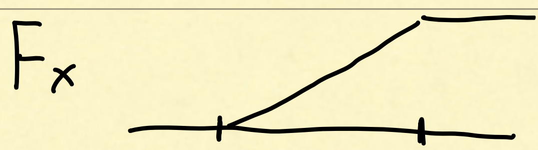


FAMOUS CONTINUOUS RANDOM VARIABLES

- UNIFORM $\quad X \sim \text{UNIF}(a, b)$



$$p_X(x) = \begin{cases} \frac{1}{b-a} & a<x<b \\ 0 & \text{ELSE} \end{cases}$$

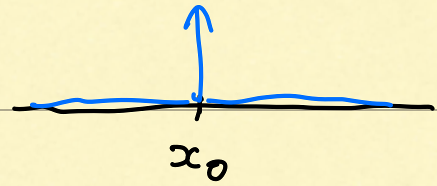


$$F_X(x) = \begin{cases} 0 & x<a \\ \frac{x-a}{b-a} & a<x<b \\ 1 & b<x \end{cases}$$

- DELTA "DIRAC"

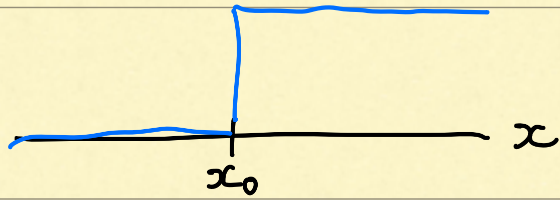
p_x

$p_X(x)$



$p_X(x) = \delta(x - x_0)$

$F_X(x)$

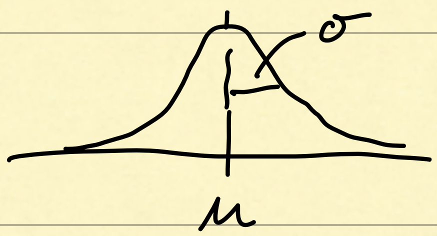


$$F_X(x) = \begin{cases} 0 & x < x_0 \\ 1 & x > x_0 \end{cases}$$

- GAUSSIAN

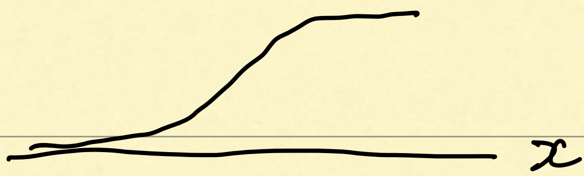
$X \sim \text{NORMAL}(\mu, \sigma)$

$p_X(x)$



$$p_X(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-\mu)^2}{\sigma^2}}$$

$F_X(x)$



$F_X(x) = \text{ERF}(x)$

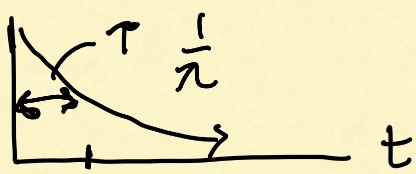
Z - STANDARD NORMAL $(\mu = 0, \ \sigma = 1)$

- EXPONENTIAL $\quad T \sim \text{EXP}(\lambda) = \text{EXP}(\tau)$

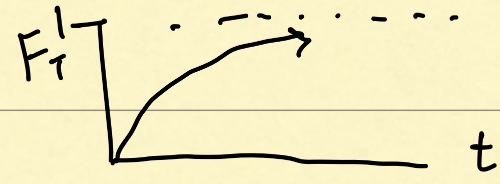
$p_T(t) = \lambda e^{-\lambda t}$ $\qquad \tau = 1/\lambda$

$= \frac{1}{\tau} e^{-t/\tau}$

p_T

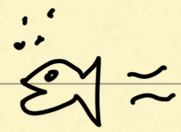


$$F_T(t) = 1 - e^{\lambda t}$$
$$= 1 - e^{-t/\tau}$$

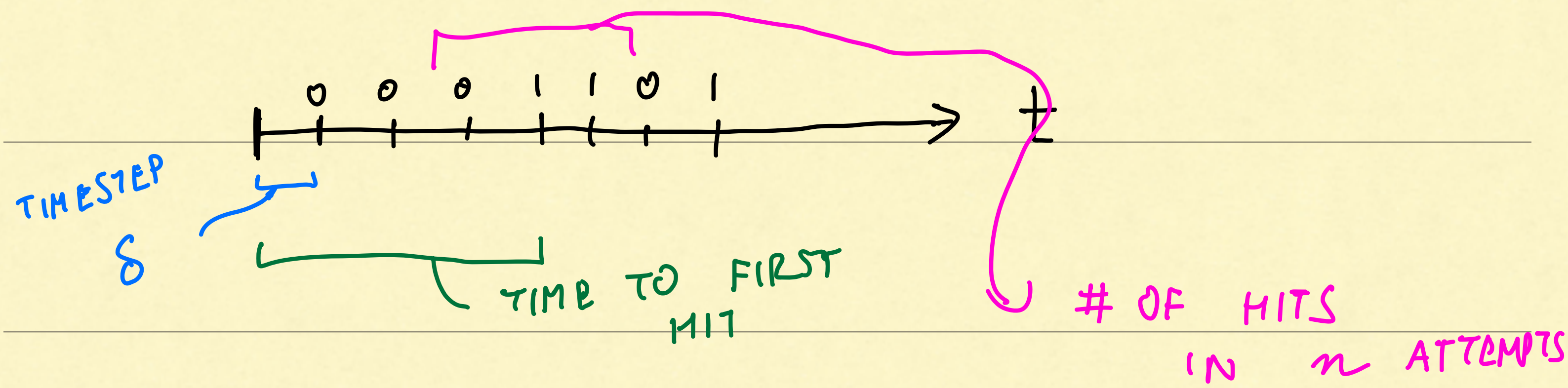


$$E[T] = \frac{1}{\lambda} = \tau$$

POISSON PROCESS

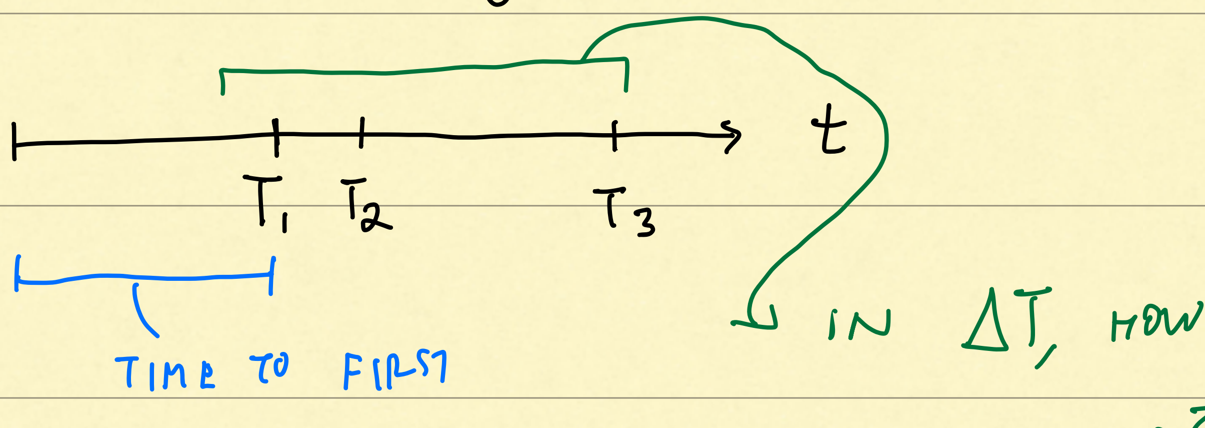


RECALL A SEQUENCE OF BERNOULLI TRIALS



A POISSON PROCESS IS A CONTINUOUS TIME STOCHASTIC PROCESS THAT IS THE LIMIT OF BERNOULLI TRIAL SEQUENCE AS $\delta \to 0$

$$p = \lambda \delta, \qquad \lambda = \frac{p}{\delta} \text{ FIXED}$$



EVENT

$T_1 \sim EXP(\lambda)$

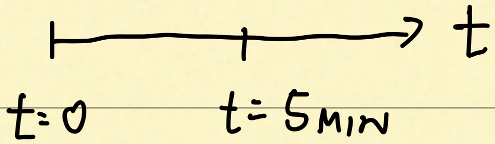
MANY EVENTS:

$N \sim$ POISSON

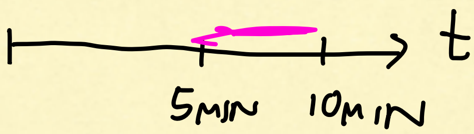
$$p_N(i) = \frac{(\lambda \Delta T)^i e^{-\lambda \Delta T}}{i!}$$

PROPERTIES

WAITING TO AN EVENT



$$\mathbb{P}(T_1 < 5\,MIN) = 1 - e^{-\lambda \cdot 5MIN}$$



$$\mathbb{P}(T_1 < 10MIN \mid T_1 > 5MIN) = \frac{\mathbb{P}(T_1 < 10MIN \cap T_1 > 5MIN)}{\mathbb{P}(T_1 > MIN)}$$

$$= \frac{\int_5^{10} \lambda e^{-\lambda t}\,dt}{1-(1-e^{-\lambda 5})} = \ldots$$

$$= 1 - e^{-\lambda \cdot 5MIN}$$

THIS IS THE MEMORYLESS PROPERTY.

POISSON PROCESS IS THE UNIQUE MEMORYLESS CONTINUOUS TIME STOCHASTIC

PROCESS