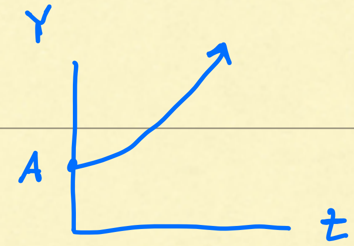
EX $\quad \dfrac{dY}{dt} = R\,Y$



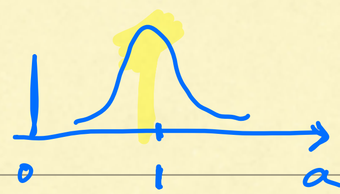
$Y(0) = A$

SOLUTION

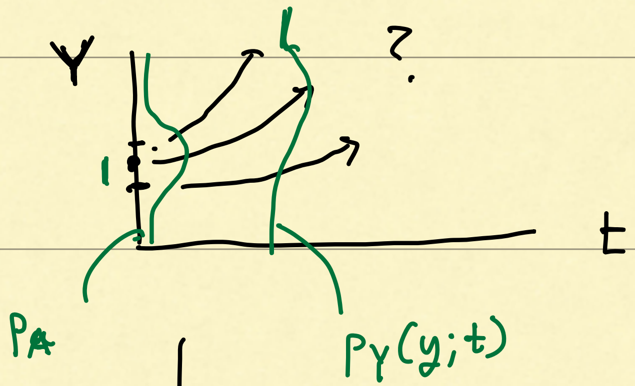
$$Y(t) = A\,e^{Rt}$$

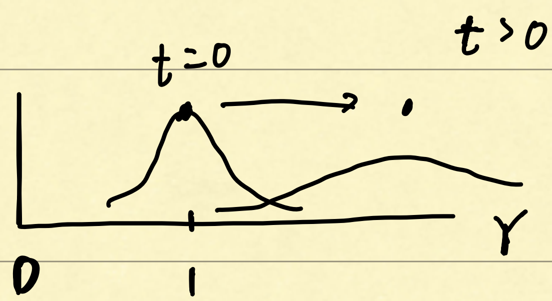
SUPPOSE $\quad A \sim p_A(a)$

$$p_A(a) = \frac{1}{\sqrt{2\pi}\,\sigma}\, e^{-\frac{(a-1)^2}{2\sigma^2}}$$



WHAT IS $\quad p_Y(y;t)$





NOTE $\quad Y = g(A)$

$$g(a) = a\,e^{Rt}$$

$y = ae^{Rt}$

$$p_Y(y) = p_A\left(g^{-1}(y)\right) \cdot \left|\left(\frac{dg}{da}\right)^{-1}\right|$$

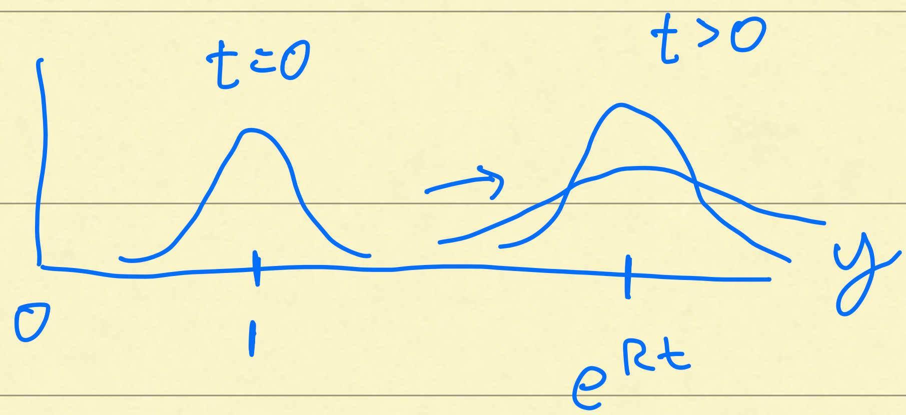
$$g^{-1} = \frac{y}{e^{Rt}} = ye^{-Rt}$$

$$\frac{dg}{da} = e^{Rt}$$

$$\boxed{p_Y(y) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(ye^{-Rt}-1)^2}{2\sigma^2}} \cdot e^{-Rt}}$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma e^{+Rt}} e^{-\frac{(y-e^{+Rt})^2}{2(\sigma e^{+Rt})^2}}$$

$$= \frac{1}{\sqrt{2\pi}\,\sigma e^{+Rt}} \cdot \exp\left(-\frac{(y-e^{+Rt})^2}{2(\sigma e^{+Rt})^2}\right)$$

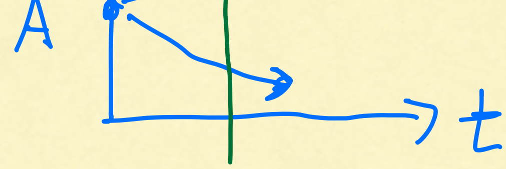


PS6

$$\frac{dY}{dt} = RY$$

$p_R(r)$

$$Y(0) = A$$



$$P_Y(y:t) = ?$$

$$p_Y(y) = p_R\Big(\underbrace{g^{-1}(y)}_{\text{INVERSE}}\Big) \cdot \left| \frac{1}{\frac{dg}{dr}} \right|$$

g

$$\underbrace{y = e^{(r-1)t}} \qquad y = g(r)$$

$$g(r) = e^{(r-1)t}$$

$$\rightarrow \quad \frac{dg}{dr} \stackrel{?}{=} t\, e^{(r-1)t}$$

$$g^{-1}(y) = \frac{\ln(y)}{t} + 1$$

$$p_Y(y) \overset{?}{=} p_R(g^{-1}(y)) \cdot \left| \frac{d(g^{-1})}{dy} \right|$$