THE ENTIRE LIKELIHOOD LANDSCAPE

EX $\quad T \sim p_T(t) = \lambda e^{-\lambda t}$ $\qquad \lambda$ - RATE

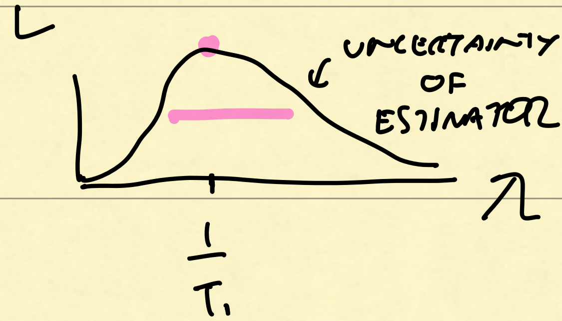
EXPONENTIAL

$$= \frac{1}{\tau} e^{-t/\tau}$$

$\tau = \frac{1}{\lambda}$ MEAN TIME

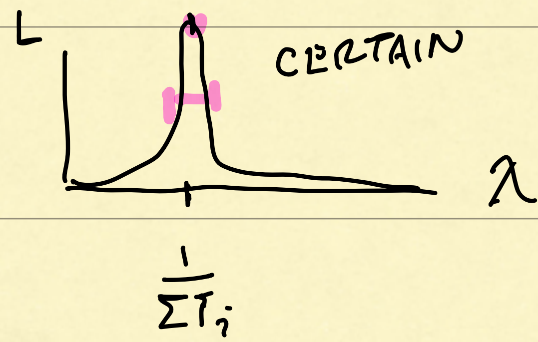
DATA $\quad T_1$

$$L(\lambda) = \lambda e^{-\lambda \cancel{t}}$$
$$= \lambda e^{-\lambda T_1}$$



DATA $\quad T_1 \ldots T_N$

$$L(\lambda) = \prod_{i=1}^{N} \lambda e^{-\lambda T_i}$$

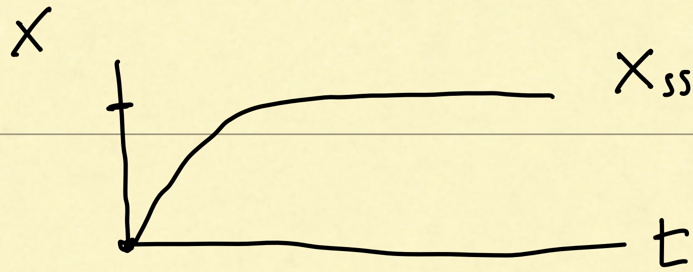


EX $\quad \frac{dX}{dt} = k_{on} - k_{off} X \quad \leftarrow \quad X_{ss} = \frac{k_{on}}{k_{off}}$

$$X(0) = 0$$

$$Y = X + \epsilon$$

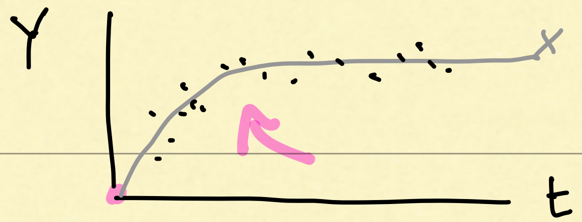
RANDOM iid NORMAL



$X_{ss} = k_{on}/k_{off}$

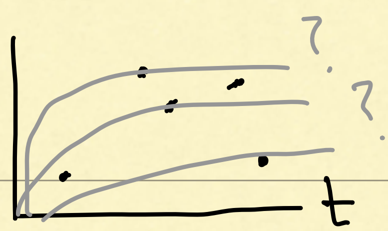
$$X(t) = \frac{k_{on}}{k_{off}}\left(1 - e^{-k_{off}t}\right)$$

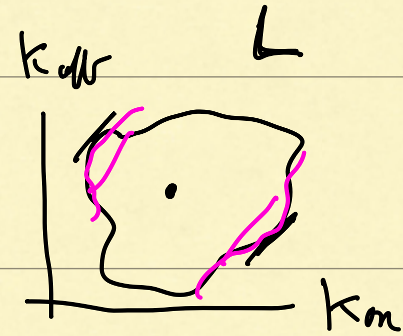
$$= X_{ss}\left(1 - e^{-t/\tau}\right) \qquad \tau = \frac{1}{k_{off}}$$



HIGH σ

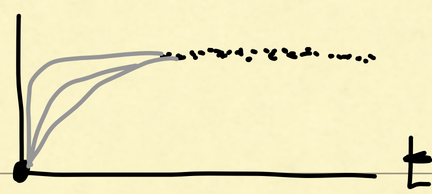
LOW DATA

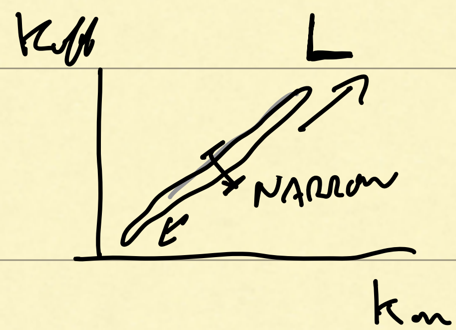




LOW σ

LATE DATA





ISSUE WITH LIKELIHOOD LANDSCAPE: NOT A PROBABILITY DISTRIBUTION

- DOES NOT INTEGRATE TO UNITY
- NO MEAN, NO STANDARD DEVIATION, CONFIDENCE INTERVALS

BAYESIAN STATISTICS

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)}$$

FOR ANY A, B

$P(x;\theta) = L(\theta)$

$$p(\theta\,|\,x) = \frac{\boxed{P(x|\theta)}\;\underbrace{P(\theta)}_{\text{PRIOR}}}{P(x)}$$

PARAMETERS, DATA

POSTERIOR

$\int p(x)\,dx = 1$

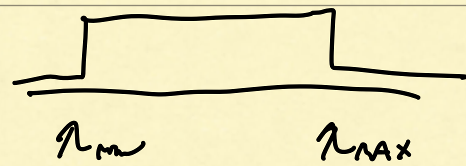
$$= \frac{L(\theta)\;P(\theta)}{P(x)}$$

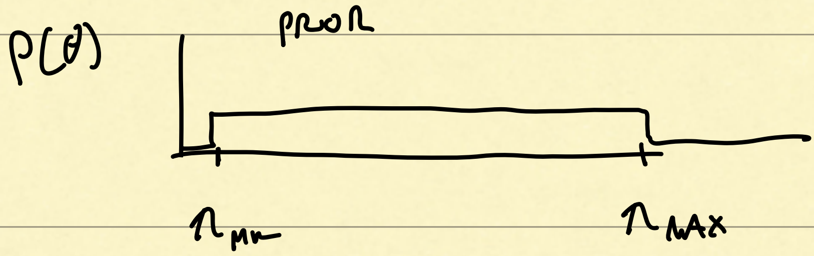
INSTEAD OF $L(\theta)$, LOOK AT $P(\theta|x)$

EX $\quad T \sim p_T(t) = \lambda e^{-\lambda t}$

PRIOR $\lambda \sim$ UNIFORM IN $(\lambda_{MIN}, \lambda_{MAX})$

$$P(\theta) = \frac{1}{\lambda_{MAX} - \lambda_{MIN}}$$





NOTES

1) AS $\lambda_{MIN} \to 0$, $\lambda_{MAX} \to \infty$

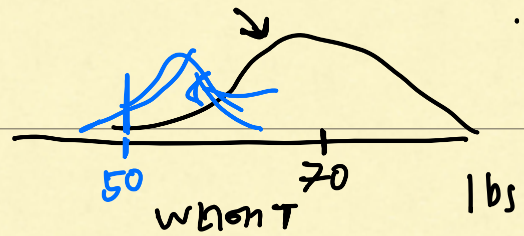
$$p(\lambda|t) \to \frac{\lambda e^{-\lambda t}}{(1/t^2)}$$

POSTERIOR IS STILL WELL-DEFINED EVEN THOUGH THE PRIOR IS ILL-DEFINED

→ "IRREGULAR PRIOR"

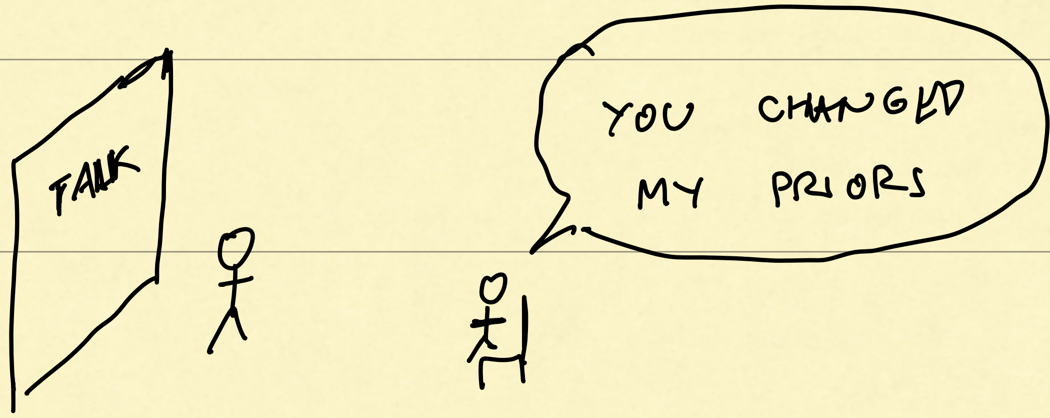
2 NON-UNIFORM PRIORS

- MY DOG WEIGHS 70 lbs
- VET SCALE : 50 lbs



PRIOR - PRIOR BELIEF

POSTERIOR - UPDATED BELIEF



2b RIDGE: PRIOR $p(\beta_i) = e^{-\frac{\sum \beta_i^2}{\lambda}}$

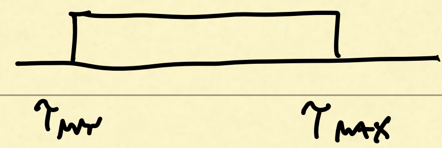
LASSO: PRIOR $p(\beta_i) = e^{-\frac{\sum |\beta_i|}{\lambda}}$

$\boxed{3}$ T - EXPONENTIAL

$$p_T(t) = \frac{1}{\tau} e^{-t/\tau}$$

τ - PARAMETER

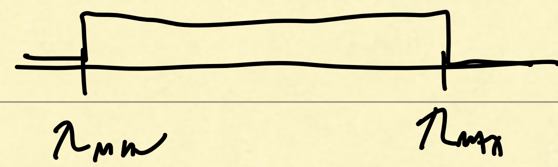
PRIOR: $P(\tau) \sim$ UNIF



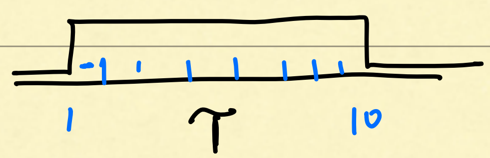
$$P_\tau(t) = \lambda e^{-\lambda t}$$

λ - PARAMETER

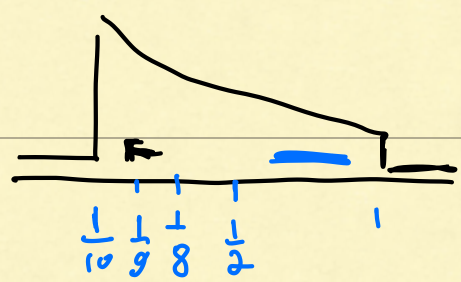
PRIOR $P(\lambda) \sim$ UNIF



$$\lambda = \frac{1}{\tau}$$



$\Rightarrow$



$\Rightarrow$ NO SUCH THING AS FLAT PRIORS