1 POISSON PROCESS

1 EVENT

$E[T] = E[T \mid A]\,\mathbb{P}(A) + E[T \mid B]\,\mathbb{P}(B)$

TIMING? RACING?

A. Your advisor has scheduled two appointments with two graduate students, one at 1pm and the other at 1:30pm. The amounts of time that appointments last are independent exponential random variables with mean of 30 minutes. Assuming both graduate students arrive on time, find the expected amount of time that the 2nd grad student spends waiting outside the advisor's office (and/or Zoom meeting waiting room).

B. The "two-hit hypothesis" is a model of cancer that assumes that two mutations are required for a normal cell to exhibit cancerous behavior. Although the two-hit hypothesis is an oversimplification, it led indirectly to the discovery of tumor suppressor genes.

Suppose there are two genes, Gene A and Gene B, each of which undergoes mutation at rate λ (in units of mutations per year).

i. How long, on average, until (either) one of the genes has mutated?

$E[T]: \frac{1}{2\lambda}$

ii. How long, on average, until both genes have mutated? (Hint: Another question one might ask is, unrealistically, if Gene B could only mutate after Gene A had mutated, how long on average until both genes have mutated?)

Now suppose the mutation rates for the two genes are different, λ_A and λ_B.

iii. How long, on average, until (either) one of the genes has mutated?

iv. How long, on average, until both genes have mutated?

? $E[T] = \frac{1}{\lambda_A + \lambda_B}$

$\boxed{\frac{3}{2}\frac{1}{\lambda}} \overset{?}{\approx} 1.5\,\frac{1}{\lambda}$ $< 2\left(\frac{1}{\lambda}\right)$

A FIRST

B FIRST

$E[\text{BOTH}] = E[\text{BOTH} \mid \text{A FIRST}] \cdot \mathbb{P}(\text{A FIRST}) + E[\text{BOTH} \mid \text{B FIRST}] \cdot \mathbb{P}(\text{B FIRST})$