

Many processes, including the spread of an infectious disease through a small community, can be modeled as first-order exponential processes like

$$\frac{dV}{dt} = \frac{1}{\tau} (R - 1) V, \quad V(0) = 1 \quad (1)$$

where V is the tumor volume, measured in number of cells, and R is a constant.

This will either lead to exponential growth or exponential decay.

The constant R is different for every patient. Assume it has Gaussian distribution with mean 1 and standard deviation σ ,

$$p_R(r) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(r-1)^2/2\sigma^2}.$$

- i. Find the probability density function $p_V(v, t)$ of $V(t)$.

Intuitively, we expect half of the trajectories to grow exponentially, and half of the trajectories to decay exponentially.

- ii. Sketch or plot the probability density you found for $p_V(v, t)$.
- iii. What is the probability that a trajectory is above the initial condition at $V = 1$? In other words, what is $\mathbb{P}(V(t) > 1)$? Is it true that half the trajectories remain above the initial condition $V = 1$, and half remain below the initial condition $V = 1$?
- iv. Suppose $\tau = 1$ months and $\sigma = 0.1$. What percent of patients have a tumor with more than 1000 cells after 10 months?

A slightly more complicated model that is a modified version of Equation 1, called the Gompertz model, is used to fit patient data.