Camera Pose from Homography Matrix

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Let $R \in \mathbf{R}^{3\times 3}$ and $T \in \mathbf{R}^3$ be the rotation and translation of the camera. Let $f_x = fm_x, f_y = fm_y$ be the focal length of the camera.

Then the equation for

$$w\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & 0 & 0 \\ 0 & f_y & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R_{11} & R_{12} & R_{13} & T_1 \\ R_{21} & R_{22} & R_{23} & T_2 \\ R_{31} & R_{32} & R_{33} & T_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} M_{11}x_1 + M_{12}x_2 + M_{13} \\ M_{21}x_1 + M_{22}x_2 + M_{23} \\ M_{31}x_1 + M_{32}x_2 + M_{33} \end{bmatrix} = \begin{bmatrix} f_x & 0 & 0 & 0 \\ 0 & f_y & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R_{11}x_1 + R_{12}x_2 + T_1 \\ R_{21}x_1 + R_{22}x_2 + T_2 \\ R_{31}x_1 + R_{32}x_2 + T_3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} M_{11}x_1 + M_{12}x_2 + M_{13} \\ M_{21}x_1 + M_{22}x_2 + M_{23} \\ M_{31}x_1 + M_{32}x_2 + M_{33} \end{bmatrix} = \begin{bmatrix} f_x(R_{11}x_1 + R_{12}x_2 + T_1) \\ f_y(R_{21}x_1 + R_{22}x_2 + T_2) \\ R_{31}x_1 + R_{32}x_2 + T_3 \end{bmatrix}$$

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} = \begin{bmatrix} f_xR_{11} & f_xR_{12} & f_xT_1 \\ f_yR_{21} & f_yR_{22} & f_yT_2 \\ R_{31} & R_{32} & T_3 \end{bmatrix}$$

$$\begin{aligned} \text{find } & f_x, f_y, R \\ \text{such that } & \\ & R \text{ is a rotation matrix} \\ & R_{31} = M_{31} \\ & R_{32} = M_{32} \\ & T_3 = M_{33} \end{aligned}$$