Camera Pose from Homography Matrix

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November 17, 2020

Let $R \in \mathbf{R}^{3 \times 3}$ and $T \in \mathbf{R}^3$ be the rotation and translation of the camera. Let $f_x = f m_x, f_y = f m_y$ be the focal length of the camera.

Then the equation for

$$
w \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 1 \end{bmatrix} = \begin{bmatrix} f_x & 0 & 0 & 0 \\ 0 & f_y & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R_{11} & R_{12} & R_{13} & T_1 \\ R_{21} & R_{22} & R_{23} & T_2 \\ R_{31} & R_{32} & R_{33} & T_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ 0 \\ 1 \end{bmatrix}
$$

$$
\begin{bmatrix} M_{11}x_1 + M_{12}x_2 + M_{13} \\ M_{21}x_1 + M_{22}x_2 + M_{23} \\ M_{31}x_1 + M_{32}x_2 + M_{33} \end{bmatrix} = \begin{bmatrix} f_x & 0 & 0 & 0 \\ 0 & f_y & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} R_{11}x_1 + R_{12}x_2 + T_1 \\ R_{21}x_1 + R_{22}x_2 + T_2 \\ R_{31}x_1 + R_{32}x_2 + T_3 \\ 1 \end{bmatrix}
$$

$$
\begin{bmatrix} M_{11}x_1 + M_{12}x_2 + M_{13} \\ M_{21}x_1 + M_{22}x_2 + M_{23} \\ M_{31}x_1 + M_{32}x_2 + M_{33} \end{bmatrix} = \begin{bmatrix} f_x(R_{11}x_1 + R_{12}x_2 + T_1) \\ f_y(R_{21}x_1 + R_{22}x_2 + T_2) \\ R_{31}x_1 + R_{32}x_2 + T_3 \end{bmatrix}
$$

$$
\begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix} = \begin{bmatrix} f_x R_{11} & f_x R_{12} & f_x T_1 \\ f_y R_{21} & f_y R_{22} & f_y T_2 \\ R_{31} & R_{32} & T_3 \end{bmatrix}
$$

find f_x, f_y, R

such that

R is a rotation matrix

$R_{31} = M_{31}$

$R_{32} = M_{32}$

$T_3 = M_{33}$