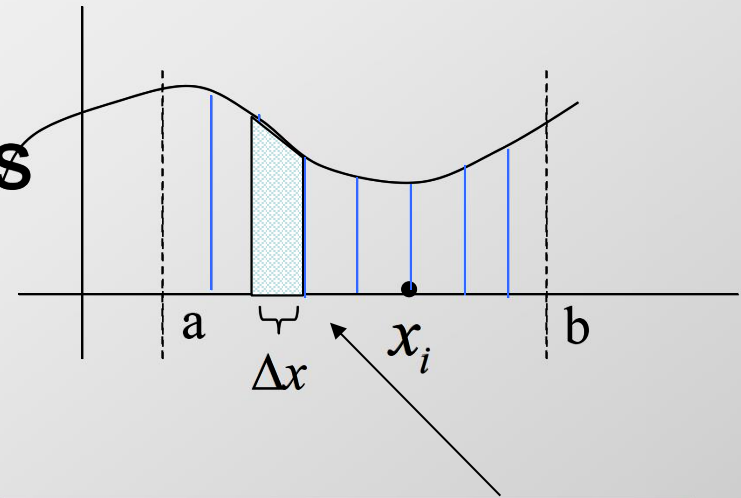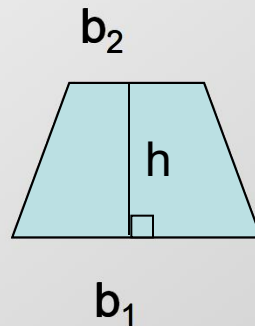# Numerical Integration

Trapezoidal and Simpson Rules

Trapezoidal Rule

- Instead of calculating approximation rectangles we will use trapezoids
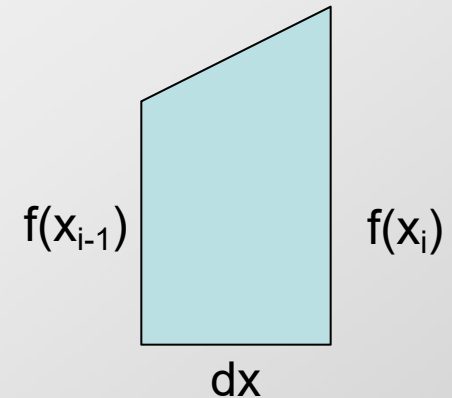 - More accuracy

- Area of a trapezoid

$$A = \frac{1}{2}\left(b_1 + b_2\right) \cdot h$$

b_2

b_1

h

a

Δx

x_i

b

- Which dimension is the h?
- Which is the b_1 and the b_2

Trapezoidal Rule

- Trapezoidal rule approximates the integral

f(x_{i-1})   f(x_i)

dx

$$\int_a^b f(x)dx \approx \frac{dx}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \dots 2f(x_{n-1}) + f(x_n)\right]$$

where $dx = \dfrac{b-a}{n}$

- Calculator function for f(x)

$(\sum(2*f(a+k*(b-a)/n),k,1,n-1)+f(a)+f(b))*(b-a)/(n*2)$
$\rightarrow$trap(a,b,n)

Algorithm (Trapezoidal Rule)

1. Define f(x)
2. Enter the values of lower and upper limit of x (x0, xn)
3. Enter number of intervals N.
4. h = {(xn-x0)/N }
5. sum = 0
6. Repeat until x0>=xn

       a) sum = sum + (h/2).[f(x0) + f(x0 + h)]
       b) x0 = x0 +h

7. Print sum
8. Compute absolute error
E = |true error/true value| x 100 %

Trapezoidal Rule

- Try $\displaystyle\int_{0}^{2} 2x^5 \, dx \quad n = 8$ using the trapezoidal rule

- Check with integration true value
- Calculate absolute error

Simpson's Rule

Snidly Fizbane Simpson

- ## As before, we divide the interval into *n* parts
  - ▪ *n* must be even

- ## Instead of straight lines we draw parabolas through each group of three consecutive points
  - ▪ This approximates the original curve for finding definite integral – formula shown below
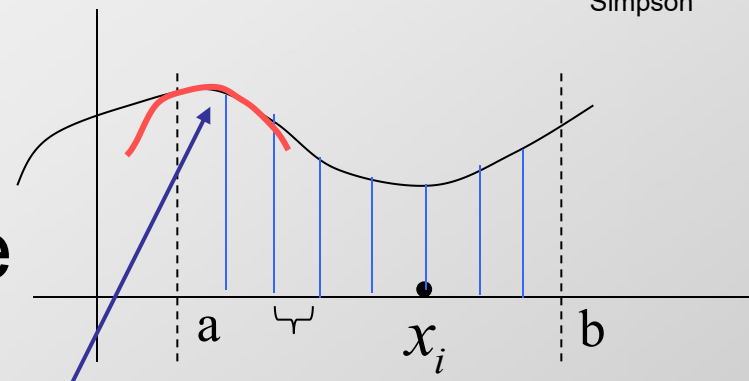
$$\int_a^b f(x)dx \approx \frac{dx}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + 2f(x_4)$$

$$+ ... + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

Algorithm (Simpson's Rule)

- 1. Define f(x)
- 2. Enter the values of lower and upper limit of x (x0, xn)
- 3. Enter number of intervals N.
- 4. h = {(xn-x0)/N }
- 5. sum = 0
- 6. Repeat until x0>=xn

- a) sum = sum + (h/3).[f(x0) +4f(x0 +h) +f(x0 + 2h)]
- b) x0 = x0 +2h
- 7. Print sum
- 8. Compute absolute error
- E = |true error/true value| x 100 %

Simpson's Rule

- Specify a function for f(x)
- When you call simp(a,b,n),
    - Make sure *n* is an <u>even</u> number

- Note the accuracy of the approximation

Error Estimation

- Trapezoidal error for f on [a, b]

$$\left|E_n\right| \leq \frac{(b-a)^3}{12n^2} \cdot M$$

  - Where M = max value of |f ''(x)| on [a, b]

- Simpson's error for f on [a, b]

$$\left|E_n\right| \leq \frac{(b-a)^5}{180n^4} \cdot K$$

  - Where K = max value of |f $^{(4)}$(x)| on [a, b]