Tutorial Sheet-4

Sol 1 $T(n) = 3T(n/2) + n^2$

$a = 3,\ b = 2,\ f(n) = n^2$

$\because$ a & b are const. & f(n) is a +ve function

Masters theorem is applicable

$c = \log_b a = \log_2 3 = 1.58$

$\Rightarrow n^c = n^{1.58}$

which is $n^2 > n^{1.58}$

$\therefore$ case 3 is applied here

$$\boxed{T(n) = \Theta(n^2)}$$

Sol 2: $T(n) = 4T(n/2) + n^2$

$a = 4\ \ b = 2,\ f(n) = n^2$

$\because$ a & b are const and f(n) is a +ve function

$\therefore$ Master's theorem is applicable

$c = \log_b a$

$= \log_2 4 = \log_2 2^2 = 2\log_2 2 = 2$

$\Rightarrow n^c = n^2$

which is $n^2 = f(n)$

Case 2 is applied here

$T_n = \Theta(n^2 \log n)$

Sol3 $T(n) = T(n/2) + 2^n$

$a=1 \quad b=2 \quad f(n) = 2^n$

a & b are const. and f(n) is a +ve func.

Masters theorem is applicable

$c = \log_b a = \log_2 1$

$= 0$

$\Rightarrow n^c = n^0 = 1$

$f(n) > n^c$

case 3 is applied here

$\boxed{T(n) = \Theta(2^n)}$

Sol4: $T(n) = 2^n T(n/2) + n^n$

$a = 2^n \quad b = 2 \quad f(n) = n^n$

$\because$ a not const, it's value depends on n

$\therefore$ Masters theorem is not applicable.

Sol5: $T(n) = 16T(n/4) + n$

$a = 16, \; b = 4 \quad f(n) = n$

a & b are const. and f(n) is a +ve func

$\therefore$ Masters theorem is applicable here.

$c = \log_b a = \log_4 16 = \log_4 (4)^2$

$= 2 \log_4 4 = 2$

$\Rightarrow n^c = n^2$

$\because f(n) < n^c$

$\therefore$ Case 1 is applied here

$\boxed{T(n) = \Theta(n^2)}$

Sol6: $T(n) = 2T(n/2) + n\log n$

$a = 2 \quad b = 2 \quad f(n) = n\log n$

$\because$ a & b are const. & $f(n)$ is a +ve function

$c = \log_b a$

$\log_2 2 = 1$

$n^c = n$

$n < n\log n \Rightarrow f(n) > n^c$

$\therefore$ case 3 is applied

$$\boxed{T(n) = \Theta(n\log n)}$$

Sol7 $T(n) = 2T(n/2) + n/\log n$

$a = 2 \quad b = 2 \quad f(n) = n/\log n$

$\because$ a & b are const. & $f(n)$ is a +ve function

$c = \log_b a$

$= \log_2 2 = 1$

$n^c = n^1 = n$

$\because$ non-polynomial difference b/w $f(n)$ & n^c

$\therefore$ Master's theorem is not applicable.

Sol8: $T(n) = 2T(n/4) + n^{0.51}$

$a = 2 \quad b = 4 \quad f(n) = n^{0.51}$

a & b are consts and $f(n)$ is a +ve function

$\therefore$ Master theorem is applicable

$c = \log_b a = \log_4 2 = 0.50$

$n^c = n^{0.50}$

$f(n) > n^c$

Case 3 is applicable $\Rightarrow \boxed{T(n) = \Theta(n^{0.50})}$

Sol 9: $T(n) = 0.5\, T(n/2) + 1/n$

$a = 0.5 \quad b = 2 \quad f(n) = 1/n$

$\because a < 1$

$\therefore$ Master's theorem is not applicable.

Sol 10 $T(n) = 16\, T(n/4) + n!$

$a = 16 \quad b = 4 \quad f(n) = n!$

a & b are const and f(n) is a +ve function

$\therefore$ Master's theorem is applicable

$c = \log_b a$

$= \log_4 16 = \log_4 4^2 = 2 \log_4 4 = 2$

$n^c = n^2$

$f(n) > n^c$

case 3 is applied here

$\boxed{T(n) = \Theta(n!)}$

Sol 11 $T(n) = 4T(n/2) + \log n$

$a = 4 \quad b = 2 \quad f(n) = \log n$

a and b are constant and f(n) is a +ve func

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_2 4 = \log_2 2^2 = 2 \log_2 2 = 2$

$n^c = n^2$

$f(n) < n^c$

case 1 is applied

$\boxed{T(n) = \Theta(n^2)}$

Sol12 $\sqrt{n}\, T(n/2) + \log n$

$a = \sqrt{n}$, $b = 2$ $f(n) = \log n$

$\because$ a is not constant

$\therefore$ Master's theorem is not applicable.

Sol13: $T(n) = 3T(n/2) + n$

$a = 3$ $b = 2$ $f(n) = n$

$\because$ a & b are const. and $f(n)$ is a +ve func

$\therefore$ Master's theorem is applicable

$c = \log_b a = \log_2 3 = 0.1.58$

$n^c = n^{1.58}$

$\because f(n) < n^c$

$\therefore$ case 1 is applied here

$T(n) = \Theta(n^{1.58})$

Q14 $T(n) = 3T(n/3) = \sqrt{n}$

$a = 3$ $b = 3$, $f(n) = \sqrt{n}$

a and b are constant and $f(n)$ is a +ve func.

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$\because f(n) < n^c$

$\therefore$ case 1 is applicable

$$\boxed{T(n) = \Theta(n)}$$

Sol 16 $T(n) = 4T(n/2) + c \cdot n$

$a = 4$, $b = 2$, $f(n) = c \cdot n$

$\because$ a & b are constant & $f(n)$ is a +ve f^n

$\therefore$ Master's theorem is applicable here.

$c = \log_b a = \log_2 4 = \log_2 2^2 = 2 \log_2 2 = 2$

$n^c = n^2$

$\because f(n) = n^c$

$\therefore$ case 1 is applied here.

$\boxed{T(n) = \theta(n^2)}$

Sol 16 $T(n) = 3T(n/4) + n \log n$

$a = 3$, $b = 4$ $f(n) = n \log n$

$\because$ a & b are const. & $f(n)$ is a +ve function

$\therefore$ Master's theorem is applicable here.

$c = \log_b a = \log_4 3 = 0.79$

$n^c = n^{0.79}$

$\because f(n) > n^c$

case 3 is applicable here

$\boxed{T(n) = \theta(n \log n)}$

Sol 17 $T(n) = 3T(n/3) + n/2$

$a = 3$, $b = 3$, $f(n) = n/2$

$\because$ a & b are const. & $f(n)$ is a +ve func.

$\therefore$ Masters theorem is applicable here

$c = \log_b a = \log_3 3 = 1$

$n^c = n^1 = n$

$\therefore f(n) = n^c$

$\therefore$ case 2 is applied here

$$\boxed{T(n) = n \log n}$$

Sol 18 : $T(n) = 6T(n/3) + n^2 \log n$

$a = 6, \quad b = 3 \quad f(n) = n^2 \log n$

$\because$ a & b are const and f(n) is a +ve func

Master's theorem is applicable here

$c = \log_b a = \log_3 6 = 1.63$

$n^c = n^{1.63}$

$\because f(n) > n^c$

case 3 is applied here.

$$\boxed{T(n) = \Theta(n^2 \log n)}$$

Sol 19 $T(n) = 4T(n/2) + n/\log n$

$a = 4, \ b = 2 \quad f(n) = n/\log n$

$\because$ a & b are const. and f(n) is a +ve function

$\therefore$ Master's theorem is applicable here

$c = \log_b a$

$= \log_2 4$

$= \log_2 2^2 = 2\log_2 2 = 2$

$n^c = n^2$

$\because f(n) < n^c$

$\therefore$ case 1 is applied here

$$\boxed{T(n) = \Theta(n^2)}$$

Sol 20 $T(n) = 64T(n/8) - n^2 \log n$

$\because$ a & b are const but f(n) is a –ve fn

$\therefore$ Masters theorem is applied here.

Sol 21 $T(n) = 7T(n/3) + n^2$

$a = 7 \quad b = 3 \quad f(n) = n^2$

$\because$ a, b are const & f(n) is a +ve fn

$\therefore$ Master's Theorem is applied here.

$\Rightarrow c = \log_b a = \log_3 7 = 1.77$

$n^c = n^{1.77}$

$\because f(n) > n^c$

$\therefore$ Case 3 is applied here.

$\boxed{T(n) = \Theta(n^2)}$

Sol 22) $T(n) = T(n/2) + n(2 - \cos n)$

$\because$ f(n) is not regular func.

$\therefore$ Master's theorem can not be applied here.