Logic: Passing Criteria:

(i) Score at least 30% aggregate (p)
and
(ii) Score at least 25% in midsem (q)
or
score at least 25% in endsem (r)

$\boxed{p \wedge (q \vee r)}$

Every computer is connected to the university network.

CS3 is connected to the university network.

How to represent using logical expression]

Domain → the set of all computers from the university network

Example: Domain = set of all integers.

variable x

"$x > 3$". $\boxed{P(x)}$: $x > 3$

There exists x | $\exists x$

For all x | $\forall x$

There is a computer of the university network that is under malware attack.

the set of computers from the university network

Computer x is under malware attack

SUBJECT

PREDICATE

a property that the subject of the state can have

Domain

$P(x)$ → Computer x is under malware attack ✓

There is x s.t. $P(x)$

For all x $P(x)$

For all x : $x > 3$ ✓

$\forall x\ P(x)$ → false

$P(5) : 5 > 3$ → true

$P(1) : 1 > 3$ → false

$P(x) : x^2 \geq 0$ where the domain is the set of all real numbers

→ square of every real number is non-negative (≥ 0) ✓

$P(0) : 0^2 \geq 0$ → true

$P(1) : 1^2 \geq 0$ → true

$P(-2) : 4 \geq 0$

true

$\mathbb{N} = \{0, 1, 2, \ldots\}$ = natural numbers = nonnegative integers

$\forall x\ P(x)$ $P(x): x^2 \geq 0$ → true

Quantifiers:

$Q(x) : x < 0$

$Q(2) : 2 < 0$ → false

$Q(-3) : -3 < 0$

$\exists x\ Q(x)$ → true because

$\forall x \rightarrow$ universal quantifier

$\exists x \rightarrow$ existential quantifier.

$P(x, y): x = y + 3$, domain is the set of all real numbers.

$\boxed{x = 7 \\ y = 4}$ $P(7,4)$: $\boxed{7 = 4+3}$ $\downarrow$ true

$P(7,4)$ is true

$P(x)$ is a predicate where x is a variable from some domain.

Then, $\boxed{\forall x\, P(x)}$ is true when $P(x)$ is true for all values of x in the domain

Example: $P(x): x > 2$ domain is the set of all real numbers.

Why called first order logic.

Predicate logic quantification

$\forall x$ $\exists x$ are over the variables from some domain

$\boxed{\forall x\, P(x) \text{ is true}}$ if and only if $\boxed{\exists x \neg P(x)}$ is false

$\exists x \neg P(x)$ is false. No matter of which what x is considered $\neg P(x)$ is false

For all x $\boxed{\neg P(x) \text{ is false}}$

if $x = -3$, then $Q(3)$ is true

$P(2,3)$ $\begin{matrix} x=2 \\ y=3 \end{matrix}$ $\boxed{2 = 3+3}$ is false false.

The truth value of $P(x, y)$ depends on the values of x and y.

$\boxed{\forall x\, P(x)}$ is false when $\boxed{\text{there is}}$ an element x in the domain such that $P(x)$ is false

equivalently, $\boxed{\exists x \neg P(x)}$ is true

$\boxed{P(x): x > 2} \rightarrow \boxed{\neg P(x): x \leq 2}$

$\forall x\, P(x)$ is false

But $\boxed{\exists x\, P(x) \text{ is true}}$ $\downarrow$ $x = 4$ is a value such that $P(4)$ is true

$\boxed{\exists x \neg P(x) \text{ is true}}$ $\downarrow$ $x = 0$ is a value $\neg P(x)$ is true.

$\forall x\, P(x)$ is false if and only if $\exists x \neg P(x)$ is true

$$\boxed{\forall x\, P(x) \equiv \neg(\exists x \neg P(x))}$$

$\boxed{\exists x \neg P(x) \text{ is true}}$ when there $\downarrow$ are some elements $a_1, a_2, \dots a_n \in$ of the domain such that $\neg P(x)$ is true.

$\boxed{\exists x \neg P(x) \text{ is false}}$ then even

$P(x)$ is true

For the elements other than $a_1, a_2, \ldots, a_n$ $\neg P(x)$ is false

$$\forall x\,(P(x)) \equiv \neg(\exists x(\neg P(x)))$$

$$\neg(\forall x\, P(x)) \equiv \exists x\,(\neg P(x))$$

for the elements $a_1, a_2, \ldots, a_n$ from the domain $\neg P(x)$ is false

For the elements other than $a_1, a_2, \ldots, a_n$ from the domain $\neg P(x)$ is false.

For all elements in the domain $\neg P(x)$ is false. → for all elements in the domain $P(x)$ is true.

$\forall x\, P(x)$ is false when there is x in the domain s.t. $P(x)$ is false.

$\exists x\, P(x)$ is true when there is some element x in the domain for which $P(x)$ is true.

$\exists x\, P(x)$ is false when $P(x)$ is false for all elements in the domain.

$\forall x$ = universal quantifier

$\exists x$ = existential quantifier.

$A(x)$: x is divisible by 2

the domain is set of all integers

$B(x)$: x is an even number.

$\forall x\,(A(x) \to B(x))$ → True

For every integer x, if x is divisible by 2, then x is even

$\forall x\,(B(x) \to A(x))$ → is True

For every integer x, if x is even then x is divisible by 2

a is divisible by b if there is an integer k such that $a = kb$

$b \mid a$. → b divides a

→ a is divisible by b

$$\forall x\,((A(x) \to B(x)) \wedge (B(x) \to A(x)))$$

$$\equiv \forall x\,(A(x) \leftrightarrow B(x))$$

$\forall x\, P(x)$

$\forall x\, Q(x)$

The domain is the same.

If $\forall x\, P(x)$ is true and $\forall x\, Q(x)$ is true then can we say $\forall x\,(P(x) \wedge Q(x))$

is true?

Yes, because for every possible element x in the domain, $P(x)$ is true and $Q(x)$ is true.
Therefore $P(x) \land Q(x)$ is true.
Hence $\forall x\,(P(x) \land Q(x))$ is true.

domain is the same

$\exists x\, P(x)$ is true

$\exists x\, Q(x)$ is true

can we say that $\exists x\,(P(x) \land Q(x))$ is true? NO

The particular element for which $P(x)$ is true is a

The particular element for which $P(x)$ is true is b.

That does not imply that $P(x)$ and $Q(x)$ both are true for the same element in the domain.

That is why $\exists x\,(P(x) \land Q(x))$ cannot be true.

Suppose $\exists x\,(P(x) \land Q(x))$ is true.

Then, $(\exists x\, P(x)) \land (\exists x\, Q(x))$ is true

Because there element b such that $P(b) \land Q(b)$ is true. $\exists x\, P(x)$ is true

$P(b)$ is true and $Q(b)$ is true.

$P(x)$: x is an even integer

$Q(x)$: x is an odd integer

Domain: set of all integers.

$\exists x\, P(x)$ is true

$\exists x\, Q(x)$ is true

$\exists x\,(P(x) \land Q(x))$ is false

Section 1.4 + Rosen's Book

Predicate Logic