Discrete Structures-2025: Quiz-3

Sets and Functions

Full Marks: 20
Time: 40 minutes

October 6, 2025

(1) Let A and $D_1, \ldots, D_n$ denote a collection of sets. Then prove that

$$A \times \left(\bigcup_{i=1}^{n} D_i \right) = \bigcup_{i=1}^{n} (A \times D_i)$$

(8 Marks)

(2) A function $f : \mathbb{Z} \to \mathbb{Z}$ is *strictly increasing* if for every $a, b \in \mathbb{Z}$, if $a < b$, then $f(a) < f(b)$. Then prove that: if a function $g : \mathbb{Z} \to \mathbb{Z}$ is strictly increasing, then g is an injective function.

(6 Marks)

(3) Let $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is defined as $f(m, n) = 2m - n$. **Prove or disprove that:** f is a surjective function.
(6 Marks)