Discrete Structures-2025: Quiz-1

Propositional and First Order Logic

Full Marks: 15
Duration: 30 minutes

September 1, 2025

(1) Let p, q, r and s be propositions. Then prove or disprove that $(p \rightarrow q) \rightarrow (r \rightarrow s)$ and $(p \rightarrow r) \rightarrow (q \rightarrow s)$ are logically equivalent.

(7 Marks)

(2) Let $P(x, y)$ be the statement "y is divisible by x" where the domain for both x and y is the set of all positive integers. Determine the truth values of each of these statements with justification.

  (a) $\forall x \, \exists y \, P(x, y)$.

  (b) $\forall x \, \forall y \, P(x, y)$.

(1 Marks + 1 Marks)

(3) Express each of the English sentences using predicates and quantifiers.

  (a) Let $C(x)$ be the statement "x has a cat", $D(x)$ be the statement "x has a dog", and $F(x)$ is a statement "x has a ferret". The domain is the set of all students in your class.

  Express this sentence in predicate and quantifiers.

  "For each of the animals cats, dogs, and ferrets, there is a student in your class who has this as a pet".

  (b) Let the domain be the set of all car drivers in Delhi-NCR region.

  For the following sentence, define a predicate, and express the sentence using that predicate and quantifier.

  "Some drivers do not obey speed limits".

  (c) Suppose that the domain is the set of all students in your class.

  For the following sentence, define predicate(s), and express using predicates and quantifiers.

  "No student in your class owns both a motorcycle, and a 4-wheeler car".

(2 Marks + 2 Marks + 2 Marks)