Propositional Logic

p, q

$p \vee q$, $p \wedge q$, $p \rightarrow q$, $\neg p$

logical connectives

$p \oplus q$ p XOR q

$p \oplus q$ is true when either p or q is true but not both. otherwise false

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

Biconditional statement

$p \leftrightarrow q$

p if and only if q

"p iff q"

$p \leftrightarrow q$ is true (when both p, q are true or both p, q are false.)

p	q	$p \leftrightarrow q$	$\neg(p \oplus q)$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	T	T

$p \oplus q$ is true when $p \leftrightarrow q$ is false

$p \oplus q$ is false when $p \leftrightarrow q$ is true

$\neg(p \oplus q) \equiv (p \leftrightarrow q)$

equivalent to

$\neg(p \oplus q)$ and $(p \leftrightarrow q)$ give identical truth values for every possible combinat of truth values of p and q.

That is why $\neg(p \oplus q)$ and $(p \leftrightarrow q)$ are logically equivalent.

Tautology: If a compound proposition evaluates to true for all possible combinations of propositional variables. then it is called a tautology

Hence, $\neg(p \rightarrow q) \rightarrow \neg q$ is a tautology.

Example: $\neg(p \rightarrow q) \rightarrow \neg q$

p	q	$\neg q$	$p \rightarrow q$	$\neg(p \rightarrow q)$	$\neg(p \rightarrow q) \rightarrow \neg q$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	T	F	T
F	F	T	T	F	T

every possible combination of p and q, evaluates to true

$\neg(p \rightarrow q) \rightarrow \neg q$

A compound proposition is a contradiction if it evaluates to false for every possible combination of truth values of propositional variables

$\neg(\neg(p \rightarrow q) \rightarrow \neg q)$ is a contradiction.

How to prove that a compound proposition is a tautology?

Construct the truth table for that compound proposition.

How to disprove that a compound proposition is a tautology?

Construct truth table.
Mention which particular values of propositional variables evaluate to false.

How to prove that two compound propositions are logically equivalent?

Construct truth tables for both the propositions.
In truth table, it must ensure that every possible truth values evaluates the same.

$\neg(p \vee q) \equiv \neg p \wedge \neg q$

Truth table for $\neg(p \vee q)$

p	q	$\neg p$	$\neg q$	$p \vee q$	$\neg p \wedge \neg q$	$\neg(p \vee q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

As $\neg(p \vee q)$ and $(\neg p \wedge \neg q)$ evaluate same for every combination of truth values.

$\neg(p \vee q)$ and $(\neg p \wedge \neg q)$ are logically equivalent

$\neg(p \vee q) \equiv (\neg p \wedge \neg q)$
$\neg(p \wedge q) \equiv (\neg p \vee \neg q)$
→ De-Morgan's Law

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Exercise? use the truth table to prove

Distributive Laws

Exercise: Prove that $(\neg p \vee q) \equiv (p \to q)$

$p \wedge \neg p$

p	$\neg p$	$p \wedge \neg p$
T	F	F
F	T	F

If p and q are logically equivalent then $(p \leftrightarrow q)$ is a tautology.

$p \equiv q$

Conditional disjunction equivalence.

What we know?

(i) $(p \to q) \equiv \neg p \vee q$

(ii) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

(iii) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(iv) $\neg(p \vee q) \equiv \neg p \wedge \neg q$

(v) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

Using (i), (ii), (iii), (iv), (v) prove that

$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$

Proof: $(p \to q) \equiv (\neg p \vee q)$ due to (i)

$(p \to r) \equiv (\neg p \vee r)$ due to (i)

$$\begin{aligned}
LHS &= (p \to q) \wedge (p \to r) \\
&\equiv (\neg p \vee q) \wedge (\neg p \vee r) \\
&\equiv \neg p \vee (q \wedge r) \quad \text{due to (ii)} \\
&\equiv p \to (q \wedge r) \quad \text{due to (i)} = RHS
\end{aligned}$$

A compound proposition is satisfiable if there is an assignment of truth values of propositional variables that evalues to true.

Every tautology is satisfiable. But converse is not true.

SATISFIABILITY:

Input: A compound proposition p

Question: Is there an assignment of propositional variables that evaluates p to true?

An algorithm involves checking all possible combination of truth values of its propositional variables

n variables $\Rightarrow$ 2^n possible assignments $\Rightarrow$ each have to be evaluated.

① $(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$

AND OF OR — CONJUNCTION (and) of disjunction (or)

CONJUNCTIVE NORMAL FORM (CNF) → "CNF boolean formula" "boolean formula in CNF".

→ **Fact:** Every compound proposition can be represented as conjunctive normal form.

Is it satisfiable? all variables true then it evalues to true.

$(q \to p) \wedge (r \to q) \wedge (p \to r)$ → CLAUSE

all variables false

② $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r) \wedge (q \vee \neg r) \wedge (r \vee \neg p) \wedge (p \vee \neg q) \equiv \phi$

If all p, q, r are true then $(\neg p \vee \neg q \vee \neg r)$ is false

If all p, q, r are false, then $(p \vee q \vee r)$ is false.

Is ϕ satisfiable?

all p, q, r true

all p, q, r false

Hence, no matter of the assignment of values, ϕ is false.

Hence, ϕ is $\boxed{\text{unsatisfiable}}$. $\equiv$ Contradiction

Section 1.3 Rosen's book

6 Groups of Tutorial

25-30

Next Class: 23rd August 9:30 AM.
(Monday time table)

Tutorial-1 : 23rd Aug 1:30 pm