Discrete Structures-2025: Tutorial-7

Countable Sets, Uncountable Sets, and Cardinality of Sets

(1) Let A be an uncountable set and B be a countable set. Then, is $A - B$ (also denoted as $A \setminus B$) countable? or uncountable? Formally, prove your answer.

(2) Prove that the following sets are uncountable.

(a) $(0, \frac{1}{2})$ = the set of all real numbers strictly between 0 and $1/2$.

(b) the real numbers not containing 0 in their decimal representation. (Hint: use diagonalization method)

(c) the real numbers with decimal representations containing finite number of 1s. (Hint: use diagonalization method)

(3) Give an example of two uncountable sets A and B such that $A \cap B$ is

(a) finite.

(b) countably infinite.

(c) uncountable.

For each of the above mentioned cases, justify your answers.

(4) Give an example of two uncountable sets A and B such that $A - B$ is

(i) nonempty and finite.

(ii) countably infinite.

(iii) uncountable.

For each of the above mentioned cases, justify your answers.

(5) Let S be a relation on the set $\mathbb{N} \setminus \{0\} \times \mathbb{N} \setminus \{0\}$ such that $((a, b), (c, d)) \in S$ if and only if $a + d = b + c$. Prove that S is an equivalence relation.

(6) Let S be a relation on the set $\mathbb{N} \setminus \{0\} \times \mathbb{N} \setminus \{0\}$ such that $((a, b), (c, d)) \in S$ if $ad = bc$. Prove that S is an equivalence relation.