MTH-210: Mid-Semester Examination

Monsoon-2025

September 27, 2025

Full Marks: 60 **Time: 2:00 hours**

General Instructions: This is a closed book examination. Please be precise, brief and to the point in your answer. Unnecessary blabbers or unreadable handwriting will fetch deduction of marks.

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1. Prove that $(p \vee q) \wedge (\neg p \vee r) \rightarrow (q \vee r)$ is a tautology. **(8 Marks)**

2. For each of the statements, determine whether they are true or false. Justify your answer for each the statements. **($3 \times 4 = 12$ Marks)**

   (a) The domain for x and y is the set of all real numbers.
   $\exists x \, \forall y \, ((y \neq 0) \; \rightarrow \; (xy = 1))$

   (b) The domain for x and y is the set of all real numbers.
   $\exists x \, \forall y \, (x \geq y^2)$

   (c) The domain for x and y is the set of all integers.
   $\exists x \, \forall y \, (x \leq y^2)$

   (d) The domain for x, y and z is the set of all integers.
   $\forall x \, \exists y \, \forall z \, (x + z = y)$

3. For every positive prime number p, prove the statement: "If n^3 is divisible by p, then n is divisible by p." **(6 Marks)**

4. Prove that $\sqrt[3]{25}$ is irrational. **(8 Marks)**

5. If A, B, D, and E are sets, then prove that
   $A \times (B \cup (D \cap E)) = (A \times (B \cup D)) \cap (A \times (B \cup E))$. **(8 Marks)**

6. Prove that for every positive integer n, $16^{n+1} + 17^{2n-1}$ is divisible by 273. **(8 Marks)**

7. Suppose that all we have are 3-cent and 10-cent stamps. Then, prove that we can make any postage of 18-cents or more. **(10 Marks)**