

MTH-210: Assignment-2

Due Date: 18th November, 2025

Full Marks: 20

Instructions: Please note that the Page-Limit is at most 5 pages in A4-sized page. This page limit is very strict. Use of extra pages will incur penalty of marks. If you want to prove that two infinite sets have the same cardinality, then you must formally prove the existence of a bijection. Vague reasonings will not be accepted.

- (1) Let \mathcal{A} be the set of all functions $f : \mathbb{N} \rightarrow \{0, 1\}$ and \mathcal{B} be the set of all functions $g : \mathbb{N} \rightarrow \{0, 1, 2\}$. Then, prove or disprove that $|\mathcal{A}| = |\mathcal{B}|$.

Please write a formal answer in either cases.

(10 Marks)

- (2) Prove the following identity using combinatorial arguments (bijective proof or double-counting arguments). If you give proof by algebraic manipulation, then you will get zero marks.

$$\sum_{k=0}^r \binom{n+k}{k} = \binom{n+r+1}{r}$$

(10 Marks)