CSE140: Introduction to Intelligent Systems
Quiz 3 - Rubric

Date: 03/04/2025 **Total Time:** 45 minutes **Total Marks:** 10

Marking Scheme and Solutions

Q 1: A robot vacuum correctly identifies obstacles 90% of the time. If it encounters 5 obstacles in a row, what is the probability that it correctly avoids all of the obstacles? **(1 Mark)**

(A) 0.9

(B) 0.59

(C) 0.41

(D) 0.81

Answer: (B) 0.59
Explanation: The probability of correctly identifying each obstacle is 0.9. For the robot to correctly avoid all 5 obstacles, it must correctly identify each one. Using the multiplication rule for independent events:
$P(\text{all correct}) = 0.9 \times 0.9 \times 0.9 \times 0.9 \times 0.9 = 0.9^5 = 0.59049 \approx 0.59$

Marking:

- Correct answer with proper explanation: 1 mark
- Incorrect answer: 0 marks

Q 2: Which statement is true about configuration space and workspace? **(1 Mark)**

(A) A single configuration in configuration space always corresponds to a unique position and orientation in the workspace.

(B) A single position and orientation in the workspace can be achieved by multiple configurations in configuration space.

(C) The dimensionality of both configuration space and workspace is always the same.

(D) Workspace is a subset of configuration space.

Answer: (B) A single position and orientation in the workspace can be achieved by multiple configurations in configuration space.
Explanation: In robotics, multiple joint configurations can produce the same end-effector position and orientation. For example, a robot arm can reach the same point using different joint angles (e.g., elbow-up vs. elbow-down configurations). Statement A is incorrect because one-to-many mapping exists from configuration to workspace. Statement C is incorrect because configuration space typically has higher dimensionality than workspace. Statement D is incorrect as these are fundamentally different spaces.

Marking:

- Correct answer with proper explanation: 1 mark
- Incorrect answer: 0 marks

Q 3: Consider the robot arm shown in Figure 1. Assume that the robot's base element is 60 cm long and that its upper arm is 40 cm long. Provide an explicit closed-form solution of the inverse kinematics for this arm. **(3 Marks)**

Answer:
For the robot arm shown in Figure 1, with link lengths L1 and L2:

Let θ_1 be the angle of the first link from the horizontal axis and θ_2 be the angle of the second link relative to the first link. We need to find θ_1 and θ_2 given the end-effector position (x, y).

Step 1: The forward kinematics equations for this robot are:

$$x = L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2) \tag{1}$$
$$y = L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2) \tag{2}$$

Step 2: For inverse kinematics, we first calculate θ_2:

$$x^2 + y^2 = (L_1 \cos(\theta_1) + L_2 \cos(\theta_1 + \theta_2))^2 + (L_1 \sin(\theta_1) + L_2 \sin(\theta_1 + \theta_2))^2 \tag{3}$$
$$= L_1^2 + L_2^2 + 2L_1L_2 [\cos(\theta_1)\cos(\theta_1 + \theta_2) + \sin(\theta_1)\sin(\theta_1 + \theta_2)] \tag{4}$$
$$= L_1^2 + L_2^2 + 2L_1L_2 \cos(\theta_2) \tag{5}$$

Therefore, solving for $\cos(\theta_2)$:

$$\cos(\theta_2) = \frac{x^2 + y^2 - L_1^2 - L_2^2}{2L_1L_2} \tag{6}$$

And the angle:

$$\theta_2 = \pm \arccos\left(\frac{x^2 + y^2 - L_1^2 - L_2^2}{2L_1L_2}\right) \tag{7}$$

The $\pm$ indicates two possible solutions (elbow-up and elbow-down configurations).

Step 4: Derivation of θ_1:

Let ϕ be the angle between the positive x-axis and the vector from the base to the end-effector:

$$\phi = \arctan 2(y, x) \tag{8}$$

Now, define the auxiliary angle β between the first link and the line to the end-effector. Using triangle geometry:

$$\beta = \arctan 2\left(L_2 \sin(\theta_2), L_1 + L_2 \cos(\theta_2)\right) \tag{9}$$

Then, the angle of the first joint is:

$$\begin{aligned}
\theta_1 &= \phi - \beta && (10)\\
&= \arctan 2(y, x) - \arctan 2\left(L_2 \sin(\theta_2), L_1 + L_2 \cos(\theta_2)\right) && (11)\\
&= \arctan 2(y, x) - \arctan 2\left(\frac{L_2 \sin(\theta_2)}{L_1 + L_2 \cos(\theta_2)}\right) && (12)
\end{aligned}$$

Note: This solution assumes the point (x, y) is reachable, which requires:

$$|L_1 - L_2| \leq \sqrt{x^2 + y^2} \leq L_1 + L_2 \tag{13}$$

Marking:

- Correct formula for θ_2: 1.5 mark
- Correct formula for θ_1: 1.5 mark

Q 4: Explain the following concepts: **(3 Marks)**

(a) Forward Kinematics and Inverse Kinematics

(b) Types of Actuators based on energy

(c) Open-Loop Control and Closed-Loop Control

Answer:

a) Forward Kinematics and Inverse Kinematics: (1 mark)

- **Forward Kinematics:** Determines end-effector position/orientation from joint variables. Maps from joint space to workspace with unique solutions.
- **Inverse Kinematics:** Determines joint variables needed for desired end-effector position/orientation. Maps from workspace to joint space with potentially multiple, zero, or infinite solutions.

b) Types of Actuators based on energy: (1 mark)

- **Electric actuators:** Motors, servos, solenoids. Precise control, common in robotics.
- **Hydraulic actuators:** Use pressurized fluid. High force output for heavy applications.
- **Pneumatic actuators:** Use compressed air. Lightweight, good for quick movements.

c) Open-Loop Control and Closed-Loop Control: (1 mark)

- **Open-Loop Control:** No feedback. Performs based on predetermined inputs without monitoring results. Simpler but less accurate with disturbances.
- **Closed-Loop Control:** Uses sensors to compare actual output with desired output. Adjusts based on error. More complex but more accurate and robust.

Marking:

- Each sub-question: 1 mark
- Complete explanation: 1 mark
- Incorrect/missing explanation: 0 marks

Q 5: Consider a mobile robot moving on a horizontal surface. Suppose that the robot can execute two types of motion: **(2 Marks)**

- Rolling forward a specified distance.
- Rotating in place through a specified angle.

The state of such a robot can be characterized in terms of three parameters (x, y, ϕ), where x and y are the coordinates of the robot's center of rotation, and ϕ is the orientation angle from the positive x-axis. The action *Roll(D)* changes the state

(x, y, ϕ) to $(x + D\cos(\phi), y + D\sin(\phi), \phi)$, and the action *Rotate(θ)* changes the state (x, y, ϕ) to $(x, y, \phi + \theta)$.

Suppose the robot is initially at $(0, 0, 0)$ and executes the actions *Rotate(*$60°$*)*, *Roll(1)*, *Rotate(*$25°$*)*, *Roll(2)*. What is the final state of the robot?

Answer:

Initial state: $(0, 0, 0)$

1. *Rotate(*$60°$*)*: $(0, 0, 60°)$
2. *Roll(1)*: $x = 0 + 1 \cdot \cos(60°) = 0.5$
   $y = 0 + 1 \cdot \sin(60°) = 0.866$
   New state: $(0.5, 0.866, 60°)$
3. *Rotate(*$25°$*)*: $(0.5, 0.866, 85°)$
4. *Roll(2)*: $x = 0.5 + 2 \cdot \cos(85°) = 0.5 + 2 \cdot 0.087 = 0.674$
   $y = 0.866 + 2 \cdot \sin(85°) = 0.866 + 2 \cdot 0.996 = 2.858$

Final state: $(x = 0.674, y = 2.858, \phi = 85°)$

Marking:

- action1[Rotate(60°)]: 0.25 marks
- action2[Roll(1)]: 0.75 marks
- action3[Rotate(25°)]: 0.25 mark
- action4[Roll(2): 0.75 mark