Discrete Structures-2025: Tutorial-4

Proof-II and Sets

(1) Prove or disprove the following statements for three sets A, B, C.

(a) If $A \cup C = B \cup C$, then $A = B$.

(b) If $A \cap C = B \cap C$, then $A = B$.

(c) If $A \cup C = B \cup C$ and $A \cap C = B \cap C$, then $A = B$.

(2) Prove or disprove the following two statements.

(a) $A \times (B \cup C) = (A \times B) \cup (A \times C)$.

(b) $A \times (B \cap C) = (A \times B) \cap (A \times C)$.

(3) Prove that for every positive integer n, $9^n - 2^n$ is divisible by 7.

(4) Prove that for every positive integer n,

$$\frac{1}{1 \cdot 4} + \frac{1}{4 \cdot 7} + \ldots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$