Discrete Structures-2025: Tutorial-2

Propositional and First Order Logic

(1) Let p and q be two propositions. Prove that the following compound propositions are tautologies.

   (i) $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

  (ii) $(p \wedge q) \rightarrow \neg(p \oplus q)$

(2) Prove or disprove that the following propositions are logically equivalent

   (i) $p \rightarrow (q \rightarrow r)$ and $(p \rightarrow q) \rightarrow r$

  (ii) $\neg p \rightarrow (q \rightarrow r)$ and $q \rightarrow (p \vee r)$

(3) Let $N(x)$ be the statement "x has visited North Dakota" where domain for x consists the set of all students. Express the following quantifications in English.

   (i) $\exists x \, N(x)$

  (ii) $(\exists x \, \neg N(x)) \wedge (\exists x \, N(x))$

(4) Express the following sentences using first order logic using appropriate quantifications and appropriate domains.

   (i) A student in PQR-school has lived in Vietnam. – what is the domain here? Using this domain, express using first order logic expressions.

  (ii) Every bird can fly. – what is the domain here? Using this domain, express this sentence using first order logic expression.

 (iii) Let $I(x)$ be the statement "x has an internet connection" and $C(x,y)$ be the statement "x and y have chatted over the internet. Domain is the set of students in your class. Then, express the following using first order logic formula.

    (a) Sanjay has chatted with everyone except Joseph.

    (b) There is a student in the class chatted with every other student over internet.

    (c) Every student in the class has an internet connection but each of them has not chatted with some student over the internet.

(5) Suppose that the domain of these variables is the set of all integers. Then, for each of the statements, explain whether the statement is true or false.

(i) $\forall x \, \exists y \, (y^2 - x < 100)$.

(ii) $\forall x \, \forall y \, (x^2 \neq y^3)$.

(iii) $\forall x \, \forall y \, (x^2 = y^2 \rightarrow x = y)$.

(iv) $\forall x \, \exists y \, (x^2 = y)$.

(v) $\forall x \, \exists y \, (x^2 \neq y)$.

(6) Prove or disprove that the following propositions are satisfiable.

(i) $(p \vee q \vee \neg r) \wedge (p \vee \neg q \vee \neg s) \wedge (p \vee r \vee \neg s) \vee (p \vee q \vee \neg s) \wedge (p \vee q \vee \neg s)$

(ii) $((p \wedge q) \rightarrow r) \leftrightarrow ((p \rightarrow r) \wedge (q \rightarrow r))$