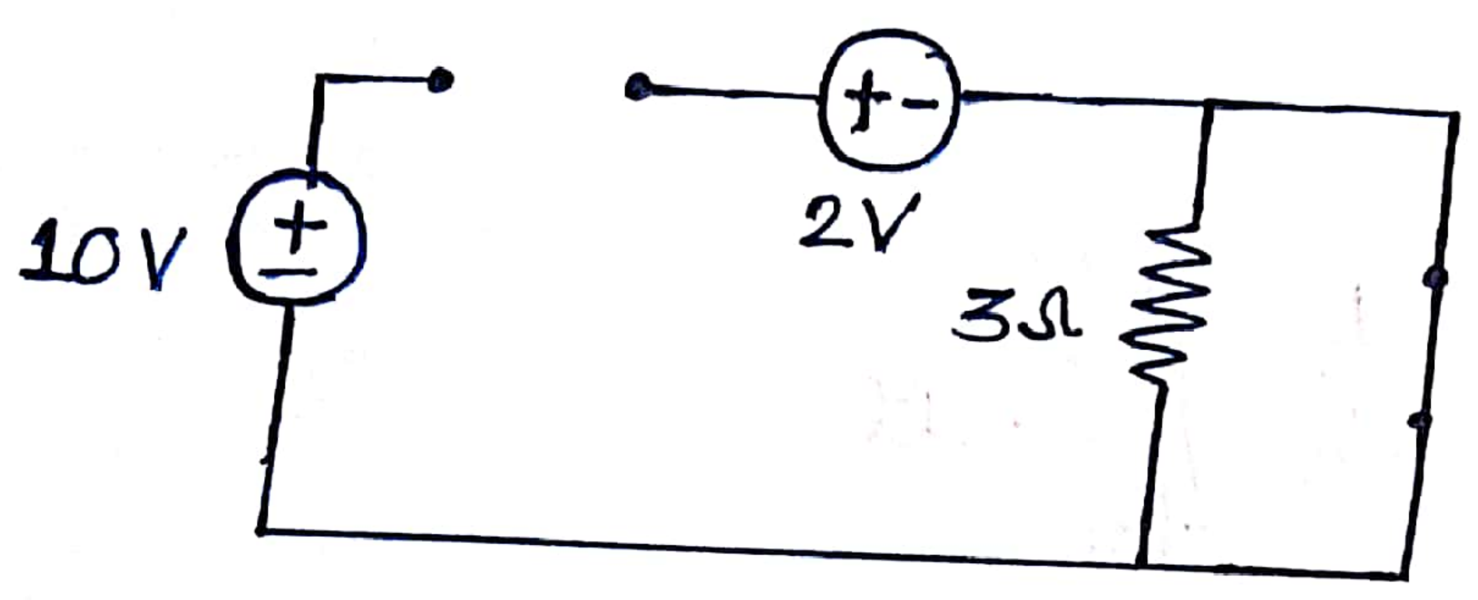
BE ASSIGNMENT – 1
SOLUTION

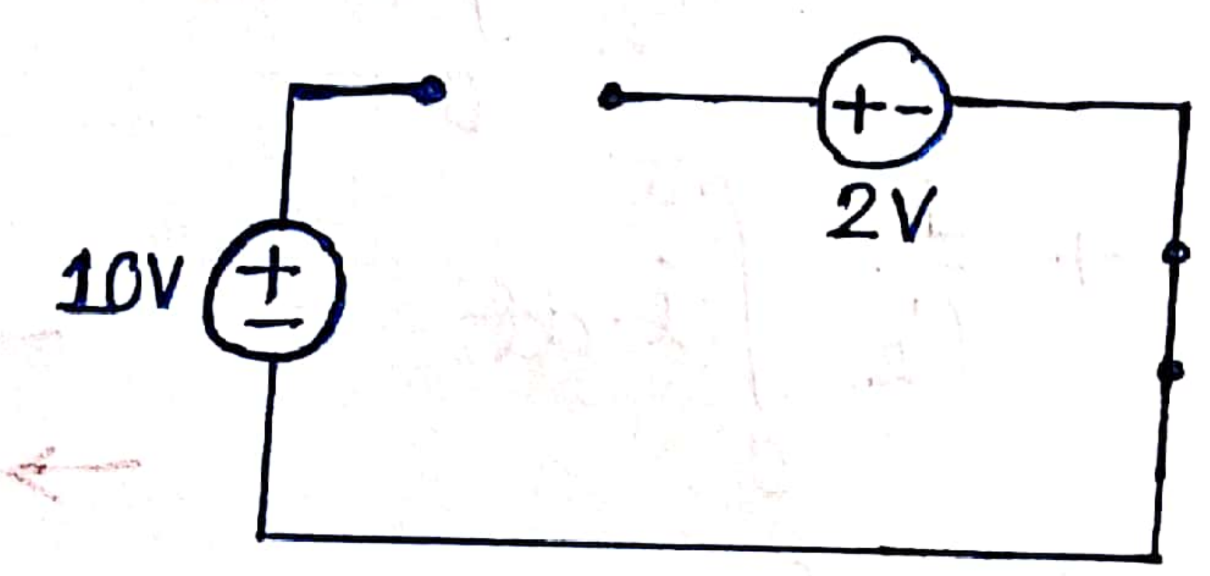
SOL (1):

Case (I): circuit analysis at $t = 0^-$

Given that – $V_{C_1}(0^-) = 2V$, $V_{C_2}(0^-) = 0V$



⇓

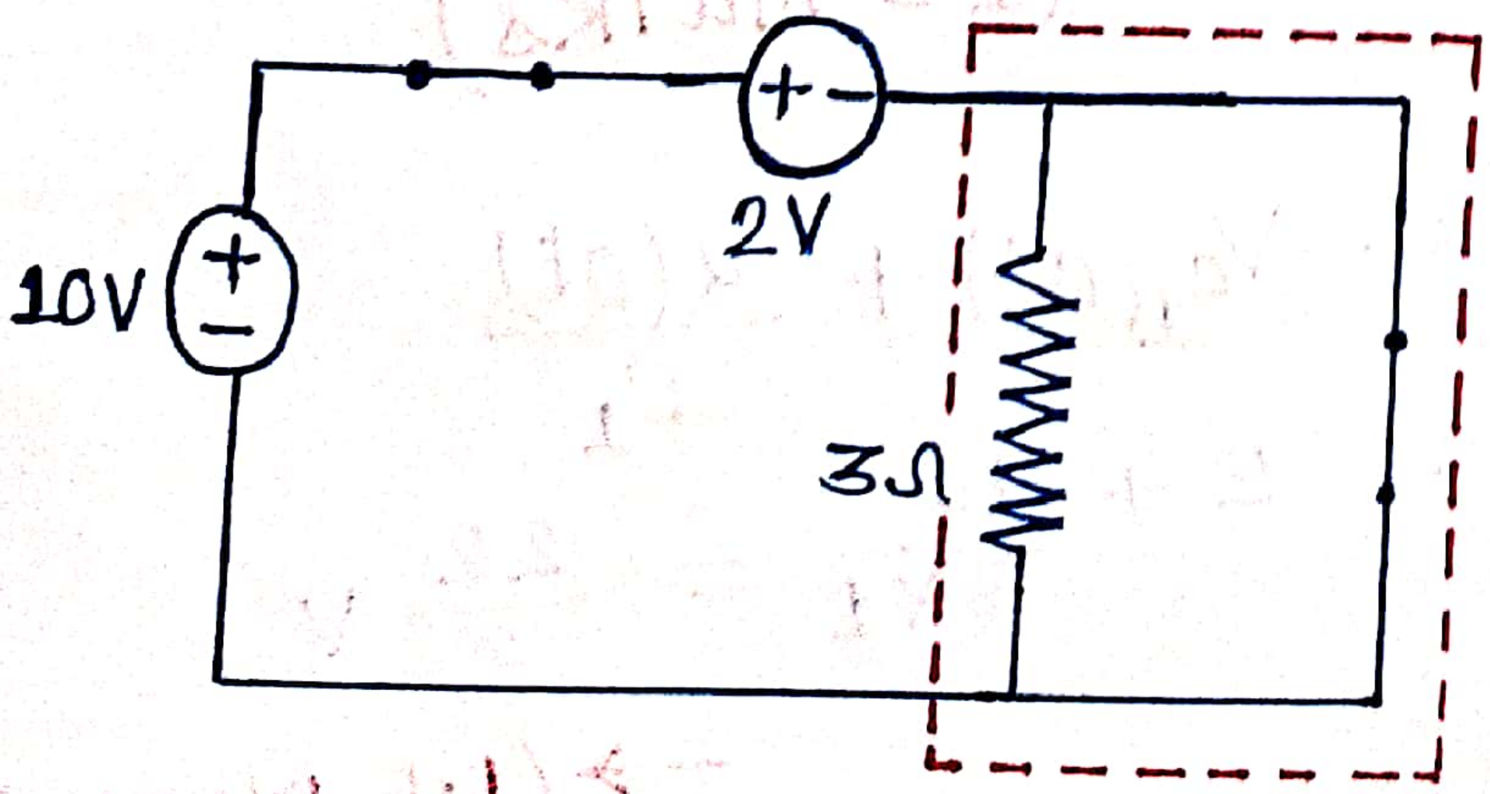


Case (II): circuit analysis at $t = 0^+$

As we know that – capacitor does not allow sudden change of voltage. Hence –

$$V_{C_1}(0^+) = V_{C_1}(0^-) = 2V \quad \text{— (1)}$$

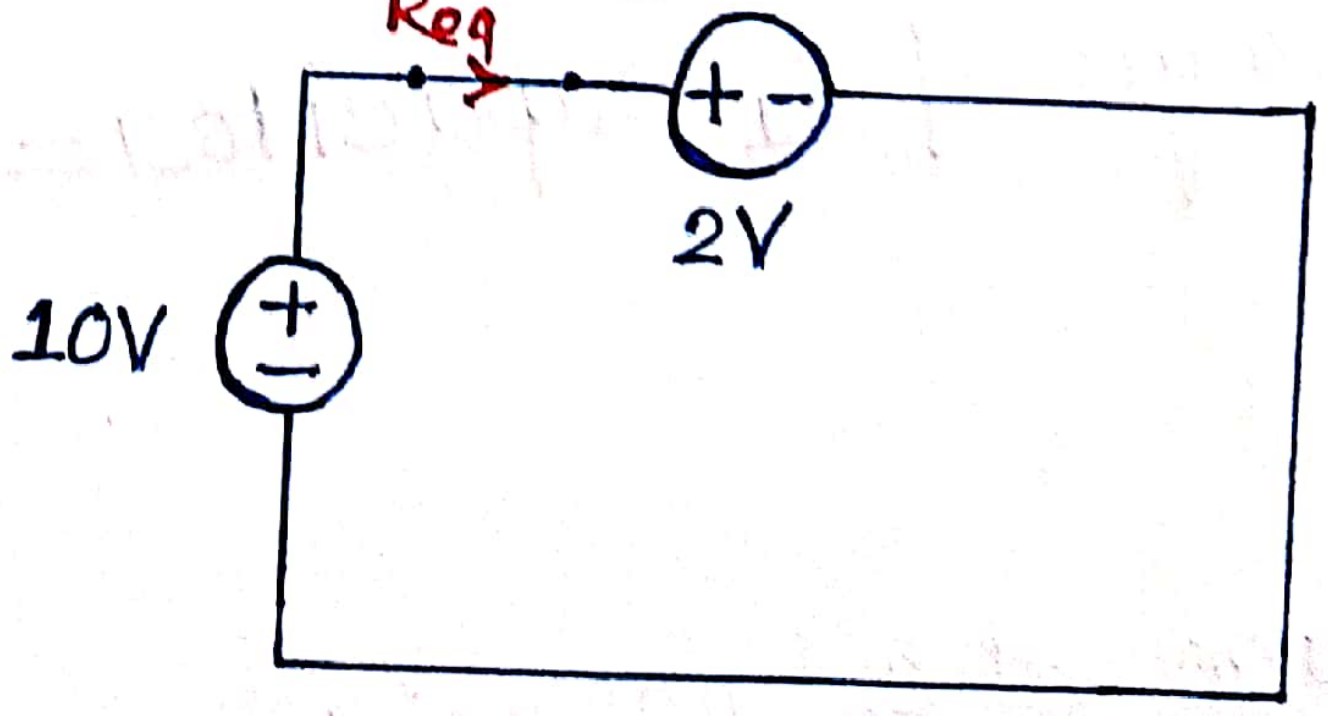
$$V_{C_2}(0^+) = V_{C_2}(0^-) = 0V \quad \text{— (2)}$$



$R_{eq} = 0\,\Omega$

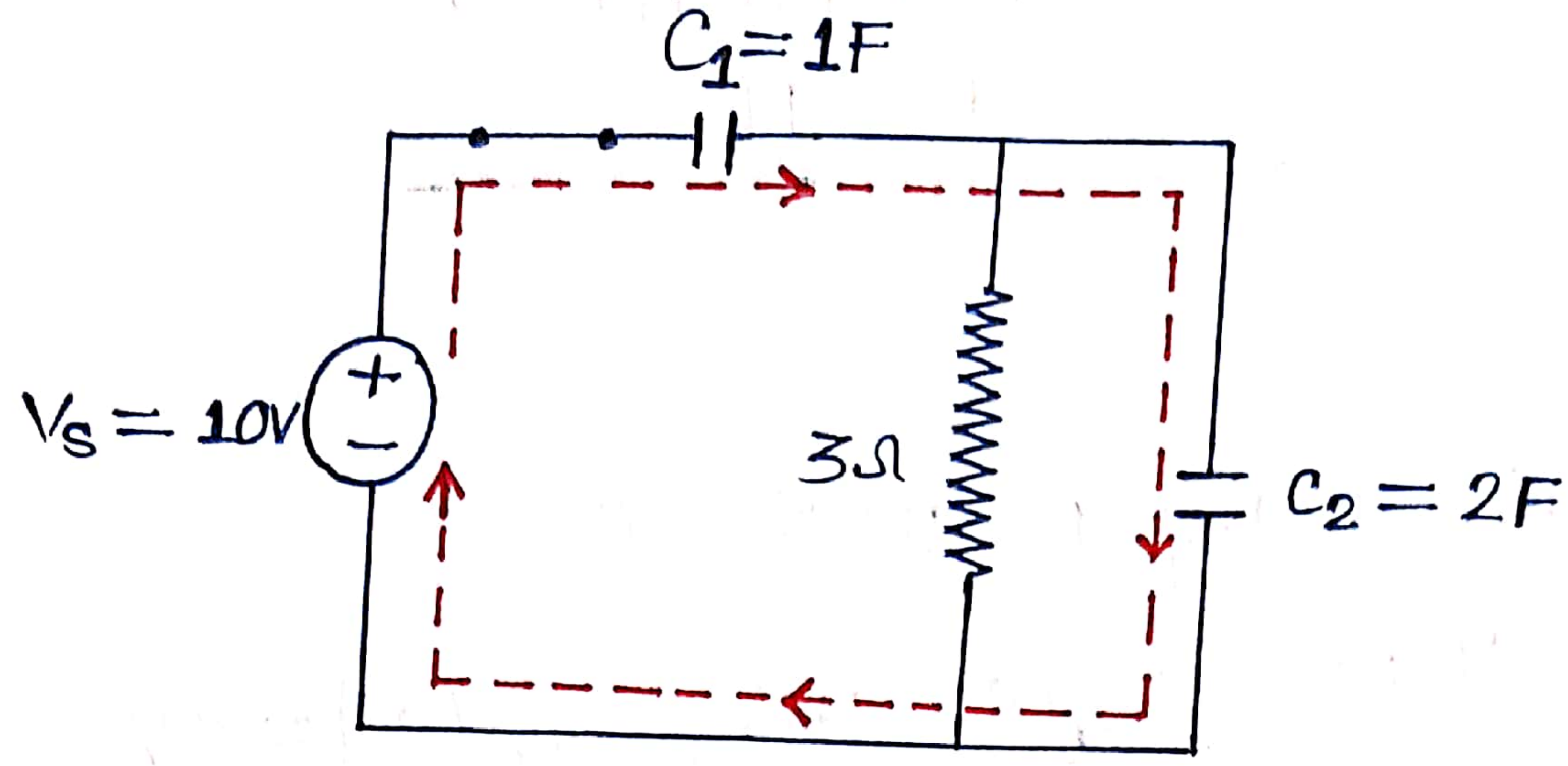
⇒

$i = \frac{V_{eq}}{R_{eq}} = \infty$



This circuit not satisfy KVL. Hence we can't use $eq^n(1)$ & $eq^n(2)$.

→ (1.5 Mark)



Apply KVL at the loop–

$$V_s = \frac{1}{C_1}\int_{-\infty}^{t} i\cdot dt + \frac{1}{C_2}\int_{0}^{t} i\cdot dt$$

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$$V_s = V_{C_1}(0^-) + \frac{1}{C_1}\int_{0}^{t} i\cdot dt + \frac{1}{C_2}\int_{0}^{t} i\cdot dt \quad \text{— (3)}$$

→ (1.5 Mark)

Now put $t = 0^+$ in eq^n (3), we get –

$$V_s = V_{C_1}(0^-) + \frac{1}{C_1}\int_{0}^{0^+} i\, dt + \frac{1}{C_2}\int_{0}^{0^+} i\, dt$$

$$10 = 2 + \frac{Q(0^+)}{C_1} + \frac{Q(0^+)}{C_2}$$

$$8 = \frac{Q(0^+)}{1} + \frac{Q(0^+)}{2}$$

$$\left\{ \frac{dQ}{dt} = i \qquad Q = \int i\cdot dt \right.$$

$$Q(0^+) = \frac{16}{3}\, C \quad \text{— (4)}$$

→ (1.5 Marks)

∴ Voltage of C_1 capacitor $= V_{C_1}(0^+) = V_{C_1}(0^-) + \frac{Q(0^+)}{C_1}$

$$= 2 + \frac{16}{3\times 1} = \frac{22}{3}\, V$$

→ (1.5 Marks)

∴ Voltage of C_2 capacitor $= V_{C_2}(0^+) = \frac{Q(0^+)}{C_2}$

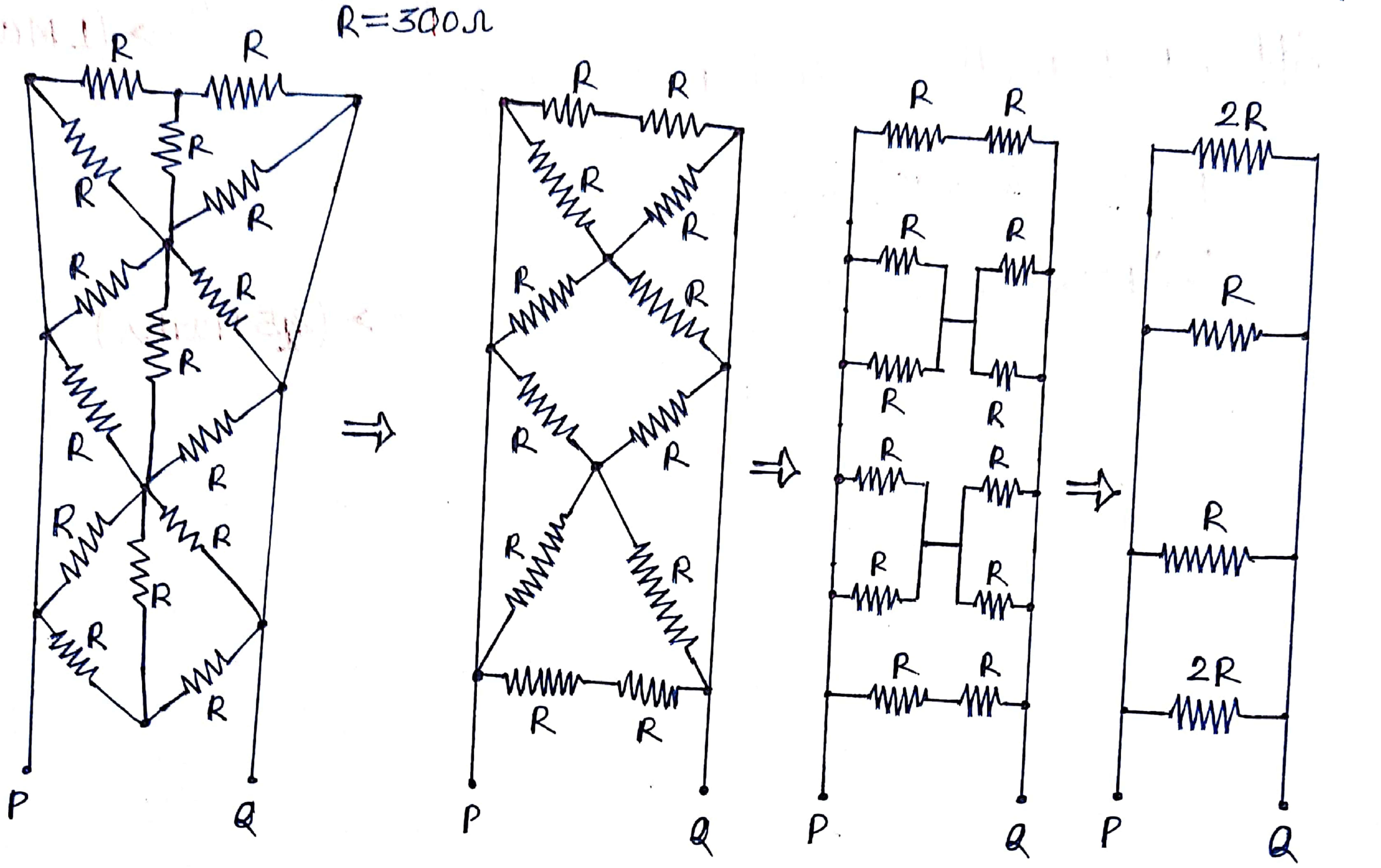
$$= \frac{16}{3\times 2} = \frac{8}{3}\, V$$

→ (1.5 Marks)

SOL(2):-

Step(I): To calculate equivalent resistance between P & Q

R=300Ω



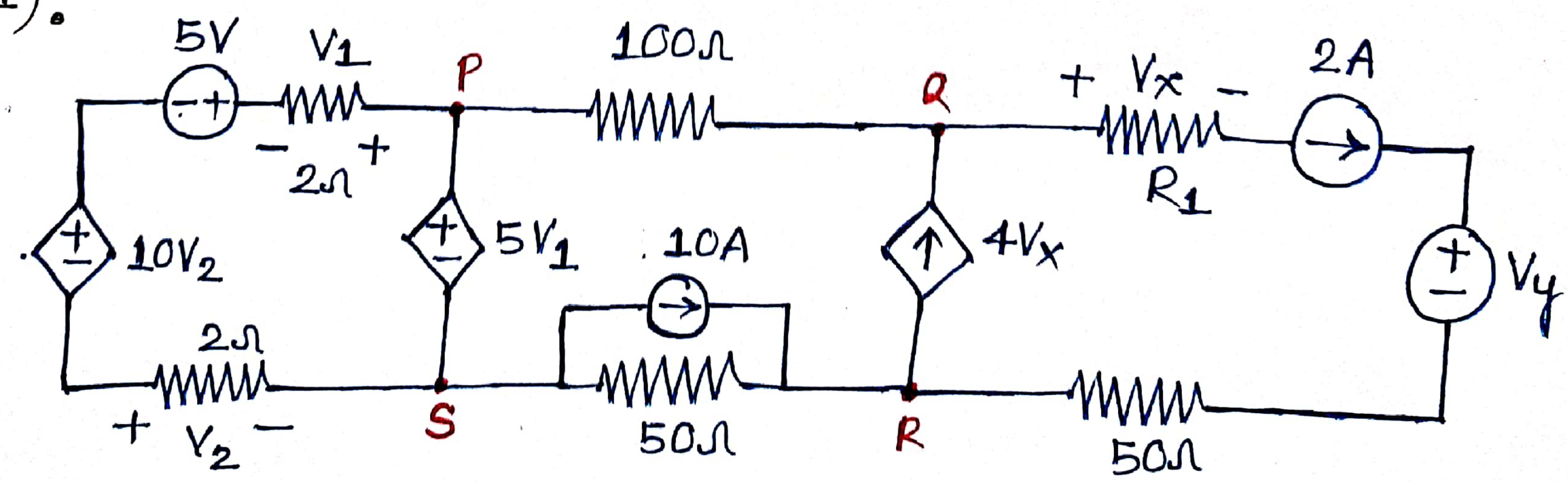
∴ Equivalent resistance between P & Q = R_{PQ}

$$= \left(\frac{1}{2R} + \frac{1}{R} + \frac{1}{R} + \frac{1}{2R}\right)^{-1}$$

$$= \frac{R}{3} = \frac{300}{3} = 100\,\Omega$$

→ (3 Marks)

Step(II):



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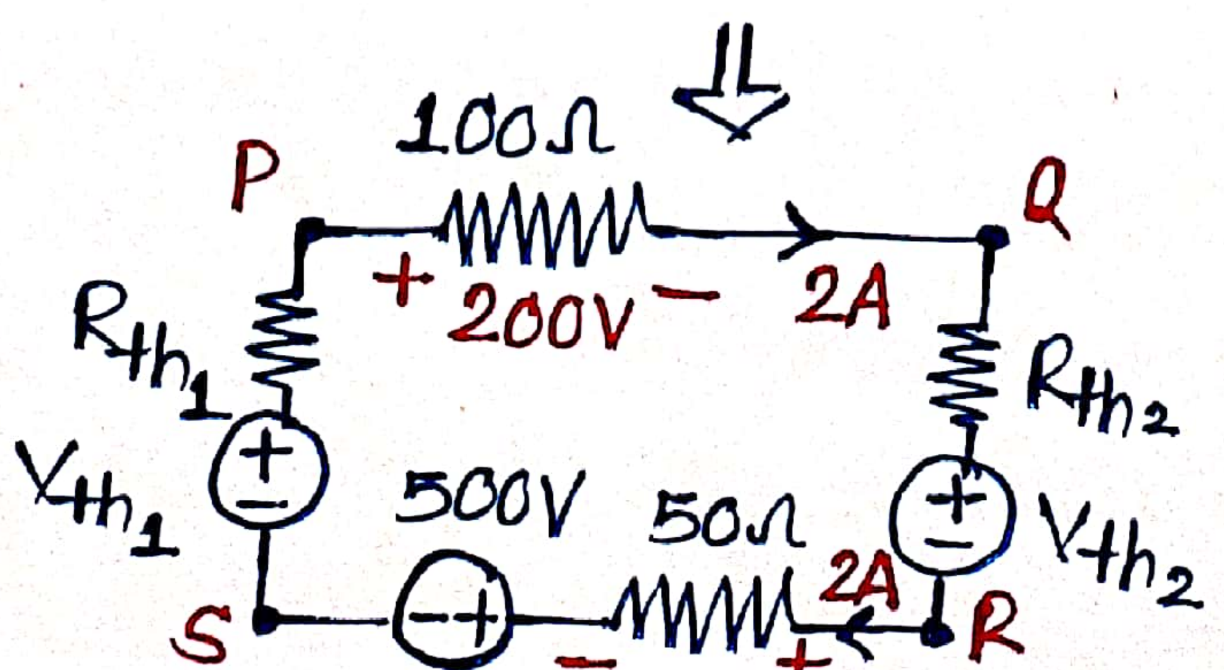


Current through 100Ω resistance $= \frac{V_P - V_Q}{100} = \frac{200}{100} = 2A$ → (1 Mark)

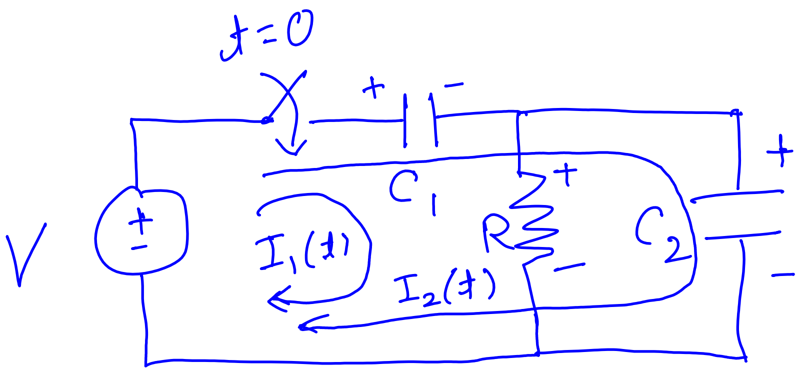
∴ Current through 50Ω resistance will also be 2A. → (1 Mark)

Apply KVL in the branch R–S, we get –

$$-V_R + 500 + 50 \times 2 + V_S = 0$$

$$\therefore (V_R - V_S) = 600\ V$$ → (2.5 Marks)

Alternate Solution of Q1



$V_{c_1}(0^-) = V_0$

$V_{c_2}(0^-) = 0$

$I_1(t)$, $I_2(t)$ are loop currents.

KVL for the loop having I_1 current

$$V = V_{c_1}(t) + V_R(t) \quad , \forall\, t \geq 0 \qquad \text{(I)}$$

KVL for the loop having I_2 current

$$V = V_{c_1}(t) + V_{c_2}(t) \quad , \forall\, t \geq 0 \qquad \text{(II)}$$

Now

$$V_{c_1}(t) = V_{c_1}(0^-) + \frac{1}{C_1}\int_{0^-}^{t} I_{c_1}(\tau)\, d\tau$$

$$V_{c_2}(t) = V_{c_2}(0^-) + \frac{1}{C_2}\int_{0^-}^{t} I_{c_2}(\tau)\, d\tau$$

$$I_{c_1}(t) = I_1(t) + I_2(t)$$

$$I_{c_2}(t) = I_2(t) \quad , \quad V_R(t) = I_1(t)R$$

(*) (*) since V (the source voltage) and V_0 (the initial voltage of capacitor C_1) are finite constants, we can infer all the branch voltages are finite for all time.

Hence $I_1(t) = \dfrac{V_R(t)}{R}$ is also finite for all time.

<u>From eq. (11)</u>,

$$V = V_{C_1}(0^+) + V_{C_2}(0^+)$$

$$= V_{C_1}(0^-) + \frac{1}{C_1}\int_{0^-}^{0^+}\big(I_1(\tau) + I_2(\tau)\big)d\tau$$

$$+ V_{C_2}(0^-) + \frac{1}{C_2}\int_{0^-}^{0^+} I_2(\tau)d\tau$$

$$= V_0 + \frac{1}{C_1}\int_{0^-}^{0^+} I_2(\tau)d\tau + \frac{1}{C_2}\int_{0^-}^{0^+} I_2(\tau)d\tau$$

since $\displaystyle\int_{0^-}^{0^+} I_1(\tau)\,d\tau = 0$ due to $I_1(t)$ being finite at $t=0$

So,

$$V = V_0 + \frac{1}{C_1} q(0^+) + \frac{1}{C_2} q(0^+)$$

where $q(0^+) \triangleq \int_{0^-}^{0^+} I_2(\tau)\, d\tau$

$$\Rightarrow \left(\frac{1}{C_1} + \frac{1}{C_2}\right) q(0^+) = V - V_0$$

$$q(0^+) = \frac{C_1 C_2 (V - V_0)}{C_1 + C_2}$$

$$V_{C_1}(0^+) = V_{C_1}(0^-) + \frac{1}{C_1} q(0^+)$$

$$= V_0 + \frac{C_2 (V - V_0)}{C_1 + C_2}$$

$$= \frac{C_1 V_0 + C_2 V}{C_1 + C_2}$$

$$V_{C_2}(0^+) = \frac{1}{C_2} q(0^+) = \frac{C_1 (V - V_0)}{C_1 + C_2}$$

In the given problem,

$$V = 10V, \quad V_0 = 2 V$$

$$C_1 = 1F, \quad C_2 = 2F$$

$$\Rightarrow V_{C_1}(0^+) = \frac{C_1 V_0 + C_2 V}{C_1 + C_2} = \frac{2 + 20}{3} V$$

$$= \frac{22}{3} V$$

$$V_{C_2}(0^+) = \frac{C_1 (V - V_0)}{C_1 + C_2} = \frac{8}{3} V$$