Probability and Statistics: Worksheet 2

January 27, 2025

Important Formulas:

$p(x)$ to be a valid p.m.f if $\sum_x p(x) = 1$ and $p(x) \geq 0$

$$E(X) = \sum_x x \cdot p(x).$$

$$\mathrm{E}(a \cdot X + b) = a \cdot \mathrm{E}(X) + b$$

1. Determine whether the given distributions represent a valid probability distribution. If not, state why?.

   (a)

X	5	10	15
$P(X)$	0.3	0.4	0.1

   (b)

X	8	12	16	20
$P(X)$	$\frac{5}{6}$	$\frac{1}{12}$	$\frac{1}{12}$	$\frac{1}{12}$

   (c)

X	1	2	3	4	5
$P(X)$	$\frac{1}{10}$	$\frac{3}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$

   (3)

2. Consider a random variable Y with the probability mass function

   $$f(y) = c \cdot \frac{2^y}{y!}, \quad y = 2, 3, 4, 5, \ldots$$

   where $c = \frac{1}{e^2 - 3}$. calculate the expected value of Y.

   (4)

3. We already know that:

   $$\sum_{k=0}^{n} \binom{n}{k} = 2^n$$

   prove that:

   $$\sum_{k=0}^{n} k \binom{n}{k} = n \cdot 2^{n-1}$$

   (3)