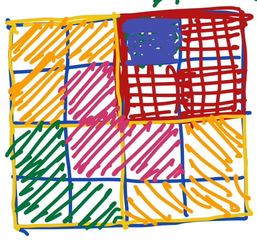
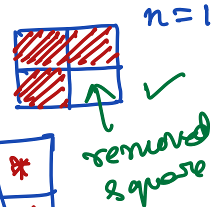
Proof by induction:

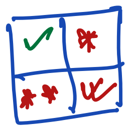
$2^n \times 2^n$ checkerboard $\boxed{\begin{array}{l} n=1 \\ n=2 \end{array}}$

$n \geq 1$

$n \in \mathbb{Z}$







$n=2$ $\quad 2^2 \times 2^2$ checkerboard

Can a $2^2 \times 2^2$ checkerboard with one square removed be filled with a collection of triminoe

Theorem 1: Let n be a positive integer. Then every $\underline{2^n \times 2^n}$ checkerboard with one square removed can be filled with triminoes.

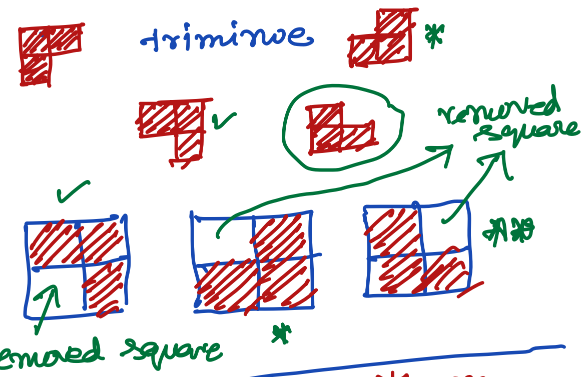
Induction Hypothesis: $P(n)$: the sentence/statement as given

Let the statement be true for $n=k$. A $2^k \times 2^k$ checkerboard with one square removed can be filled by a collection of triminoes.

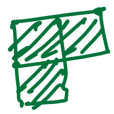
Induction Step: $n = k+1$.

Consider a $2^{k+1} \times 2^{k+1}$ checkerboard with one square removed.

This $2^{k+1} \times 2^{k+1}$ checkerboard



2×2 checkerboard with one square removed can be filled with a triminoe.

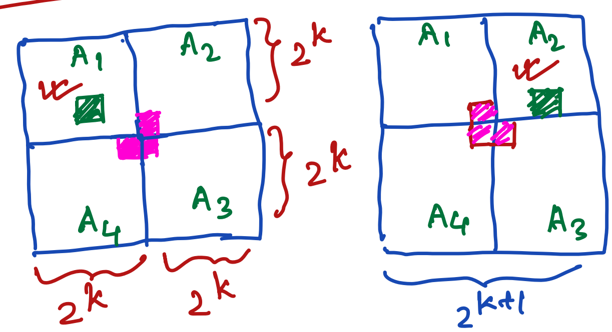


Basis Step: $n=1$.

Draw 4 different pictures each of 2×2 checkerboard clearly highlighting which particular square removed.

Then show pictorially how to place one triminoe.

[Fill this gap]



A_1 contains the

can be partioned into 4 checkerboards each having size $2^k \times 2^k$.

These 4 checkerboards are A_1, A_2, A_3, A_4.

Without loss of generality, suppose that the removed square of this $2^{k+1} \times 2^{k+1}$ checkerboard is in A_1.

By induction hypothesis, A_1 has $2^k \times 2^k$ size, and one square removed.

After putting a triminoe with these 3 squares, from A_2, A_3 and A_4 each, one square will be removed.

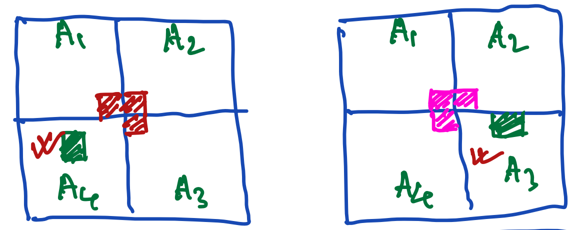
Due to induction hypothesis, each of A_2, A_3, A_4 having one square removed can be filled with triminoes.

Therefore, a $2^{k+1} \times 2^{k+1}$ checkerboard with one square removed can be filled with triminoes.

Strong Mathematical Induction: $\boxed{P(n)}$

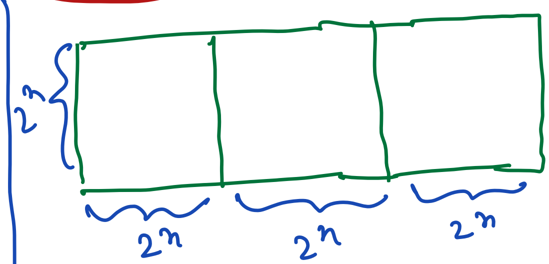
Basis Step: Prove that the statement is true for one or a few smallest

removed square



Then A_1 can be filled with triminoes (by induction hypothesis)

Observe that there is one square from A_2, one square from A_3, and one square from A_4 such that those 3 squares putting together forms a triminoe.



Exercise: Can a 3×2^n checkerboard be filled with triminoes. (no square removed)

Proof by induction: $P(n)$

Basis Step: Prove the statement for smallest possible value n

Induction hypothesis: Assum

possible values. Ex: $P(1)$, $P(2)$, $P(3)$.

Induction Hypothesis: Assume that $(P(1) \wedge P(2) \wedge \ldots \wedge P(k))$ is true. Assume that the statement is true for all $n = 1, 2, \ldots, k$.

Induction Step: Prove that $P(k+1)$ is true using $(P(1) \wedge \ldots \wedge P(k))$

Proof: Basis Step: $n = 2$,

Then, $2 = 2^1$, $p_1 = 2$, $a_1 = 1$ and $2 = p_1^{a_1}$. As 2 is a prime, therefore the statement is true

$n = 1$: Vacously true. as $n \geq 2$ is false.

Induction Hypothesis: For every positive integer $n = 2, 3, \ldots, k$ n can be written as product of primes.

Induction Step: Consider $n = k+1$.

Case (i): $(k+1)$ is prime. Then, choose $p_1 = (k+1)$ and $a_1 = 1$ such that $p_1^{a_1} = k+1$. Hence, the statement is true.

$p(k)$ is true.

Induction Step: Use induction hypotheses to prove that $P(k+1)$ is true.

Theorem 2: If n is a positive integer and $n \geq 2$, then n is a prime or n can be written as a product of primes.

Examples: $3^1 \rightarrow p_1 = 3$, $a_1 = 1$

$52 = 2^2 \times 13$ 2 prime 13 prime

For every positive integer n if $n \geq 2$, then there exist primes $p_1, p_2, \ldots, p_k$ and positive integers $a_1, a_2, \ldots a_k$ such that

$n = p_1^{a_1} p_2^{a_2} \ldots p_k^{a_k}$.

Equivalent statement

Case (ii): $(k+1)$ is composite.

Then, there exists two numbers $x, y \geq 2$ such that $k+1 = xy$.

Observe that $x, y \le k$.

Hence $2 \le x \le k$ and $2 \le y \le k$.

Due to induction hypothesis on x, there exist primes $p_1, p_2, \dots, p_r$ and positive integers $a_1, a_2, \dots, a_r$ such that $x = p_1^{a_1} \cdots\cdots p_r^{a_r}$

Similarly due to induction hypothesis on y, there exist primes $q_1, \dots, q_\ell$ and positive integers $b_1, b_2, \dots, b_\ell$ such that

$$y = q_1^{b_1} q_2^{b_2} \cdots q_\ell^{b_\ell}$$

Then
$$k+1 = xy = p_1^{a_1} \cdots p_r^{a_r} \cdot q_1^{b_1} \cdots q_\ell^{b_\ell}$$

$$xy = s_1^{d_1} s_2^{d_2} \cdots s_m^{d_m}$$

Since each $p_1, p_2, \dots, p_r$ and each $q_1, q_2, \dots, q_\ell$ are primes, hence $(k+1)$ can be written as a product of primes.

Hence, the statement is true.

As the cases are mutually exhaustive this completes the proof.

FUNDAMENTAL THEOREM OF ARITHMETIC

FACTORIZATION OF A NUMBER into PRIMES.

$n = p_1^{a_1} \cdots p_k^{a_k}$ uniquely factorization into primes.

UNIQUENESS GUARANTEE ON A STATEMENT:

Let $a, b \in \mathbb{R}$ (real numbers) such that $a \neq 0$.

Then there exists unique r such that $ar + b = 0$.

If there are two numbers r_1 and r_2 satisfying the desired property, then $r_1 = r_2$.

Proof: a, b are real numbers and $a \neq 0$.

Then consider $r = \left(-b/a\right)$

Clearly $ar + b = 0$.

Existence guarantee.

Uniqueness: Let there are two numbers x and y such that $ax + b = 0$ and $ay + b = 0$.

Then $ax + b = ay + b$

Hence, $x = y$.

Hence, the number is unique.