Propositional Logic:

A proposition is a sentence that declares a fact. (declarative sentence)

A proposition can be either true or false but not both.

Examples:

(i) Washington D.C. is the capital of USA. → propositions true (T)

(ii) Toronto is the capital of Canada. → proposition → (F)

(iii) $2+4=6$ → proposition → (T)

False (F)

(iv) What is the time now?

Not a proposition

Read this carefully. → Not a proposition.

(ii) Toronto is the Capital of Canada → p

Opposite proposition: Toronto is not the capital of Canada → true (T)

"not p"

$\neg p$ → a proposition

truth table

$p \equiv$ Maria has a dell laptop

$q \equiv$ Maria's laptop has 32 GB RAM

Maria has a Dell laptop → p and her laptop has 32 GB RAM. → q

$\mathbb{N} = \{0, 1, 2, 3, 4, 5, 6, \ldots\}$

= natural numbers

(used to count discrete objects)

$\mathbb{Z} = \{0, -1, 1, -2, 2, -3, 3, \ldots\}$

= set of all integers

$\mathbb{R}$ = real numbers.

(used to measure one-dimensional quantity)

temperature, height, distance

$\sqrt{2}$

$\mathbb{Q}$ = rational numbers

$= \frac{p}{q}$ $\quad p, q \in \mathbb{Z}$, $q \neq 0$

prime numbers = positive integers x greater than 1 and divisible by 1 and x only.

Propositional variable:

p, q, r — atomic proposition

truth values

Truth table:

p	$\neg p$
T	F
F	T

"p and q" $(p \wedge q)$ $\underline{\wedge}$ → and

Compound proposition

Logical connectives.

$(p \wedge q)$ is true when both p

Truth table for $p \wedge q$

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

$p \vee q$

"p or q"

"p is true or q is true or both"

$(p \vee q)$ is true

$(p \vee q) \rightarrow$ disjunction of p and q

$(p \wedge q) \rightarrow$ conjunction of p and q

Conditional statements:

Example: If it rains today, then England will win this test series

$(p \Rightarrow q)$ $(p \rightarrow q)$

"p implies q"

Truth table

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

If premise (p) is false, then $(p \rightarrow q)$ is true vacously true.

Contrapositive of $p \rightarrow q$: $(\neg q \rightarrow \neg p)$

Converse of $p \rightarrow q$: $(q \rightarrow p)$

Inverse of $p \rightarrow q$: $\neg p \rightarrow \neg q$

The truth tables for $p \rightarrow q$ and $\neg q \Rightarrow \neg p$ give same values.

and q are true. False otherwise

Maria has a Dell laptop or her laptop has 32 GB RAM.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

It rains today $= p$

England will win this test series $= q$

"If p, then q" is false when p is true but q is false. Otherwise this satement is true.

If the assumption is false, then the entire is true (vacously true)

premise $\equiv$ assumption

Truth table for contrapositive of $p \rightarrow q$ $(\neg q \rightarrow \neg p)$

p	q	$\neg q$	$\neg p$	$\neg q \Rightarrow \neg p$
T	T	F	F	T
T	F	T	F	F
F	T	F	T	T
F	F	T	T	T

$(p \to q)$ and $(\neg q \to \neg p)$ are propositionally equivalent

Exercise: Construct truth tables for (converse of $p \to q$)

(inverse of $p \to q$)

$(p \to q)$ "If p, then q"

"p implies q"

"q is necessary condition for p"

"p is sufficient condition for q"

"q is true whenever p is true"

$p \wedge q$

Conjunction of p and q

$p \vee q$

disjunction of p and q

Biconditional statement:

"p if and only if q"

Example: $p \equiv$ You take the flight

$q \equiv$ You purchase a ticket.

p if and if q

You take the flight if and only if You purchase a ticket

$(p \Leftrightarrow q)$ is true when

either both p and q are true

or both p and q are false

Truth table: $(p \leftrightarrow q)$

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

(If p, then q) and (If q, then p)

$(p \to q)$ $(q \to p)$

Example: $\neg p \to (q \vee r)$

Truth table

p	q	r	$\neg p$	$q \vee r$	$\neg p \to (q \vee r)$
T	T	T	F	T	T
T	T	F	F	T	T
T	F	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	T	F	T	T	T
F	F	T	T	T	T
F	F	F	T	F	F

If a compound proposition is true for all possible combination of truth values of propositional variables then it is called $\boxed{\text{tautology}}$

$$(p \to q) \to (\neg q \to \neg p) \equiv r$$

p	q	$p \to q$	$\neg q \to \neg p$	r
T	T	T	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

Tautology,

A compound proposition is contradiction when for all combination of truth values of variables, the compound proposition is false.

$\neg p \to (q \vee r)$ is a contingency.

If a compound proposition is neither a tautology, nor a contradiction then it is called a contingency.

Reference: Section 1.1 and ~~Section 1.3~~ (Chapter-1 of Rosen's book)

Truth table construction