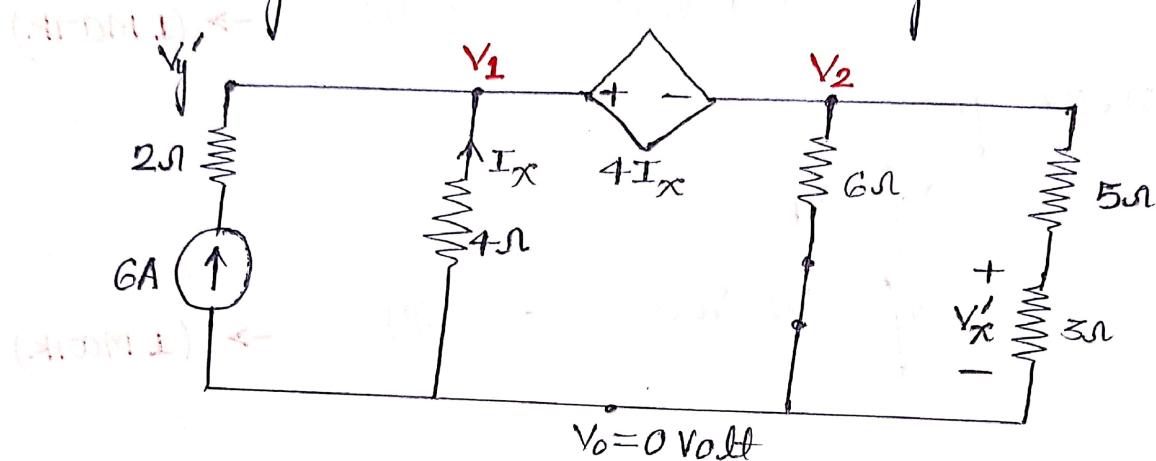


Mid Semester Exam Solution

Sol(1) :- By using Superposition theorem -

Case(I) : Taking 6A current source only,



Apply KCL at node V_1 & V_2 , we get -

$$\frac{V_1}{4} + \frac{V_2}{6} + \frac{V_2}{8} = 6$$

$$6V_1 + 7V_2 = 144 \quad \text{--- (1)}$$

$$V_1 - V_2 = 4I_x \quad \text{--- (2)}$$

$$\frac{V_1}{4} = -I_x \quad \text{--- (3)}$$

$$\frac{V_2}{8} = \frac{V_x'}{3} \quad \text{--- (4)}$$

$$V_1 = V_y' \quad \text{--- (5)}$$

By eqn (2) & eqn (3), we get -

$$V_1 - V_2 = -V_1$$

$$2V_1 = V_2 \quad \text{--- (6)}$$

By eqn (1) & eqn (6), we get -

$$V_1 = 7.2 \text{ Volt}$$

$$\therefore V_2 = 14.4 \text{ Volt} \quad (\text{By eqn (6)})$$

By eqⁿ(4), we get -

$$V_x' = \frac{3}{8} V_2$$

$$V_x' = \frac{3}{8} (14.4)$$

$$V_x' = 5.4 \text{ Volt} \quad \text{--- (7)}$$

→ (1 MARK)

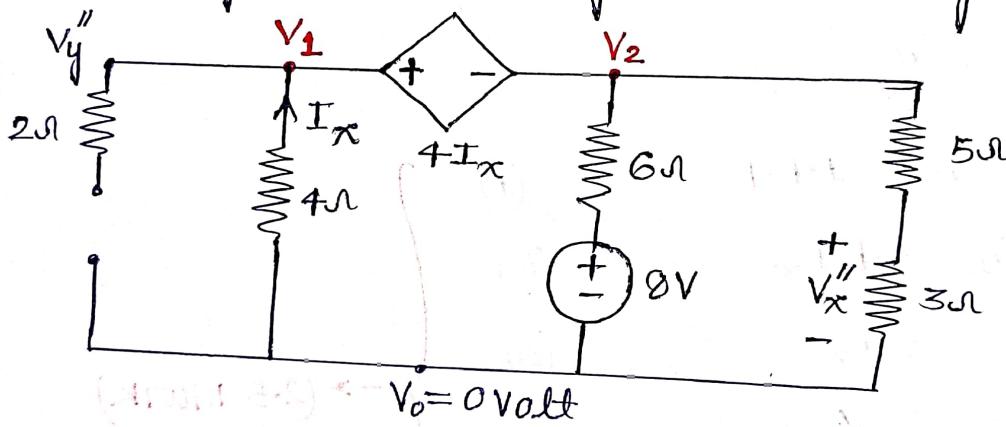
By eqⁿ(5), we get -

$$V_y' = V_1$$

$$V_y' = 7.2 \text{ Volt} \quad \text{--- (8)}$$

→ (1 MARK)

Case(II): Taking 8 volt voltage source only



Apply KCL at node V_1 & V_2 , we get -

$$\frac{V_1}{4} + \frac{V_2 - 8}{6} + \frac{V_2}{8} = 0$$

$$6V_1 + 7V_2 = 32 \quad \text{--- (a)}$$

$$V_1 - V_2 = 4I_x \quad \text{--- (b)}$$

$$\frac{V_1}{4} = -I_x \quad \text{--- (c)}$$

$$\frac{V_2}{8} = \frac{V_x''}{3} \quad \text{--- (d)}$$

$$V_y'' = V_1 \quad \text{--- (e)}$$

→ (2.5 MARK)

By eqn (b) & eqn (c), we get -

$$2V_1 = V_2 \quad \text{--- (f)}$$

By eqn (a) & eqn (f), we get -

$$V_1 = 1.6 \text{ Volt}$$

$$\therefore V_2 = 3.2 \text{ Volt}$$

(By eqn (f))

By eqn (d), we get -

$$V_x'' = \frac{3}{8} V_2$$

$$V_x'' = \frac{3}{8} (3.2)$$

$$V_x'' = 1.2 \text{ Volt} \quad \text{--- (g)}$$

→ (1 MARK)

By eqn (e), we get -

$$V_y'' = V_1$$

$$V_y'' = 1.6 \text{ Volt} \quad \text{--- (h)}$$

→ (1 MARK)

By Superposition theorem,

$$V_x = V_x' + V_x''$$

$$= 5.4 + 1.2$$

$$= 6.6 \text{ Volt}$$

(By eqn (r) & eqn (g))

→ (0.5 MARK)

$$V_y = V_y' + V_y''$$

$$= 7.2 + 1.6$$

$$= 8.8 \text{ Volt}$$

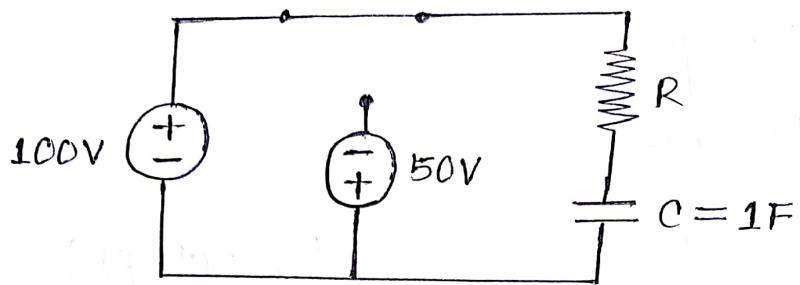
(By eqn (o) & eqn (h))

→ (0.5 MARK)



Sol(2) :-

Case(I): At time $t=0$ (switch connected to Position-A)



Given that — Time constant (τ) = 2 sec

$$RC = 2$$

$$\therefore R = \left(\frac{2}{1}\right) = 2 \Omega \rightarrow (1 \text{ MARK})$$

General eqⁿ of RC circuit with source,
(Ans 1) \leftarrow

$$V_c(t) = [V_c(0^+) - V_c(\infty)] e^{-t/\tau} + V_c(\infty) \quad \dots (1)$$

$$V_c(0^+) = V_c(0^-)$$

[given that $V_c(0^-) = 0 \text{ volt}$]

$$\therefore V_c(0^+) = 0 \text{ volt} \quad \dots (2)$$

$\rightarrow (1 \text{ MARK})$

$$\therefore V_c(\infty) = 100 \text{ volt} \quad \dots (3)$$

$\rightarrow (1 \text{ MARK})$

By eqⁿ (1), eqⁿ (2) & eqⁿ (3), Voltage response across capacitor,

$$V_c(t) = (0 - 100) e^{-t/2} + 100$$

$$(Ans 2) \quad V_c(t) = 100(1 - e^{-t/2}) \quad \dots (4)$$

$\rightarrow (2.5 \text{ MARK})$

Case(II): At time $t \geq \tau$ (switch connected to position-B)

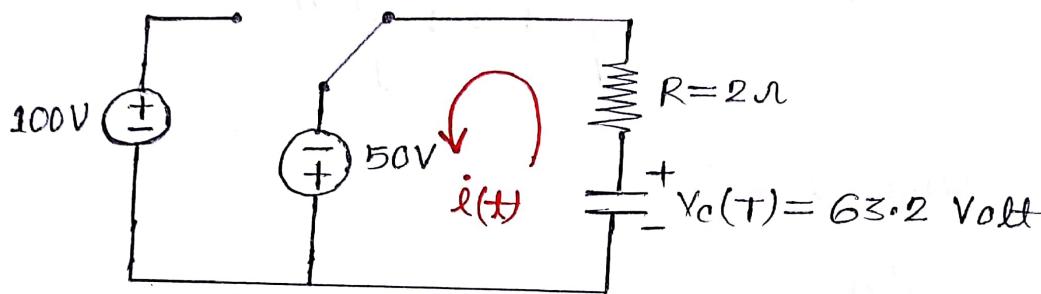
(Ans 3) \leftarrow

Voltage across capacitor (at $t=T$),

$$V_c(T) = 100(1 - e^{-T/T})$$

$$V_c(T) \approx 63.2 \text{ Volt} \quad \text{--- (5)}$$

$\rightarrow (0.5 \text{ MARK})$



$$\therefore i(t=T) = \frac{(63.2 + 50)}{2} = 56.6 \text{ A} \quad \text{--- (6)} \rightarrow (1 \text{ MARK})$$

$$\therefore i(t=\infty) = 0 \text{ A} \quad \text{--- (7)}$$

$\rightarrow (1 \text{ MARK})$

General eqn of current through capacitor,

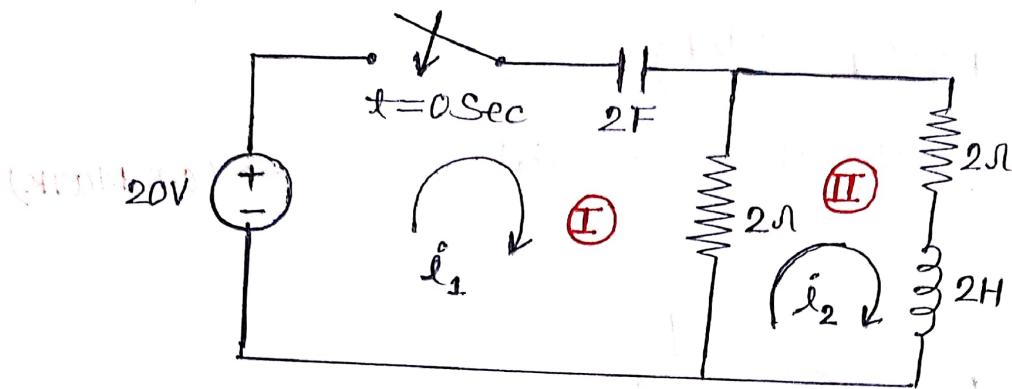
$$i(t) = [i(T) + i(\infty)] e^{-(t-T)/T} + i(\infty)$$

$$i(t) = (56.6 - 0) e^{-(t-T)/T} + 0$$

$$i(t) = 56.6 e^{-(t-2)/2}$$

$\rightarrow (2 \text{ MARK})$

Sol(3) :-



Case(I): At time $t=0^-$

$$i_1(0^-) = 0 \text{ A}$$

→ (0.5 Mark)

$$i_2(0^-) = i_1(0^-) = 0 \text{ A}$$

→ (0.5 Mark)

$$V_C(0^-) = 0 \text{ volt}$$

Case(II): At time $t=0^+$

$$i_1(0^+) = \frac{20}{2} = 10 \text{ A} \quad (1)$$

→ (1 Mark)

$$i_2(0^+) = i_2(0^-) = 0 \text{ A} \quad (2)$$

→ (1 Mark)

$$V_C(0^+) = V_C(0^-) = 0 \text{ volt} \quad (3)$$

→ (1 Mark)

Apply KVL in mesh-II, we get -

$$2[i_2(t) - i_1(t)] + 2i_2(t) + 2 \cdot \frac{di_2(t)}{dt} = 0$$

Put $t=0^+$, we get -

$$2(0 - 10) + 2 \cdot 0 + 2 \cdot \frac{di_2(0^+)}{dt} = 0 \quad (4)$$

$$2(0 - 10) + 2 \cdot 0 + 2 \cdot \frac{d i_2(0^+)}{dt} = 0$$

$$\therefore \frac{d i_2(0^+)}{dt} = 10 \text{ A/sec}$$

→ (1.5 Mark)

Apply KVL in mesh-I, we get -

$$-20 + \frac{1}{2} \int i_1(t) dt + 2[i_1(t) - i_2(t)] = 0$$

→ (6)

Differentiate eqn(6), with respect to 't', we get -

$$0 + \frac{1}{2} i_1(t) + 2 \left[\frac{d i_1(t)}{dt} - \frac{d i_2(t)}{dt} \right] = 0 \quad \text{--- (7)}$$

Put $t=0^+$, we get -

$$\frac{1}{2} \times 10 + 2 \left[\frac{d i_1(0^+)}{dt} - 10 \right] = 0$$

$$\therefore \frac{d i_1(0^+)}{dt} = 7.5 \text{ A/sec} \quad \text{--- (8)} \rightarrow (1.5 \text{ Mark})$$

Differentiate eqn(4) with respect to 't', we get -

$$2 \left[\frac{d i_2(t)}{dt} - \frac{d i_1(t)}{dt} \right] + 2 \frac{d^2 i_2(t)}{dt^2} + 2 \frac{d^2 i_1(t)}{dt^2} = 0$$

Put $t=0^+$, we get -

$$2 \left[\frac{d i_2(0^+)}{dt} - \frac{d i_1(0^+)}{dt} \right] + 2 \frac{d^2 i_2(0^+)}{dt^2} + 2 \frac{d^2 i_1(0^+)}{dt^2} = 0$$

$$2(10 - 7.5) + 2 \times 10 + 2 \frac{d^2 i_2(0^+)}{dt^2} = 0$$

$$\therefore \frac{d^2 i_2(0^+)}{dt^2} = -12.5 \text{ A}^2/\text{sec}^2 \quad \text{--- (9)} \rightarrow (1.5 \text{ Mark})$$

Differentiate eqn(7) with respect to 't', we get -

$$\frac{1}{2} \frac{d i_1(t)}{dt} + 2 \left[\frac{d^2 i_1(t)}{dt^2} - \frac{d^2 i_2(t)}{dt^2} \right] = 0$$

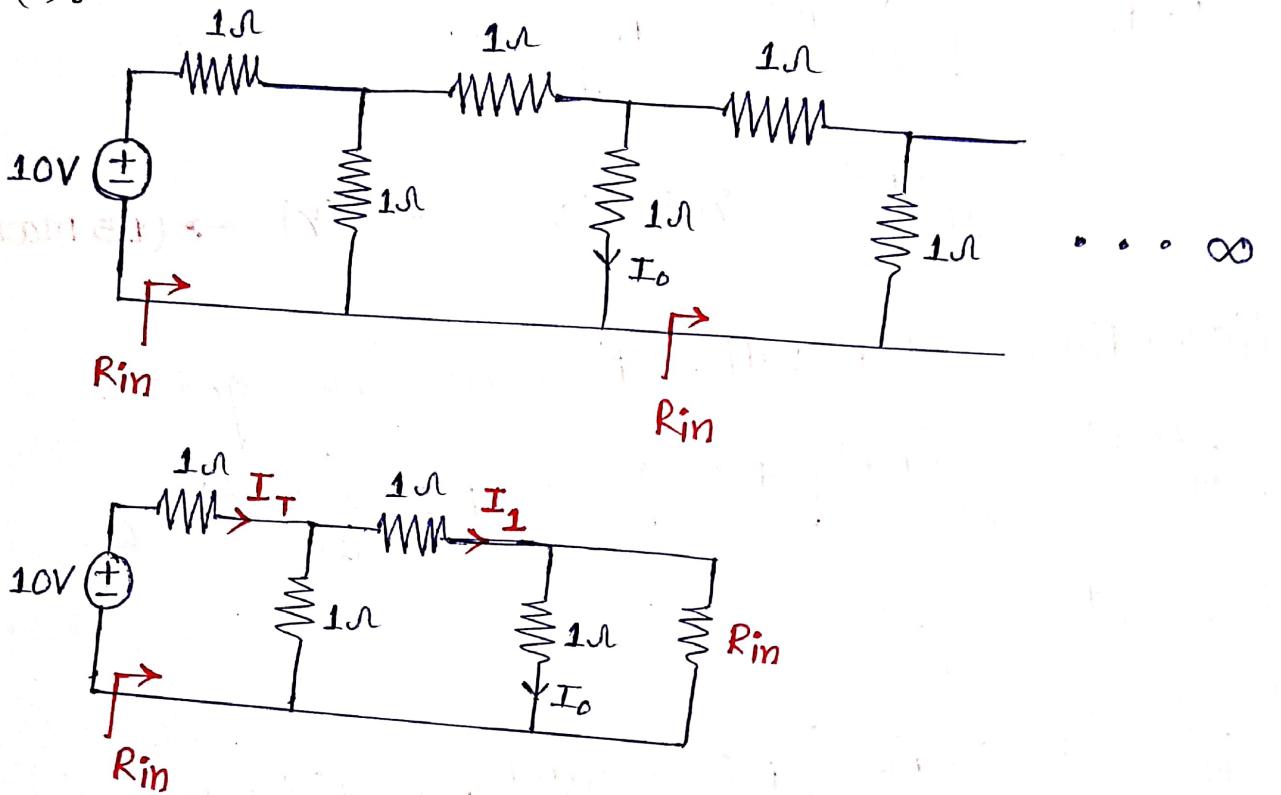
Put $t=0^+$, we get -

$$\frac{1}{2} \times 7.5 + 2 \left[\frac{d^2 i_1(0^+)}{dt^2} + 12.5 \right] = 0$$

$$\therefore \frac{d^2 i_1(0^+)}{dt^2} = -14.395 \text{ A}^2/\text{sec}^2 \quad \text{--- (10)}$$

→ (1.5 MARK)

Sol(4) :-



$$\therefore R_{in} = \left(\frac{R_{in}}{1+R_{in}} + 1 \right) \parallel 1 + 1$$

$$R_{in} = \left(\frac{2R_{in}+1}{R_{in}+1} \right) \parallel 1 + 1$$

$$R_{in} = \frac{\left(\frac{2R_{in}+1}{R_{in}+1} \right)}{\left(\frac{2R_{in}+1}{R_{in}+1} \right) + 1} + 1$$

$$R_{in} = \left(\frac{2R_{in}+1}{3R_{in}+2} \right) + 1$$

$$R_{in}(3R_{in}+2) = 2R_{in}+1 + 3R_{in}+2$$

$$3R_{in}^2 + 2R_{in} = 5R_{in} + 3$$

$$3R_{in}^2 - 3R_{in} - 3 = 0$$

$$R_{in} = (1 \pm \sqrt{5})/2$$

$$R_{in} = 1.62\Omega \quad \checkmark \quad -0.62\Omega \quad \times$$

$$\therefore R_{in} = 1.62\Omega$$

→ (2 MARK)

$$\therefore \text{Current, } I_T = \frac{10}{1.62} = 6.17A$$

→ (1 MARK)

$$\therefore \text{Current, } I_1 = \frac{1}{1 + (1 + 1/1.62)} \times I_T$$

$$= \frac{1}{1 + (1 + 0.62)} \times 6.17$$

$$= \frac{1}{2.62} \times 6.17$$

$$= 2.35 A$$

→ (1 MARK)

$$\therefore \text{Current, } I_o = \left(\frac{1.62}{1 + 1.62} \right) \times I_1$$

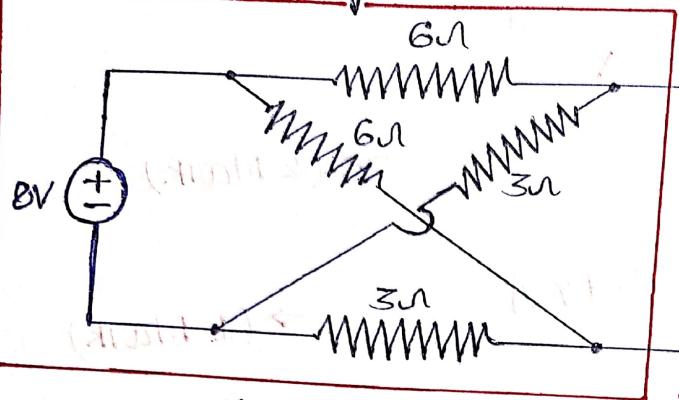
$$= \frac{1.62}{2.62} \times 2.35$$

$$= 1.45 A$$

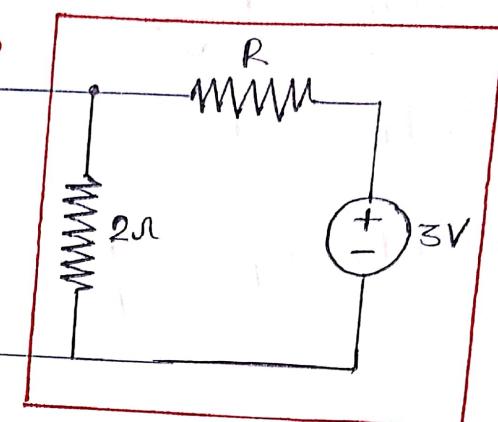
→ (1 MARK)

Sol(5):

At steady state capacitors behave as an open circuit.



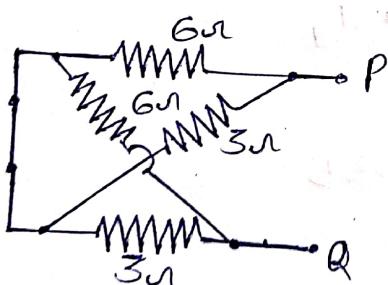
Network-A



Network-B

For maximum power transfer from Network-A (work as a source) to Network-B (work as a load), the value of equivalent resistance of Network-B is must be equal to source internal resistance (Thevenin resistance of Network-A).

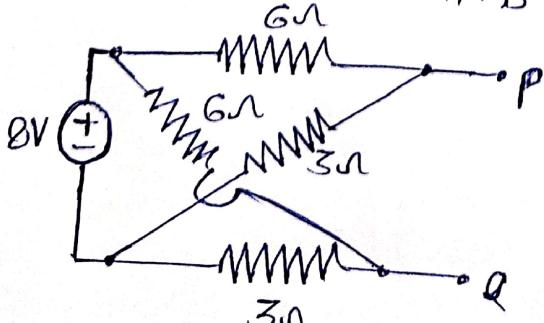
Step(I): For R_{th} of Network-A



$$\therefore R_{th} = 4\Omega$$

→ (2 MARK)

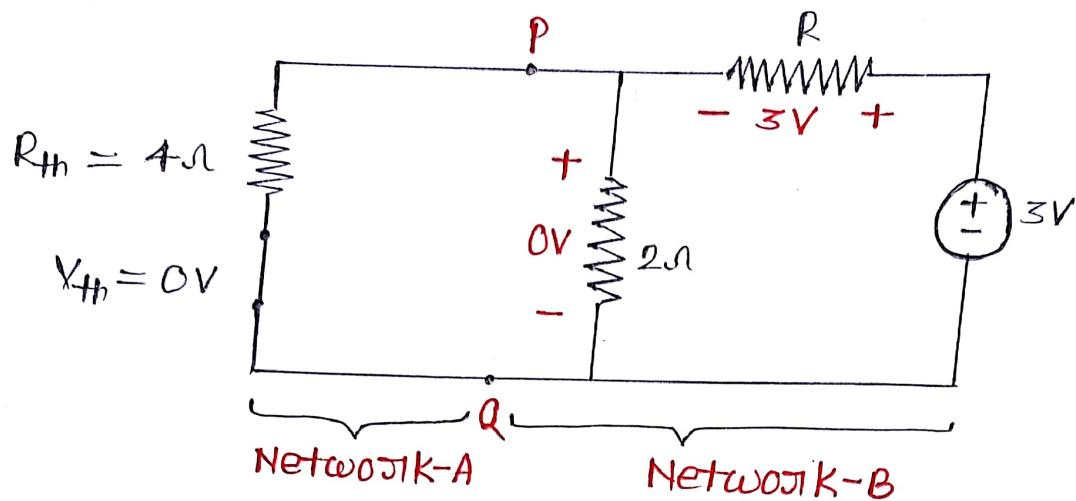
Step(II): For X_{th} of Network-B



$$\therefore X_{th} = (V_p - V_q) = \left(\frac{3}{3+6}\right)8 - \left(\frac{3}{3+6}\right)8 = 0 \text{ volt}$$

→ (2 MARK)

Now reduced circuit -



Here, Network-A can't be work as a source because $V_{Th} = 0$ Volt. Hence we can not determine the value of ' R '.

→ (1 MARK)