Discrete Structures-2025: Tutorial-6

Functions and Countable Sets

(1) Let two sets A and B satisfy the property that $A \subset B$. Then, prove that $|A| \leq |B|$ by exhibiting an explicit injective function $f : A \to B$.

(2) If there are three infinite sets A, B and C such that $|A| = |B|$ and $|B| = |C|$, then prove that there exists a bijection between A and C.

(3) Consider the set $A \times \mathbb{Z}$ when $A = \{2, 3, 6\}$. Prove that $A \times \mathbb{Z}$ is countable by exhibiting a precise bijection between $\mathbb{N}$ and $A \times \mathbb{Z}$.

(4) Prove that a set X is countable if and only if $|X| \leq |\mathbb{Z}|$. Here $\mathbb{Z}$ is the set of all integers.

(5) If $|X| = |Y|$, then prove that there is a bijection between $\mathcal{P}(X)$ and $\mathcal{P}(Y)$. We use $\mathcal{P}(A)$ to denote the power-set of A.

(6) Suppose that A is a countable set. If there is a surjective function $f : A \to B$, then prove that B is countable.