Discrete Structures-2025: Quiz-1 (Model Solution)

Propositional and First Order Logic

Total Marks: 15

September 1, 2025

(1) Let p, q, r and s be propositions. Then prove or disprove that $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$ are logically equivalent.

(7 Marks)

Solution: Two formulas are not logically equivalent if they take different values on same valuations.
Let v be a truth value assignment such that

$$v(p) = F, \ \ v(q) = T, \ \ v(r) = F, \ \ v(s) = F$$

$$\begin{aligned}
&\bar{v}(p \to q) = T \quad (\text{since } F \to T = T), \\
&\bar{v}(r \to s) = T \quad (\text{since } F \to F = T), \\
&\bar{v}((p \to q) \to (r \to s)) = \bar{v}(T \to T) = T;
\end{aligned}$$

$$\begin{aligned}
Similarly,& \\
&\bar{v}(p \to r) = T \quad (\text{since } F \to F = T), \\
&\bar{v}(q \to s) = F \quad (\text{since } T \to F = F), \\
&\bar{v}((p \to r) \to (q \to s)) = \bar{v}(T \to F) = F.
\end{aligned}$$

Hence the two formulas have different truth values under v, so they are not logically equivalent.

$\mathbb{NOTE}$: One can also check by truth table.

p	q	r	s	$p \to q$	$r \to s$	$(p \to q) \to (r \to s)$	$p \to r$	$q \to s$	$(p \to r) \to (q \to s)$
T	T	T	T	T	T	T	T	T	T
T	T	T	F	T	F	F	T	F	F
T	T	F	T	T	T	T	F	T	T
T	T	F	F	T	T	T	F	F	T
T	F	T	T	F	T	T	T	T	T
T	F	T	F	F	F	T	T	T	T
T	F	F	T	F	T	T	F	T	T
T	F	F	F	F	T	T	F	T	T
F	T	T	T	T	T	T	T	T	T
F	T	T	F	T	F	F	T	F	F
F	T	F	T	T	T	T	T	T	T
F	T	F	F	T	T	T	T	F	F
F	F	T	T	T	T	T	T	T	T
F	F	T	F	T	F	F	T	T	T
F	F	F	T	T	T	T	T	T	T
F	F	F	F	T	T	T	T	T	T

(2) Let $P(x, y)$ be the statement "y is divisible by x" where the domain for both x and y is the set of all positive integers. Determine the truth values of each of these statements with justification.

(a) $\forall x \, \exists y \, P(x, y)$.

(b) $\forall x \, \forall y \, P(x, y)$.

(1 Marks + 1 Marks)

Solution:

(a) $\forall x \, \exists \, y \, P(x, y)$

The statement says that for every positive integer x, there exists a positive integer y such that y is divisible by x.
Given statement is **true** since every positive integer is divisible by itself.
So we take $y = x$ and the statement holds.

(b) $\forall x \, \forall y \, P(x, y)$

Given statement says that for every positive integer x and y, y is divisible by x.
Clearly, this is **false**. Take $x = 2$ and $y = 3$, but 3 is not divisible by 2.
Hence, the statement does not holds.

(3) Express each of the English sentences using predicates and quantifiers.

(a) Let $C(x)$ be the statement "x has a cat", $D(x)$ be the statement "x has a dog", and $F(x)$ is a statement "x has a ferret". The domain is the set of all students in your class.

Express this sentence in predicate and quantifiers.

"For each of the animals cats, dogs, and ferrets, there is a student in your class who has this as a pet".

(2 Marks)

Solution:

$$(\exists x C(x)) \wedge (\exists x D(x)) \wedge (\exists x F(x))$$

(b) Let the domain be the set of all car drivers in Delhi-NCR region.

For the following sentence, define a predicate, and express the sentence using that predicate and quantifier.

"Some drivers do not obey speed limits".

(2 Marks)

Solution: Let $S(x)$ be the statement "x obeys speed limits".
The given sentence can be expressed as:

$$\exists x\ (\neg S(x))$$

(c) Suppose that the domain is the set of all students in your class.

For the following sentence, define predicate(s), and express using predicates and quantifiers.

"No student in your class own both a motorcycle, and a 4-wheeler car".

(2 Marks)

Solution: Let $M(x)$ be the statement "x owns a motorcycle" and $C(x)$ be the statement "x owns a 4-wheeler car".
The given sentence can be expressed as:

$$\forall x\ (\neg(M(x) \wedge C(x)))$$

or equivalently,

$$\neg\ (\exists\ x\ (M(x) \wedge C(x)))$$