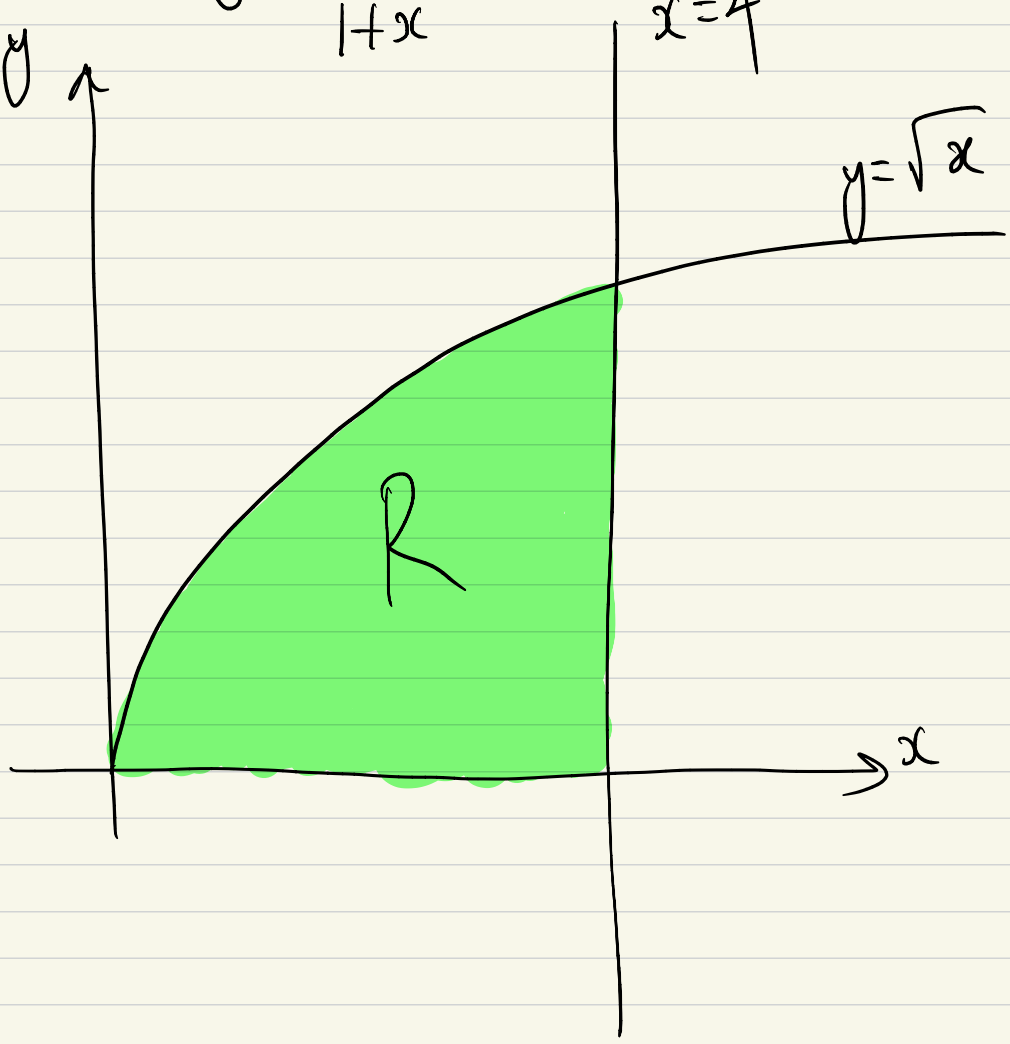
4) a)

$$f(x,y) = \frac{y}{1+x}$$



$$\iint_R f(x,y)\,dA = \int_0^4 \int_0^{\sqrt{x}} f(x,y)\,dy\,dx$$

$$= \int_0^4 \int_0^{\sqrt{x}} \frac{y}{1+x}\,dy\,dx$$

$$= \int_0^4 \left[\frac{y^2}{2(1+x)}\right]_{y=0}^{y=\sqrt{x}} dx$$

$$= \int_0^4 \frac{x}{2(1+x)}\,dx$$

$$= \frac{1}{2}\int_0^4 \frac{x+1-1}{1+x}\, dx$$

$$= \frac{1}{2}\int_0^4 1 - \frac{1}{1+x}\, dx$$

$$= \frac{1}{2}\Big[x - \ln(1+x)\Big]_0^4$$

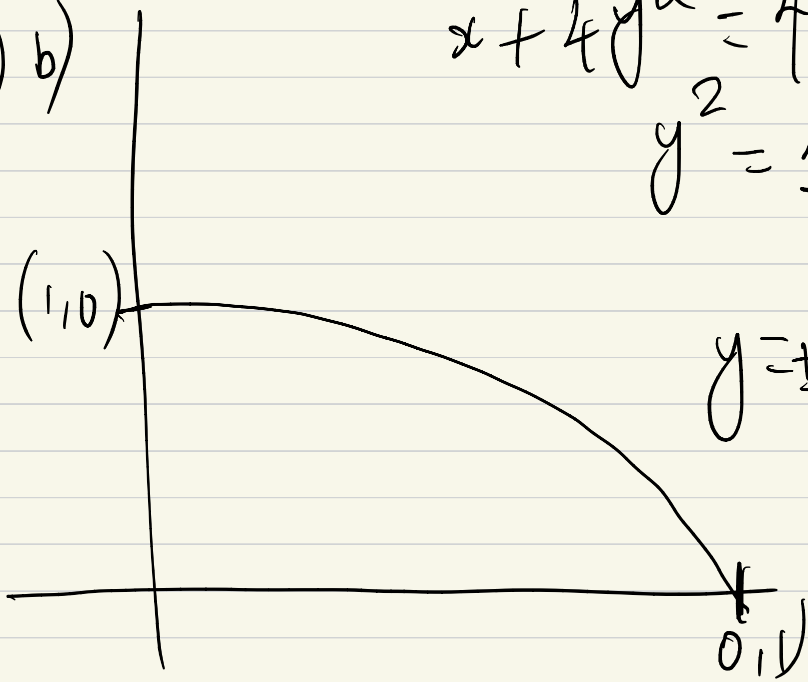
$$= \frac{1}{2}(4 - \ln 5)$$

6) $$\sum_{K=1}^{3} \frac{1}{K} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3}$$

$$= \frac{6+3+2}{6} = \frac{11}{6}$$

(b) $$\sum_{K=1}^{n} 1 = n.$$

11) b)



$$x^2 + 4y^2 = 4$$

$$y^2 = \frac{4 - x^2}{4}$$

$$y = \pm\sqrt{\frac{4 - x^2}{2}}$$

Can be parametrized by

$$f : [0,2] \to \mathbb{R}$$

$$f(t) = \left(t, \frac{\sqrt{(2+x)(2-x)}}{2}\right)$$

12) d)

$$g(x,y,z) = (2xy, x^2+z, y)$$

line segment from $(1,0,2)$ to $(3,4,1)$

Consider equations

$$x = 1 + at$$

$$y = 0 + bt$$

$$z = 2 + ct$$

Suppose at $t=1$, and $t \in [0,1]$

$x = 3 \Rightarrow \quad 3 = 1 + a \Rightarrow \quad a = 2$

$y = 4 \Rightarrow \quad 4 = 0 + b \Rightarrow \quad 4 = b$

$z = 1 \Rightarrow \quad 1 = 2 + c \Rightarrow \quad c = -1$

Possible parametrization:

$$x = 1+2t$$

$$y = 4t$$

$$z = 2-t$$

So

$$p : [0,1] \to \mathbb{R}$$

$$p(t) = (1+2t,\ 4t,\ 2-t)$$

$$\dot{p}(t) = (2, 4, -1)$$

$$\int_C \underline{g}(\underline{x}) \cdot d\underline{x} = \int_0^1 \underline{g}(\underline{R}(t)) \cdot \dot{\underline{R}}(t)\, dt$$

$$= \int_0^1 \left(2(1+2t)(4t), (1+2t)^2 + 2 - t, 4t\right) \cdot \underset{(2,4,-1)}{} \, dt$$

$$= \int_0^1 \left(8t + 16t^2, 4t^2 + 3t + 3, 4t\right) \cdot (2,4,-1)\, dt$$

$$= \int_0^1 16t + 32t^2 + 16t^2 + 12t + 12 - 4t \, dt$$

$$= \int_0^1 48t^2 + 24t + 12\, dt$$

$$= \left[16t^3 + 12t^2 + 12t\right]_0^1 = 40$$